Finding a Good Deal: Stable Prices, Costly Search, and the Effect of Entry

David P. Myatt (London Business School)
David Ronayne (ESMT Berlin)

Discussion Paper No. 524

January 28, 2025

Collaborative Research Center Transregio 190 | www.rationality-and-competition.de
Ludwig-Maximilians-Universität München | Humboldt-Universität zu Berlin
Spokesperson: Prof. Georg Weizsäcker, Ph.D., Humboldt University Berlin, 10117 Berlin, Germany
info@rationality-and-competition.de

Finding a Good Deal:
Stable Prices, Costly Search, and the Effect of Entry

David P. Myatt
London Business School
`www.david-p-myatt.org`
`dmyatt@london.edu`

David Ronayne
ESMT Berlin
`www.davidronayne.com`
`david.ronayne@esmt.org`

January 2025

Main paper: pp. 1–28 · Appendices: pp. 28–42 · References: pp. 42–44

Abstract. We study markets in which potential buyers engage in costly search to find a good deal. Our novel solution concept for prices builds upon the idea that any movement in a firm's price is followed by an opportunity for its competitors to respond with special offers. This mechanism selects the highest prices such that no firm wishes to undercut a competitor. We identify a distinctive closed-form pattern of disperse prices that uniquely satisfy our pricing solution, and pair that price profile with optimal fixed-sample search. In a stable equilibrium with active search, the intensity of search and consumer surplus are lower and industry profit is higher with more competitors. In a concentrated oligopoly, complete search in equilibrium can eliminate industry profit.

If firms' prices differ then buyers face a search problem: how many (costly) quotations should be obtained in order to find a good deal? If search is active (some buyers gather multiple quotations) but not exhaustive (others approach only a single firm) then there cannot be a single price: prices exceed marginal cost given that some buyers are captive to a single firm but active comparisons rule out tied prices. This generates the price dispersion which itself poses a search problem for buyers. In this context, we seek to characterize jointly search behavior and pricing, and to ask how they respond to changes in the market and in particular to entry.

Our modeling novelty is the nature of pricing. We seek "stable" prices that satisfy two criteria. Firstly, no firm can profit from undercutting a cheaper competitor. Secondly, no firm can gain from a price rise given that firms would then have an opportunity to cut prices in response. The profile of maximal (that is, highest) "undercut proof" prices uniquely satisfies these criteria.

Fixing search behavior, we find that adding a competitor lowers the average price charged by firms. This weakens the incentive for buyers to obtain additional price quotations, and so (in an equilibrium with active search) search endogenously falls in response to firm entry. This reduced search pushes upward the set of stable prices and so raises the average price paid by buyers. A notable result is that more competition is actually good for aggregate industry profit.

Acknowledgements. We were helped by and are grateful for generous comments on this work and other related papers (Myatt and Ronayne, 2024a,b) from Simon Anderson, Mark Armstrong, Dan Bernhardt, Alessandro Bonatti, Tim Cason, Yongmin Chen, Alex de Cornière, Andrea Galeotti, Paul Heidhues, Roman Inderst, Maarten Janssen, Philippe Jehiel, Justin Johnson, Jeanine Miklós-Thal, José Moraga-González, Volker Nocke, Martin Pesendorfer, Régis Renault, Michael Riordan, Robert Somogyi, Greg Taylor, Juuso Välimäki, Hal Varian, Chris Wilson, Jidong Zhou, and the participants of conferences and seminars. Ronayne is grateful for support from Deutsche Forschungsgemeinschaft through CRC TRR 190 (project 280092119).

We specify a model which brings together a conventional demand side with a novel supply side.

On the demand side, we adopt a fixed-sample search technology: buyers simultaneously choose how many costly price quotations to obtain. This corresponds to situations in which buyers use a "gather then evaluate" protocol, which can necessarily be the case, for example, when there is a non-negligible lag between requesting and receiving a quotation.

On the supply side, a conventional modeling approach in Bertrand environments is for firms to play a simultaneous-move pricing game in which disperse prices are interpreted as the realizations from mixed-strategy play. Of course, such prices are susceptible to ex post deviations.

In contrast, our own pricing mechanism exploits the idea that firms can find it relatively easy to offer special deals, perhaps using cut-price "sale" offers, to respond to competitors. Our first criterion requires prices to be robust to such poaching from rivals and so to prevent spirals of mutual undercuts. Our second criterion means that raising an ask above the prescribed stable price is dissuaded by the knowledge that it will provoke rivals to unleash a price cut.

Stable prices readily emerge from the play of natural price-formation games. In one such game, firms choose their regular price positions and then have the opportunity to offer special deals. In another game, a facilitating industry association specifies (collusive) prices but firms are then able to offer secret price cuts to buyers. In both cases (and in several other games) there is a subgame-perfect equilibrium outcome in which maximal undercut-proof prices are charged.

Fixing search behavior exogenously (within the class that appears in equilibrium: buyers request one or two quotations) we find that entry to the industry "fills in" a distribution of stable prices that (in the limit) approaches the one seen in the symmetric mixed-strategy Nash equilibrium of a single-stage pricing game. The average price charged by firms falls as the number of competitors rises, which is helpful for buyers who obtain a single quotation. Nevertheless, the average price paid across all buyers (this is lower than the average price charged by firms, given that some buyers obtain a second quotation) remains constant. An accounting identity implies a rise in the average price paid by any buyer who obtains two quotations. The overall impact is to reduce the incentive to seek that second quotation. This (as we show) means that entry can prompt an endogenous reduction in buyer search and so raise industry profit.

A complete picture unifies our stable prices with optimal search. We study equilibria in which search is stable in the sense that a small perturbation in search behavior does not result in prices that drive buyers further away from the original equilibrium. As usual, there is a "no search" equilibrium in which prohibitively high prices deter any search. If buyers enjoy sufficient surplus at the monopoly price then this becomes a "minimal search" equilibrium in which all buyers obtain a single quotation and firms charge that monopoly price. If search costs are not too high then there is also a unique "active search" equilibrium in which at least some buyers obtain two quotations, and so meaningful price comparisons occur and prices are disperse. Notably, this active-search equilibrium can (and must, for sufficiently concentrated industries) involve complete search in the sense that all buyers obtain two quotations.

We convey the intuition in a duopoly setting. In equilibrium buyers obtain either one quotation (and so are “captive” to a single firm) or two (they are “shoppers” who see both) and so an increase in search turns captives into shoppers. Stable prices are such that one firm charges the monopoly price while its competitor charges a limit price that dissuades an undercut: the high-price firm is indifferent between fully exploiting captive customers (at the monopoly price) and capturing shoppers (by undercutting that limit price). As more buyers become shoppers, the low (limit) price moves downward while the high price is static. A larger price gap encourages buyers to obtain a second quotation. The complementarities in search result in two stable equilibria: one with minimal search (or no search) and one with complete (here, two-quotation) search. This contrasts markedly with the canonical treatment that uses symmetric mixed-strategy pricing: in that world, heightened search pushes the distribution of prices used by all firms down toward zero, which dampens incentives and prevents a complete-search equilibrium.

To extend the analysis to oligopolies of any size, we derive the search benefit schedule for buyers and its properties. New auxiliary results are required to do this because buyers draw from a discrete pool of stable prices without replacement and so well-known results on sampling from continuous populations do not apply. We derive and provide the analogous results for the discrete case, and we obtain concise closed-form solutions for prices and search benefits.

Our analysis reveals that our duopoly story extends to small oligopolies: in a stable equilibrium with active search and with four or fewer firms, we find that complete search eliminates industry profit. For less concentrated industries, search choices become strategic substitutes as search intensity rises. For moderate search costs we find that incomplete search and industry profit return as buyers find it less worthwhile to look for a good deal. Additional entry results in less search, a higher average price paid, and lower buyer surplus. The fall in search means that entry favors the industry. The lower bound to active search in an oligopoly has approximately two-thirds of buyers search actively by requesting a second price quotation.

These key findings also hold for an extended model in which buyers’ search costs are heterogenous, where we can identify situations in which there is a unique equilibrium with active but incomplete search where (once again) more competition depresses search and raises industry profit. We also study situations with downward-sloping demand from each buyer, we evaluate the impact of allowing some buyers to see every price, and we identify conditions with a constant marginal cost of search that can generate a “model of sales” environment.

Guide to the Paper. After relating our work (Section 2) to some of the (extensive) literature on price dispersion and search, we begin (Section 3) by describing our model and solution concept. For fixed search behavior we characterize (Sections 4 and 5) the properties of stable prices. We then (Sections 6 and 7) pair stable prices with optimal search to characterize equilibria and to examine the effect of entry. We study (Section 8) non-cooperative foundations for stable prices before (Section 9) describing several extensions. Various supplements contain our formal proofs and further extensions to our model (Appendices A to D).

2. Literature

The canonical model of pricing with costly fixed-sample search (Burdett and Judd, 1983) has been extended extensively to include multiple products (McAfee, 1994), product differentiation (Anderson, De Palma, and Thisse, 1992; Moraga-González, Sándor, and Wildenbeest, 2021), heterogeneous search costs (Moraga-González, Sándor, and Wildenbeest, 2017), exogenous search (Janssen and Moraga-González, 2004), and many other factors.[1]

The continuum of firms in Burdett and Judd (1983) means that a symmetric mixed pricing strategy can be interpreted as a pure-strategy profile of disperse prices. We provide such prices in an oligopoly with a finite set of firms. To do this we replace the usual Bertrand game with a notion of price stability that we study in more depth elsewhere (Myatt and Ronayne, 2024b).[2] We emphasize a distinctive effect of entry that is absent under mixed-strategy pricing.[3]

Our focus on entry to an oligopoly is shared with Janssen and Moraga-González (2004). Using a conventional pricing game, they incorporated exogenous "shopper" types, and they dealt with the substantial analytic complexities of a symmetric mixed-strategy solution. For concentrated oligopolies they predicted that entry generates greater search and lower prices, whereas we obtain the opposite result. We provide (in Section 9) a detailed discussion of their work.

Entry pushes up industry profit in Rosenthal (1980) because the mass of captive customers expands exogenously with such entry. Here, that mass is endogenous. For fixed search behavior, the average price charged falls with entry (as is conventional) which depresses search and so indirectly drives up the average price paid. Our use of a conventional mechanism differs from other explanations for price-increasing competition (for example, Chen and Riordan, 2008).[4]

Non-standard entry effects are a feature of some sequential-search models (Janssen, Pichler, and Weidenholzer, 2011; Chen and Zhang, 2018a). A key component here and elsewhere (Janssen, Moraga-González, and Wildenbeest, 2005, for example) is a search-depression effect from factors that might appear pro-competitive such as price caps (Armstrong, Vickers, and Zhou, 2009a), price-party clauses (Wang and Wright, 2020), price-comparison websites (Ronayne, 2021), or the steering of buyer consideration (Chen and Zhang, 2018b; De Cornière and Taylor, 2019; Teh and Wright, 2022). Relatedly, in some models (Zhou, 2014; Rhodes and Zhou, 2019; Chen, Li, and Zhang, 2022) heightened search costs can raise rather than lower consumer surplus.

[1]Anderson and Renault (2018) surveyed price dispersion with search. We exclusively study fixed-sample search. Sequential-search models (Wolinsky, 1983, 1986; Anderson and Renault, 1999) have been extensively studied and developed to include elements such as shoppers (Stahl, 1989), captives (Chen and Zhang, 2011), prominence (Armstrong, Vickers, and Zhou, 2009b), multiproduct search (Zhou, 2014), categorization (Fershtman, Fishman, and Zhou, 2018), search targeting (De Cornière, 2016), and mergers (Moraga-González and Petrikaitė, 2013).
[2]Models of ordered search with uncertain product match (Zhou, 2011) and models with non-optimizing buyers (Anderson and De Palma, 2005) can also generate disperse prices via pure-strategy play. In another setting (Chen and He, 2011) firms charge the same price but bid differently for search positions.
[3]In conventional analyses (just as in within our framework) homogeneous buyers obtain at most two quotations in equilibrium. Symmetric mixed-strategy pricing means that each firm effectively competes against a single random competitor. The equilibrium pricing distribution is independent of the number of firms.
[4]In recent work with exogenous buyer consideration, Armstrong and Vickers (2022) found a conventional effect of entry under symmetry, but that entry can harm consumers under asymmetric consideration specifications.

3. A Model of Pricing with Endogenous Search

3.1. **Supply and Demand.** There are $n \geq 2$ symmetric firms with constant marginal costs which (without further loss of generality) we normalize to zero. Firm $i \in \{1, \ldots, n\}$ offers (via a channel to be described below) a price $p_i \in [0, v]$ to any buyer who requests a quotation, where we interpret $v > 0$ as the price that a monopolist would charge for a single unit. More generally, this is the profit-per-customer that such a monopolist earns.

Each buyer from a unit mass is willing to pay at most v for a single unit. A buyer also gains extra surplus $\underline{u} \geq 0$ when purchasing, even at the monopoly price. The usual case under unit demand is when $\underline{u} = 0$, but we explain below (in Section 3.3) why positive surplus might arise.

A buyer who requests and pays for $q \in \{1, \ldots, n\}$ price quotations sees a random sample, without replacement, of the offers from q firms and buys from the cheapest. Ties are broken in any interior way. We write κ_q for the marginal cost of the qth quotation and we assume (for most of the paper, and to streamline exposition) increasing marginal cost: $0 < \kappa_1 < \cdots < \kappa_n$. However, we also study (in Section 9) the constant-marginal-cost case where $\kappa_q = \kappa$ for all q as well as a model variation (in Appendix B) in which buyers' search costs are heterogeneous. To avoid uninteresting "knife edge" cases we assume that $\kappa_1 \neq \underline{u}$.

We write m_q for the (endogenously determined) mass of buyers who obtain q quotations so that the remaining mass $m_0 = 1 - \sum_{q=1}^{n} m_q$ of buyers do not search at all.

3.2. **Solution Concept.** Our solution concept (an "equilibrium") comprises (i) a profile of prices and (ii) a specification of search behavior. Fixing those prices, our requirement is conventional: buyers engage in optimal fixed-sample search given the expected profile of prices.

Our pricing concept is novel. It captures the idea that it can be relatively easy for a firm to offer a promotion or special deal in response to any price adjustments by competitors. Prices are "stable" if no firm can gain from such a deal by undercutting a cheaper competitor, and if no firm wishes to raise its price given that others may respond with special deals.

