

Insourcing Vs Outsourcing in Vertical Structure¹

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ABSTRACT

We study an agency model with vertical hierarchy—the principal, the prime-agent and the sub-agent. The principal faces a project that needs both agents' services. Due to costly communication, the principal receives a report only from the prime-agent, who receives a report from the sub-agent. The principal can directly incentivize each agent by setting individual transfers (insourcing), or sets only one overall transfer to an independent organization in which the prime-agent hires the sub-agent (outsourcing). We show that insourcing is always optimal when the principal can perfectly process the prime-agent's report. When the principal's information process is limited, however, outsourcing can be the prevailing mode of operation. In addition, insourcing under limited information process is prone to collusion between the agents, whereas no possibility of collusion arises with outsourcing.

JEL Classification: D86, L23, L25

Key words: Information Process, Sourcing Policy, Vertical Structure

1 Introduction

A fundamental question concerning the organizational structure of a firm is whether to in- or outsource the production of its inputs.¹ At first sight, the answer to this question seems straightforward. To maximize its profit, the sourcing policy that is most cost-efficient should be chosen. This simple answer however leaves open the following question—how does the mere act of placing a productive unit within or outside the firm’s boundary affect efficiency? That is, what difference does it make whether the unit is within or outside the firm?

For example, Apple and Samsung, two major players in the smartphone industry, both have their production bases in China, but adopt opposite strategies when it comes to the sourcing policy.² Apple outsources its inputs from its local supplier, Foxconn, by leaving the supplier as a separate entity, whereas Samsung employs an insourcing policy by vertically integrating itself with the local supplier. Since both firms are sourcing from the same location, what are the benefits that one sourcing policy can bring and the other policy cannot? Simply put, what makes one operational mode prevails over the other?

Following the viewpoint of firms as a nexus of contracts,³ we study the optimality of in- or outsourcing from a pure contractual perspective. In particular, we study a setting in which the firm’s sourcing policy does not directly affect the process of operation in any way. That is, different sourcing policies only change the contractual arrangement. We view an insourcing firm as one that sets its contractual terms to each agent, thus directly incentivizing them individually. In contrast, we view an outsourcing firm as one that sets the contracting terms only to an externally productive unit from whom the firm sources its inputs—and the external unit contracts with its own agents to incentivize them.

From this contractual perspective, a firm simply has more control with insourcing. Hence, if incentive provision is the only economic friction, the firm cannot be worse off from insourcing, as the designed incentives with outsourcing can always be replicated with insourcing. Our paper’s key insight is that economic frictions concerning the firm’s ability to process information upsets its ability to replicate the outsourcing outcome by insourcing. This is in line with Stucky and White (1993) who report that information structure can determine an organization’s degree of sourcing control. We show that limited information process may lead to outsourcing as the more cost-effective mode of operation.

In an agency framework with multiple agents, we compare two contractual relationships

¹See Venkatesan (1992) and Quinn and Hilmer (1994).

²See Ross (2021) and Gee (2022).

³See Jensen and Meckling (1976) and Easterbrook and Fischel (1989) for example.

in vertical structure, one corresponding to insourcing, and the other to outsourcing. In particular, we model the situation with a firm (the principal) and two agents in a vertical relationship in that communications go through a “chain of command”—the principal receives a report only from one of the agents (the prime-agent), who receives a report from the other agent (the sub-agent). While existing literature motivates such vertical structures by costly communication,⁴ our focus on a fixed vertical structure also allows us to contrast two operational modes purely on the basis of different contractual relationships.

With insourcing, both agents are on the principal’s payroll as they belong to the principal’s firm. As such, although not all agents can directly communicate with her, the principal can directly incentivize both agents—each agent receives a transfer directly from the principal. With outsourcing, on the other hand, the principal pays only one overall transfer to the external organization that provides the required input to the firm—the principal pays only to the prime-agent, who in turn incentivizes the sub-agent.

As mentioned above, we introduce limited information process as an economic friction. With no limits on information process, the principal can perfectly process decomposed information about the two agents from the prime-agent’s report. When the principal’s information process is limited, in contrast, she can only process coarse information—hence, the prime-agent only reports aggregate information about the agents.⁵ As Arrow (1974) points out, all organizations have their “limits as information processors.” It is for this reason that a firm’s top management often makes its decisions based on “executive summaries” or “briefing notes,” instead of detailed information about day-to-day business. The main insight of our study is that such limited information process may be the very cause why firms prefer outsourcing to insourcing. We obtain this insight by comparing in- and outsourcing under unlimited and limited information process.

Preview of the results is as follows. Under unlimited information process, although the principal must provide information rents to the agents for their truthful behavior, insourcing brings about no hierarchical efficiency loss.⁶ In other words, although the vertical structure provides the prime-agent with a superior position to manipulate information (as he reports both his and the sub-agent’s information), he is unable to take advantage of his position when the principal’s information process is unlimited. As a result, the optimal insourcing

⁴See Radner (1993) and Melumad et al. (1995), for example, for such justifications for vertical structures.

⁵We follow Laffont and Martimort (1997, 1998) in modeling this part. Under limited information process, the principal can only process a report on the aggregate information of the agents’ types.

⁶That is, under unlimited information process, the optimal insourcing outcome coincides with the optimal outcome from the structure where each agent directly reports to the principal.

contract treats the agents symmetrically under unlimited information process.

Outsourcing, by contrast, is accompanied by a loss of control—the prime-agent can take advantage of his position in the vertical structure. Since the principal pays the overall transfer to the prime-agent, who subsequently incentivizes the sub-agent, the principal’s rent provision is “double-marginalized.”⁷ The principal not only needs to incentivize the prime-agent for his truthful report, but she also needs to incentivize him to incentivize the sub-agent for a truthful report. As a result, the principal must provide a strictly positive information rent to the prime-agent regardless of his information (his type) in the optimal outsourcing contract. This rent expresses the principal’s hierarchical efficiency loss from outsourcing and implies that insourcing is the prevailing mode of operation when the principal can perfectly process information.

Under limited information process, a new incentive problem arises with insourcing. Since the principal can process only the aggregate information about the agents, the prime-agent has extra room for manipulating information. In particular, he can now, regardless of the sub-agent’s report to him, misreport such that he can reap the largest amount of rent. As a result, the optimal insourcing contract no longer treats the agents symmetrically. In particular, whereas the sub-agent’s information rent depends on the prime-agent’s report to the principal, the prime-agent’s rent is independent of the sub-agent’s report to the prime-agent. In the end, the prime-agent’s rent is not only independent of the sub-agent’s type, but also strictly larger compared to the case under unlimited information process.

Because of this, the principal cannot use the available (aggregate) information as effectively in that for some parameter values, the optimal project sizes become less distinct. Again, under limited information process, while the prime-agent is fully informed when reporting to the principal, the principal can only process the aggregate information and thus cannot tell each agent’s individual type. The principal, however, must incentivize each agent separately based on her coarse information—inducing the agents’ truthful behavior becomes significantly costlier than when her information process is unlimited. As a result, the optimal insourcing contract uses the available information less effectively to incentivize the agents, thus reducing the degree of separation in the project sizes.

Because with outsourcing the principal pays only the prime-agent, who then incentivizes the sub-agent based on the sub-agent’s individual information, the principal’s limited information process under outsourcing does not suffer from this ineffective use of information. Hence, under limited information process the principal faces a clear trade-off between in-

⁷See also McAfee and McMillan (1995) for double marginalization of rent in vertical structure.

sourcing and outsourcing. Insourcing has the disadvantage that she cannot use the aggregate information as effectively, while outsourcing has the aforementioned disadvantage of the double-marginalization effect. We show that this trade-off renders outsourcing optimal when the likelihood that agents are efficient is small.

In an extension, we show that the possibility of collusion between the agents reinforces our result. Under unlimited information process, neither mode of operation is prone to collusion, and therefore insourcing still dominates outsourcing. Under limited information process, on the other hand, the agents' incentive to collude becomes an issue with insourcing, whereas no such incentive arises with outsourcing.⁸ As a result, the parameter range in which outsourcing is optimal is enlarged when the agents can collude.

2 Review of the Literature

Distinguishing in- and outsourcing from a contractual perspective, our paper belongs to the literature on the optimal structure of organizations. Much of the literature, however, compares horizontal and vertical structure, whereas our paper compares two operational modes within the vertical structure.⁹ In earlier works, such as Baron and Besanko (1992) and Melumad et al. (1995) identify conditions under which the vertical structure performs as well as the horizontal structure. They show that, if the principal can perfectly monitor transactions between the agents and information process within the organization is unlimited, the optimal outcome is independent of the structure.

In practice, neither perfect monitoring nor unlimited information process may be feasible. In such cases, as noted by Mookherjee (2006) in his extensive survey of the literature, agents within an organization may have incentives to engage in undesirable manipulation from an efficiency perspective, and different organizational structures may be adopted by the top management to deal with the incentive problems.

Laffont and Martimort (1998) shows that even when the principal's information process is limited, the horizontal and the vertical structure can perform equally well, but only without the possibility of collusion between the agents. They show that, under collusion,

⁸Che and Kim (2006) show that the prospect of collusion between the agents imposes no cost to the principal in a large class of models. Our results, however, identify that preventing collusion may no longer be costless when the principal's information process is limited.

⁹While there are papers on optimal structures based on cost of information process (e.g. Radner 1993, Bolton and Dewatripont 1994) or coordination (e.g. Harris and Raviv 2002, Hart and Moore 2005), the closest papers to ours study organizational structures where the agents are endowed with private information.

the vertical structure dominates the horizontal structure. Mookherjee and Tsumagari (2004) demonstrate that, if the principal can manipulate the agents' reservation payoffs then the horizontal structure always dominates the vertical one.¹⁰ In both of these papers, the vertical structure exhibits only contractual delegation, whereas in ours, it encompasses both communicational and contractual delegation. Choe and Ishiguro (2012) show that the optimal hierarchy depends on the extent to which externalities among a firm's projects require coordination and effort incentives. Their paper, however, has no hidden information component in incentive problems, thus bypassing the issue of communicational delegation.

Celik et al. (2023) show that the vertical structure is optimal when the principal can manipulate information from the agents. Their study, like ours, assumes that the principal's information process is limited, and identifies that, under horizontal structure (all agents directly report to the principal), the principal has an incentive to manipulate aggregated information, and mitigating her incentive requires a pooling outcome. They show that, an optimal way to remove the principal's manipulation incentive is to delegate communication to one of the agents at the cost of loss of control. Unlike theirs, the principal's manipulation incentive is not an issue in our study since information is aggregated by an agent in our model. Our focus in this paper is to compare different contractual relationships in the vertical structure.¹¹

Whereas all studies mentioned above compare the horizontal (centralization) and the vertical structure (delegation), our study takes the vertical structure as given, to examine the optimality of different operational modes within the structure.¹² That is, aggregation of information is always delegated to an agent in the current paper, and thus communication within the organization is always sequential. In fact, as noted by Brooke (1984), no operation in practice is fully centralized in any organization, and in that sense, our study also casts light on optimal choice between *partial* and *full delegation* within an organization.

There are also studies on the optimal organizational structures in the incomplete contract paradigm. Alonso et al. (2008) and Rentakari (2008) show that, when cheap talk communication between agents is the medium through which coordination is achieved, a decentralized structure can dominate a centralized structure. Unlike theirs, our paper employs the complete contract approach in which all choice variables are contracted upon.

Empirical studies related to ours include the following papers. Pierce (2012) shows

¹⁰For studies advocating horizontal structures by highlighting a loss of control in vertical structure, see Williamson (1967), McAfee and McMillan (1995) and Gilbert and Riordan (1995) among others.

¹¹The "hierarchical" or delegated communication in their study coincides with "insourcing" in our paper.

¹²Jansen et al. (2008) study vertical separation vs. vertical integration. Their focus, however, is collusion.

that enhancing its organizational boundary does not necessarily reduce the agency cost in a firm’s operation. That is, vertically integrated organizations do not always outperform disintegrated organizations. Weigelt and Miller (2013) demonstrate that an organization’s boundary is influenced by its internal structure—e.g., when a subunit proposes to integrate a new task, the top management considers how to handle communication with other subunits within the organization and units outside the boundary. In a computational simulation, Clement and Puranam (2018) demonstrate that, in the absence of a formal vertical structure, coordination within an organization is prone to decline over time, implying that organizations in the long run may need to be vertically structured. A more recent study by Lee (2022) suggests that extending delegation by adding formal hierarchical levels of managers can substantially help even small organizations such as start-ups achieve commercial success, unless their outputs heavily rely on creativity.

Lastly, there are studies analyzing in- and outsourcing, especially in the field of industrial organization and international trade. We, however, adopt a different approach from those studies. Grossman and Helpman (2002), for example, address the choice between in- and outsourcing in a general equilibrium framework, to identify sectoral characteristics that lead to one or the other equilibrium structure. Their study is extended to an international trade setting by Antras (2003) who shows that while insourcing provides well-defined property rights, such rights may not give insourcing an advantage over outsourcing.¹³ Unlike these papers, we look at sourcing policies purely from a contractual perspective.

The rest of the paper is organized as follows. The model is presented in the following section, followed by the results under unlimited and limited information process presented in Section 4 and 5 respectively. Section 6 extends our results. Section 7 concludes. All proofs are relegated to the Appendix.