Definition (Stable Prices). *Fix a profile of prices where $p_i \in [0, v]$ is the price of firm i.*

We define a price-cutting game *as a simultaneous-move game in which risk-neutral profit-maximizing firms simultaneously offer special deals by setting a final price $\tilde{p}_i \in [0, p_i]$.*

Given this definition, we say that the profile of prices is stable *if it satisfies these two criteria.*

(1) No firm strictly gains with a special deal that undercuts a cheaper opponent. Formally, the associated price-cutting game has a Nash equilibrium in which all firms set $\tilde{p}_i = p_i$.

(2) No firm gains from a price rise, given that all firms enjoy a subsequent opportunity to offer special deals. Formally, following an adjustment in the price of a firm i, a Nash equilibrium of a price-cutting game gives a weakly lower expected profit to firm i.

Later in the paper (in Section 4) we refine our notion of price stability to deal with situations in which stable prices are not uniquely defined. There prices are "robustly stable" if they are compatible with an arbitrarily small probability that a price quotation is not received.

3.3. **Commentary.** In many settings prices are not immediately accessible and buyers must search to reveal them. If there is a lag between requesting and receiving a quotation then a buyer needs to make multiple requests in advance (a "fixed sample" technology) rather than taking the "wait and see" approach of sequential search. Quotation lags arise in many industries and contexts, including construction and large-project tendering; Morgan and Manning (1985) characterized conditions under which fixed-sample search is optimal. In broader settings, researchers have documented consumer behavior that appears equivalent to such fixed-sample search. For example De los Santos, Hortaçsu, and Wildenbeest (2012) and Honka and Chintagunta (2016) studied data from markets for books and automobile insurance, respectively. Both studies found their data to be consistent with fixed-sample, or "simultaneous," search, rather than a sequential evaluation of the choice alternatives. This "gather then evaluate" protocol is exactly the behavior captured by assuming buyers use a fixed-sample search technology.

We assume that the marginal cost of search is increasing, but we can cope readily with decreasing marginal cost if it does not fall too quickly. The reason is that, and as we derive, the marginal benefit of additional searches is decreasing, and typically strictly so. We consider an extension (in Section 9) in which the marginal cost of search is constant.

If $\kappa_1 > \underline{u}$ (which holds when $\underline{u} = 0$) then there is a "no search" equilibrium in which firms charge the monopoly price, buyers stay at home, and trade collapses. However, we also allow for $\underline{u} > \kappa_1 > 0$ so that buyers obtain at least a single quotation and there is a "minimal search" equilibrium in which buyers become captive and exploited à la Diamond (1971). Strictly positive consumer surplus under monopoly pricing ($\underline{u} > 0$) arises if we replace "unit demand" with a downward-sloping demand curve for each buyer. We consider this explicitly (in Section 9) where we also allow firms to offer general deals in the sense of Armstrong and Vickers (2001), and where we discuss other mechanisms (such as "add on" opportunities) to generate $\underline{u} > 0$.

Finally, the important novelty of our model is our pricing solution concept. Condition (1) of our price-stability definition (which says that the profile of prices is "undercut proof") also holds for a pure-strategy Nash solution to a simultaneous-move pricing game.[5] Nash prices also have the property that there is no profitable upward movement in price. Our condition (2) modifies this by supposing that any upward price movement is followed by a further opportunity for firms to cut prices using special offers.[6] We document in Section 8 (and also in our supplementary Appendix C) several natural non-cooperative games that predict the play of stable prices.

---

[5]A pure-strategy Nash profile of prices (if it were to exist) would also satisfy our second criterion.

[6]We use a related notation of stable prices in our other work (Myatt and Ronayne, 2024b), but weaken the criterion (2) by replacing the requirement that "no firm gains from a price rise" with a condition ("creep resistance") that asks that "no firms gains from an arbitrarily small price rise." For the current paper, both definitions generate the same set of stable prices and so we use a simpler (but stricter) definition here.

4. Stable Prices

Here we characterize the prices that satisfy our price-stability criteria. Given the symmetry of firms, and without loss of generality, we label firms so that $p_1 \geq \cdots \geq p_n$.

4.1. **Stable Prices with (Some) Captive Customers.** Recall that m_q buyers obtain q quotations. We assume for now that $m_1 > 0$ so that some buyers are "captive" to a single firm. (We return at the end of this section to consider $m_1 = 0$.) Any positive price earns a positive profit (from at least the captive customers) and so a zero-price firm strictly gains from a price rise, even if others cut their prices. We conclude that stable prices must be strictly positive.

Suppose (temporarily) that $m_2 > 0$. Undercut-proofness eliminates tied prices: a firm knows that it may be pairwise compared to any competitor and so would it undercut that competitor's price.[7] We conclude that stable prices are entirely distinct, $p_1 > \cdots > p_n > 0$, and we proceed to calculate firms' sales. Firm i sells only to buyers with $q \in \{1, \ldots, i\}$ quotations because those who obtain more than i quotations are sure to find a better offer. A mass m_q of buyers acquire q quotations. There are $\binom{n}{q}$ equally likely consideration sets for such a buyer. (A "consideration set" lists firms from whom a buyer obtains quotations.) Firm i makes a sale only if compared to firms from within $\{1, \ldots, i-1\}$ that are more expensive. There are $\binom{i-1}{q-1}$ sets of size q that combine firm i with $q-1$ higher-price competitors. Summing over relevant buyer types,

$$\text{sales of firm } i \equiv X_i = \sum_{q=1}^{i} m_q \left[\binom{i-1}{q-1} \Big/ \binom{n}{q} \right]. \tag{1}$$

The expression X_i is strictly increasing in i, and so cheaper firms enjoy greater sales. For $i = 1$, eq. (1) reduces to $X_1 = m_1/n$, which corresponds to a firm's captive buyers.

A higher-priced firm i with profit $p_i X_i$ can "steal" the profit (or arbitrarily close to it) of a cheaper firm $j > i$ by undercutting p_j, and so the "no undercutting" conditions take the form $p_i X_i \geq p_j X_j$ for $j > i$. These constraints can be expressed jointly as $p_1 X_1 \geq \cdots \geq p_n X_n$, so that lower-indexed (and higher priced) firms earn (weakly) more profit.

We obtain the same conclusion if $m_2 = 0$. Suppose that $m_q = 0$ for $q \in \{2, \ldots, \bar{q}\}$ but that $m_{\bar{q}+1} > 0$, so that any comparisons are made between at least $\bar{q} + 1$ firms. A firm $i \in \{\bar{q}, \ldots, n-1\}$ knows that it might be compared to the (weakly more expensive) firms $\{1, \ldots, \bar{q}-1\}$ and also the (weakly cheaper) firm $i+1$. The "no undercutting" criterion rules out a tie in prices, and so $p_i > p_{i+1}$. We conclude that $p_1 \geq \cdots \geq p_{\bar{q}} > \cdots > p_n$. This leaves open the possibility of ties in the prices of firms from $i \in \{1, \ldots, \bar{q}\}$. Such firms makes sales only if they are considered uniquely (by captive buyers) and so $X_1 = \cdots = X_{\bar{q}} = m_1/n$. These $\bar{q}$ solutions all satisfy eq. (1). The same equation for X_i also holds for $i \in \{\bar{q}+1, \ldots, n\}$, and the previous "no undercutting" constraints give us $p_1 X_1 \geq \cdots \geq p_n X_n$ as before.

[7] Johnen and Ronayne (2021) explored related logic, which leads to the conclusion that the traditional single-stage pricing game has a unique equilibrium if and only if $m_2 > 0$.

Our definition of price stability asks us to consider a price-cutting game in which firms make special offers relative to their regular prices. If those regular prices are undercut-proof then there is a (unique) a pure-strategy Nash equilibrium in which there are no price cuts, and firm i earns profit $p_i X_i$. For other prices we can bound the expected profit earned by firms.

Lemma 1 (Profits from Price-Cutting Opportunities). *For prices $p_1 \geq \ldots \geq p_n$, consider the simultaneous-move price-cutting game in which each firm i chooses a price $\tilde{p}_i \in [0, p_i]$.*

(1) In equilibrium, the expected profit of each firm is bounded above by $p_1 m_1/n \leq v m_1/n$.

(2) If the prices are undercut proof, so that $p_1 X_1 \geq \cdots \geq p_n X_n$, then the game is dominance solvable. In the unique equilibrium firm i chooses $\tilde{p}_i = p_i$ and earns profit $p_i X_i$.

We sketch the proof of the first claim here. In an equilibrium consider a firm that charges (using an atom, if an atom-playing firm exists) the highest price from the joint support of all firms. Such a firm setting this price sells only to "captive" customers and so earns at most $p_1 m_1/n$. This is weakly more than any other competitor, owing to a "profit stealing" argument: the highest-price firm can always earn (at least) the profit of a competitor by undercutting the lower bound of the mixed-strategy support of that other firm.

From the second claim of this lemma we can immediately identify (subject to the order of firms' labels) a unique candidate for a stable price profile. If $v > p_1$, then the highest-price firm can strictly increase its price and profit without violating a no-undercutting constraint; claim (2) guarantees that its profit increases. Similarly, if $p_{i-1} X_{i-1} > p_i X_i$ for $i > 1$, so that there is slack in a constraint, then firm i can also strictly gain from a local price rise. We conclude that $p_1 = v$ and that stable prices satisfy $p_1 X_1 = \ldots = p_n X_n$. These are maximal undercut-proof prices: the highest prices (and profit) that can be achieved subject to avoiding price cuts.

Under maximal undercut-proof prices, each firm earns $v m_1/n$ (the profit from exploiting captive customers) and so attains the upper bound reported in claim (1) of Lemma 1. This also shows that any further rise in a firm's price (for any firm $i \in \{2, \ldots, n\}$) cannot result in a gain.[8]

Proposition 1 (Stable Prices). *Fix an arbitrary labeling of firms. The unique stable price profile comprises the maximal undercut-proof prices. The price and profit of each firm are*

$$p_i = \frac{v m_1/n}{X_i} \quad \textit{and} \quad \pi_i = \frac{v m_1}{n}. \tag{2}$$

4.2. **Prices in the Absence of Market Power.** We return now to consider the case where $m_1 = 0$. There are no captive buyers: a firm's price is always compared to a competitor. Undercut-proof prices are forced down to eliminate profits à la Bertrand, and each buyer has a

[8]For example, following a shift by $i > 1$ to $\hat{p}_i \in (p_i, p_{i-1})$, there is a (unique) equilibrium of the follow-on price-cutting game in which firms $i-1$ and i mix for $p \in [\underline{p}, \hat{p}_i)$, and then place residual mass at p_{i-1} and $\hat{p}_i$ respectively. All other firms do not engage in price cuts. This equilibrium gives expected profit $v m_1/n$ to every firm. In our related work (Myatt and Ronayne, 2024b) we characterize the (sometimes more intricate) equilibria of price-cutting games when a firm deviates upward to a price $\hat{p}_i > p_j$ for some $j < i - 1$.

zero price in their consideration set. However, prices are not tied down fully: there are multiple stable profiles, each with a minimal number of zero prices.[9]

Suppose that $m_1 = \cdots = m_{\bar{q}} = 0$ and $m_{\bar{q}+1} > 0$ for $\bar{q} \in \{1\ldots, n-1\}$, so that buyers consider at least $\bar{q}+1$ prices. Any undercut-proof profile contains at least $n - \bar{q}$ zero prices and so $p_{\bar{q}+1} = \ldots p_n = 0$. (If there were $\bar{q}+1$ or more strictly positive prices then there would be an undercutting opportunity: a firm $i \in \{1, \ldots, \bar{q}\}$ would know that to make a sale it must win a comparison against at least some firm from $\{\bar{q}+1, \ldots, n\}$ and so would face an incentive to undercut (at least) the price $p_{\bar{q}+1}$.) The $\bar{q}$ highest prices can take any value, but no firm from this set makes a profit, either because it charges a zero price or it is inevitably compared to a zero-price competitor. Price stability places no further structure on those $\bar{q}$ prices.

The multiplicity of stable prices can be avoided pragmatically by adding some exogenous buyer types or trembles (even if arbitrarily small) so that some buyers are restricted to only one firm. Another reasonable modification is obtained if price quotations are not always delivered.

To explore more fully this modification, suppose that a solicited quotation fails to arrive with probability $\varepsilon > 0$. Given that m_q buyers request q quotations for $i \in \{1, \ldots, n\}$, the mass who receive i quotations is m_i^ε where

$$m_i^\varepsilon = \sum_{q=i}^{n} m_q \binom{q}{i} (1-\varepsilon)^i \varepsilon^{q-i}. \tag{3}$$

Given that $m_1^\varepsilon > 0$, a unique stable profile of prices $\{p_1^\varepsilon, \ldots, p_n^\varepsilon\}$ is obtained. We take the limit as the "delivery failure" parameter ε vanishes to define "robust price stability."

Definition (Robustly Stable Prices). *Fix the mass of buyers who request i quotations. Suppose that a quotation is received with probability $1-\varepsilon$, and consider the uniquely stable prices p_i^ε for $i \in \{1, \ldots, n\}$. A price profile is* robustly stable *if $p_i = \lim_{\varepsilon \to 0} p_i^\varepsilon$.*

If $m_1 > 0$ the unique stable profile of (2) is robustly stable. If $m_1 = 0$ then it is

$$\lim_{\varepsilon \downarrow 0} p_i^\varepsilon = \begin{cases} v & i = 1 \\ 0 & i \in \{2, \ldots, n\} \end{cases} \tag{4}$$

and so a request for two quotations is sufficient to uncover a zero-price offer.

Robustly stable prices give us a unique characterization of prices conditional on buyers' search behavior. The logic of Lemma 1, which shows that these prices maximize profits when firms face a subsequent opportunity to offer special deals in a price-cutting game, also suggests that the same prices are likely to arise from non-cooperative play. We confirm that this is true later in the paper (in Section 8). For now, we proceed to describe the pattern of prices that emerge from equilibrium-compatible search behavior.

[9]Recall that Nash equilibrium prices are also not uniquely defined in classic Bertrand games when $n \geq 3$.

5. Pricing Patterns under Equilibrium-Compatible Search Behavior

We now illustrate stable prices when buyers obtain either one or two quotations, so that $m_1 > 0$ (some buyers are captive) and $m_2 > 0$ (there are pairwise comparisons) but $m_q = 0$ otherwise, and so $m_1+m_2 = 1$. We will confirm (in Section 6) that this type of search occurs in equilibrium.

5.1. The Distribution of Prices.

Because $m_2 > 0$, the quantity of sales for each successively lower-priced firm is increasing: $X_1 < \cdots < X_n$. Each (strict) step down the ladder of disperse prices is exactly compensated for by a corresponding increase in sales.