3 Model

Project and Information A principal decides on the size of a project $x \in \mathbb{R}_+$, yielding a return $v(x)$. The project size x is publicly verifiable and the principal’s value function $v(x)$ satisfies that: $v(0) = 0$, $v(\infty) = \infty$, $\lim_{x \rightarrow 0} v'(x) = \infty$, $\lim_{x \rightarrow \infty} v'(x) = 0$, $v''(x) < 0$ and $v'''(x) \geq 0$.¹⁴ For the project of size x , the principal needs an input of amount x . The

¹³See also Ethier (1986) and Ethier and Markusen (1996) for earlier studies on multinationals’ decisions on insourcing (FDI) and outsourcing.

¹⁴Commonly used utility functions such as those exhibiting constant absolute risk aversion (CARA) or constant relative risk aversion (CRRA) satisfy these conditions.

input is produced by two agents, α and β , who provide complementary contributions.

For input of x , each agent $k \in \{\alpha, \beta\}$ receives a transfer t^k and bears the cost $\theta_i^k x$, where $\theta_i^k \in \{\theta_g, \theta_b\}$ is his private information, and $\Delta\theta \equiv \theta_b - \theta_g > 0$.¹⁵ An agent with θ_g (θ_b) is said to be type- g (type- b). Because θ_i^k represents an agent’s marginal cost and $\theta_b > \theta_g$, we refer to type- g as “efficient,” and type- b as “inefficient.” The probability distributions of the agents’ types are independent and identical—an agent is type- g with probability $\varphi \in (0, 1)$, and thus type- b with $1 - \varphi$. The probability distribution is public knowledge.

It follows that the aggregate marginal cost of the project can be one of the values below:

$$\Theta_G \equiv 2\theta_g, \quad \Theta_M \equiv \theta_g + \theta_b, \quad \Theta_B \equiv 2\theta_b.$$

Thus, the first-best size of the project, denoted by x^* , is characterized by:

$$v'(x_\gamma^*) = \Theta_\gamma, \quad \gamma \in \{G, M, B\},$$

where the marginal value equals the aggregate marginal cost. The first-best outcome depends on the aggregate type, $\{G, M, B\}$, rather than the decomposed types, $\{gg, gb, bg, bb\}$.

Limited Liability While the principal is fully committed to her contract offer, we assume that the agents are protected by limited liability in that each agent can quit and walk away from the contract at any point. An agent will do so if he expects his payoff to be less than his reservation payoff level, which is normalized to zero. If any agent quits, then no payoff is realized from the project for all players and the game ends at that point—that is, $x_\emptyset = t_\emptyset^k = 0$, where the subscript \emptyset denotes an agent’s message that he quits.¹⁶

Vertical Structure and Information Process We follow Radner (1993) and Melumad et al. (1995) to motivate the vertical structure by costly communication. Communication is costly so that the principal receives a report only from agent α (the prime-agent). Therefore, the agents operate in a vertical structure where agent β (the sub-agent) sends his report $j \in \{g, b\}$, to agent α , who in turn communicates with the principal about his own type $i \in \{g, b\}$ and agent β ’s report $j \in \{g, b\}$.

We consider two forms of information process by the principal—*unlimited* and *limited information process*. Under *unlimited information process*, agent α sends to the principal a

¹⁵See Martimort (1997) and Shin and Yun (2013) for studies in which the agents learn each other’s type before any reports are made.

¹⁶The limited liability reflects a “non-slavery condition.” See Sappington (1983) for more on this issue.

decomposed report consisting of a report about both his own and agent β 's type. In this case, agent α 's report is $\gamma \in \bar{\Gamma} \equiv \{gg, gb, bg, bb, \emptyset\}$, where \emptyset denotes the “quit” message.¹⁷

Under *limited information process*, the principal's opportunity cost allows her to assimilate only a rough information (such as an executive summary). Specifically, when the principal's ability to process information is limited, she can only process aggregate information on the agents' types. Therefore, agent α 's report under such a limit is $\gamma \in \Gamma \equiv \{G, M, B, \emptyset\}$, only stating information on the agents' aggregate type, not individual types.

Sourcing and Contracts Our objective is to compare two organizational modes in the vertical structure.¹⁸ We interpret these modes as “insourcing” and “outsourcing” as explained as follows. Insourcing is viewed as a mode where agent α and β are both internal agents of the principal's organization. As such, both agents are on the principal's payroll, and therefore the contract has the form $\mathcal{C}^I \equiv (x_\gamma, t_\gamma^\alpha, t_\gamma^\beta)$, specifying an individual transfer to agent α and β , where $\gamma \in \bar{\Gamma}$ or $\gamma \in \Gamma$ depending on the information process by the principal. Hence, with insourcing, the principal can directly incentivize each agent.

By contrast, outsourcing is viewed as an organizational mode where agent α and β are external to the principal's organization, and the principal therefore can set only one overall transfer t to the external organization. That is, the principal offers a prime-contract to agent α , who hires and incentivizes agent β by sub-contracting with him. With outsourcing, the principal's prime-contract to agent α specifies $\mathcal{C}^O \equiv (x_\gamma, t_\gamma)$, where $\gamma \in \bar{\Gamma}$ or $\gamma \in \Gamma$ depending on the information process by the principal. Agent α 's sub-contract to agent β specifies $\mathcal{C}_s^O \equiv (\gamma(ij), t_{ij}^\beta)$, where $j \in \{g, b\}$ is agent β 's report on his type to agent α , and $\gamma(ij)$ is agent α 's report to the principal.¹⁹ We note that $\gamma(ij)$ represents how agent α responds to the prime-contract contract \mathcal{C}^O .

Table 1. summarizes the contracts offered by the principal depending on her ability to process information and on the sourcing policy.

¹⁷On the equilibrium path, $\gamma = \emptyset$ does not take place as it is prevented in the optimal contract. Due to the Inada condition of the value function $v(x)$, the marginal value of the project at $x = 0$ is infinite and thus $x = 0$ cannot be optimal for the principal.

¹⁸In a related study, Celik et al. (2023) compare “centralized communication” where all agents report directly to the principal, with “hierarchical communication” where the setting is the same as “insourcing” in this paper. In Celik et al. (2023), the highlight is to identify the principal's manipulation incentive under centralized communication. In this paper, we compare different contractual relationships (interpreted as in- and outsourcing) under the same communicational structure. As aforementioned, we focus on the vertical structure by assuming that it is too costly for the principal to communicate with all agents.

¹⁹As will be explained later, we can treat the sub-contracting stage as if agent β knows agent α 's type.

	Insourcing	Outsourcing
Unlimited Process	$\mathcal{C}^I \equiv (x_\gamma, t_\gamma^\alpha, t_\gamma^\beta),$ $\gamma \in \bar{\Gamma} = \{gg, gb, bg, bb, \emptyset\}$	$\mathcal{C}^O \equiv (x_\gamma, t_\gamma),$ $\gamma \in \bar{\Gamma} = \{gg, gb, bg, bb, \emptyset\}$
Limited Process	$\mathcal{C}^I \equiv (x_\gamma, t_\gamma^\alpha, t_\gamma^\beta),$ $\gamma \in \Gamma = \{G, M, B, \emptyset\}$	$\mathcal{C}^O \equiv (x_\gamma, t_\gamma),$ $\gamma \in \Gamma = \{G, M, B, \emptyset\}$

Table 1. Categories of Contracts offered by the Principal

The timing for each sourcing policy is as follows.

• **Insourcing**

1. The principal offers the contract $\mathcal{C}^I = (x_\gamma, t_\gamma^\alpha, t_\gamma^\beta)$ to the agents.
2. Agent β sends $j \in \{g, b\}$ to agent α .
3. Agent α sends $\gamma(ij)$ to the principal.
4. The principal confirms the report by publicly announcing the reported γ .
5. The agents implement the project of size x_γ , and the principal pays t_γ^α and t_γ^β to agent α and β respectively.

Given the report γ , the principal's and the agents' ex post payoffs with insourcing are:

$$\pi \equiv v(x_\gamma) - t_\gamma^\alpha - t_\gamma^\beta \quad \text{and} \quad u^k \equiv t_\gamma^k - \theta^k x_\gamma.$$

• **Outsourcing**

1. The principal offers the prime-contract $\mathcal{C}^O = (x_\gamma, t_\gamma)$ to agent α .
2. Agent α offers the sub-contract $\mathcal{C}_s^O = (\gamma(ij), t_{ij}^\beta)$ to agent β .
3. Agent β sends $j \in \{g, b\}$ to agent α .
4. Agent α sends $\gamma(ij)$ to the principal.
5. The principal confirms the report by publicly announcing the reported γ .
6. The agents implement the project of size x_γ . The principal pays t_γ to agent α , who in turn pays t_{ij}^β to agent β .

Given the agent α 's report γ (to the principal) and agent β 's report j (to agent α), the principal's and the agents' ex post payoffs with outsourcing are:

$$\pi \equiv v(x_\gamma) - t_\gamma, \quad u^\alpha \equiv t_\gamma - t_{ij}^\beta - \theta^\alpha x_\gamma \quad \text{and} \quad u^\beta \equiv t_{ij}^\beta - \theta^\beta x_\gamma.$$

Recall that each agent is protected by the limited liability. At any point of the time line, an agent can quit if he wants to do so, in which case the game ends at that point with no profit.²⁰ All parties must respect the contract terms unless one or both agents quit, at which point the game ends with zero payoffs from the project for all.

The principal's sourcing policies are illustrated in Figure 1.

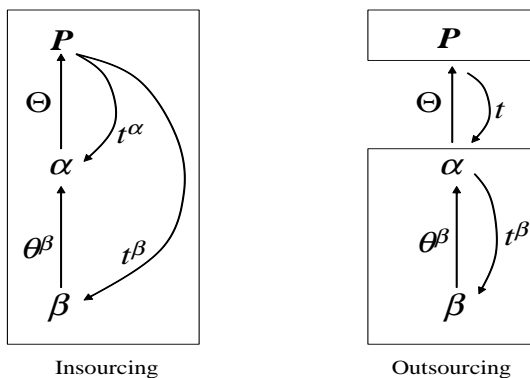


Figure 1. Insourcing and Outsourcing

4 Unlimited Information Process

We first analyze the case where the principal can perfectly process received information in that she can condition the contract on the report about each agent's individual type, $i, j \in \{g, b\}$, where i and j are agent α 's and β 's type respectively.

4.1 Insourcing

With insourcing, the principal directly pays a transfer to each agent. As mentioned in the previous section, the contract specifies:

$$\mathcal{C}^I = \left(x_\gamma, t_\gamma^\alpha, t_\gamma^\beta \right), \quad \gamma \in \bar{\Gamma} = \{gg, gb, bg, bb, \emptyset\},$$

²⁰Under insourcing, agent α can be seen as the head of a division and agent β as the head of sub-division, each of who can break off his employment relationship with the principal. Under outsourcing, agent α can be seen as the head/owner of the supplier, who can declare bankruptcy, and agent β as the operational manager who can break off his employment relationship with agent α .

and the principal gets reported only from agent α , who receives a report from agent β .

The Inada condition of the principal's value function $v(x)$ implies that she wants to have a strictly positive project size regardless of the agents' types. Because an agent can quit anytime, the contract must provide a non-negative ex post rent to each agent for all combinations of i and j . The contract must therefore satisfy the following participation constraints for agent α and β respectively:

$$t_{ij}^\alpha - \theta_i x_{ij} \geq 0, \quad i, j \in \{g, b\}, \quad (1)$$

$$t_{ij}^\beta - \theta_j x_{ij} \geq 0, \quad i, j \in \{g, b\}. \quad (2)$$

Since agent β does not know agent α 's type when reporting to agent α , the optimal contract satisfies the following (interim) Bayesian incentive compatibility constraints for agent β :

$$\begin{aligned} & \varphi \left[t_{gj}^\beta - \theta_j x_{gj} \right] + (1 - \varphi) \left[t_{bj}^\beta - \theta_j x_{bj} \right] \\ & \geq \varphi \left[\max\{t_{gj'}^\beta - \theta_j x_{gj'}, 0\} \right] + (1 - \varphi) \left[\max\{t_{bj'}^\beta - \theta_j x_{bj'}, 0\} \right], \quad j, j' \in \{g, b\}. \end{aligned} \quad (3)$$

The $\max\{\cdot, 0\}$ operators in the RHS of the constraint reflect agent β 's option to quit after misreporting his type to agent α . While it is implied by (2) that agent β will not quit on the equilibrium path, he may consider it off the equilibrium path, depending on agent α 's report to the principal.

The incentive conditions for agent β above imply that agent α , when he makes a report to the principal, has learned agent β 's type. Inducing agent α 's truthful report about both θ^α and θ^β requires that the optimal contract satisfies the following (ex post) incentive compatibility conditions:

$$t_{ij}^\alpha - \theta_i x_{ij} \geq \begin{cases} t_{i'j'}^\alpha - \theta_i x_{i'j'} & \text{if } t_{i'j'}^\beta - \theta_j x_{i'j'} \geq 0 \\ 0 & \text{otherwise} \end{cases}, \quad i, j, i', j' \in \{g, b\}. \quad (4)$$

The incentive constraints for agent α are more restrictive than those for agent β . First, agent α knows agent β 's type when making his report to the principal, thus may have more flexibility for manipulation. Hence, the constraints for agent α have to hold state-by-state, rather than only in expected terms as in the case of agent β . Second, the RHSs of the incentive constraints for agent α are conditioned on agent β 's resulting payoff, since agent β may choose to “quit” if he faces a negative payoff from agent α 's misreport. Note that the

participation constraint (2) does not preclude such quitting, since it only prevents quitting on the equilibrium path.²¹

Under unlimited information process, the outcome of the following problem is the optimal contract with insourcing:

$$\bar{\mathcal{P}}^I: \max_{\mathcal{C}^I} \pi(\mathcal{C}^I) = \left\{ \begin{array}{l} \varphi^2[v(x_{gg}) - \sum_k t_{gg}^k] + \varphi(1 - \varphi)[v(x_{gb}) - \sum_k t_{gb}^k] \\ + \varphi(1 - \varphi)[v(x_{bg}) - \sum_k t_{bg}^k] + (1 - \varphi)^2[v(x_{bb}) - \sum_k t_{bb}^k] \end{array} \right\},$$

subject to the agents' participation and incentive compatibility constraints: (1)–(4). The solution to $\bar{\mathcal{P}}^I$ is presented in the following proposition.