Proposition 2 (Stable Prices with One or Two Quotations). *Suppose that buyers obtain either one or two quotations so that $m_1 > 0$ and $m_2 > 0$ but $m_q = 0$ for $q \in \{3, \ldots, n\}$,*

(1) The unique profile of stable prices is entirely dispersed and satisfies, for each $i \in \{1, \ldots, n\}$,

$$p_i = \frac{vm_1}{m_1 + 2m_2Z_i} \text{ where } Z_i \equiv \frac{i-1}{n-1}. \quad \text{In particular, } p_1 = v \text{ and } p_n = \frac{vm_1}{m_1 + 2m_2}, \tag{5}$$

so that the highest and lowest prices do not depend on the number of firms.

(2) As $n \to \infty$ these prices converge to a continuous distribution

$$F(p) = 1 - \frac{(v-p)m_1}{2pm_2} \quad \text{for} \quad p \in \left[\frac{vm_1}{m_1 + 2m_2}, v\right]. \tag{6}$$

(3) Prices, for $i > 1$, are decreasing in the proportion of buyers who obtain two quotations.

(4) Each firm earns profit of vm_1/n, and so the average paid price by a buyer is equal to vm_1.

In Figure 1 we plot the prices given in eq. (5) for a given search intensity (values for m_1 and m_2). By inspection (and as noted in Proposition 2) the highest and lowest prices are invariant to n. As firms enter, the new price positions "fill in" between these extremes.

Indeed, quantiles other than the highest and lowest prices are also (when they exist in the finite set of price positions) invariant to n. Consider, for example, the median price; this exists, of course, whenever n is odd, and corresponds $Z_i = \frac{1}{2}$ and so $i = (n-1)/2 + 1$. Straightforwardly,

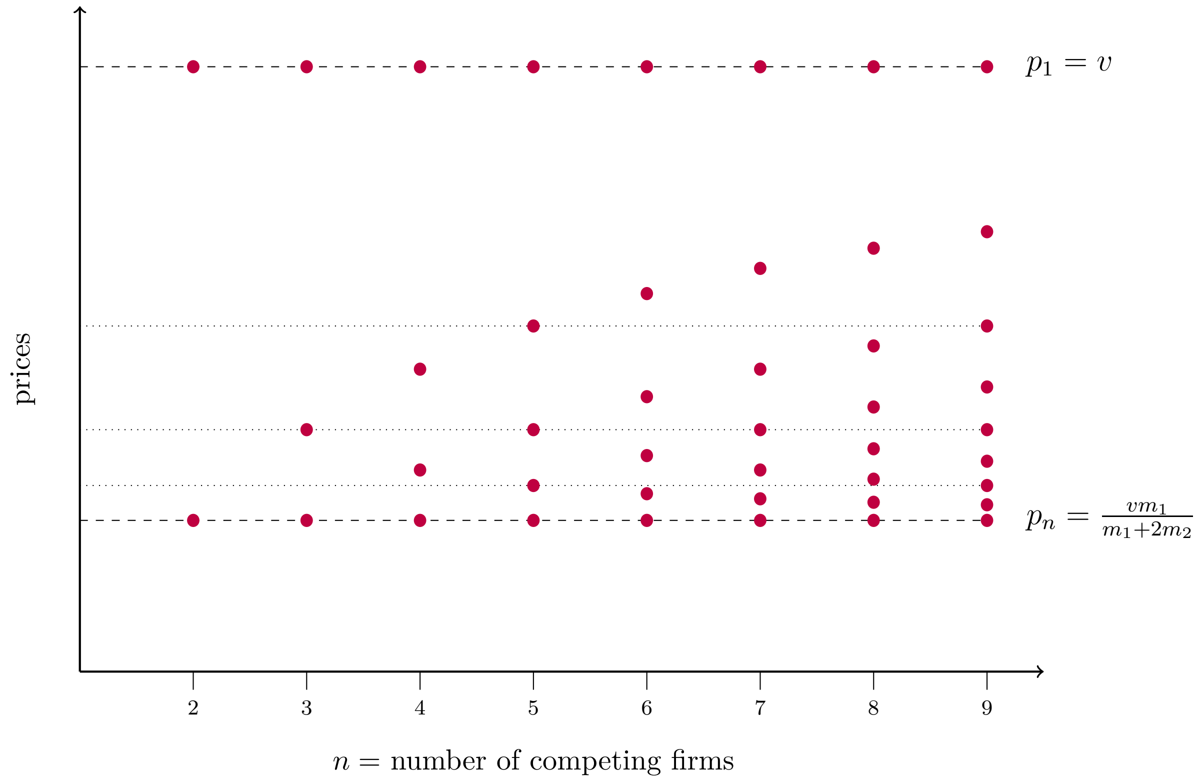
$$p_{(n-1)/2+1} = \frac{vm_1}{m_1 + m_2}. \tag{7}$$

Similarly, if the quartile price positions exist, which is true whenever $n-1$ is a multiple of four, then those quartile positions do not depend on n. The lowest quartile position for an n satisfying this is $i = 1 + (n-1)/4$ and so $Z_i = \frac{1}{4}$, and so straightforwardly

$$p_{(n-1)/4+1} = \frac{vm_1}{m_1 + (m_2/2)}. \tag{8}$$

The limiting case (as $n \to \infty$) connects our prices to traditional single-stage pricing models with mixed-strategy equilibria: the distribution $F(\cdot)$ corresponds to such a mixed equilibrium.[10]

[10] See, for example, Burdett and Judd (1983, eq. (2)): with single-stage pricing, the same CDF in symmetric mixed-strategies is recovered. As $n \to \infty$ the mixed equilibria of a large class of games become ex post Nash (Kalai, 2004, Theorem 1). When n is assumed to be finite, strategies are necessarily not ex post Nash.



Notes. This figure illustrates the distribution of prices, reported in eq. (5), while varying the number of competing firms for a specification in which buyers obtain one or two quotations. The parameter choices are $m_2 = \frac{3}{5}$ and $m_1 = \frac{2}{5}$.

The highest (p_1) and lowest (p_n) prices do not vary with the number of firms. The dotted lines indicate the median (which applies when n is odd) and the upper and lower quartile (which apply when $n-1$ is a multiple of 4) positions. These also do not vary whenever they exist. Additional competitors "fill in" a distribution of prices.

FIGURE 1. The Response of Prices to Entry for Fixed Search Behavior

The final claims concern profitability or (equivalently) the average price paid. We recall that $m_1 + m_2 = 1$, and so $(1-m_2)/n$ customers are captive to each firm. The most expensive firm loses any comparisons, and so earns its profit only from exploitation of captive customers. The price construction (from the binding no-undercutting constraints) means that firms earn equal expected profit. The average price paid (and so the industry profit) is equal to $(1-m_2)v$ and does not depend directly on the number of competitors. Instead, it depends on buyers' actions: more intensive search (an increase in m_2) lowers the distribution of prices offered. In fact,

$$\text{industry profit} = (1-m_2)v \tag{9}$$

$$\text{welfare} = v + \underline{u} - \kappa_1 - \kappa_2 m_2 \tag{10}$$

$$\text{consumer surplus} = \underline{u} + m_2(v-\kappa_2) - \kappa_1 \tag{11}$$

The welfare expression is obtained by noting that everyone is served (generating $v + \underline{u}$) and deducting search costs. The difference between welfare and industry profit gives the expression for consumer surplus which, by inspection, increases with the intensity of search.

5.2. **Average Prices.** The average price paid (this is industry profit) is independent of n, but the distribution of prices charged is not. This is relevant for a buyer who obtains one quotation, and so pays (in expectation) the average price charged. As n increases, prices fill in between the highest and lowest prices. Figure 1 suggests that this average falls as we move from duopoly to triopoly: the new (central) price is below the average of the two others. We extend this observation to find the reaction of the average price to a general increase in n. The ith price is

$$p_i = P(Z_i) \quad \text{for} \quad Z_i = \frac{i-1}{n-1} \quad \text{where} \quad P(Z) \equiv \frac{vm_1}{m_1 + 2m_2 Z}, \tag{12}$$

where we note that $P(Z)$ is a convex function of Z. The average price charged is then

$$\frac{\sum_{i=1}^{n} p_i}{n} = \mathrm{E}[P(\tilde{Z}_{(n)})] \quad \text{where} \quad \tilde{Z}_{(n)} \sim U\left\{\frac{0}{n-1}, \frac{1}{n-1}, \ldots, \frac{n-1}{n-1}\right\}, \tag{13}$$

meaning that $\tilde{Z}_{(n)}$ is a discrete uniform distribution with cardinality n across evenly spaced positions on $[0,1]$.[11] This satisfies $\mathrm{E}[\tilde{Z}_{(n)}] = \frac{1}{2}$ which is independent of n. For a duopoly ($n = 2$) it divides mass equally between the end points at 0 and 1, but the move to triopoly ($n = 3$) shifts some mass from those endpoints to the center at $\frac{1}{2}$. This is a reduction in risk in the usual second-order sense.[12] More generally, and as we prove, an increase in the cardinality of a discrete uniform distribution on a fixed interval reduces the riskiness of that distribution.

Lemma 2 (Riskiness of Discrete Uniform Distributions). *Consider a discrete uniform distribution on the unit interval. An increase in $n \geq 2$ results in a second-order reduction in riskiness. Formally, $\tilde{Z}_{(n+1)} \succ_{SOSD} \tilde{Z}_{(n)}$ where "$\succ_{SOSD}$" means "second order dominates."*

Inspecting eq. (13), the average price charged, $\mathrm{E}[P(\tilde{Z}_{(n)})]$, is the expectation of a convex function of a random variable that becomes less risky as n increases. We obtain Proposition 3.

Proposition 3 (Average Price with One or Two Quotations). *If buyers obtain either one or two quotations, then the average price charged is decreasing in the number of firms.*

Proposition 3 contrasts with the symmetric mixed-strategy predictions from standard single-stage pricing, in which firms' equilibrium pricing cdf does not depend on the number of firms.[13] The difference here is the nature of pricing. Instead of each firm continuously mixing in an i.i.d. fashion over an interval of prices, we predict that each firm charges one price.

The key result (Proposition 3) does seem conventional: fixing the nature of demand (determined by search behavior) an increase in competition lowers the average price charged. This conventional feature is ultimately a stepping stone towards a central result in Section 7, where it leads to the conclusion that the increase in competition raises the average price paid.

[11] Equivalently, $\tilde{Z}_{(n)} = Z_{(n)}/(n-1)$ where $Z_{(n)}$ is the uniform distribution across integers from 0 to $n-1$.

[12] This is illustrated by examining the variance: $\mathrm{var}[\tilde{Z}_{(n)}] = \frac{n^2-1}{12(n-1)^2} = \frac{n+1}{12(n-1)}$, which is decreasing in n.

[13] This is true when buyers search once or twice. The symmetric mixed-strategy prediction does depend on n if there are "shoppers" who consider all n prices by assumption (Janssen and Moraga-González, 2004).

6. Optimal and Equilibrium Search

So far we have calculated stable prices and the reaction of those prices to firm entry, but have done so while fixing the search behavior of potential buyers. We now characterize optimal search and bring that optimality together with price stability to find an equilibrium.

6.1. **The Benefits of Search.**

Here we fix the profile of firms' prices and consider the optimal search response of buyers. Take an arbitrary price profile such that $v \geq p_1 \geq \cdots \geq p_n$. We evaluate the expected marginal benefit $B_n^{(q)}$ of the qth quotation, for $q \in \{1, \ldots, n\}$.

The benefit of the first quotation is straightforward: there is a $1/n$ chance of each price and so

$$B_n^{(1)} = \underline{u} + v - \frac{\sum_{i=1}^{n} p_i}{n}. \tag{14}$$

Now consider the second quotation. With probability $1/n$ the first quotation is p_i from firm i. The gain from the second search is then $p_i - p_j$ only if that second search finds a cheaper firm $j > i$. Each such cheaper firm is found with probability $1/(n-1)$. We find that:

$$B_n^{(2)} = \frac{\sum_{i=1}^{n} \sum_{j>i}(p_i - p_j)}{n(n-1)}. \tag{15}$$

These solutions are reported, in re-arranged form, as the first claim of Lemma 3 below. This logic extends to enable calculation of the expected marginal benefit of the qth search, and this is reported as the second claim of Lemma 3, which provides explicit solutions.

This provision of closed-form solutions may itself be useful to researchers. The literature, which has thus far adopted a single-stage of pricing using mixed strategies. Convenient analytic expressions for these are typically unavailable. Even when they are, the calculations of search benefits become intractable, involving polynomials with multiple distinct power terms.

The benefit of search is also related to the properties of first-order statistics. Let us write $\mathrm{E}[p_{(q)}]$ for the expected first-order statistic (that is, the expected lowest price) when sampling $q \in \{1, \ldots, n\}$ quotations without replacement, and for convenience define $\mathrm{E}[p_{(0)}] \equiv \underline{u} + v$. Using this notation, the expected marginal benefit of the qth search is

$$B_n^{(q)} = \mathrm{E}[p_{(q-1)}] - \mathrm{E}[p_{(q)}]. \tag{16}$$

It is well-known that the expected lowest order statistic when drawing from continuous populations (where the replacement assumption is immaterial, and so we think of this as sampling with replacement) is decreasing but at a slowing rate in the number of draws; abusing terminology slightly, it is convex. It seems (to our knowledge) that the corresponding result for sampling from discrete populations without replacement is not readily available. We provide a discussion and our own derivations in Appendix D for the case of random sampling. Applying our results there (Propositions D1 and D2), which show that this well-known result holds also when sampling is without replacement, we obtain the third claim of Lemma 3

Lemma 3 (Returns to Search). *Consider a price profile $p_1 \geq \cdots \geq p_n$.*

(1) The expected marginal benefits of the first and second quotations are

$$B_n^{(1)} = \underline{u} + v - \frac{\sum_{i=1}^n p_i}{n} \text{ and } B_n^{(2)} = \frac{\sum_{i=1}^n (1-2Z_i)p_i}{n} \text{ where } Z_i = \frac{i-1}{n-1}. \tag{17}$$

(2) More generally, the expected marginal benefit of the qth quotation, for $q \in \{2, \ldots, n\}$, is

$$B_n^{(q)} = \left[q\binom{n}{q}\right]^{-1} \sum_{i=q-1}^{n} \binom{i-1}{q-2} \left[n+1-\frac{iq}{q-1}\right] p_i. \tag{18}$$

(3) The returns to search are decreasing, that is, the marginal benefit of search is decreasing:

$$B_n^{(q)} - B_n^{(q-1)} \leq 0 \qquad \text{for } q \in \{2, \ldots, n\}, \tag{19}$$

where this holds strictly if prices are entirely distinct so that $p_1 > \cdots > p_n$.