Proposition 1 *Under unlimited information process, the optimal contract with insourcing entails the project sizes $x_{gg}^I > x_{gb}^I = x_{bg}^I > x_{bb}^I$ characterized by the following:*

$$\begin{aligned} v'(x_{gg}^I) &= \Theta_G, \\ v'(x_{gb}^I) &= v'(x_{bg}^I) = \Theta_M + \frac{\varphi}{1 - \varphi} \Delta\theta, \\ v'(x_{bb}^I) &= \Theta_B + \frac{2\varphi}{1 - \varphi} \Delta\theta. \end{aligned}$$

It yields an agent a strictly positive rent only when he is efficient.

Under unlimited information process, the principal treats the agents symmetrically even though agent α has more flexibility to manipulate information. The reason is that agent α 's information manipulation is constrained by agent β 's quit option. For example, when $ij = gb$, agent α 's information rent from the truthful report is $t_{gb}^\alpha - \theta_g x_{gb}^I = \Delta\theta x_{bb}^I$, and he wants to misreport the types as $ij = bg$ because: $t_{bg}^\alpha - \theta_g x_{bg}^I = \Delta\theta x_{bg}^I > \Delta\theta x_{bb}^I$. Such misreporting by agent α , however, would induce agent β to quit since his payoff from participating in the project becomes negative as: $t_{bg}^\beta - \theta_b x_{bg}^I = \Delta\theta(x_{bb}^I - x_{bg}^I) < 0$. As a result, agent α will not misreport their types as bg when the true types are $ij = gb$ in the optimal contract.

The point here is that, despite the chain of command in information flow, agent β 's limited liability implies that there is no hierarchical efficiency loss when the project is implemented entirely in-house if the principal's information process is unlimited.²² As will be shown next, that is no longer true when the project's input is outsourced.

²¹Notice that agent β 's report does not affect agent α 's quit option. Since his report takes place before agent α 's report to the principal, agent β 's report is on the equilibrium path from agent α 's viewpoint.

²²This also implies that the optimal outcome presented in Proposition 1 is the same outcome under pure centralization where each agent directly reports to the principal.

4.2 Outsourcing

With outsourcing, the principal deals only with agent α , who hires and incentivizes agent β . By backward induction, we first look at agent α 's sub-contracting problem before solving the principal's problem. When offering the sub-contract to agent β , agent α is privately informed about his own type. In the sense of Maskin and Tirole (1990), the sub-contracting stage therefore conforms to an "informed principal problem" but with "private values," because agent α 's private information does not directly enter agent β 's payoff function.

Following Maskin and Tirole (1990), we can analyze the sub-contracting stage as if agent β were fully informed about agent α 's private information.²³ Because agent β is privately informed about his type, it is optimal for agent α to offer agent β a direct revelation contract that induces agent β to report $j \in \{g, b\}$ truthfully.

The sub-contract is contingent on agent β 's report $j \in \{g, b\}$, and specifies the message $\gamma(ij) \in \bar{\Gamma} = \{gg, gb, bg, bb, \emptyset\}$ that agent α is to report to the principal based on his own type $i \in \{g, b\}$ and agent β 's report, together with the transfer to agent β :

$$\mathcal{C}_s^O = \left(\gamma(ij), t_{ij}^\beta \right).$$

Given the prime-contract $\mathcal{C}^O = (x_\gamma, t_\gamma)$ from the principal, agent α of type- $i \in \{g, b\}$ solves the following problem:

$$\bar{\mathcal{P}}_s^O : \max_{\mathcal{C}_s^O} u^\alpha(\mathcal{C}_s^O) = \varphi[t_{\gamma(ig)} - t_{ig}^\beta - \theta_i x_{\gamma(ig)}] + (1 - \varphi)[t_{\gamma(ib)} - t_{ib}^\beta - \theta_i x_{\gamma(ib)}],$$

subject to:

$$t_{ij}^\beta - \theta_j x_{\gamma(ij)} \geq 0, \quad j \in \{g, b\}, \quad (5)$$

for agent β 's participation, and

$$t_{ij}^\beta - \theta_j x_{\gamma(ij)} \geq \max\{t_{ij'}^\beta - \theta_j x_{\gamma(ij')}, 0\}, \quad j, j' \in \{g, b\}, \quad (6)$$

for his truthful report on his type. Again, the $\max\{\cdot, 0\}$ operator in the RHS of the constraint reflects agent β 's option to quit after misreporting his type to agent α if agent α 's report to the principal would yield him a strictly negative payoff.

As usual in a model of this type, the participation constraints for the inefficient type- b and incentive constraints for the efficient type- g are binding. Hence, agent α of type- i 's

²³Intuitively, this is so because the optimal sub-contract for the case where agent α 's type $i \in \{g, b\}$ is observable to agent β is automatically incentive compatible for the case where agent α 's type is not observable, as agent β 's payoff does not depend on the information about agent α 's type.

optimal transfers to agent β are:

$$t_{ib}^\beta = \theta_b x_{\gamma(ib)} \quad \text{and} \quad t_{ig}^\beta = \theta_g x_{\gamma(ig)} + \Delta\theta x_{\gamma(ib)}. \quad (7)$$

The expression shows that agent β of type- g receives a strictly positive rent, as long as $x_{\gamma(ib)} > 0$ (i.e., as long as the project with a strictly positive size is implemented when agent β is type- b). Agent α 's problem, therefore, is a standard screening problem, except for the following subtle but important point. Notice that, agent β 's rent becomes zero if agent α 's message to the principal is $\gamma(ib) = \emptyset$ (i.e., if agent α quits when agent β reports $j = b$). As we now explain, this plays an important role as agent α may in fact have an incentive to do so if the prime-contract from the principal fails to provide him with a large enough rent.

Indeed, to see such an incentive of agent α , we now move on to the principal's problem. The prime-contract offered from the principal to agent α is:

$$\mathcal{C}^O \equiv (x_\gamma, t_\gamma), \quad \gamma \in \bar{\Gamma} = \{gg, gb, bg, bb, \emptyset\}.$$

When making her offer, the principal anticipates agent α 's problem $\bar{\mathcal{P}}_s^O$ in the next stage. By the revelation principle, there is no loss in assuming that the optimal prime-contract induces agent α to choose his sub-contract with $\gamma(ij) = ij$, where i is agent α 's type and j is agent β 's report. The principal's optimal prime-contract offered to agent α therefore satisfies the incentive constraints that are implied by the next lemma.

Lemma 1 *Under unlimited information process, agent α chooses \mathcal{C}_s^O with $\gamma(ij) = ij$ if and only if the prime-contract \mathcal{C}^O satisfies:*

$$t_{ig} - (\theta_i + \theta_g)x_{ig} \geq \max\{t_{i'j'} - (\theta_i + \theta_g)x_{i'j'}, 0\}, \quad (8)$$

$$t_{ib} - \left(\theta_i + \theta_b + \frac{\varphi}{1-\varphi} \Delta\theta \right) x_{ib} \geq \max\{t_{i'j'} - \left(\theta_i + \theta_b + \frac{\varphi}{1-\varphi} \Delta\theta \right) x_{i'j'}, 0\}, \quad (9)$$

where $i, i', j' \in \{g, b\}$.

The $\max\{\cdot, 0\}$ operators in the RHSs of (8) and (9) reflect once more agent α 's potential choice of $\gamma = \emptyset$. The constraints in the lemma also imply agent α 's participation on the equilibrium path.²⁴

²⁴By (7) and agent α 's truthful report (Lemma 1), the participation constraints for him are written as: $t_{ig} - (\theta_i + \theta_g)x_{ig} - \Delta\theta x_{ib} \geq 0$ and $t_{ib} - (\theta_i + \theta_b)x_{ib} \geq 0$, $i \in \{g, b\}$. The first participation constraint is satisfied since $t_{ig} - (\theta_i + \theta_g)x_{ig} - \Delta\theta x_{ib} \geq t_{ib} - (\theta_i + \theta_g)x_{ib} - \Delta\theta x_{ib} = t_{ib} - \left(\theta_i + \theta_b + \frac{\varphi}{1-\varphi} \Delta\theta \right) x_{ib} + \frac{\varphi}{1-\varphi} \Delta\theta x_{ib} > 0$, where the weak and the strict inequality are implied by (8) and (9) respectively. The second participation constraint is implied by (9).

Under unlimited information process, the principal's problem with outsourcing is:

$$\bar{\mathcal{P}}^O : \max_{\bar{\mathcal{C}}^O} \pi(\bar{\mathcal{C}}^O) = \left\{ \begin{array}{l} \varphi^2 [v(x_{gg}) - t_{gg}] + \varphi(1 - \varphi) [v(x_{gb}) - t_{gb}] \\ + \varphi(1 - \varphi) [v(x_{bg}) - t_{bg}] + (1 - \varphi)^2 [v(x_{bb}) - t_{bb}] \end{array} \right\},$$

subject to (8) and (9) in Lemma 1.

The following proposition presents the optimal outcome in $\bar{\mathcal{P}}^O$, where the principal can process the aggregate information from agent α with no limits.

Proposition 2 *Under unlimited information process, the optimal contract with outsourcing entails the project sizes $x_{gg}^O > x_{bg}^O > x_{gb}^O > x_{bb}^O$ characterized by the following:*

$$\begin{aligned} v'(x_{gg}^O) &= \Theta_G, \\ v'(x_{bg}^O) &= \Theta_M + \frac{\varphi}{1 - \varphi} \Delta\theta, \\ v'(x_{gb}^O) &= \Theta_M + \frac{\varphi}{1 - \varphi} \left(2 + \frac{\varphi}{1 - \varphi} \right) \Delta\theta, \\ v'(x_{bb}^O) &= \Theta_B + \frac{\varphi}{1 - \varphi} \left(3 + \frac{\varphi}{1 - \varphi} \right) \Delta\theta. \end{aligned}$$

It yields agent α a strictly positive rent regardless of his type, and agent β a strictly positive rent only when he is efficient.

Comparing the outcomes in Propositions 1 and 2 shows that x_{gg}^O and x_{bg}^O coincide to the optimal project sizes with insourcing. By contrast, x_{gb}^O and x_{bb}^O are distorted further downward ($x_{gb}^O < x_{gb}^I$ and $x_{bb}^O < x_{bb}^I$). These additional distortions stem from the fact that, unlike in the case with insourcing, the prime-agent receives strictly positive rent regardless of his type, whereas the sub-agent's rent is positive only when he is type- g . This hinges upon the limited liability of the agents that each agent can quit any time he wants.

To see this, recall that, since agent β is not on the principal's payroll, agent α may have an incentive to shut down the project if agent β is of a certain type and walk away (by setting $\gamma(i, j) = \emptyset$ in the sub-contract). In fact, this incentive arises when agent α is type- b . Suppose no rent is given to agent α of type- b . Then, agent α 's sub-contract will optimally shut down the project in the case agent β reports that he is type- b (that is, agent α will set $\gamma(b, b) = \emptyset$ in the sub-contract). This way, agent α 's expected rent becomes strictly positive since he is effectively pocketing agent β of type- g 's information rent. To see this, recall from (7) that agent β of type- g receives a strictly positive rent as long as type- b is not excluded by agent α (i.e., $x_{\gamma(i,b)} > 0$). By setting $\gamma(b, b) = \emptyset$ (and walking away when

both agents are type- b), agent α of type- b 's expected rent becomes $\varphi\Delta\theta x_{bb}$ (he grabs rent of $\Delta\theta x_{bb}$ only if agent β is type- g), whereas agent β 's rent is zero.(and he is hired only when he is type- g). To prevent this opportunistic shutdown by agent α , the optimal prime-contract must provide a strictly positive rent to agent α even when he is type- b .

4.3 Comparison

We now compare the sourcing policies under unlimited information process. The optimal outcomes presented in Propositions 1 and 2 lead to the following corollary.

Corollary 1 *Suppose the principal's information process is unlimited. Then, insourcing strictly dominates outsourcing.*

The reason behind this result is the “double marginalization of rent” by agent α , which takes place only with outsourcing. With insourcing, although the prime-agent can misrepresent his own and/or the sub-agent's type, the latter is not an issue once the sub-agent is incentivized to report his type truthfully to the prime-agent. Because each agent's transfer payment comes directly from the principal, together with the limited liability, the prime-agent's incentive for manipulation is bounded so that he would not misrepresent the sub-agent's type. At the end, with insourcing, the principal does not lose control to the prime-agent who may manipulate information in the middle. That is, the principal's problem is not restricted by the prime-agent's position that provides the agent an additional advantage of information manipulation in the organization. Consequently, there is no double marginalization of rent due to hierarchical inefficiency with insourcing.

With outsourcing, the sub-agent gets paid by the prime-agent, which allows the prime-agent to take an advantage of his position between the principal and the sub-agent. Whereas the principal wants to have a strictly positive project size regardless of the agents' types, the prime-agent, when he is inefficient, has an incentive to hire only the efficient sub-agent, thus walking away from the project when the sub-agent is inefficient. By such exclusion, the inefficient prime-agent can keep the efficient sub-agent's information rent from the principal in his pocket. To prevent this, the principal must also provide a strictly positive rent to the prime-agent when he is inefficient. In equilibrium, double marginalization of rent takes place with outsourcing since the prime-agent has full flexibility in the middle. As a result, the optimal project sizes are distorted downwards further from the optimal levels with insourcing.

5 Limited Information Process

In this section, we show that when the principal's information process is limited, the principal has a further informational disadvantage, leading to a genuine trade-off between in- and outsourcing. Whereas agent α makes his choice based on decomposed information of individual types, the principal determines the optimal project sizes based only on coarse information. As will be shown, agent α has more flexibility to manipulate his information in this case, and to alleviate the additional manipulation incentive, the optimal contract must create additional distortions in the project sizes.

5.1 Insourcing

Under limited information process, the insourcing contract offered to the agents specifies:

$$\mathcal{C}^I = (x_\gamma, t_\gamma^\alpha, t_\gamma^\beta), \quad \gamma \in \Gamma = \{G, M, B, \emptyset\}.$$

As mentioned before, due to the Inada conditions of the principal's value function, $\gamma = \emptyset$ leading to $x_\emptyset = t_\emptyset = 0$ cannot be optimal. The contract must provide a non-negative ex post rent to the agents for any $\gamma \in \{G, M, B\}$, and thus must satisfy the following participation constraints:

$$t_\gamma^k - \theta_g x_\gamma \geq 0, \quad \gamma \in \{G, M\}, \quad (10)$$

$$t_\gamma^k - \theta_b x_\gamma \geq 0, \quad \gamma \in \{M, B\}. \quad (11)$$

The first set of constraints are for agent $k \in \{\alpha, \beta\}$ of type- g , and from his point of view, the state can be either $\gamma = G$ or $\gamma = M$ depending on the other agent's type. Likewise, for a type- b agent, $\gamma = M$ or $\gamma = B$.

As before, the following Bayesian incentive compatibility constraints for agent β must be satisfied in the optimal contract:

$$\begin{aligned} & \varphi \left[t_G^\beta - \theta_g x_G \right] + (1 - \varphi) \left[t_M^\beta - \theta_g x_M \right] \\ & \geq \varphi \left[\max\{t_M^\beta - \theta_g x_M, 0\} \right] + (1 - \varphi) \left[\max\{t_B^\beta - \theta_g x_B, 0\} \right], \end{aligned} \quad (12)$$

$$\begin{aligned} & \varphi \left[t_M^\beta - \theta_b x_M \right] + (1 - \varphi) \left[t_B^\beta - \theta_b x_B \right] \\ & \geq \varphi \left[\max\{t_G^\beta - \theta_b x_G, 0\} \right] + (1 - \varphi) \left[\max\{t_M^\beta - \theta_b x_M, 0\} \right], \end{aligned} \quad (13)$$

where the $\max\{\cdot, 0\}$ operators reflect the agent's quit option off the equilibrium path.

Again, agent α , when making a report to the principal, knows agent β 's type. This yields the following incentive constraints for agent α 's truthful reporting:

$$t_G^\alpha - \theta_g x_G \geq \begin{cases} t_\gamma^\alpha - \theta_g x_\gamma & \text{if } t_\gamma^\beta - \theta_g x_\gamma \geq 0 \\ 0 & \text{otherwise} \end{cases}, \quad \gamma \in \{G, M, B\}, \quad (14)$$

$$t_M^\alpha - \theta_g x_M \geq \begin{cases} t_\gamma^\alpha - \theta_g x_\gamma & \text{if } t_\gamma^\beta - \theta_b x_\gamma \geq 0 \\ 0 & \text{otherwise} \end{cases}, \quad \gamma \in \{G, M, B\}, \quad (15)$$

$$t_M^\alpha - \theta_b x_M \geq \begin{cases} t_\gamma^\alpha - \theta_b x_\gamma & \text{if } t_\gamma^\beta - \theta_g x_\gamma \geq 0 \\ 0 & \text{otherwise} \end{cases}, \quad \gamma \in \{G, M, B\}, \quad (16)$$

$$t_B^\alpha - \theta_b x_B \geq \begin{cases} t_\gamma^\alpha - \theta_b x_\gamma & \text{if } t_\gamma^\beta - \theta_b x_\gamma \geq 0 \\ 0 & \text{otherwise} \end{cases}, \quad \gamma \in \{G, M, B\}. \quad (17)$$

The RHS of the incentive constraints, as before, reflects the idea that agent α 's misreporting strategy is bounded by agent β 's limited liability.

Under limited information process, the optimal contract with insourcing is the outcome of the following problem:

$$\mathcal{P}^I: \max_{\mathcal{C}^I} \pi(\mathcal{C}^I) = \varphi^2[v(x_G) - \sum_k t_G^k] + 2\varphi(1-\varphi)[v(x_M) - \sum_k t_M^k] + (1-\varphi)^2[v(x_B) - \sum_k t_B^k],$$

subject to the agents' participation and incentive compatibility constraints: (10)–(17).

Notice that, here the principal essentially solves $\bar{\mathcal{P}}^I$ of the previous section with the following extra constraints: $x_{gb} = x_{bg} = x_M$ and $t_{gb}^k = t_{bg}^k = t_M^k$, $k \in \{\alpha, \beta\}$. The additional constraint on the project sizes has no bite since $x_{gb}^I = x_{bg}^I$ (Proposition 1), but the constraint on the transfers affects the optimal outcome since $t_{gb}^k \neq t_{bg}^k$, $k \in \{\alpha, \beta\}$, under unlimited information process. Hence, the optimal outcome in $\bar{\mathcal{P}}^I$ cannot be implemented in \mathcal{P}^I . The additional constraint makes it costlier for the principal to induce participation of the agents under limited information process. This, in turn, also makes it costlier to induce truthful reports from the agents, as will be shown below.

The following proposition presents the optimal outcome in \mathcal{P}^I .

Proposition 3 (Celik et al. 2023) *Under limited information process, there exists $\hat{\varphi} \in (0, 1/2)$ such that the optimal contract with insourcing entails the project sizes $x_G^I > x_M^I > x_B^I$ characterized by the following:*

- For $\varphi \geq \hat{\varphi}$,

$$v'(x_G^I) = \Theta_G, \quad v'(x_M^I) = \Theta_M + \frac{1+\varphi}{2(1-\varphi)}\Delta\theta, \quad v'(x_B^I) = \Theta_B + \frac{\varphi}{1-\varphi}\Delta\theta.$$

It yields an agent strictly positive rent only when he is efficient.

- For $\varphi < \widehat{\varphi}$,

$$v'(x_G^I) = \Theta_G, \quad 2\varphi v'(x_M^I) + (1 - 2\varphi)v'(x_B^I) = \Theta_B + \frac{\varphi^2}{1 - \varphi}\Delta\theta, \quad x_B^I = \frac{1 - 2\varphi}{1 - \varphi}x_M^I.$$

It yields agent α a strictly positive rent only when he is efficient, and agent β a strictly positive rent only when he is efficient and agent α is inefficient.

When the principal's information process is limited, rent provision to the agents becomes different, and as a result, the optimal project sizes change. For large φ ($> \widehat{\varphi}$), for example, one can see that the optimal x_M is distorted more compared to the optimal project size under unlimited information process ($x_M^I < x_{bg}^I = x_{gb}^I < x_M^*$), while the optimal x_B is distorted less ($x_B^* > x_B^I > x_{bb}^I$). In other words, the optimal project sizes for $\gamma = M$ and $\gamma = B$ are closer to each other under limited information process.

The intuition behind the changes in project sizes is as follows. First, under unlimited information process, agents α and β of type- g command the following information rents:

$$\begin{aligned} t_{gg}^\alpha - \theta_g x_{gg}^I &= t_{gg}^\beta - \theta_g x_{gg}^I = \Delta\theta x_{gb}^I (= \Delta\theta x_{bg}^I) \quad \text{and} \\ t_{gb}^\alpha - \theta_g x_{gb}^I &= t_{bg}^\beta - \theta_g x_{bg}^I = \Delta\theta x_{bb}^I. \end{aligned}$$

As aforementioned, under unlimited information process, the agents receive their rents symmetrically while their hierarchical positions are different. Under limited information process, the agents' rents are as follows:

$$\begin{aligned} t_G^\alpha - \theta_g x_G^I &= \Delta\theta x_M^I > t_G^\beta - \theta_g x_G^I = \frac{2\varphi - 1}{\varphi}\Delta\theta x_M^I + \frac{1 - \varphi}{\varphi}\Delta\theta x_B^I \quad \text{and} \\ t_M^\alpha - \theta_g x_M^I &= t_M^\beta - \theta_g x_M^I = \Delta\theta x_M^I. \end{aligned}$$

A direct comparison of the rents under unlimited and limited information process shows that the agent's rents are no longer symmetric in the latter case. When information process is limited, the project size for $\gamma = M$ ($\gamma = B$) is more (less) likely to be the source of rents. In particular, agent α 's information rent is now only linked to x_M , which makes his rent larger compared to when the principal's information process is unlimited. As a result, the principal distorts the project size for $\gamma = M$ further downwards, while partially recovering the distortion in the project size for $\gamma = B$.

It is noteworthy that what makes the prime-agent get paid more than the sub-agent is the principal's limited information process. Again, under unlimited information process, the prime-agent cannot take advantage of his hierarchical position over the sub-agent. Under limited information process, however, the prime-agent's superior information gives him an

advantage. Because the principal cannot distinguish individual types of the agents, the prime-agent has extra room to manipulate his information once he is informed of the sub-agent's type. The extra flexibility of manipulation allows the prime-agent to increase his information rent by tying his rent only to x_M , whereas the sub-agent's rent is linked to both x_M and x_B since he faces uncertainty when reporting his type.

The asymmetric treatment becomes extreme for small φ ($< \widehat{\varphi}$). Again, agent α of type- g , with full flexibility of manipulation and facing no uncertainty when making his report, is guaranteed to receive a rent of $\Delta\theta x_M$ irrespective of agent β 's type. Intuitively, agent α is indirectly subsidized by agent β in this regime. That is, the optimal contract provides agent α with more rent, but it provides agent β of type- g with a strictly positive rent only when he is paired with agent α of type- b . Notice also that as φ becomes smaller, the organization gets to use the agents' information less effectively. In particular, the project sizes for $\gamma = M$ and $\gamma = B$ become closer to each other in the optimal contract as φ decreases. As mentioned above, agent α 's manipulation flexibility under limited information process makes x_M the more likely source of rent, but as φ decreases, it becomes more (less) likely that $\gamma = B$ ($\gamma = M$). As a result, x_M gets distorted further downward, while the distortion in x_B gets mitigated, thus becoming closer to each other in the optimal contract.²⁵

5.2 Outsourcing

As before, we first deal with agent α 's sub-contracting problem before analyzing the principal's problem. Again, analyzing the sub-contract offered from agent α to agent β allows us to treat the problem as if agent α has no private information (an informed principal problem with private values). The sub-contract offered from agent α to agent β specifies $\gamma \in \Gamma = \{G, M, B, \emptyset\}$ that agent α is to report to the principal based on his own type $i \in \{g, b\}$ and agent β 's report $j \in \{g, b\}$, together with a transfer to agent β :

$$\mathcal{C}_s^O = \left(\gamma(ij), t_{ij}^\beta \right).$$

Notice that agent α 's problem at the sub-contracting stage is identical to his problem under unlimited information process. Given the prime-contract from the principal, agent α of type- i 's problem is:

$$\mathcal{P}_s^O : \max_{\mathcal{C}_s^O} u^\alpha(\mathcal{C}_s^O) = \varphi[t_{\gamma(ig)} - t_{ig}^\beta - \theta_i x_{\gamma(ig)}] + (1 - \varphi)[t_{\gamma(ib)} - t_{ib}^\beta - \theta_i x_{\gamma(ib)}],$$

²⁵From Proposition 3, $x_M^I \rightarrow x_B^I$ (and $x_B^I \rightarrow x_B^*$) as $\varphi \rightarrow 0$.

subject to:

$$t_{ij}^\beta - \theta_j x_{\gamma(ij)} \geq 0, \quad j \in \{g, b\}, \quad (18)$$

$$t_{ij}^\beta - \theta_j x_{\gamma(ij)} \geq t_{ij'}^\beta - \theta_j x_{\gamma(ij')}, \quad j, j' \in \{g, b\}. \quad (19)$$

Only the participation constraints for type- b and incentive constraints for type- g are binding and therefore:

$$t_{\gamma(ib)}^\beta = \theta_b x_{\gamma(ib)} \quad \text{and} \quad t_{\gamma(ig)}^\beta = \theta_g x_{\gamma(ig)} + \Delta\theta x_{\gamma(ib)}, \quad i \in \{g, b\}. \quad (20)$$

As in the case of unlimited information process, agent β 's rent becomes zero if agent α 's message to the principal is $\gamma(ib) = \emptyset$ (i.e., if agent α excludes agent β of type- b), and agent α may have such an incentive.