We note that the benefit of the qth quotation does not depend on the $q-2$ highest prices, given that the first $q-1$ quotations uncover a price p_{q-1} or below. For the nth quotation, for example, the only possible improvement is if p_n is the last remaining price, which occurs with probability $1/n$. In that case, the final quotation finds that lowest price and so $B_n^{(n)} = (p_{n-1} - p_n)/n$.

6.2. **Basic Equilibrium Properties.** We now examine the different categories of equilibrium search behavior. We recall that an equilibrium combines (i) a stable profile of prices, for given search behavior; and (ii) search behavior that is optimal given that profile of prices.

We can immediately identify a limited number of cases to consider. Buyers must be indifferent between any search sizes used in equilibrium. This rules out the use of more than two adjacent search sizes: the marginal benefit of a quotation is at least weakly decreasing in the number of quotations, and (by assumption) the marginal cost is strictly increasing. We can also show that if two different search sizes are used then they must consist of requesting either one or two quotations. In other cases, all buyers (in equilibrium) must make the same search choice.

Lemma 4 (Properties of Equilibrium Search). *Write $q^\star = \min\{q : m_q > 0\}$ for the lowest number of quotations requested by buyers in equilibrium.*

(1) If $q^\star = 1$ then buyers search once or twice: $m_1 + m_2 = 1$.

(2) If $q^\star = 0$ or $q^\star \in \{2, \ldots, n\}$ then all buyers obtain the same number of quotations: $m_{q^\star} = 1$.

(3) If prices are robustly stable then $q^\star \in \{0, 1, 2\}$ and so no buyer searches more than twice.

Claim (1) follows from the argument above which concluded that search must consist of using (at most) two adjacent search sizes. This argument uses the fact that the marginal cost of search is strictly increasing. If marginal cost is exactly constant then (as we show in Section 9) we can construct unstable equilibria in which two non-adjacent sizes are used.

For claim (2), suppose that $q^\star = 0$. At least some buyers do not search ($m_0 > 0$) but it could be that that $m_1 > 0$ so that other buyers obtain a single quotation. All firms charge $p_i = v$ and so buyers' unique (and common) best reply is either to eschew search altogether (if $\kappa_1 > \underline{u}$) or to obtain a single quotation (if $\kappa_1 < \underline{u}$). Given that $m_0 > 0$, we conclude that the former case applies and so that $m_0 = 1$. This is a "no search" equilibrium reminiscent of Diamond (1971) in which buyers anticipate a monopoly price and so search is not worthwhile.[14]

Continuing with claim (2), suppose instead that $q^\star \in \{2, \dots, n\}$ so that buyers obtain at least two quotations. Stable prices necessarily satisfy $p_i = 0$ for $i \in \{q^\star, \dots, n\}$. A buyer who obtains $q^\star$ quotations is guaranteed to find a zero price, and so there is no need to pay any more for an additional quotation: $m_{q^\star+1} = 0$. Pushing further toward claim (3), if $q^\star \in \{2, \dots, n\}$ and if prices are robustly stable then they satisfy $p_i = 0$ for $i \in \{2, \dots, n\}$ and so a zero-price offer can be obtained simply by requesting two quotations, which implies claim (3) of the lemma.

Summarizing, we note that if $\underline{u} = 0$ (so that a buyer enjoys no surplus when paying a monopoly price) or if (more generally) $\kappa_1 > \underline{u}$ then there is a "no search" equilibrium. Acknowledging this standard case, we now suppose that $\underline{u} > 0$ and that $\kappa_1 < \underline{u}$. This means that there is instead a "minimal search" equilibrium in which all buyers obtain and accept a single quotation ($m_1 = 1$) and all firms charge $p_i = v$. There can be other equilibria with "active search" in the sense that (at least some) buyers obtain more than one quotation. With robustly stable prices, all such equilibria satisfy $m_1 + m_2 = 1$ and so an equilibrium can be usefully summarized by $m_2 \in [0, 1]$, which represents the fraction of buyers who engage in active search.

7. Robustly Stable Equilibria and the Effect of Entry

We now focus on equilibria with robustly stable prices, so that buyers search at most twice. We also focus on situations in which buyers search at least once, by assuming that the cost of the first quotation is less than the consumer surplus obtained when paying a monopoly price.

7.1. Robustly Stable Equilibria.

For equilibria with $m_1 + m_2 = 1$, we think of $m_2 \in [0, 1]$ (the proportion of buyers who request a second quotation) as the *intensity of search* in equilibrium. Search is *minimal* if $m_2 = 0$, but otherwise there is *active search* when $m_2 > 0$. A special case of active search is when $m_2 = 1$ so that search is *complete.*

To characterize an equilibrium we compare the marginal cost and marginal benefit of a second quotation eq. (17) of Lemma 3. Given that $m_1 + m_2 = 1$, the stable prices that form part of the expression for that marginal benefit come from eq. (5). We use the notation $B_n^{(2)}(m_2)$ and $P(Z, m_2)$ to reflect the dependence on the intensity of buyer search.

$$B_n^{(2)}(m_2) = \frac{1}{n}\sum_{i=1}^{n}(1-2Z_i)P(Z_i, m_2) \text{ where } P(Z, m_2) \equiv \frac{v(1-m_2)}{1+(2Z-1)m_2} \text{ and } Z_i \equiv \frac{i-1}{n-1}. \quad (20)$$

[14]Such an equilibrium is supported by a price profile that satisfies $\frac{1}{n}\sum_{i=1}^{n} p_i \geq v + \underline{u} - \kappa_1$.

This is the essence of the incentive to search. The summation places positive weight on above-median prices ($1 - 2Z_i > 0$ if and only if $i < n/2$) and negative weight on below-median prices. We report its properties in the next lemma.

Lemma 5 (The Incentive to Search). *Suppose that buyers obtain one or two quotations, and firms set robustly stable prices. Let $m_1 = 1 - m_2$, so that m_2 is the* intensity of search. *Define the* incentive to search *as the marginal benefit of a second quotation.*

(1) The incentive to search is strictly positive for $m_2 > 0$, is strictly concave in m_2, and satisfies

$$B_n^{(2)}(0) = 0 \quad \textit{and} \quad B_n^{(2)}(1) = \frac{v}{n}. \tag{21}$$

(2) For $m_2 > 0$ the incentive is strictly decreasing in n, and satisfies

$$B_\infty^{(2)}(m_2) \equiv \lim_{n\uparrow\infty} B_n^{(2)}(m_2) = \frac{v}{2}\frac{1-m_2}{m_2}\left(\frac{1}{m_2}\log\left(\frac{1+m_2}{1-m_2}\right) - 2\right). \tag{22}$$

(3) If $n \leq 4$ then the incentive is increasing in m_2. Its maximum v/n is attained at $m_2 = 1$.

(4) If $n \geq 5$ then the incentive is hill-shaped, with a maximum attained at some $m_2 \in (0, 1)$.

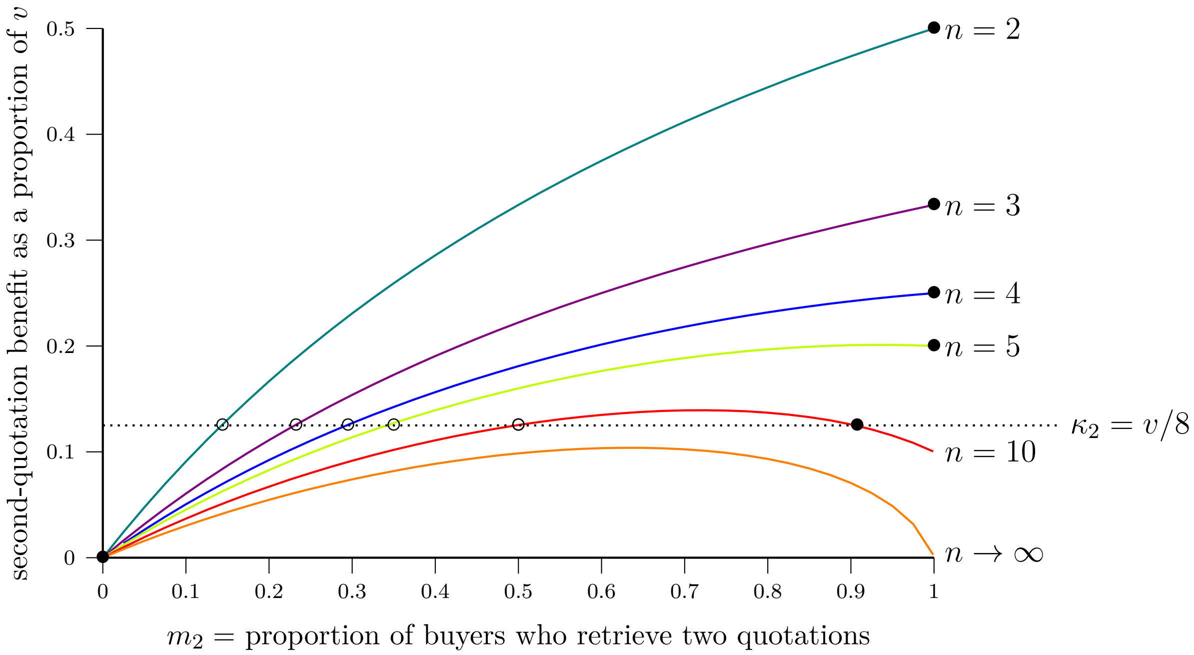
Figure 2 illustrates all of these claims, which we discuss extensively here.

The boundary cases in claim (1) arise naturally. If $m_2 = 0$ then nobody searches actively, firms all charge the monopoly price v, and so there is no benefit to a second quotation. However, the incentive to search remains strictly positive even if $m_2 = 1$ so that search is complete. With complete search, firms' profits are forced down to zero. However, under stable prices, one firm (indexed by $i = 1$ following our labeling of firms) sets the monopoly price. This means that a single-quotation buyer faces a $1/n$ risk of obtaining the monopoly price v rather than a zero-price offer. This means that there is a positive gain to obtaining the second quotation.

Claim (2) reports that an increase in competition depresses the incentive to search. It lies at the heart of our results on firm entry, and readily emerges from an application of Proposition 3. Recall that $\mathrm{E}[p_{(q)}]$ is the expected first-order statistic of prices following the retrieval of q quotations. Including the dependence of this on the intensity of buyer search, we use $\mathrm{E}[p_{(1)}\,|\,m_2]$ to denote the average price paid by a buyer who does obtains a single quotation, which is equal to the average price charged by the n firms. Similarly, $\mathrm{E}[p_{(2)}\,|\,m_2]$ is the average price paid by buyers who search actively by requesting two quotations. Expected industry profit is, of course, the appropriately weighted average of these two terms. From eq. (9), that expected industry profit is also equal to the profit obtained from fully exploiting the fraction $m_1 = 1 - m_2$ of "captive" buyers. From accounting, therefore,

$$\begin{aligned} \text{industry profit} = (1-m_2)v &= (1-m_2)\,\mathrm{E}[p_{(1)}\,|\,m_2] + m_2\,\mathrm{E}[p_{(2)}\,|\,m_2] && (23)\\ &= \mathrm{E}[p_{(1)}\,|\,m_2] - m_2(\mathrm{E}[p_{(1)}\,|\,m_2] - \mathrm{E}[p_{(2)}\,|\,m_2]) && (24)\\ &= \mathrm{E}[p_{(1)}\,|\,m_2] - m_2 B_n^{(2)}(m_2) && (25) \end{aligned}$$

$$\Rightarrow \quad \text{incentive to search} = B_n^{(2)}(m_2) = v - \frac{1}{m_2}\left(v - \frac{\sum_{i=1}^n p_i}{n}\right) \tag{26}$$



Notes. The second search benefit, $B_n^{(2)}(m_2)$ for $n \in \{2, 3, 4, 5, 9, \infty\}$, is shown with solid lines. An example second search cost is also shown, at $\kappa_2 = v/8$.

For this example search-cost value, there is a robustly stable equilibrium with active search for $n \leq 5$ in which $m_2 = 1$. With $n = 10$ there is a robustly stable equilibrium with active search in which a majority gather two quotations and the rest retrieve one. For n sufficiently high, the only robustly stable equilibrium is that with no search. We use "•" to indicate a robustly stable equilibrium, and "∘" for equilibria that are unstable in the sense of Fershtman and Fishman (1992).

FIGURE 2. (i) The Incentive to Search and (ii) Robustly Stable Equilibrium

The number of firms enters only via the expected price charged by firms. This (from Proposition 3) is decreasing in the number of competitors. The (conventional) effect of competition on the average price charged weakens the incentive for a buyer to engage in search beyond the acquisition of the first quotation. Figure 2 illustrates this effect.

Search incentives are minimized in the limit as the number of firms grows arbitrarily large.[15] That limiting case has the property that $B_\infty^{(2)}(1) = 0$, and so the benefit of search vanishes if the intensity of search by others is maximized. This is because the $n - 1$ lowest prices collapse to zero as $m_2 \uparrow 1$, and this means that a buyer almost always finds a price arbitrarily close to zero by obtaining only one quotation.

More generally, the incentive to search is "hill shaped" (as in Burdett and Judd, 1983) in the intensity of search so long as there are five or more competitors, as confirmed by claim (4) of Lemma 5 and illustrated in Figure 2. For lower levels of m_2 the search decisions of buyers are strategic complements, but at higher levels those decisions become strategic substitutes.

However, in more concentrated oligopolies with four or fewer firms, search decisions are strategic complements throughout: the incentive to search is always increasing in the intensity of search

[15] That limiting case reported in eq. (22) corresponds to the search incentive in Burdett and Judd (1983).

by others. This is particularly easy to see in the duopoly case of $n = 2$. For this case,

$$p_1 = v \quad \text{and} \quad p_2 = \frac{v(1-m_2)}{1+m_2} \quad \text{and so} \quad B_2^{(2)}(m_2) = \frac{p_1 - p_2}{2} = \frac{vm_2}{1+m_2}. \tag{27}$$

As search intensity increases, the gap between the high and low prices increases, which enhances the incentive to obtain the second quotation in exchange for the chance of capturing this gap.

To find an equilibrium we now compare the incentive to search against its cost.

It is immediate that there is an equilibrium in which $m_2 = 0$: the incentive to acquire a second quotation is zero, and the cost is $\kappa_2 > 0$. The strict incentive for a buyer to avoid searching actively remains even if we shift a few other buyers toward acquiring a second quotation (a local increase in m_2 away from zero) and this "minimal search" equilibrium is robustly stable, in a sense to be defined just below. Similarly, if $\kappa_2 < v/n$ then there is a "complete search" equilibrium in which $m_2 = 1$. (Search is complete because each buyer successfully retrieves a lowest (zero) price.) This is also robustly stable, in the sense (defined formally below) that an exogenous local decrease in m_2 maintains a strict incentive to acquire a second quotation.