The principal anticipates agent α 's problem when making an offer to him. The prime-contract offered from the principal to agent α is:

$$\mathcal{C}^O \equiv (x_\gamma, t_\gamma), \quad \gamma \in \Gamma = \{G, M, B, \emptyset\}.$$

The optimal prime-contract induces agent α to choose his sub-contract with truthful reports of γ to the principal. The following lemma presents the constraints faced by the principal when she offers the prime-contract.

Lemma 2 *Under limited information process, agent α chooses \mathcal{C}_s^O with $\gamma(gg) = G$, $\gamma(gb) = \gamma(bg) = M$, and $\gamma(bb) = B$ if and only if the prime-contract \mathcal{C}^O satisfies:*

$$t_\gamma - \Theta_\gamma x_\gamma \geq \max\{t_{\gamma'} - \Theta_\gamma x_{\gamma'}, 0\}, \quad (21)$$

where $\gamma \in \{G, M\}$, $\gamma' \in \{G, M, B\}$, and

$$t_\gamma - \left(\Theta_\gamma + \frac{\varphi}{1-\varphi} \Delta\theta \right) x_\gamma \geq \max\{t_{\gamma'} - \left(\Theta_\gamma + \frac{\varphi}{1-\varphi} \Delta\theta \right) x_{\gamma'}, 0\}, \quad (22)$$

where $\gamma \in \{M, B\}$, $\gamma' \in \{G, M, B\}$.

Agent α 's participation on the equilibrium path is implied by (21) and (22).²⁶ A strictly positive “rent at the bottom” again takes place with outsourcing, as suggested by (22) for

²⁶By (20) and agent α 's truthful report (Lemma 2), the participation constraints for him are written as: $t_G - \Theta_G x_G - \Delta\theta x_M \geq 0$, $t_M - \Theta_M x_M - \Delta\theta x_B \geq 0$ and $t_B - \Theta_B x_B \geq 0$. The first two constraints are satisfied since $t_G - \Theta_G x_G - \Delta\theta x_M \geq t_M - \Theta_G x_M - \Delta\theta x_M = t_M - \left(\Theta_M + \frac{\varphi}{1-\varphi} \Delta\theta \right) x_M + \frac{\varphi}{1-\varphi} \Delta\theta x_M > 0$ and $t_M - \Theta_M x_M - \Delta\theta x_B \geq t_B - \Theta_M x_B - \Delta\theta x_B = t_B - \left(\Theta_B + \frac{\varphi}{1-\varphi} \Delta\theta \right) x_B + \frac{\varphi}{1-\varphi} \Delta\theta x_B > 0$, where the weak and the strict inequalities are implied by (21) and (22) respectively. The third participation constraint, $t_B - \Theta_B x_B \geq 0$, is implied by (22).

$\gamma = B$. The multiple-layer of contracting results in a “double marginalization” of rent such that agent α 's rent is strictly positive even when he is type- b . The next proposition presents the optimal outcome in \mathcal{P}^O .

Proposition 4 *Under limited information process, the optimal contract with outsourcing entails the project sizes $x_G^O > x_M^O > x_B^O$ characterized by the following:*

$$\begin{aligned} v'(x_G^O) &= \Theta_G, \\ v'(x_M^O) &= \Theta_M + \frac{\varphi}{1-\varphi} \left(1 + \frac{1}{2(1-\varphi)} \right) \Delta\theta, \\ v'(x_B^O) &= \Theta_B + \frac{\varphi}{1-\varphi} \left(3 + \frac{\varphi}{1-\varphi} \right) \Delta\theta. \end{aligned}$$

It yields agent α a strictly positive rent regardless of his type, and agent β a strictly positive rent only when he is efficient.

The intuition behind Proposition 4 is the same as the one behind Proposition 2, except that $x_{gb} = x_{bg} = x_M$ here due to the principal's limited information process. As in the case of unlimited information process, agent α of type- b has an incentive to hire agent β of type- g only, thus excluding agent β of type- b . This way, agent α of type- b 's expected rent becomes strictly positive, and the principal must provide rent to him to prevent such excluding behavior. In turn, agent α of type- g 's rent must increase as well, since he now has a stronger incentive to misrepresent his type as type- b .

5.3 Comparison

We now compare the sourcing policies under limited information process. Recall that, under unlimited information process, the project sizes with outsourcing are distorted more than those with insourcing—there is no trade-off between the two sourcing policies, and insourcing always dominates. That, as the following corollary shows, is not the case under limited information process.

Corollary 2 *Under limited information process, the optimal project sizes satisfy that:*

$$x_G^O = x_G^I, \quad x_M^O \gtrless x_M^I, \quad x_B^O < x_B^I.$$

When the principal's information process is limited, the optimal x_M with outsourcing is distorted less than with insourcing for φ small enough (see Appendix). This hints the potential trade-off between insourcing and outsourcing. That is, although x_B is distorted

further with outsourcing due to the double marginalization of rent, there may be an advantage the principal can take of only with outsourcing. The next corollary presents our main message.

Corollary 3 *Suppose the principal's information process is limited. Then, there exists $\bar{\varphi} \in (0, \hat{\varphi})$ and $\underline{\varphi} \leq \bar{\varphi}$ such that:*

- For $\varphi > \bar{\varphi}$, insourcing dominates outsourcing.
- For $\varphi < \underline{\varphi}$, outsourcing dominates insourcing when the following holds:

$$2 [v(x_M^O) - \Theta_M x_M^O - (v(x_B^O) - \Theta_M x_B^O)] - \Delta\theta x_B^O > 0. \quad (23)$$

For φ large, outsourcing is again dominated by insourcing for the same reason as under unlimited information processing. With outsourcing in this range, the principal only provides the prime-agent with an additional source to take advantage of his superior position to the sub-agent. Again, the prime-agent can pocket agent β 's rent in the middle by shutting down and walking away when agent β reports that $j = b$.

For φ small, however, a trade-off arises between the sourcing modes. Recall from Proposition 3 that, for φ small enough, the optimal project sizes with insourcing must satisfy:

$$x_B^I = \frac{1 - 2\varphi}{1 - \varphi} x_M^I.$$

As can be seen from the equation, as φ decreases, x_M^I and x_B^I become closer to each other—the reason being that if agent β of type- g under insourcing receives no rent when φ is small, the principal cannot induce the agent's participation with truthful behavior. That is, as φ decreases, full separation becomes harder with insourcing. With outsourcing, in contrast, the fully separated project sizes become closer to their first-best levels as φ decreases (see Proposition 4). Although the principal suffers the double marginalization of rent provision, useful information for the project is more effectively used with outsourcing. As a result, outsourcing can dominate insourcing when φ is sufficiently small ($\varphi < \underline{\varphi}$), and explicitly does so if the condition in Corollary 3 is satisfied.

The intuition behind the argument that outsourcing can yield a more effective use of information is as follows. Because the principal's ability to process information is limited, she cannot contract upon the precise decomposition of the agents' individual types but only on a rough report from the prime-agent, that is, the aggregate information on their types. In other words, with insourcing, the transfers from the principal cannot reflect one agent's

relative efficiency to the other agent's. With outsourcing, while the principal still pays based on the aggregate information, he pays only to the prime-agent. And the prime agent, who has the precise decomposed information, can distribute the transfer between himself and the sub-agent. This allows the principal to use the overall information more effectively in determining the project sizes, thereby incentivizing each agent in a more fine-tuned way.

More formally, (23) in Corollary 3 ensures that the principal's profit with outsourcing (denoted by $\tilde{\pi}^O$: see Proof of Corollary 3) is strictly increasing at $\varphi = 0$, while the principal's profit with insourcing (denoted by $\tilde{\pi}^I$) stays constant at $\varphi = 0$:

$$\left. \frac{\partial \tilde{\pi}^O}{\partial \varphi} \right|_{\varphi=0} > 0 = \left. \frac{\partial \tilde{\pi}^I}{\partial \varphi} \right|_{\varphi=0}. \quad (24)$$

This is sufficient for implying that outsourcing dominates insourcing for φ small, because at $\varphi = 0$, we have that profits from out- and insourcing coincide ($\tilde{\pi}^O = \tilde{\pi}^I$). In fact, In fact, as shown in our proof of Corollary 3, the inequality in (23) represents a *tight* condition for outsourcing to dominate insourcing for φ small, because the inequality in (24) holds *if and only if* (23) holds.

While condition (23) has the advantage that it is tight, it has the drawback that it depends on the endogenous variables x_M^O and x_B^O . Expressed in the model's exogenous terms, the superiority of outsourcing holds for small φ if the principal's value function is relatively concave. To see this, note that (23) can be rewritten as:

$$2 [v(x_M^O) - \Theta_M x_M^O - (v(x_B^O) - \Theta_B x_B^O)] - 3\Delta\theta x_B^O < 0.$$

Thus, the condition is satisfied if the following stricter condition holds:

$$\begin{aligned} & 2 [v(x_M^O) - \Theta_M x_M^O - (v(x_B^O) - \Theta_B x_B^O)] - 4\Delta\theta x_B^O \\ & = 2 [v(x_M^O) - v(x_B^O) - \Theta_M x_M^O + \Theta_B x_B^O] \leq 0. \end{aligned} \quad (25)$$

For $\varphi = 0$, the values of x_M^O and x_B^O coincide with the first best x_M^* and x_B^* and we therefore have, $v'(x_M^O) = \Theta_M = \theta_b$ for $\theta_g = 0$. Hence, for $\varphi = \theta_g = 0$, the inequality in (25) reduces to the following:

$$v(x_M^*) - v(x_B^*) - v'(x_M^*)x_M^* \leq 0, \quad (26)$$

where the left hand side is reminiscent of the Bregman (1967) divergence measure for the curvature of a function.²⁷ This indicates that the original condition (23) holds for φ small and $\Delta\theta$ large if the value function $v(\cdot)$ is sufficiently concave.

²⁷The Bregman divergence between two points x and y of a differentiable function $v : \mathbb{R} \rightarrow \mathbb{R}$ is $v(x) - v(y) - v'(x)(x - y)$ so that the left-hand side of (26) differs from the Bregman divergence of v at (x_M^*, x_B^*) by $v'(x_M^*)x_B^*$.

Indeed, for the functional form $v(x) = \mu x^\sigma$, where $\sigma \in [0, 1]$ expresses the concavity of $v(\cdot)$, condition (26) holds in equality for $\sigma = 1/2$ and strictly for $\sigma < 1/2$. Thus, with $v(x) = \mu x^\sigma$ and $\sigma \leq 1/2$, outsourcing is optimal for φ close to 0 and $\Delta\theta$ close to θ_b . We illustrate this with the following numerical example, where $v(x) = \mu\sqrt{x}$ (i.e., $\sigma = 1/2$).

Examples Let $v(x) \equiv 10\sqrt{x}$, $\theta_g = 0.1$, $\theta_b = 2$, and denote by $\tilde{\pi}^I$ and $\tilde{\pi}^O$ the expected payoffs with insourcing and outsourcing respectively. With these parameters, the cutoff levels of φ in Corollary 3 are: $\underline{\varphi} = \bar{\varphi} \simeq 0.133$.

- For $\varphi = 0.2 (> \underline{\varphi})$, $\tilde{\pi}^I = 10.833 > \tilde{\pi}^O = 10.672$ and thus insourcing is optimal.
- For $\varphi = 0.1 (< \underline{\varphi})$, $\tilde{\pi}^I = 7.401 < \tilde{\pi}^O = 7.452$, and thus outsourcing is optimal.

According to condition (23) in Corollary 3, the optimality of outsourcing depends not only on φ , but on $\Delta\theta$ as well. With $v(x) \equiv 10\sqrt{x}$ and $\theta_b = 2$ as in the example above, we fix the value of φ at $\varphi = 0.1$ and vary the value of θ_g (thus the value of $\Delta\theta$) as follows:

- For $\theta_g = 1 (\Delta\theta = 1)$, $\tilde{\pi}^I = 6.688 > \tilde{\pi}^O = 6.203$, and thus insourcing is optimal.
- For $\theta_g = 0.1 (\Delta\theta = 1.9)$, $\tilde{\pi}^I = 7.401 < \tilde{\pi}^O = 7.452$, and thus outsourcing is optimal.

Figure 2 illustrates the optimal sourcing policies in φ and $\Delta\theta$ (with fixing $\theta_b = 2$).²⁸

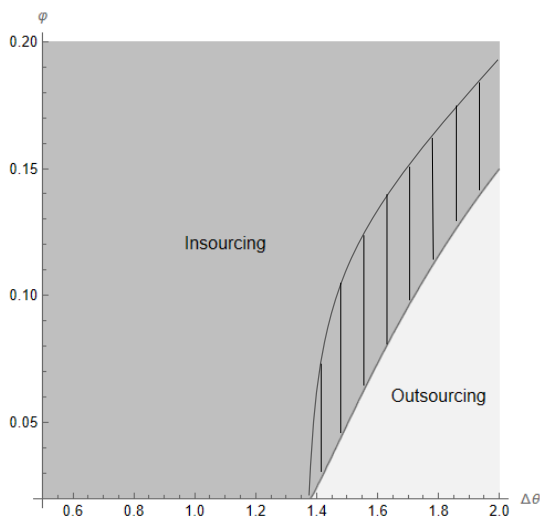


Figure 2. Optimal Sourcing Policy

²⁸In Figure 2, the shaded area with lines represents the expanded area of outsourcing as the optimal mode when the agents can collude, which will be discussed in the next section.