The remaining case is an intermediate value of active but incomplete search, so that $m_2 \in (0, 1)$. The condition required for this is $B_n^{(2)}(m_2) = \kappa_2$. We can find solutions to this that meet our equilibrium definitions, but are not robustly stable in the sense discussed informally just above. To be more precise we adopt a criterion for equilibrium stability that is well-known in the literature and introduced by Fershtman and Fishman (1992, p. 1225).[16]

Definition (Robust Stability). *Consider an equilibrium in which buyers obtain one or two quotations. If a local rise (respectively, fall) in search intensity that is accompanied by revised robustly stable prices results in the strict optimality of requesting one (respectively, two) quotations, then equilibrium search is* stable *in the sense of Fershtman and Fishman (1992).*

An equilibrium is robustly stable *if it specifies robustly stable prices and stable search.*

The "minimal search" ($m_2 = 0$) and "complete search" ($m_2 = 1$) equilibria discussed above (where we recall that the latter equilibrium exists only if the cost of search is sufficiently low) are robustly stable in the sense now defined. For equilibria with active but incomplete search, so that $m_2 \in (0, 1)$, to meet this definition we require $B_n^{(n)}(m_2)$ to be locally decreasing. Inspecting Figure 2 and claim (3) of Lemma 5 this necessarily requires $n \geq 5$.

Our discussion above focuses on situations in which buyers obtain one or two quotations, which holds under the maintained assumption that $\underline{u} > \kappa_1$, so that there is enough consumer surplus for a buyer who pays a monopoly price. If $\underline{u} < \kappa_1$, there are "no search" equilibria with $m_0 = 1$.

Putting these observations together, we see that search in a robustly stable equilibrium is either "minimal" (buyers obtain at most one quotation so that no prices are compared) or "active" (at least some buyers retrieve two quotations). We now summarize this formally.

[16] They credit their notion to a suggestion of Dale Mortensen (see their Footnote 4).

Proposition 4 (Robustly Stable Equilibria). *Define* $\bar{\kappa} \equiv \max_{m_2 \in [0,1]} B_n^{(2)}(m_2)$.

(1) If $\underline{u} > \kappa_1$ *then all buyers obtain at least one quotation. There is a robustly stable "minimal search" equilibrium in which every buyer obtains only a single quotation, and all firms charge the monopoly price. If* $\kappa_2 > \bar{\kappa}$ *then this equilibrium is unique.*

(2) If $\underline{u} < \kappa_1$ *then the robustly stable "minimal search" equilibrium is replaced with robustly stable "no search" equilibria where buyers get no quotations and prices are sufficiently high.*

(3) If $\kappa_2 < \bar{\kappa}$ *then there is a robustly stable "active search" equilibrium, and two sub-cases:*

(a) If $\kappa_2 < v/n \leq \bar{\kappa}$ *then active search is complete. Every buyer obtains two quotations, one firm charges the monopoly price, others charge zero, and industry profits are zero.*

(b) If $v/n < \kappa_2 < \bar{\kappa}$*, which can only hold if* $n \geq 5$*, then active search is incomplete. All firms set distinct stable prices. Industry profit corresponds to the captive-only profit that would be earned from charging the monopoly price to buyers who request a single quotation.*

7.2. **Entry.** For small oligopolies we see that any active search equilibrium involves complete search and so zero profits. To obtain positive profits requires five or more firms, and so entry to a relatively concentrated industry must (at least weakly) raise industry profit.

This effect continues for further entry. The key to the positive effect of entry on firms' profits is Proposition 3, which in turn underpins claim (2) of Lemma 5. Entry (for fixed search behavior) lowers the average price charged (Proposition 3) which reduces the incentive to search (Lemma 5). This suppresses the intensity of search in equilibrium, and so ultimately raises the average price paid. The effect is easy to see visually (via Figure 2) and exit yields the reverse.

Proposition 5 (The Effects of Entry). *Consider a robustly stable equilibrium with active search. Following the entry of a firm to the industry, there remains a robustly stable equilibrium with active search if the second-search cost is not too high. That equilibrium has (at least weakly) higher industry profit, a higher average price paid by buyers, and lower consumer surplus.*

We summarize this result now in more intuitive terms. Consider an oligopoly and buyer search costs low enough so that we have a robustly stable equilibrium with active and incomplete search. Now add one more firm and consider the post-entry prices holding search behavior constant. The average price falls; a natural price-competition effect. However and crucially, this fall causes the return to a buyer from a single search to rise, which implies the return to a second search falls. The upshot is that in a robustly stable equilibrium with active and incomplete search post entry, there is less search. That search depression increases firms' monopoly power, increasing the average price buyers actually pay, which reduces their surplus.

This reasoning is novel in the classic fixed-sample search setting. A crucial point of departure is the (ceteris paribus) effect of entry on prices. That is so important because, as we show, it determines search incentives, and search intensity gives the equilibrium distribution of surplus.

8. Foundations for Stable Prices

Our key novelty is our notion of stable prices. Several non-cooperative scenarios predict exactly these prices. Here we study two natural extensive-form games in which a firm's choice (or the choice of a firm's representative) at each relevant information set can be expressed as a price. We seek subgame perfect equilibria with pure strategies on the equilibrium path.

For both of the games that follow (and others, offered in Appendix C) we use this definition.

Definition (Pure-Strategy Play). *A profile of prices is* supported by the equilibrium play of pure strategies *if and only if there is a subgame-perfect equilibrium in which, along the equilibrium path, a price from that profile is played at each relevant information set.*

Here we describe two games: in a *price formation game* firms choose regular price positions that can then be modified using special-offer deals or sales, and a *collusive pricing game* in which a facilitating industry association sets regular prices but where firms can subsequently offer secret price cuts. In a supplement (Appendix C) we also study a *sniper* game in which one firm has a last-mover advantage and a *Stackelberg* game with sequential pricing. In our supplemental treatments we obtain results that reinforce the main findings obtained here.

8.1. **A Price Formation Game.**

We note again that firms might choose regular price positions, but then have an opportunity to offer discounted special deals to those who seek price quotations.[17] A simple representation of this situation is a two-stage game in which:

(i) firms simultaneously choose and observe their regular price positions; and then
(ii) firms simultaneously choose whether to revise downward their prices, prior to purchases.

Firms' payoffs in this game are profits and they are assumed to be risk neutral.

Prices that are supported by the equilibrium play of pure strategies must be undercut proof (otherwise a firm would deviate at the second stage) and be such that there are no profitable upward price movements given firms have a subsequent opportunity to undercut (otherwise a firm could deviate upward at the first stage). Necessarily these are the stable prices of eq. (2).

To construct an equilibrium we specify a strategy profile in which the prices of eq. (2) are chosen at the first stage. These are maintained (as a unique equilibrium of the subgame, from claim (2) of Lemma 1) on the equilibrium path at the second stage. Turning to the first stage, on the equilibrium path each firm earns a profit of vm_1/n and so attains the upper bound (from claim (1) of Lemma 1) of its expected profit from any second-stage subgame, and so it has no profitable deviation at the first stage. We obtain this proposition.[18]

[17] We considered this situation in depth in other work (Myatt and Ronayne, 2024b).

[18] The arguments here hold under the assumption that $m_1 > 0$. For $m_1 = 0$ (which we covered in detail in Section 4), and as in standard Bertrand competition, firms make zero profit in any equilibrium and those zero profits lead to multiple equilibrium profiles. Although a unique profile is unavailable, it is straightforward to show that all maximal undercut-proof profiles are supported by the equilibrium play of pure strategies.

Proposition 6 (Stable Dispersed Prices from a Price Formation Game). *Consider the price formation game. A profile of prices is supported by the equilibrium play of pure strategies if and only if it is the unique stable profile given by eq.* (2)*.*

For a price-cutting subgame that follows any first-stage upward deviation by a firm, we can construct an equilibrium in which the expected profit of the deviator is exactly equal to the profit that it earns on the equilibrium path. This means that there is no strict incentive for a firm to play its prescribed price at the first stage. In fact, there is another subgame perfect equilibrium with the same expected profits (there are many other equilibria with the same property) in which firms all set the monopoly price ($p_i = v$ for all i) at the first stage, and then play a mixed-strategy Nash equilibrium at the (now on-path) second stage.

A key feature of the equilibrium with the on-path play of pure strategies, however, is that it generates risk-free profits. A first-stage upward deviation adds uncertainty. Risk aversion can be readily incorporated into the model specification and can imply that the on-path pure-strategy play of stable disperse prices involves strict best replies. We illustrate the effect of risk aversion next, but in the context of a situation with collusive pricing.

8.2. **A Collusive Pricing Game.** The prices in eq. (2) might emerge from (constrained) collusive behavior. Suppose, for example, that a facilitating industry association chooses regular prices for all firms but cannot prevent individual members from offering special-offer deals or secret price cuts. To represent this we study this two-stage game:

(i) an industry association sets regular prices for all firms; and then
(ii) firms simultaneously choose whether to offer secret price cuts.

Just as before, firms' payoffs are profits and they are assumed to be risk neutral. However, we enrich our specification by specifying an industry association that seeks higher industry profits but is risk averse, so that its payoff is an increasing and concave function of total profit.

From Lemma 1 the total expected industry profit is bounded above by vm_1. This bound is achieved by setting the unique profile of undercut-proof prices, and is achieved with certainty. Other profiles of regular prices that achieve the same expected profit do so with uncertainty. This readily generates a uniquely optimal choice for the industry association.

Proposition 7 (Stable Dispersed Prices in a Collusive Pricing Game). *Consider the collusive pricing game. The unique subgame perfect equilibrium outcome is for the association to specify the unique stable profile given by eq.* (2)*, and firms do not use secret price cuts.*

We can specify other games where the unique (and strict) subgame perfect equilibrium outcome predicts our stable prices. For example, if regular prices are set non-cooperatively by risk-averse representatives of the firms then we can obtain this same result.

9. Extensions

We study several extensions. We allow (Section 9.1) for downward-sloping demand from each buyer and for firms to offer general "deals" in the sense of Armstrong and Vickers (2001); we add (Section 9.2) exogenous shoppers and we compare our model and results to those of Janssen and Moraga-González (2004); and we evaluate (Section 9.3) the possibility of endogenous foundation for the captive-and-shopper "model of sales" of Varian (1980). In a supplement (Appendix B) we also allow (Appendix B.1) for heterogeneity of buyers' costs of search and we discuss (Appendix B.2) heterogeneous firms that have some exogenously captive customers.

To streamline exposition we describe our findings in the text and omit formal propositions. Nevertheless, everything we say readily translates into such propositions with full proofs.

9.1. **Downward-Sloping Demand.** Our core model specifies "unit demand" so that a buyer is willing to pay up to v for a single unit. This is central to the Diamond (1971) paradox: firms charge v to single-quotation buyers who are then unwilling to pay any search cost. We gave buyers additional surplus $\underline{u}$ from a purchase, which circumvents this problem (and so removes any "no search" equilibrium) if $\underline{u} > \kappa_1 > 0$. This leaves open the source of such surplus.

One possibility is that buyers anticipate a surplus-generating supplemental purchase, which gives them sufficient reason to search. There are many examples of this that are explored in the literature (Ellison, 2005; Johnson, 2017; Rhodes, 2015; Rhodes, Watanabe, and Zhou, 2021). Others (notably Chen and Rosenthal, 1996a,b) have posited the idea that some form of price cap can be used as a commitment device to encourage search. Without the existence of such surplus sources, and as Burdett and Judd (1983, p. 95) noted, the Diamond (1971) problem is "not an essential difficulty" so long as a uniform monopoly price (for example, from a downward-sloping demand curve) generates enough consumer surplus to justify the cost of an initial search. Here we take up this issue explicitly and study the offer of more general "deals" using the approach pioneered by Armstrong and Vickers (2001). Many results are maintained.

Under a unit-demand specification (with marginal cost normalized to zero), a price p generates profit for a firm and surplus for the buyer, and so indexes a set of deals that (for higher p) shift surplus linearly from the buyer to the firm. A similar buyer-firm trade-off holds under downward-sloping demand and in more general settings, including the offer of more complex tariffs. To develop this idea, we specify a downward-sloping demand curve $Q(p)$ for prices that lie between the zero and the monopoly price. The consumer surplus $U(p)$ and profit $V(p)$ are

$$U(p) = \int_p^{\infty} Q(x)\, dx \quad \text{and} \quad V(p) = pQ(p),$$

where $U(p)$ is decreasing and convex in price, and we adopt the regularity condition that $V(p)$ is strictly increasing, continuous, and concave over its range up until its maximum at the monopoly price. This implies that its inverse is well-defined: firm i needs to charge a price $p_i = V^{-1}(v_i)$ if it is to earn a per-transaction profit v_i from each customer that accepts its offer.

This is readily illustrated by a simple example using linear demand. Suppose that

$$Q(p) = Q_{\max}\left(1 - \frac{p}{p_{\max}}\right) \tag{28}$$

for $p \in [0, p_{\max}]$. Here $p_{\max}$ is the choke price above which demand drops to zero, and $Q_{\max}$ is the maximum per-customer demand. Under this standard textbook specification,

$$U(p) = \frac{Q_{\max}(p_{\max} - p)^2}{2p_{\max}} \quad \text{and} \quad V(p) = \frac{Q_{\max}(p_{\max} - p)p}{p_{\max}}$$
$$\text{so that} \quad p_i = V^{-1}(v_i) = \frac{p_{\max}}{2}\left(1 - \sqrt{1 - \frac{v_i}{p_{\max}Q_{\max}/4}}\right), \tag{29}$$

and where here the term $p_{\max}Q_{\max}/4$ is the maximized per-customer monopoly profit.

To proceed further we re-scale the demand curve (without loss of generality) so that the monopoly price is v (by choosing the units of price) and so that the monopoly quantity is $Q(v) = 1$ (by choosing the units of the quantity dimension). These normalizations imply that the maximized monopoly profit is $V(v) = v$ just as it is in our core model. For the linear-demand illustration, we achieve these normalizations by setting $Q_{\max} = 2$ and $p_{\max} = 2v$.

An important and elegant insight from Armstrong and Vickers (2001) is that a price offer p_i from firm i is equivalent to requesting a profit v_i from a transaction with a buyer.[19] This then generates surplus $u_i = \Psi(v_i)$ for the buyer where we define $\Psi(v_i) \equiv U(V^{-1}(v_i))$. For the illustrative linear-demand example specified above, this is straightforwardly

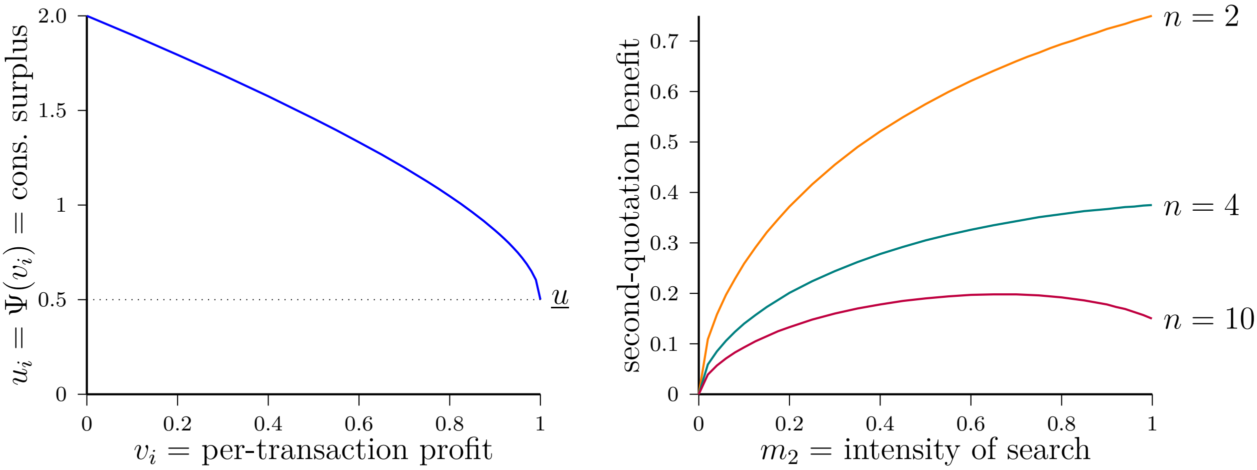
$$\Psi(v_i) = \frac{v}{2}\left(1 + \sqrt{1 - \frac{v_i}{v}}\right)^2 \quad \text{and so} \quad \underline{u} = \text{minimum consumer surplus} = \Psi(v) = \frac{v}{2}. \tag{30}$$

We illustrate this in panel (a) of Figure 3. A movement rightward (and so downward) is a better deal for the firm, but offers worse terms for the buyer. In the illustration there is a concave trade-off for the trading parties. This trade-off is linear under unit demand.

More generally, Armstrong and Vickers (2001) observed that the key primitive is the function $\Psi(\cdot) : [0, v] \mapsto [\underline{u}, \bar{u}]$ which links the profit required for a firm to the consumer surplus generated by the corresponding deal.[20] Faced with a selection of competing offers, a buyer chooses the best deal which is the one that asks for the lowest profit from a transaction. Under relatively standard regularity conditions specified for the downward-sloping demand case considered here, $\Psi(\cdot)$ is a concave function. For example, if the elasticity of $Q(p)$ is increasing in p (as it is under linear demand and many other standard specifications) then this holds. (A constant-elasticity demand specification also satisfies this condition when marginal cost is positive.)

With these elements in hand, we proceed to verify some of our core results. We begin by fixing the search behavior of buyers and (to ease exposition) we set $m_1 > 0$. Our characterization of stable prices holds straightforwardly, where we replace p_i with the profit-per-transaction v_i.

[19] This is developed more, with a comparison of linear and non-linear pricing, by Armstrong and Vickers (2010).
[20] They indexed deals by the surplus offered to buyers, and so specified the equivalent or $\Psi^{-1}(\cdot)$.



(a) Profit and Consumer Surplus from Deals (b) Incentive to Find a Good Deal

Notes. Here we specify linear demand from each customer. Without loss of generality we adopt the normalizations described in the text, and for these figures we set $v = 1$.

Panel (a) relates the per-transaction profit requested from a deal (on the horizontal axis) to the consumer surplus that this delivers to a customer (on the vertical axis).

Panel (b) reports the marginal benefit of a second quotation given that firms set stable prices, expressed as a function of the intensity of search. This is equivalent to Figure 2 but with linearly downward-sloping demand. All of the key features are maintained.

FIGURE 3. Deal Possibilities and Search Incentives with Linear Demand

This means that, modifying eq. (2), the actual price charged by firm i is

$$p_i = V^{-1}\left(\frac{vm_1/n}{X_i}\right) \quad \text{and where per-firm profit satisfies} \quad \pi_i = \frac{vm_1}{n} \quad \text{as before,} \tag{31}$$

where we have transformed the required profit-per-transaction back to the corresponding per-unit price. We can follow exactly the same procedure under the leading case of search (as we explored in Section 5) when each potential buyer obtains either one or two price quotations.

$$p_i = V^{-1}\left(\frac{vm_1}{m_1 + 2m_2 Z_i}\right) \quad \text{where just as before} \quad Z_i \equiv \frac{i-1}{n-1}. \tag{32}$$

Suitably modified (via the inverse of $V(\cdot)$) all of the claims of Propositions 1 and 2 hold. Notably, the highest and lowest prices (or corresponding deals) do not vary with n and (just as before) increased competition fills in the distribution of offers.

The techniques used for Proposition 3 also work as before, and so the expected profit-per-transaction demanded by firms is decreasing in the number of competitors. The expected price charged and the expected consumer surplus are

$$\frac{\sum_{i=1}^n p_i}{n} = \mathrm{E}[V^{-1}(P(\tilde{Z}_{(n)}))] \quad \text{and} \quad \frac{\sum_{i=1}^n u_i}{n} = \mathrm{E}[\Psi(P(\tilde{Z}_{(n)}))] \tag{33}$$

where as before $\tilde{Z}_{(n)}$ is a discrete uniform distribution. Standard properties of compositions mean that $V^{-1}(P(Z))$ and $\Psi(P(Z))$ are convex and concave in Z respectively. This implies that that the expected price charged is decreasing in the number of competitors, and the expected consumer surplus for a buyer who obtains a single quotation rises following entry.

A key feature of stable prices, therefore, is that an increase in competition results in better offers (on average) for those who request only a single quotation. Other things equal, this suppresses the incentive to search. In our unit-demand specification we used accounting identities to show this: given that the overall expected consumer surplus (across all buyers) is constant, it is necessarily true that two-quotation buyers fare worse with more firms. Under downward-sloping demand (or, more flexibly, using Armstrong-Vickers general deals) we cannot reach the same conclusion (for an oligopoly of any size) using the same simple accounting-based trick because the overall expected consumer surplus can change.

Nevertheless, the key force is for competition to force down the incentive to search, and this can be seen immediately when increasing the number of firms from $n = 2$ to $n = 3$. In a duopoly, a buyer who obtains two quotations is certain to find the best offer, and the nature of that offer (as we have noted) does not vary with the number of firms. In a triopoly, however, such a buyer may instead uncover the (less attractive) median offer. This straightforwardly implies that the addition of a competitor pushes toward less search as we move from duopoly to triopoly.

Using our illustrative linear-demand specification we can calculate the expected benefit to obtaining a second quotation: this is panel (b) of Figure 3. The relationship between the intensity of search and the benefit to obtaining a second quotation matches that seen in Figure 2.

Notably, for a duopoly the benefit of a second quotation is monotonically increasing in the search intensity of others, and so (once again) search decisions are strategic complements. This observation is true more generally under downward-sloping demand. This is because the benefit of obtaining the second quotation (in the duopoly case) is determined by the gap between the consumer surplus obtained from finding the best of two offers (this is increasing in the intensity of search by others) and that from finding the monopoly price (this remains constant). For a duopoly, therefore, any stable equilibrium with active search involves complete search so that all potential buyers retrieve two price quotations and ultimately discover the perfect deal.

Another feature from panel (b) of Figure 3 is that the benefit of search is "hill shaped" rather than increasing if the number of firms is sufficiently high. In fact, under general downward sloping demand the benefit $B_n(m_2)$ is decreasing when evaluated at $m_2 = 1$ if and only if there are five or more firms, and so (just as before, in the unit-demand case) search choices become strategic substitutes at high levels of search intensity if and only if there is sufficient entry.

9.2. **Adding Exogenous Shopper Behavior.** Some researchers have studied firm responses to exogenously specified search behavior (Varian, 1980; Baye, Kovenock, and de Vries, 1992), while others have examined the mutual best responses of firms and buyers (Burdett and Judd, 1983). Janssen and Moraga-González (2004) took a hybrid approach, twinning a mass of shoppers with a mass of endogenous searchers who optimally respond to firms' anticipated prices. Here, we show how our approach accommodates such hybrid search behavior. We also compare our results, given the difference in the approach to pricing.

Following Janssen and Moraga-González (2004), we add a mass $\lambda_S > 0$ of shoppers who see every price. On the supply side, Janssen and Moraga-González (2004) studied the symmetric mixed-strategy equilibrium of a single-stage pricing game. They revealed that equilibrium search (by optimizing non-shopper buyers) falls into one of three categories: (i) some do not search while others obtain one quotation ($m_0, m_1 > 0$ and $m_0 + m_1 = 1$); (ii) all obtain a single quotation; and (iii) buyers obtain either one or two quotations ($m_1, m_2 > 0$ and $m_1 + m_2 = 1$). They dubbed these search patterns as "low," "moderate," and "high" search intensity.

We now temporarily match their other modeling assumptions by setting $\underline{u} = 0$ (implying that a buyer that expects to face a monopoly price will not engage in search) and we set the marginal cost of search to be constant so that $\kappa_q \equiv \kappa$ for all $q \in \{1, \ldots, n\}$. We attempt to reproduce the findings of Janssen and Moraga-González (2004) for equilibrium search behavior.

For either low or moderate intensity search, stable prices, from eq. (2), simplify appreciably:

$$p_i = v \text{ for } i < n \quad \text{and} \quad p_n = \frac{vm_1}{m_1 + nm_n} \quad \text{where} \quad m_n = \lambda_S, \tag{34}$$

Recovering the low price is the only way for a buyer to benefit from search. The search for a single lowest price means that the gains from search are linearly increasing in the number of quotations sought: the probability that q quotations return the lowest price is q/n. If buyers strictly prefer to retrieve one quotation rather than none, then necessarily they prefer to maximize search by requesting a quotation from every firm. To construct a low or moderate intensity equilibrium, therefore, a buyer must be indifferent between zero and one search:

$$\frac{v - p_n}{n} = \kappa \quad \Leftrightarrow \quad m_1 = \lambda_S \left(\frac{v}{\kappa} - n \right). \tag{35}$$

We see that an equilibrium with moderate search intensity ($m_1 = 1$) does not exist for generic parameter choices (the knife-edge exception is when $1/\lambda_S = (v/\kappa) - n$). For an equilibrium with low search intensity we need $0 < m_1 < 1$, which rearranges to $n < v/\kappa < n + (1/\lambda_S)$. The set of compliant parameters shrinks as λ_S grows. However, as λ_S vanishes, so does the equilibrium solution for m_1. In essence this says that such an equilibrium is a "no search" equilibrium, as described in case (2) of our Proposition 4.

This leaves the high-intensity case. Stable prices are recovered from eq. (2) with $m_q > 0$ for $q \in \{1, 2, n\}$ and $m_q = 0$ otherwise. Owing to the recursive nature of undercutting constraints, $m_n = \lambda_S$ appears only in the lowest price, p_n. The stable prices of all other firms $i < n$ are as in claim (1) of Proposition 2 or equivalently eq. (12). Stable prices are simple closed-form expressions, just as before, with the nth price modified to become

$$p_n = P(Z_n) - \frac{vm_1 n\lambda_S}{(m_1 + 2m_2)(m_1 + 2m_2 + n\lambda_S)}. \tag{36}$$

In contrast, for the traditional single stage of pricing "an explicit solution [...] does not exist for general values of n" (Janssen and Moraga-González, 2004, p. 1103).

Note that more shoppers increases rivals' temptation to undercut the shopper-capturing p_n, forcing it lower, and this effect on the lowest price is stronger when there are more firms.

Our focal comparative-static result (reported in Proposition 5) is concerned with the effect of entry. The key element for our result is our observation (reported in Proposition 3) that if search behavior is fixed then an increase in the number of competitors lowers the average price charged. In the presence of a mass λ_S of shoppers, the average price charged becomes

$$\frac{\sum_{i=1}^{n} p_i}{n} = \mathrm{E}[P(\tilde{Z}_{(n)})] - \frac{vm_1\lambda_S}{(m_1+2m_2)(m_1+2m_2+n\lambda_S)} \tag{37}$$

where as before $\tilde{Z}_{(n)}$ is a discrete uniform distribution with cardinality n. Our Proposition 3 applies to the first term in eq. (37): it is decreasing in n. The size of the second term (which is the impact of shoppers) is also decreasing in n, but its subtraction from the first term means that its presence could possibly result in a reduction in the average price following an increase in n. Nevertheless, we can be sure that if λ_S is sufficiently small then the impact of that second term can be suppressed sufficiently to guarantee that the conclusion of Proposition 3 holds: the average price charged falls with the entry.

This is the opposite of Janssen and Moraga-González (2004, Proposition 6). There, the symmetric mixed-strategy solution spreads the influence of shoppers over the whole support of prices because any single price point in the support's interior has a chance of being the lowest and winning the shoppers. An increase in n to $n+1$ decreases the incentive to set high prices less than the incentive to set low prices. This is because the mass of a firm's captive customers, who are exploited with high prices, shrinks from m_1/n to $m_1/(n+1)$, whereas the chance of winning shoppers at any given price falls exponentially. As such, as n increases, firms give up on the latter chance, instead opting for higher prices, increasing the average price charged.[21]

Turning to consider the impact on search, we revisit our accounting exercise from Section 7:

$$\text{industry profit} = (1-m_2)v = (1-m_2)\,\mathrm{E}[p_{(1)}] + m_2\,\mathrm{E}[p_{(2)}] + \lambda_S p_n \tag{38}$$

$$= \mathrm{E}[p_{(1)}] - m_2(\mathrm{E}[p_{(1)}] - \mathrm{E}[p_{(2)}]) + \lambda_S p_n \tag{39}$$

$$= \mathrm{E}[p_{(1)}] - m_2 B_n^{(2)}(m_2) + \lambda_S p_n \tag{40}$$

$$\Rightarrow \quad \text{incentive to search} = B_n^{(2)}(m_2) = (\mathrm{E}[p_{(1)}] + \lambda_S p_n - (1-m_2)v)/m_2 \tag{41}$$

If λ_S is sufficiently small then the incentive to gather a second search falls with the number of firms because both the shopper-capturing lowest price and expected price charged (recall that Proposition 3 is maintained, at least if λ_S is sufficiently small) fall with n. In turn, this leads us to maintain also our Proposition 5: an increase in competition is met with lower search intensity, lower consumer welfare, and higher industry profit. This contrasts with the traditional approach where "consumers' search activity attains its minimum under duopoly" (Janssen and Moraga-González, 2004, p. 1108). Furthermore, their Proposition 8 reports the expectation of the equilibrium distribution of prices to be non-monotonic in the number of firms, and for the distribution itself to be the same in a duopoly as with arbitrarily many firms.[22]

[21]Similar forces are present in the classic captive-shopper framework (Varian, 1980). For the symmetric mixed strategy equilibrium, and fixing the total mass of captive customers, the expected price charged increases with entry. However, the average price paid, and so buyer welfare, as well as industry profit, remain constant.