6 Extension: Threat of Collusion

In this section, we extend our analyses by introducing the possibility of collusion between the agents.²⁹ Collusion opportunities emerge when the agents can increase their joint rent by misrepresenting one or both of their types and exchanging side transfers between them. We first present the following extended result under unlimited information process.

Corollary 4 *Under unlimited information process, collusion between the agents is no issue.*

When the principal's information process is unlimited, the optimal transfers to the agents are based on individual types. The decomposed payments create no externalities between the agents that can lead to a collusion opportunity. As a result, insourcing still dominates outsourcing under unlimited information process even if collusion is possible.

Let us now look at the cases under limited information process. Consider first the case with insourcing. With insourcing, agent α of type- i 's ex post rents are:

$$\begin{aligned} t_G^\alpha - \theta_g x_G^I &= t_M^\alpha - \theta_g x_M^I = \Delta\theta x_M^I & \text{for } i = g, \\ t_M^\alpha - \theta_b x_M^I &= t_B^\alpha - \theta_b x_B^I = 0 & \text{for } i = b. \end{aligned}$$

Given his type $i \in \{g, b\}$, therefore, agent α 's rent is independent of agent β 's type. That is, agent α of type- g (type- b) receives a rent of $\Delta\theta x_M^I$ (0) regardless of agent β 's type.

By contrast, agent β of type- g 's rent is strictly larger when agent α is type- b since:

$$t_M^\beta - \theta_g x_M^I = \Delta\theta x_M^I > \begin{cases} t_G^\beta - \theta_g x_G^I = \frac{2\varphi-1}{\varphi}\Delta\theta x_M^I + \frac{1-\varphi}{\varphi}\Delta\theta x_B^I & \text{for } \varphi \geq \hat{\varphi}, \\ t_G^\beta - \theta_g x_G^I = 0 & \text{for } \varphi < \hat{\varphi}. \end{cases}$$

Hence, whereas agent α is indifferent to how agent β 's type is reported, agent β of type- g wants agent α 's type to be reported as $i = b$. This provides the agents with an opportunity to collude. In particular, with the optimal outcome in \mathcal{P}^I , when agent β reports his type as type- g , agent α (regardless of his type) has an incentive to make the following offer to agent β : "I, agent α , will report to the principal that $\gamma = M$, and you, agent β , pay me an arbitrary small amount (say, $\varepsilon > 0$)."²⁹ From such an arrangement, the agents can increase their joint payoff by $(1 - \varphi)\Delta\theta(x_M^I - x_B^I)/\varphi$ for $\varphi \geq \hat{\varphi}$, and by $\Delta\theta x_M^I$ for $\varphi < \hat{\varphi}$.

Clearly, the optimal outcome in \mathcal{P}^I is no longer incentive compatible under collusion since agent α has an incentive to side-contract with agent β and misreport that $\gamma = M$

²⁹See Kofman and Lawarree (1993), Khalil and Lawarree (1995) and Faure-Grimaud et al. (2003) among others for collusion in vertical structure. In these papers, unlike in ours, the middle agent is non-productive.

when the true $\gamma = G$. In order to keep the agents from engaging in collusion, the following additional constraints must be satisfied in the optimal contract:

$$\sum_k t_{\gamma}^k - \Theta_{\gamma} x_{\gamma} \geq \sum_k t_{\gamma'}^k - \Theta_{\gamma'} x_{\gamma'}, \quad \gamma, \gamma' \in \{G, M, B\}. \quad (27)$$

Under limited information process, the optimal insourcing contract that prevents collusion is the outcome of the principal's problem \mathcal{P}^I subject to (27).

As for outsourcing, the transfer from the principal to agent α is the total amount, part of which is to be paid from agent α to agent β . That is, with outsourcing, agent α becomes a “residual claimant,” sub-contracting with agent β for his service. As such, the sub-contract internalizes all collusion possibilities between the agents. As a result, the optimal contract with outsourcing is free of collusion even under limited information process.

The following proposition presents the optimal outcomes with insourcing and outsourcing under collusion.

Proposition 5 *Suppose the agents can collude. Under limited information process, the optimal contract with insourcing entails the project sizes $\tilde{x}_G^I > \tilde{x}_M^I = \tilde{x}_B^I$ characterized by the following:*

$$\begin{aligned} v'(\tilde{x}_G^I) &= \Theta_G, \\ v'(\tilde{x}_M^I) &= v'(\tilde{x}_B^I) = \Theta_B + \frac{2\varphi^2}{1 - \varphi^2} \Delta\theta. \end{aligned}$$

It yields an agent a strictly positive rent only when he is efficient. With outsourcing, collusion between the agents is no issue.

Under limited information process, the optimal prevention of collusion with insourcing leads to a “pooling” of information in determining the project sizes. To understand the pooling in the optimal contract, recall that, under limited information process, the principal treats the agents asymmetrically because agent α has more room for manipulation.³⁰ That is, agent α can make x_M the source of his rent regardless of agent β 's type, while agent β 's rent depend on x_B as well since he does not know agent α 's type when reporting. These asymmetric sources of rent lead to the collusion opportunity, and preventing collusion requires that agent β 's sources of rent be changed—from x_M and x_B depending on agent α 's type to x_M regardless of his type. As a result, the two project sizes get bunched ($x_M = x_B$) in the optimal contract.

³⁰Recall that under unlimited information process, the agents are treated symmetrically in the optimal insourcing contract although their hierarchical positions are different.

Our main result in this section is summarized in the following corollary.

Corollary 5 *Suppose the principal’s information process is limited. Under collusion, there exists $\varphi^+ > \bar{\varphi}$ and $\varphi^- > \underline{\varphi}$, where $\varphi^+ \geq \varphi^-$, such that:*

- *For $\varphi > \varphi^+$, insourcing dominates outsourcing.*
- *For $\varphi < \varphi^-$, outsourcing dominates insourcing when the following holds:*

$$2 [v(x_M^O) - \Theta_M x_M^O - (v(x_B^O) - \Theta_M x_B^O)] - \Delta\theta x_B^O > 0.$$

The corollary states that, under limited information process, potential collusion between the agents shifts the trade-off between “effective use of information vs. double marginalization of rent” more in favor of outsourcing. With insourcing, the threat of collusion prevents an effective use of information whether the project’s aggregate cost is $\gamma = M$ or $\gamma = B$, as both project sizes are pooled in the optimal contract. This is not the case with outsourcing, where the optimal contract distinguishes among all potential project sizes. As a result, as illustrated in Figure 2, the area of outsourcing as the optimal mode of operation becomes larger when the agents can engage in collusion.

7 Concluding Remarks

There have been extensive discussions on optimal sourcing policies from different perspectives, as a firm’s decision on whether to in- or outsource its inputs depends on the angle from which its operation is viewed. One of the widely studied view point since Coase (1937) looks at a firm’s operation as a nexus of contracts. Given this view point, one can ask why a firm’s placing a productive unit within or outside the firm’s boundary makes any notable difference in the outcome. Answering the question calls for an examination of insourcing and outsourcing as different contractual arrangements, *ceteris paribus*.

Using an agency model with multiple agents, we have done so by comparing two operational modes, one of which is interpreted as “insourcing” and the other “outsourcing,” both in the vertical structure. The structure is vertical in the sense that the principal receives a report only from one of the agents (the prime-agent), who receives a report from the other agent (the sub-agent). The sub-agent has no direct communication channel to the principal. With insourcing, both agents are on the principal’s payroll, and each of them receives a transfer payment directly from the principal. With outsourcing, on the other hand,

the principal makes a payment to a separate organization where the prime-agent hires and incentivizes the sub-agent.

We have shown that, when the principal can perfectly process information from the prime-agent, there is no trade-off between in- and outsourcing—the optimal outcome with insourcing is not accompanied by any hierarchical inefficiency although the principal gets a report only from the prime-agent. With outsourcing, however, rent provision to the prime-agent is “double marginalized,” which allows the agent to receive a strictly positive rent even when he is inefficient. As a result, insourcing always dominates outsourcing under unlimited information process.

By contrast, a meaningful trade-off between in- and outsourcing arises when the principal’s information process is limited in that she can process only the aggregate information from the prime-agent’s report. Under limited information process, insourcing brings about a hierarchical inefficiency because the prime-agent has more flexibility to manipulate in the middle, resulting in less effective use of information. With outsourcing, although the prime-agent’s double marginalization of rent is still an issue, the principal is making her payment only to the prime agent who has better and detailed information. And using his detailed information, the prime-agent internalizes the payment from the principal between himself and the sub-agent. That is, information is used more effectively with outsourcing. As a result, outsourcing can be the prevailing sourcing policy under limited information process. We have also shown that insourcing under limited information process is prone to collusion between the agents, whereas no possibility of collusion arises with outsourcing.

We close the paper with clarifying the role of limited liability in our analysis. Without the limited liability, the agents cannot quit once participated in the project. With outsourcing, this has two effects. First, the double marginalization effect on rents vanishes. Second, the participation constraints of the agents enter in expected terms since neither agent knows the other agent’s type when deciding whether or not to participate in the project. Both effects are strictly beneficial to the principal. With insourcing, by contrast, unlimited liability brings only the latter effect and moreover, its benefit is weaker compared to the outsourcing—the principal’s optimal contract would still effectively respect agent α ’s limited liability even when not imposing it as a constraint. This would suggest that an optimality of outsourcing depends crucially on the absence of limited liability as it holds for agent α with insourcing while violating it with outsourcing. By including limited liability constraints directly into our model and still deriving the optimality of outsourcing, we show that its optimality does not crucially depend on any limited liability concerns.

Appendix

Proof of Proposition 1

We first conjecture that the binding constraints in $\overline{\mathcal{P}}^I$ are (1) for bj , (2) for ib , (3) for $j = g$, and (4) for $ij = gg$ with $i'j' = bg$ and (4) for $ij = gb$ with $i'j' = bb$. The following expressions for the transfers follow from the binding constraints:

$$t_{gg}^\alpha = \theta_g x_{gg} + \Delta\theta x_{bg}, \quad t_{gb}^\alpha = \theta_g x_{gb} + \Delta\theta x_{bb}, \quad t_{bg}^\alpha = \theta_b x_{bg}, \quad t_{bb}^\alpha = \theta_b x_{bb}, \quad (28)$$

$$\begin{aligned} \varphi t_{gg}^\beta + (1 - \varphi)t_{bg}^\beta &= \varphi [\theta_g x_{gg} + \Delta\theta x_{gb}] + (1 - \varphi) [\theta_g x_{bg} + \Delta\theta x_{bb}], \\ t_{gb}^\beta &= \theta_b x_{bg}, \quad t_{bb}^\beta = \theta_b x_{bb}, \end{aligned} \quad (29)$$

In (29), there is some degree of freedom in choosing t_{gg}^β and t_{bg}^β . One way is to set:

$$t_{gg}^\beta = \theta_g x_{gg} + \Delta\theta x_{gb}, \quad t_{bg}^\beta = \theta_g x_{bg} + \Delta\theta x_{bb}.$$

Substituting for the transfers in the objective function and optimizing in x_{ij} gives the equations in Proposition 1. The monotonicity of x_{ij} follows from comparing the RHSs of the equations. Each agent's rent is obtained from the binding constraints. It is straightforward to check that the solution with the binding constraints satisfies all the other constraints. ■

Proof of Lemma 1

By substituting for (7) in agent α 's objective function in $\overline{\mathcal{P}}_s^O$, we have:

$$\varphi [t_{\gamma(ij)} - \theta_i x_{\gamma(ij)} - \theta_g x_{\gamma(ij)} - \Delta\theta x_{\gamma(ib)}] + (1 - \varphi) [t_{\gamma(ib)} - \theta_i x_{\gamma(ib)} - \theta_b x_{\gamma(ib)}],$$

which is rewritten as:

$$\varphi [t_{\gamma(ij)} - (\theta_i + \theta_g)x_{\gamma(ij)}] + (1 - \varphi) \left[t_{\gamma(ib)} - \left(\theta_i + \theta_b + \frac{\varphi}{1 - \varphi} \Delta\theta \right) x_{\gamma(ib)} \right]. \quad (30)$$

Hence, if agent α of type- $i \in \{g, b\}$ is to choose an incentive compatible sub-contract \mathcal{C}_s^O with $\gamma(ij) = ij$, expression (30) must be maximized with $\gamma(ij) = ig$ rather than any combination of $\gamma(ij) \in \{i'b, i'g, ib, \emptyset\}$, and also with $\gamma(ib) = ib$ rather than any other combination of $\gamma(ib) \in \{i'g, i'b, ig, \emptyset\}$. Thus, to be incentive compatible for agent α , the principal's prime-contract \mathcal{C}^O must satisfy:

$$\begin{aligned} t_{ig} - (\theta_i + \theta_g)x_{ig} &\geq \max\{t_{i'j'} - (\theta_i + \theta_g)x_{i'j'}, 0\}, \\ t_{ib} - \left(\theta_i + \theta_b + \frac{\varphi}{1 - \varphi} \Delta\theta \right) x_{ib} &\geq \max\{t_{i'j'} - \left(\theta_i + \theta_b + \frac{\varphi}{1 - \varphi} \Delta\theta \right) x_{i'j'}, 0\}, \end{aligned}$$

where $i, i', j' \in \{g, b\}$. ■

Proof of Proposition 2

The following constraints are conjectured as the binding ones: (8) for $ij = gg$ with $i'j' = bg$, (8) for $ij = bg$ with $i'j' = gb$, (9) for $ij = gb$ with $i'j' = bb$, and (9) for $ij = bb$ with 0 in the RHS of the constraint. It can be easily checked that the optimal outcome satisfies the other constraints. The binding constraints give the expressions for transfers and:

$$t_{gg} = \Theta_G x_{gg} + \Delta\theta x_{bg} + \frac{\varphi}{1-\varphi} \Delta\theta x_{gb} + \Delta\theta x_{bb}, \quad t_{gb} = \left(\Theta_M + \frac{\varphi}{1-\varphi} \Delta\theta \right) x_{gb} + \Delta\theta x_{bb},$$

$$t_{bg} = \Theta_M x_{bg} + \frac{\varphi}{1-\varphi} \Delta\theta x_{gb} + \Delta\theta x_{bb}, \quad t_{bb} = \left(\Theta_B + \frac{\varphi}{1-\varphi} \Delta\theta \right) x_{bb}.$$