[22]In a duopoly, shopper behavior coincides with optimizing buyers who request two quotations and the latter internalize (free-ride on) shopper behavior opting more often to gather only a single quotation; with many firms,

9.3. **Endogenous Captives and Shoppers.** A popular assumption in models with exogenous buyer behavior is that there are "captives" with access only to a single firm's price and "shoppers" who consider every price (Varian, 1980). Captives are a source of market power and tend to push prices up while shoppers are a source of competition, driving prices down. In our setting, this behavior is captured by $m_1, m_n > 0$ and $m_q = 0$ for each $q \in \{2, \ldots, n-1\}$. Can such behavior arise as an equilibrium phenomenon? We address that question here.

Duopoly is special because shoppers gather two quotations; this coincides with our main analysis. Now suppose $n \geq 3$. For our core model there is no equilibrium with $m_1, m_n > 0$ because there are weakly decreasing returns to search (Lemma 3) yet a strictly increasing marginal search cost. We now specify a constant marginal search cost: $\kappa_q \equiv \kappa > 0$ for all $q \in \{1, \ldots, n\}$ and seek "captive-shopper" equilibrium behavior. We note that the mixed-strategy solution of the traditional approach implies that returns to search are strictly increasing and so cannot generate this behavior. We also set $\underline{u} > 0$ for simplicity. (It kills an uninteresting multiplicity.)

If some buyers gather one quotation and others all, then stable prices are those of eq. (34).

Given those prices, buyers make a first search because $\underline{u} > 0$, so $m_0 = 0$. Because the marginal search cost is constant over q, a captive-shopper equilibrium protocol requires the expected marginal benefit of each subsequent search, up to and including the nth, to be equal. Given the prices of eq. (34), each additional quotation gathered has an expected benefit of $(v - p_n)/n$.[23] Setting that equal to κ gives eq. (35) but with endogenous shopper behavior:

$$\frac{m_1}{m_n} = \frac{v}{\kappa} - n. \tag{42}$$

Equilibrium enforces a captive-shopper ratio that is increasing in v, and decreasing in κ and n. An increase in v or a decrease in n raises the benefit to each search, all else equal. To restore the indifference, m_1/m_n rises, pushing p_n back up, and the benefit to each search back down.

For this solution for m_1/m_n to be positive, we require $\kappa < v/n$. As such, for a smaller v/n, there is a smaller set of search cost functions that permit such an equilibrium. We conclude that if $\kappa \in (0, v/n)$, then we have an equilibrium with $m_1, m_n > 0$ and $m_q = 0$ for $q \in \{2, \ldots, n-1\}$.

Although it is remarkable to find that captive-shopper buyer behavior can arise endogenously, the conditions required are strong. Firstly, any such equilibrium is not robustly stable. If we extend the notion of stability from Fershtman and Fishman (1992) to the division of buyers between captives and shoppers, then a local shift upward (downward) in m_n pushes p_n down (up) and so makes requesting all prices (one price) strictly optimal, and the equilibrium unravels.[24] Secondly, it requires $\kappa < v/n$, which is harder to satisfy for larger n, and impossible as n grows large. Thirdly, introducing even a slight convexity to the search cost schedule precludes captive-shopper behavior in equilibrium, as in our main analysis.

---

the symmetric mixed strategy solution takes the probability of any one firm winning the shoppers to zero and so the force they exert on equilibrium prices disappears.

[23] We can readily show that only profiles with $n-1$ monopoly prices equate marginal benefits across all $q > 1$.

[24] The only robustly stable equilibria when buyers choose between $q = 1$ and $q = n$ involve either minimal search (all buyers are captive) or complete search (all buyers are shoppers).

Appendix A. Omitted Proofs

Proof of Lemma 1. We note that the existence of a Nash equilibrium in each relevant price-cutting games is guaranteed by Theorem 5 of Dasgupta and Maskin (1986).

Write $p^\dagger$ for the upper bound to the joint support of firms' pricing strategies.

Suppose that firm i places an atom at $p^\dagger$. If $m_2 > 0$ then i is unique, because otherwise it would undercut an atom-playing competitor. If there is some $q \in \{2, \ldots, n-1\}$ such that $0 = m_2 = \cdots = m_q < m_{q+1}$ then the same argument says that at most q firms play an atom at $p^\dagger$. In either case, firm i sells only when uniquely considered, and so earns $p^\dagger m_1/n \leq p_1 m_1/n$.

Suppose instead that no firm places an atom at $p^\dagger$. Find a firm i with $p^\dagger$ as the upper bound to its support. This firm is willing to price arbitrarily close to $p^\dagger$ which (by continuity, given the lack of atoms at $p^\dagger$) gives the same profit as charging $p^\dagger$. Doing so, such a firm loses all price comparisons and so again achieves the captive-only profit $p^\dagger m_1/n \leq p_1 m_1/n$.

We have shown that the profit of a "highest price" firm i satisfies claim (1). Suppose that another firm $j \neq i$ earns a strictly higher expected profit than firm i. Take the lower bound $\underline{p}_j$ of the support for firm j. Firm i can undercut that price and perform at least as well as firm j. (It would perform strictly better than j if $\underline{p}_i < \underline{p}_j$ and otherwise steals the profit of j.) This contradicts the assumption that firms were playing an equilibrium.

We turn now to claim (2). We write π_i for the profit of firm i. This claim also appears as claim (iii) of Lemma 1 of Myatt and Ronayne (2024b) but we repeat the proof for completeness.

By charging $\tilde{p}_1 = p_1$ the firm with the highest regular price earns π_1 no matter what the choices of other firms. For any other price strictly below p_1, its profit is highest if all others maintain their initial prices. The profile of initial prices is undercut-proof, and so this firm earns strictly less than π_1 by strictly undercutting any other firm. If it matches another firm, then (given that ties are broken in an interior way) it also earns strictly less. We conclude that all prices strictly below p_1 are strictly dominated. We conclude (as an induction basis) that $\tilde{p}_1 = p_1$.

Consider $i > 1$, where $\tilde{p}_j = p_j$ for all $j < i$. Firm i can guarantee a profit of π_i by charging its regular price. Continuing to recycle the argument above, even if others maintain their regular prices (so maximizing the profit of firm i) then firm i earns strictly less by undercutting others than by setting $\tilde{p}_i = p_i$. By the principle of induction, this holds for all firms. □

Proof of Proposition 1. This follows from arguments in the text using Lemma 1. □

Proof of Proposition 2. Because $m_q = 0$ for $q > 2$, eq. (1) simplifies to

$$X_i = \frac{m_1}{n} + \frac{2m_2(i-1)}{n(n-1)} = \frac{m_1 + 2m_2 Z_i}{n}, \tag{A1}$$

which we can substitute into eq. (2) to find the prices stated in eq. (5) of claim (1).

Now, for claim (2), consider $n \to \infty$. Take any price p within the interval bounded by the highest and lowest prices, p_1 and p_n, and write $F_n(p)$ for the cdf of prices. For finite n,

$$F_n(p) = \frac{n-i}{n-1} \quad \Leftrightarrow \quad p_{i-1} > p \geq p_i \quad \Leftrightarrow \quad \frac{m_1 v}{m_1 + 2m_2 \frac{(i-2)}{n-1}} > p \geq \frac{m_1 v}{m_1 + 2m_2 \frac{(i-1)}{n-1}}$$
$$\Leftrightarrow \quad \frac{i-2}{n-1} < \frac{m_1}{m_2}\frac{v-p}{2p} \leq \frac{i-1}{n-1} \quad \Leftrightarrow \quad i = \left\lceil (n-1)\frac{m_1}{m_2}\frac{v-p}{2p} \right\rceil + 1, \tag{A2}$$

where "$\lceil\cdot\rceil$" means "the least integer weakly greater than." Hence

$$F_n(p) = 1 - \frac{1}{n-1}\left\lceil (n-1)\frac{m_1}{m_2}\frac{v-p}{2p} \right\rceil, \tag{A3}$$

which converges to $F(p)$ as reported, as $n \to \infty$. Claims (3) and (4) hold by inspection. □

Proof of Lemma 2. An equivalent statement is that $\tilde{Z}_{(n)}$ is riskier than $\tilde{Z}_{(n+1)}$. This means that $\tilde{Z}_{(n)}$ can be obtained by adding a sequence of mean-preserving spreads to $\tilde{Z}_{(n+1)}$.

We will do this by adding $n-1$ mean preserving spreads which are applied to the $n-1$ points of support of $\tilde{Z}_{(n+1)}$ that exclude the endpoints of the unit interval.

Fix the distribution $\tilde{Z}_{(n+1)}$. For each $i \in \{1, \ldots, n-1\}$ consider a mean preserving spread that (i) eliminates the realization i/n, which equivalently lowers its probability by $1/(n+1)$; (ii) increases the probability of $(i-1)/(n-1)$ by $i/(n(n+1))$; and (iii) increases the probability of $i/(n-1)$ by $(n-i)/(n(n+1))$. Conditional on the realisation $\tilde{Z}_{(n+1)} = i/n$ this is equivalent to adding the garbling $\tilde{\Delta}_i$ (à la Blackwell) where

$$\tilde{\Delta}_i = \begin{cases} -\frac{n-i}{n(n-1)} & \text{with probability } \frac{i}{n} \\ \frac{i}{n(n-1)} & \text{with probability } \frac{n-i}{n} \end{cases} \tag{A4}$$

It is straightforward to see that $\mathrm{E}[\tilde{\Delta}_i] = 0$. Furthermore,

$$\frac{i}{n} - \frac{n-i}{n(n-1)} = \frac{i-1}{n-1} \quad \text{and} \quad \frac{i}{n} + \frac{i}{n(n-1)} = \frac{i}{n-1}. \tag{A5}$$

This confirms that $\tilde{\Delta}_i$ is a zero-mean shock that shifts the point i/n in the support of $\tilde{Z}_{(n+1)}$ to the neighboring points $(i-1)/(n-1)$ and $i/(n-1)$ that lie with the support of $\tilde{Z}_{(n)}$.

It is straightforward to confirm that these $n-1$ mean-preserving spreads generate a uniform distribution with cardinality n on the unit interval. To see this, abuse notation by writing $\tilde{Z}_{(n)}$ for the distribution obtained by adding the $n-1$ mean-preserving spreads to $\tilde{Z}_{(n+1)}$. We have

$$\Pr\left[\tilde{Z}_{(n)} = \frac{i}{n-1}\right] = \Pr\left[\tilde{Z}_{(n+1)} = \frac{i}{n}\right]\Pr[\Delta_i > 0] + \Pr\left[\tilde{Z}_{(n+1)} = \frac{i+1}{n}\right]\Pr[\Delta_{i+1} < 0] \tag{A6}$$
$$= \frac{1}{n+1}\frac{n-i}{n} + \frac{1}{n+1}\frac{i+1}{n} = \frac{1}{n}, \tag{A7}$$

where this applies for the $n-2$ points on the interior of the support of $\tilde{Z}_{(n)}$. Similar calculations confirm that the two endpoints also have probability $1/n$. □

Proof of Proposition 3. We can write $v_n = \mathrm{E}[P(\tilde{Z}_{(n)})]$ where $\tilde{Z}_{(n)}$ is the discrete uniform distribution on the unit interval with cardinality n, and where (as earlier) $P(z)$ is defined as

$$P(z) \equiv \frac{v}{1+\frac{2m_2 z}{m_1}} \Rightarrow P'(z) = -\frac{2m_2}{m_1}\frac{v}{\left(1+\frac{2m_2 z}{m_1}\right)^2} \Rightarrow P''(z) = 2\left(\frac{2m_2}{m_1}\right)^2 \frac{v}{\left(1+\frac{2m_2 z}{m_1}\right)^3} > 0. \tag{A8}$$

$P(z)$ is convex in z. We know, from Lemma 2, that $\tilde{Z}_{(n)}$ is riskier (in the second-order sense) than $\tilde{Z}_{(n+1)}$, and so $\tilde{Z}_{(n+1)} \succ_{\mathrm{SOSD}} \tilde{Z}_{(n)}$. This implies that $\mathrm{E}[P(\tilde{Z}_{(n)})] > \mathrm{E}[P(\tilde{Z}_{(n+1)})]$. □

Proof of Lemma 3. The benefit of a first and second search are derived in the main text, and support claim (1). Here we consider, for claim (2), the benefit of the qth search.

The worst realization of the first $q-1$ draws would be the highest $q-1$ prices (so that p_{q-1} is the best price in hand). There is only one combination of such prices, but they can be drawn in any of $(q-1)!$ possible orders, each of which occurs with the probability $\frac{1}{n}\frac{1}{n-1}\cdots\frac{n}{n-(q-2)}$. When a remaining lower price, $\bar{p}_i$ such that $i \in \{q,\ldots,n\}$, is drawn, which happens with probability $\frac{1}{n-(q-1)}$, the benefit is $p_{q-1} - p_i$. This is reflected in the first line of (A9) below.