After substituting for the transfers in the principal's objective function in $\bar{\mathcal{P}}^O$, optimization in x_{ij} yields the equations in Proposition 2. Comparing the RHSs of the equations gives the monotonicity of the project sizes. Each agent's rent follows from the binding constraints. ■

Proof of Corollary 1

The optimal outcome of $\bar{\mathcal{P}}^O$ is implementable in $\bar{\mathcal{P}}^I$, as it satisfies all constraints in $\bar{\mathcal{P}}^I$. It, however, is different from the optimal outcome of $\bar{\mathcal{P}}^I$. Thus, insourcing must dominate outsourcing. ■

Proof of Proposition 3

We first conjecture that, for $\varphi \geq 1/2$, the binding constraints in \mathcal{P}^I are (11), (12), and (14) for $\gamma = M$. It is straightforward to show that the solution satisfies the other constraints. With the binding constraints, we have the following expressions for the transfers:

$$t_G^\alpha = \theta_g x_G + \Delta\theta x_M, \quad t_M^\alpha = \theta_b x_M, \quad t_B^\alpha = \theta_b x_B, \quad (31)$$

$$t_G^\beta = \theta_g x_G + \frac{2\varphi-1}{\varphi} \Delta\theta x_M + \frac{1-\varphi}{\varphi} \Delta\theta x_B, \quad t_M^\beta = \theta_b x_M, \quad t_B^\beta = \theta_b x_B. \quad (32)$$

Substituting for the transfers in the objective function and optimizing in x_γ gives:

$$v'(x_G^I) = \Theta_G, \quad v'(x_M^I) = \Theta_M + \frac{1+\varphi}{2(1-\varphi)} \Delta\theta, \quad v'(x_B^I) = \Theta_B + \frac{\varphi}{1-\varphi} \Delta\theta. \quad (33)$$

The monotonicity of x_γ follows from comparing the RHSs of the equations above for $\varphi \geq 1/2$. The binding constraints give each agent's rent for this range of φ .

Notice that the rents are all non-negative for any $\varphi \in (0, 1)$, except:

$$u_G^\beta = t_G^\beta - \theta_g x_G = \frac{2\varphi-1}{\varphi} \Delta\theta x_M + \frac{1-\varphi}{\varphi} \Delta\theta x_B. \quad (34)$$

We now define $\widehat{\varphi}$ by the following equation:

$$u_G^\beta(\widehat{\varphi}) = \frac{2\widehat{\varphi} - 1}{\widehat{\varphi}} \Delta\theta x_M^I(\widehat{\varphi}) + \frac{1 - \widehat{\varphi}}{\widehat{\varphi}} \Delta\theta x_B^I(\widehat{\varphi}) = 0.$$

Next, we show that $\partial u_G^\beta / \partial \varphi > 0$ for $\varphi \geq \widehat{\varphi}$, that is, u_G^β decreases as φ gets smaller for $\varphi \geq \widehat{\varphi}$.

$$\left. \frac{\partial u_G^\beta}{\partial \varphi} \right|_{\varphi \geq \widehat{\varphi}} = \left[\frac{1}{\varphi^2} (x_M^I - x_B^I) + \frac{1}{\varphi} \omega \right] \Delta\theta,$$

where $\omega \equiv (2\varphi - 1) \frac{\partial x_M^I}{\partial \varphi} + (1 - \varphi) \frac{\partial x_B^I}{\partial \varphi}$.

For φ large enough, $u_G^\beta > 0$ in (34) holds and hence $x_M^I - x_B^I > 0$. Thus, showing $\omega > 0$ establishes that $\partial u_G^\beta / \partial \varphi > 0$ for $\varphi \geq \widehat{\varphi}$. Using the expressions for x_M^I and x_B^I in (33), we have:

$$\frac{\partial x_M^I}{\partial \varphi} = \frac{\Delta\theta}{(1 - \varphi)^2 v''(x_M^I)} \quad \text{and} \quad \frac{\partial x_B^I}{\partial \varphi} = \frac{\Delta\theta}{(1 - \varphi)^2 v''(x_B^I)},$$

and thus:

$$\omega = \frac{\Delta\theta}{(1 - \varphi)^2} \left[\frac{2\varphi - 1}{v''(x_M^I)} + \frac{1 - \varphi}{v''(x_B^I)} \right] > 0 \quad \text{since} \quad \frac{1 - \varphi}{v''(x_B^I)} > \frac{2\varphi - 1}{v''(x_M^I)} \quad \text{and} \quad v''(x_B^I) < v''(x_M^I),$$

due to $v'''(\cdot) \geq 0$. Since $\partial u_G^\beta / \partial \varphi > 0$ for φ arbitrary large and $u_G^\beta > 0$ for $\varphi \geq 1/2$ in (34), there exists $\widehat{\varphi} < 1/2$ such that for $\varphi \geq \widehat{\varphi}$, the optimal outcome in \mathcal{P}^I is characterized by (33).

Since $\partial u_G^\beta(\widehat{\varphi}) / \partial \varphi > 0$ and $u_G^\beta(\widehat{\varphi}) = 0$, when $\varphi < \widehat{\varphi}$, the optimal outcome for $\varphi \geq \widehat{\varphi}$ violates participation constraint (10) for agent β with $\gamma = G$. Thus, this constraint must be binding for $\varphi < \widehat{\varphi}$. With (34), the constraint simplifies to:

$$(1 - \varphi)x_B = (1 - 2\varphi)x_M. \tag{35}$$

With the extra constraint (35), the principal's problem is to maximize

$$\varphi^2 [v(x_G) - \Theta_G x_G - \Delta\theta x_M] + 2\varphi(1 - \varphi) [v(x_M) - \Theta_B x_M] + (1 - \varphi)^2 [v(x_B(x_M)) - \Theta_B x_B(x_M)],$$

$$\text{where } x_B(x_M) = \frac{1 - 2\varphi}{1 - \varphi} x_M.$$

Optimization gives:

$$v'(x_G^I) = \Theta_G, \quad 2\varphi v'(x_M^I) + (1 - 2\varphi)v'(x_B^I) = \Theta_B + \frac{\varphi^2}{1 - \varphi} \Delta\theta, \quad \text{where } x_B^I = \frac{1 - 2\varphi}{1 - \varphi} x_M^I. \tag{36}$$

For monotonicity of x_γ for $\varphi < \widehat{\varphi}$, we first denote \widehat{x}_G^I , \widehat{x}_M^I and \widehat{x}_B^I as the optimal project sizes for $\varphi \geq \widehat{\varphi}$. Since $x_G^I = \widehat{x}_G^I$ and $x_B^I < x_M^I$ from (35), we show $x_G^I > x_M^I > x_B^I$ for $\varphi < \widehat{\varphi}$ by showing that $x_M^I < \widehat{x}_M^I$. Suppose $x_M^I \geq \widehat{x}_M^I$, then we have:

$$\begin{aligned} 2\varphi v'(x_M^I) + (1 - 2\varphi)v'(x_B^I) &= 2\varphi v'(x_M^I) + (1 - 2\varphi)v'\left(\frac{1 - 2\varphi}{1 - \varphi}x_M^I\right) \\ &\leq 2\varphi v'(\widehat{x}_M^I) + (1 - 2\varphi)v'\left(\frac{1 - 2\varphi}{1 - \varphi}\widehat{x}_M^I\right) \\ &< 2\varphi v'(\widehat{x}_M^I) + (1 - 2\varphi)v'(\widehat{x}_B^I) \\ &= \Theta_B + \frac{\varphi^2}{1 - \varphi}\Delta\theta, \end{aligned}$$

where the strict inequality is from the fact that $(1 - 2\varphi)\widehat{x}_M^I/(1 - \varphi) > \widehat{x}_B^I$ for $\varphi < \widehat{\varphi}$, and the last equality is from the expressions for $v'(\widehat{x}_M^I)$ and $v'(\widehat{x}_B^I)$ in (33). This, however, contradicts with the expression linked to x_M^I and x_B^I . Hence, $x_M^I < \widehat{x}_M^I$ must hold. Finally, each agent's rent for $\varphi < \widehat{\varphi}$ follows from the binding constraints in this range. ■

Proof of Lemma 2

Substituting for (20) in agent α 's objective function in \mathcal{P}_s^O gives:

$$\varphi[t_{\gamma(ig)} - \theta_i x_{\gamma(ig)} - \theta_g x_{\gamma(ig)} - \Delta\theta x_{\gamma(ib)}] + (1 - \varphi)[t_{\gamma(ib)} - \theta_i x_{\gamma(ib)} - \theta_b x_{\gamma(ib)}],$$

which is rewritten as:

$$\varphi [t_{\gamma(ig)} - (\theta_i + \theta_g)x_{\gamma(ig)}] + (1 - \varphi) \left[t_{\gamma(ib)} - \left(\theta_i + \theta_b + \frac{\varphi}{1 - \varphi}\Delta\theta \right) x_{\gamma(ib)} \right]. \quad (37)$$

If agent α of type- $i \in \{g, b\}$ is to choose an incentive compatible sub-contract \mathcal{C}_s^O with $\gamma(gg) = G$, $\gamma(gb) = \gamma(bg) = M$ and $\gamma(bb) = B$, expression (37) must be maximized with the report schedule. Thus, to be incentive compatible for agent α , the principal's prime-contract \mathcal{C}^O must satisfy:

$$t_\gamma - \Theta_\gamma x_\gamma \geq \max\{t_{\gamma'} - \Theta_\gamma x_{\gamma'}, 0\}, \gamma \in \{G, M\}, \gamma' \in \{G, M, B\},$$

$$t_\gamma - \Theta_\gamma x_\gamma - \frac{\varphi}{1 - \varphi}\Delta\theta x_r \geq \max\{t_{\gamma'} - \Theta_\gamma x_{\gamma'} - \frac{\varphi}{1 - \varphi}\Delta\theta x_{\gamma'}, 0\}, \gamma \in \{M, B\}, \gamma' \in \{G, M, B\}.$$

■

Proof of Proposition 4

The following constraints are conjectured as the binding ones: (21) for $\gamma = G$ with $\gamma' = M$, (22) for $\underline{\gamma} = M$ with $\gamma' = B$, and (22) for $\underline{\gamma} = B$ with 0 in the RHS of the constraint. It can be easily checked that the optimal outcome satisfies the other constraints. The binding constraints give the expressions for transfers and:

$$t_G = \Theta_G x_G + \frac{1}{1-\varphi} \Delta\theta x_M + \Delta\theta x_B, \quad t_M = \left(\Theta_M + \frac{\varphi}{1-\varphi} \Delta\theta \right) x_M + \Delta\theta x_B,$$

$$t_B = \left(\Theta_B + \frac{\varphi}{1-\varphi} \Delta\theta \right) x_B.$$

After substituting for the transfers in the principal's objective function in \mathcal{P}^O , optimization in x_γ yields the equations in Proposition 4. The monotonicity of the project sizes follows from comparing the RHSs of the equations. Each agent's rent is obtained from the binding constraints. ■

Proof of Corollary 2

That $x_G^O = x_G^I$ follows directly from Propositions 3 and 4. To see that $x_M^O \begin{matrix} \geq \\ < \end{matrix} x_M^I$, first consider x_M^O and x_B^I for $\varphi \geq \widehat{\varphi}$:

$$v'(x_M^O) = \Theta_M + \frac{\varphi}{1-\varphi} \left(1 + \frac{1}{2(1-\varphi)} \right) \Delta\theta \quad \text{and} \quad v'(x_M^I) = \Theta_M + \frac{1+\varphi}{2(1-\varphi)} \Delta\theta.$$

Since $v'(x_M^O) > v'(x_M^I)$, we have $x_M^O < x_M^I$ for $\varphi \geq \widehat{\varphi}$. For $\varphi < \widehat{\varphi}$, we have the same expression for $v'(x_M^O)$ as above and:

$$2\varphi v'(x_M^I) + (1-2\varphi)v'(x_B^I) = \Theta_B + \frac{\varphi^2}{1-\varphi} \Delta\theta, \quad \text{where } x_B^I = \frac{1-2\varphi}{1-\varphi} x_M^I.$$

For $\varphi \simeq 0$, we have $v'(x_M^O) \simeq \Theta_M$ and $v'(x_M^I) \simeq \Theta_B$, and thus $x_M^O > x_M^I$. To see that $x_B^O < x_B^I$, first consider x_M^O and x_B^I for $\varphi \geq \widehat{\varphi}$:

$$v'(x_B^O) = \Theta_B + \frac{\varphi}{1-\varphi} \left(3 + \frac{\varphi}{1-\varphi} \right) \Delta\theta \quad \text{and} \quad v'(x_B^I) = \Theta_B + \frac{\varphi}{1-\varphi} \Delta\theta,$$

where $v'(x_B^O) > v'(x_B^I)$ and thus $x_B^O < x_B^I$.

To show that $x_B^O < x_B^I$ for $\varphi < \widehat{\varphi}$, we first note that the principal's problem for $\varphi < \widehat{\varphi}$ is equivalent to her problem for $\varphi \geq \widehat{\varphi}$ with the extra constraint $(2\varphi-1)x_M + (1-\varphi)x_B = 0$. We denote by \widehat{x}_γ the optimal project sizes for $\varphi \geq \widehat{\varphi}$. The Lagrangian for the principal's

problem for $\varphi < \widehat{\varphi}$ is:

$$\begin{aligned}
L &= \varphi^2[v(x_G) - \Theta_G x_G] \\
&+ 2\varphi(1 - \varphi) \left[v(x_M) - \left(\Theta_M + \frac{1 + \varphi}{2(1 - \varphi)} \Delta\theta \right) x_M \right] \\
&+ (1 - \varphi)^2 \left[v(x_B) - \left(\Theta_B + \frac{\varphi}{1 - \varphi} \Delta\theta \right) x_B \right] \\
&+ \lambda [(2\varphi - 1)x_M + (1 - \varphi)x_B],
\end{aligned}$$

where $\lambda > 0$. The first order condition gives:

$$v'(x_B^I) = \Theta_M + \frac{1 + \varphi}{2(1 - \varphi)} \Delta\theta - \lambda(1 - \varphi) < v'(\widehat{x}_B^I),$$

where \widehat{x}_B^I denotes the optimal x_B for $\varphi \geq \widehat{\varphi}$. The strict inequality above implies that $x_B^I > \widehat{x}_B^I$, and as shown above, $\widehat{x}_B^I > x_B^O$. It follows that $x_B^O < x_B^I$ for $\varphi < \widehat{\varphi}$. ■

Proof of Corollary 3

For $\varphi \geq \widehat{\varphi}$, the principal's objective functions with insourcing and outsourcing can be expressed as:

$$\begin{aligned}
\widehat{\pi}^I &\equiv \varphi^2 \left[\underbrace{v(x_G) - \theta x_G}_{\pi_G^I} \right] + 2\varphi(1 - \varphi) \left[\underbrace{v(x_M) - \left(\Theta_M + \frac{1 + \varphi}{2(1 - \varphi)} \Delta\theta \right) x_M}_{\pi_M^I} \right] \\
&+ (1 - \varphi)^2 \left[\underbrace{v(x_B) - \left(\Theta_B + \frac{\varphi}{1 - \varphi} \Delta\theta \right) x_B}_{\pi_B^I} \right] \quad \text{and} \\
\widehat{\pi}^O &\equiv \varphi^2 \left[\underbrace{v(x_G) - \theta x_G}_{\pi_G^O} \right] + 2\varphi(1 - \varphi) \left[\underbrace{v(x_M) - \left(\Theta_M + \frac{\varphi}{1 - \varphi} \left(1 + \frac{1}{2(1 - \varphi)} \right) \Delta\theta \right) x_M}_{\pi_M^O} \right] \\
&+ (1 - \varphi)^2 \left[\underbrace{v(x_B) - \left(\Theta_B + \frac{\varphi}{1 - \varphi} \left(3 + \frac{\varphi}{1 - \varphi} \right) \Delta\theta \right) x_B}_{\pi_B^O} \right] \quad \text{respectively,}
\end{aligned}$$

where $\pi_G^I = \pi_G^O$, $\pi_M^I > \pi_M^O$, $\pi_B^I > \pi_B^O$, and thus $\pi^I > \pi^O$ for any given x_γ , $\gamma \in \{G, M, B\}$. Since x_γ^I is the maximizer of π_γ^I , and x_γ^O is the maximizer of π_γ^O , we have $\pi_\gamma^I(x_\gamma^I) > \pi_\gamma^O(x_\gamma^O)$, $\gamma \in \{G, M, B\}$, for $\varphi \geq \widehat{\varphi}$. It then follows that there exists $\bar{\varphi} < \widehat{\varphi}$ such that for $\varphi > \bar{\varphi}$, insourcing dominates outsourcing.

To show the existence of $\underline{\varphi}$, we evaluate the marginal changes in the principal's expected payoffs with insourcing and outsourcing at $\varphi = 0$. The principal's optimal expected payoff with insourcing for $\varphi < \widehat{\varphi}$ is:

$$\begin{aligned}\tilde{\pi}^I &\equiv \varphi^2 [v(x_G^I) - \Theta_G x_G^I - \Delta\theta x_M^I] \\ &\quad + 2\varphi(1 - \varphi) [v(x_M^I) - \Theta_B x_M^I] \\ &\quad + (1 - \varphi)^2 \left[v \left(\frac{1 - 2\varphi}{1 - \varphi} x_M^I \right) - \frac{1 - 2\varphi}{1 - \varphi} \Theta_B x_M^I \right],\end{aligned}$$

since $x_B^I = (1 - 2\varphi)x_M^I/(1 - \varphi)$. Thus we have:

$$\begin{aligned}\frac{\partial \tilde{\pi}^I}{\partial \varphi} &= 2\varphi [v(x_G^I) - \Theta_G x_G^I - \Delta\theta x_M^I] + \varphi^2 \left[(v'(x_G^I) - \Theta_G) \frac{\partial x_G^I}{\partial \varphi} - \Delta\theta \frac{\partial x_M^I}{\partial \varphi} \right] \\ &\quad + 2(1 - 2\varphi) [v(x_M^I) - \Theta_B x_M^I] - 2\varphi(1 - \varphi) [v'(x_M^I) - \Theta_B] \frac{\partial x_M^I}{\partial \varphi} \\ &\quad + 2(\varphi - 1) \left[v \left(\frac{1 - 2\varphi}{1 - \varphi} x_M^I \right) - \Theta_B \frac{1 - 2\varphi}{1 - \varphi} x_M^I \right] \\ &\quad + (1 - \varphi)^2 \left[v' \left(\frac{1 - 2\varphi}{1 - \varphi} x_M^I \right) \left(\frac{2(\varphi - 1) + (1 - 2\varphi)}{(1 - \varphi)^2} x_M^I + \frac{1 - 2\varphi}{1 - \varphi} \frac{\partial x_M^I}{\partial \varphi} \right) \right] \\ &\quad - (1 - \varphi)^2 \left[\frac{2(\varphi - 1) + (1 - 2\varphi)}{(1 - \varphi)^2} \Theta_B x_M^I + \frac{1 - 2\varphi}{1 - \varphi} \Theta_B \frac{\partial x_M^I}{\partial \varphi} \right], \text{ and thus } \left. \frac{\partial \tilde{\pi}^I}{\partial \varphi} \right|_{\varphi=0} = 0,\end{aligned}\tag{38}$$

since $(1 - 2\varphi)/(1 - \varphi) = 1$ and $v'(x_M^I) = \Theta_B$ at $\varphi = 0$.

Likewise, the principal's optimal expected payoff with outsourcing is:

$$\begin{aligned}\tilde{\pi}^O &\equiv \varphi^2 \left[v(x_G^O) - \Theta_G x_G^O - \frac{1}{1 - \varphi} \Delta\theta x_M^O - \Delta\theta x_B^O \right] \\ &\quad + 2\varphi(1 - \varphi) \left[v(x_M^O) - \left(\Theta_M + \frac{\varphi}{1 - \varphi} \Delta\theta \right) x_M^O - \Delta\theta x_B^O \right] \\ &\quad + (1 - \varphi)^2 \left[v(x_B^O) - \left(\Theta_B + \frac{\varphi}{1 - \varphi} \Delta\theta \right) x_B^O \right],\end{aligned}$$

and therefore we have:

$$\begin{aligned}
\frac{\partial \tilde{\pi}^O}{\partial \varphi} &= 2\varphi \left[v(x_G^O) - \Theta_G x_G^O - \frac{1}{1-\varphi} \Delta\theta x_M^O - \Delta\theta x_B^O \right] \\
&+ \varphi^2 \left[(v'(x_G^O) - \Theta_G) \frac{\partial x_G^O}{\partial \varphi} - \frac{\Delta\theta}{(1-\varphi)^2} x_M^O - \frac{\Delta\theta}{1-\varphi} \frac{\partial x_M^O}{\partial \varphi} - \Delta\theta \frac{\partial x_B^O}{\partial \varphi} \right] \\
&+ 2(1-2\varphi) \left[v(x_M^O) - \left(\Theta_M + \frac{\varphi}{1-\varphi} \Delta\theta \right) x_M^O - \Delta\theta x_B^O \right] \\
&+ 2\varphi(1-\varphi) \left[v'(x_M^O) \frac{\partial x_M^O}{\partial \varphi} - \frac{\Delta\theta}{(1-\varphi)^2} x_M^O - \left(\Theta_M + \frac{\varphi}{1-\varphi} \Delta\theta \right) \frac{\partial x_M^O}{\partial \varphi} - \Delta\theta \frac{\partial x_B^O}{\partial \varphi} \right] \\
&+ 2(\varphi-1) \left[v(x_B^O) - \left(\Theta_B + \frac{\varphi}{1-\varphi} \Delta\theta \right) x_B^O \right] \\
&+ (1-\varphi)^2 \left[v'(x_B^O) \frac{\partial x_B^O}{\partial \varphi} - \frac{\Delta\theta}{(1-\varphi)^2} x_B^O - \left(\Theta_B + \frac{\varphi}{1-\varphi} \Delta\theta \right) \frac{\partial x_B^O}{\partial \varphi} \right], \text{ and thus} \\
\left. \frac{\partial \tilde{\pi}^O}{\partial \varphi} \right|_{\varphi=0} &= 2 [v(x_M^O) - \Theta_M x_M^O - (v(x_B^O) - \Theta_M x_B^O)] - \Delta\theta x_B^O, \tag{39}
\end{aligned}$$

since $v'(x_B^O) = \Theta_B$ at $\varphi = 0$. Thus, (38) and (39) imply that, when (39) is strictly positive, there exists $\underline{\varphi} \leq \bar{\varphi}$ such that for $\varphi < \underline{\varphi}$, outsourcing dominates insourcing. ■

Proof of Corollary 4

We first consider the case of insourcing. We check the collusion opportunities by examining the following inequalities with the optimal outcome in $\bar{\mathcal{P}}^I$:

$$u_{ij}^\alpha + u_{ij}^\beta \geq u_{i'j'}^\alpha + u_{i'j'}^\beta, \quad i, j, i', j' \in \{g, b\}.$$

It is straightforward to check that the inequalities are satisfied for all combinations of $i, j, i', j' \in \{g, b\}$. The only case that is not straightforward to see is:

$$u_{gb}^\alpha + u_{gb}^\beta \geq u_{gg}^\alpha + u_{gg}^\beta. \tag{40}$$

After substituting for the transfers, (40) is written as:

$$\begin{aligned}
&\Delta\theta x_{bb}^I \geq \Delta\theta x_{bg}^I + \Delta\theta x_{gb}^I - \Delta\theta x_{gg}^I \\
\iff &\Delta\theta x_{bb}^I \geq \Delta\theta x_{bg}^I + \Delta\theta x_{gb}^I - \Delta\theta x_{gg}^I \\
\iff &\Delta\theta x_{bb}^I \geq 2\Delta\theta x_{gb}^I - \Delta\theta x_{gg}^I \quad (\because x_{bg}^I = x_{gb}^I) \\
\iff &x_{gg}^I - x_{gb}^I \geq x_{gb}^I - x_{bb}^I.
\end{aligned} \tag{41}$$

From the expressions in Proposition 1, we have:

$$v'(x_{gb}^I) - v'(x_{gg}^I) = v'(x_{bb}^I) - v'(x_{gb}^I) = \frac{\Delta\theta}{1-\varphi}.$$

Therefore, $v''(x) < 0$ and $v'''(x) \geq 0$ imply that (41) is satisfied with a strict inequality.

In the case of outsourcing, it follows from the incentive constraints of agent α that the optimal outcome in $\bar{\mathcal{P}}^O$ is collusion proof since t_{ij} , $ij \in \{gg, gb, bg, bb\}$, is the total of the transfer payments for both agents. ■

Proof of Proposition 5

The optimal outcome in \mathcal{P}^I violates (27) for $\gamma = G$ and $\gamma' = M$:

$$\sum_k t_G^k - \Theta_G x_G \geq \sum_k t_M^k - \Theta_G x_M, \quad (42)$$

and hence (42) is binding under collusion. Binding constraints in \mathcal{P}^I with binding (42) give the following expressions for the transfers:

$$t_G^\alpha = t_G^\beta = \theta_g x_G + \Delta \theta x_M, \quad t_M^\alpha = t_M^\beta = \theta_b x_M, \quad t_B^\alpha = t_B^\beta = \theta_b x_B.$$

After substituting for the transfers, optimization yields the project sizes characterized in Proposition 5. It is straightforward to check that all the other constraints, including (27) with the other combinations of γ and γ' , are satisfied with the optimal outcome. The agents' rents follow from the expressions for the transfers. As for outsourcing, it follows from the incentive constraints of agent α that the optimal outcome in \mathcal{P}^O is collusion proof since t_γ , $\gamma \in \{G, M, B\}$, is the total of the transfer payments for both agents. ■

Proof of Corollary 5

The proof follows from Corollary 3 and Proposition 5. ■

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