The remaining cases such that the lowest price in hand is p_i with $i \in \{q,\ldots,n-1\}$ continue similarly (in the case of $i = n$, there is zero benefit to drawing the qth quotation). We obtain:

$$\begin{aligned} B_n^{(q)} =& \frac{\binom{q-2}{q-2}(q-1)!}{n(n-1)\cdots(n-(q-2))}\left[\frac{1}{n-(q-1)}((p_{q-1}-p_q)+\cdots+(p_{q-1}-p_n))\right]+\cdots+ \\ & \frac{\binom{n-2}{q-2}(q-1)!}{n(n-1)\cdots(n-(q-2))}\left[\frac{1}{n-(q-1)}(p_{n-1}-p_n)\right]. \end{aligned} \tag{A9}$$

Reorganizing terms and collecting coefficients for each price p_i,

$$B_n^{(q)} = \left[q\binom{n}{q}\right]^{-1}\sum_{i=q-1}^{n}\left(\binom{i-1}{q-2}(n-i) - \sum_{j=q-2}^{i-2}\binom{j}{q-2}\right)p_i, \tag{A10}$$

where we let $\sum_{j=q-2}^{q-3}\binom{j}{q-2} \equiv 0$. Employing some identities of the binomial coefficient,

$$B_n^{(q)} = \left[q\binom{n}{q}\right]^{-1}\sum_{i=q-1}^{n}\left(\binom{i-1}{q-2}(n-i) - \binom{i-1}{q-1}\right)p_i, \tag{A11}$$

$$B_n^{(q)} = \left[q\binom{n}{q}\right]^{-1}\sum_{i=q-1}^{n}\binom{i-1}{q-2}\left(n+1-\frac{iq}{q-1}\right)p_i, \tag{A12}$$

which is the expression given in Lemma 3.

Claim (3) follows from Propositions D1 and D2 in Appendix D, with the exception that Proposition D2 assumes $n \geq 3$ so that it omits the term associated with $B_n^{(1)}$ (because that is function of $\mathrm{E}[p_{(0)}]$, which in general is not defined). However, in our model we know what happens when there are no draws: buyers receive zero utility, which is incorporated by defining $\mathrm{E}[p_{(0)}] \equiv v + \underline{u}$. We can then extend Proposition D2 to include the case corresponding to $B_n^{(2)} - B_n^{(1)}$ and thus allowing for $n = 2$ by noticing that because (D17) holds as stated, then it also holds when for Δ_0 when we set $\mathrm{E}[X_{(1)}] \equiv v$ because in any price profile we consider, $v \geq x_n$. □

Proof of Lemma 4. The claims follow from the arguments given in the main text. □

Proof of Lemma 5. For claim (1), we note the properties of $P(Z, m_2)$ from eq. (20). By inspection is it is decreasing in m_2 (increased search pushes prices downward) and also

$$\frac{\partial^2 P(Z, m_2)}{\partial m_2^2} = \frac{4Z(2Z-1)v}{(1+(2Z-1)m_2)^3} < 0 \quad \Leftrightarrow \quad Z < \tfrac{1}{2}. \tag{A13}$$

This implies that the price of firm i is concave in m_2 for the firms indexed $i < n/2$ with above-median prices, and convex in m_2 for the firms indexed $i > n/2$ with below-median prices. The summation in eq. (20) places positive weight $(1 - 2Z_i > 0)$ on the prices of the former firms, and negative weight $(1 - 2Z_i < 0)$ on the latter firms. This implies that each non-zero term in the summation (the term is zero for a median firm satisfying $Z_i = \frac{1}{2}$) is strictly concave in m_2, and so $B_n^{(2)}(m_2)$ is the sum of strictly concave functions and is itself strictly concave.

As $m_2 \downarrow 0$, all prices go to v and there is no benefit to a second search. As $m_2 \uparrow 1$, $p_1 = v$ while $p_i \downarrow 0$ for $i > 1$. A second search is only beneficial if the first retrieves the high price (so that a second improves the buyer's surplus by v), which happens with probability $1/n$.

For claim (2), the argument in the text shows that the incentive is strictly decreasing in n. To derive the incentive in the limit as $n \to \infty$, $B_\infty^{(2)}$, we use the cdf of prices, F, reported in Proposition 2 to compute the incentive as

$$\mathrm{E}_F[p] - \mathrm{E}_F[p_{(2)}] = \int_{\underline{p}}^{v} f(p) p dp - \int_{\underline{p}}^{v} f_{(2)}(p) p dp, \tag{A14}$$

where $\underline{p} = vm_1/(m_1 + 2m_2)$ is the lowest price; f is the pdf associated with cdf F; and $f_{(2)}$ is the pdf of $1 - (1 - F(p))^2$ (the distribution of the lowest order statistic from two draws). Computing this gives the expression stated in eq. (22).

For claims (3) and (4), we know that $B_n^{(2)}(m_2) \downarrow 0$ as $m_2 \downarrow 0$ and that $B_n^{(2)}(m_2)$ is strictly positive and so is strictly increasing as m_2 increases from zero. Looking across the interval $m_2 \in [0, 1]$, and given strict concavity, the incentive must either be strictly increasing or hill-shaped. The latter holds if $B_n^{(2)}((m_2)$ slopes downward as $m_2 \uparrow 1$. Checking this:

$$\lim_{m_2 \uparrow 1} \frac{dB_n^{(2)}(m_2)}{dm_2} = \frac{v}{2}\frac{n-1}{n}(2 - H_{n-1}) < 0 \quad \Leftrightarrow \quad H_{n-1} > 2 \tag{A15}$$

where $H_{n-1} = \sum_{i=1}^{n-1} i^{-1}$ is the $(n-1)^{\text{th}}$ harmonic number. Evaluating for $n = 3$ and 4 gives $H_3 = 11/6$ and $H_4 = 25/12$, such that $n = 5$ is the first natural number to satisfy $H_{n-1} > 2$ and hence (A15). (The function H_k is strictly increasing in its argument k.) □

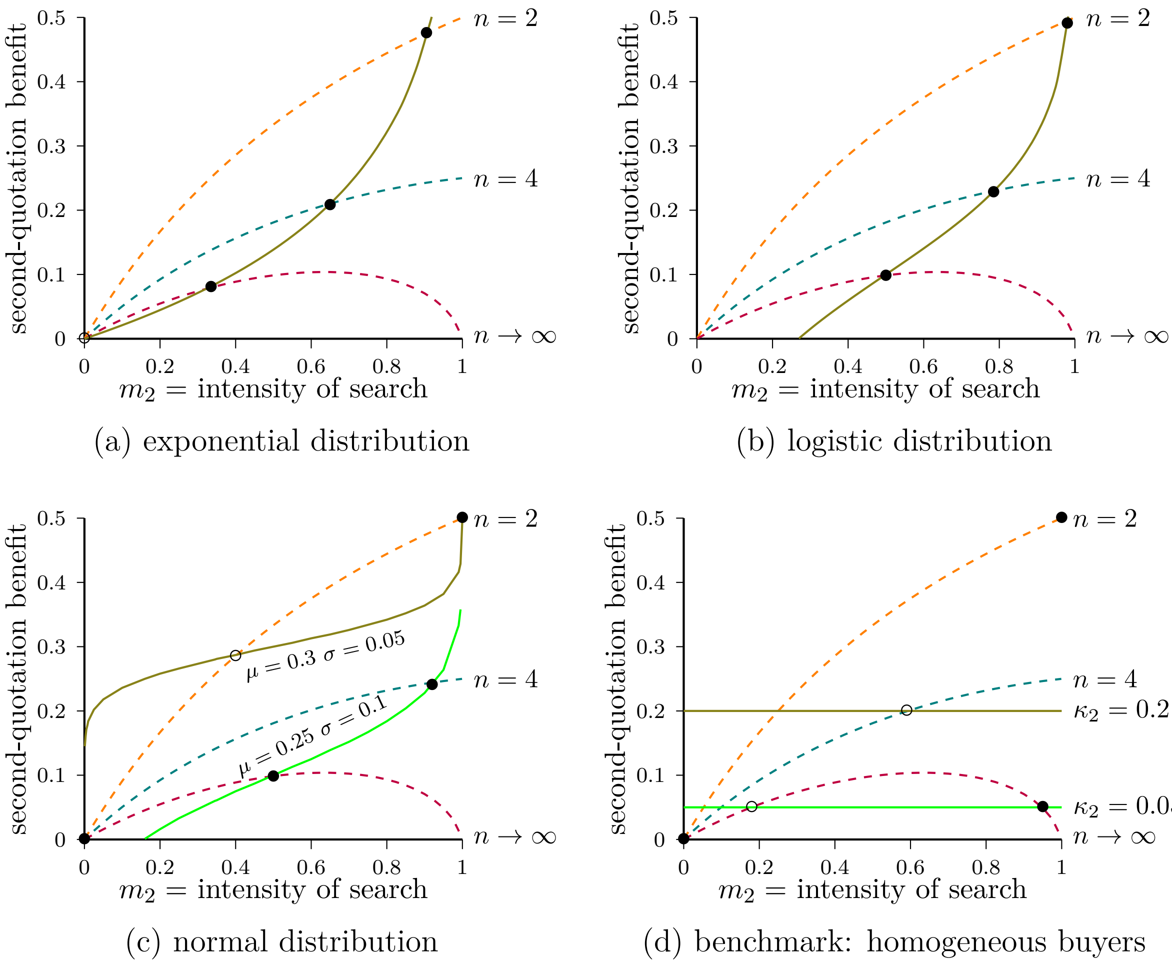
Proof of Propositions 4 and 5. These follow from arguments in the text: claims (1) and (2) of Proposition 4 are discussed directly in the text, and claim (3) uses the properties of the second-quotation incentive documented in Lemma 5. Proposition 5 is also straightforward from discussion in the text, and is also obtained by an inspection of Figure 2. □

Proof of Propositions 6 and 7. These follow from arguments in the text using Lemma 1. □

Appendix B. Further Extensions

B.1. **Heterogeneous Buyers.** The robustly stable equilibrium central to our analysis involves buyers retrieving one or two quotations and so the crucial cost parameter is the cost of that second quotation. To streamline our analysis, we proceed by supposing that $\underline{u}$ exceeds the cost of the first quotation for every buyer, and that three or more quotations are prohibitively costly. Each buyer, therefore, simply needs to decide whether or not to obtain a second price quotation in order to be able to engage in a pairwise price comparison.

The cost κ_2 of a second quotation now differs across buyers. We specify a strictly and continuously increasing distribution function $\kappa_2 \sim K(\cdot)$ for it, with support that includes $[0, v/2]$. If a proportion m_2 of buyers obtain a second quotation, then the marginal searching buyer has a cost $K^{-1}(m_2)$. An equilibrium with incomplete search $m_2 \in (0,1)$ must satisfy $B_n^{(2)}(m_2) = K^{-1}(m_2)$. This is robustly stable if the (inverse) distribution cuts the benefit schedule from below.



Notes. The search benefit, $B_n^{(2)}(m_2)$, for $n \in \{2, 4, \infty\}$, is shown with dotted lines. The inverse of three distribution specifications is shown in panels (a) to (c), with a homogeneous-buyer benchmark in panel (d). The exponential and logistic specifications have unique robustly stable equilibria with active search. The logistic and normal specifications allow for negative search costs, which eliminate minimal search equilibria.

Figure B1. Equilibria with Heterogeneous Search Costs

In Figure B1 we illustrate several specifications for the distribution of buyers' search costs. For panels (a) and (b) of the figure, which correspond to exponential and logistic specifications, there is a unique stable equilibrium for each value of n. For an exponential distribution there is always a unique stable equilibrium. This is because the density of the exponential is decreasing, which implies that its cumulative distribution function is concave. The concavity of the distribution translates to convexity of its inverse $K^{-1}(m_2)$. So long as $K^{-1}(m_2) < B_n^{(2)}(m_2)$ for all m_2 sufficiently close to zero, we are assured (because of the concavity of $B_n^{(2)}(m_2)$) that the marginal benefit and cost schedules cross only once. Similarly, the logistic specification illustrated in panel (b) has a support that extends below zero (so that some buyers enjoy searching) with the property that $K^{-1}(m_2)$ is convex over the region where it is positive.

This discussion suggests a set of conditions that are sufficient for the existing of a unique robustly stable equilibrium with incomplete active search. Suppose that the support of search costs extends strictly below zero (so that at least some buyers face a dominant incentive to request two quotations) but above $v/2$ (so that some buyers will choose not to search). Further suppose that the density of $K(\cdot)$ is decreasing when evaluated at positive search costs. Under these conditions (and as illustrated) a unique robustly stable equilibrium is obtained.

Focusing on such a unique equilibrium we can readily engage in comparative-static analysis. An increase in the number of competitors strictly lowers the benefit schedule, and so strictly depresses equilibrium search. An appropriate variant of Proposition 5 holds.

B.2. **Heterogeneous Firms.** Our pricing mechanism produces a unique profile of stable prices, which are entirely dispersed in a robustly stable equilibrium with active and incomplete search. However, firms are symmetric and so there is a multiplicity to our pricing solution because any of the n firms could take any of the n distinct price positions: fixing buyers' search behavior, the profile of stable prices is unique only if we choose firms' labels to rank their prices.

We now describe one way in which that multiplicity can be resolved. We fix the search behavior of buyers, and for now let us set $m_1 > 0$ and $m_2 > 0$ so that stable prices are distinct and (subject to firms' labels) uniquely defined. Now let us change the model specification by allocating exogenously $\lambda_i > 0$ additional captive buyers to each firm i, where $\lambda_i \neq \lambda_j$ for $i \neq j$.

In our related work (Myatt and Ronayne, 2024b) this situation falls into what we call "exchangeable" consideration of firms, which means that a buyer who sees $q > 1$ price quotations is equally likely to consider any q of the firms. However, the mass of buyers that are captive to each firm are potentially different; equivalently, a buyer who retrieves $q = 1$ quotation is steered towards firms with higher λ_i. We find (Myatt and Ronayne, 2024b, Proposition 3) a unique profile of stable prices that uniquely maps each firm to a price, which has the property that firms charge prices in ascending order of their captive audiences.[25]

[25] In that work we use a weaker version of price stability. We replace criterion (2) of our "stable prices" definition with the requirement that a firm cannot gain from a local price rise (a "creep" upward in price) given that firms then enjoy a subsequent opportunity to cut their prices via the offer of special deals.

Appendix C. Other Non-Cooperative Pricing Games

Extending our work in Section 8, we describe two further games for which there is pure-strategy equilibrium play of our stable prices. To simplify exposition here we set $m_1 > 0$ and $m_2 > 0$.

C.1. **A Sniper Game.**

Deneckere, Kovenock, and Lee (1992) studied a captive-shopper duopoly and varied the move order of the two firms. Inspired by their work, here we study a *sniper game* in which one firm has a "last mover" opportunity to revise its price in response to others:[26]

(i) firms $i \in \{2, \ldots, n\}$ each simultaneously set their prices; and then
(ii) the "sniper" firm 1 sets its own price.

The avoid complexities concerning the existence of best replies we assume that (when relevant) ties are broken in favor of the sniper, so that the sniper can undercut a price by matching it.

Proposition C1 (Stable Dispersed Prices in a Sniper Game). *Consider the sniper game. The unique profile of stable prices given by eq.* (2) *is supported by the equilibrium play of pure strategies. The sniper plays $p_1 = v$ while other firms take the other pricing positions.*

Proof. We consider a strategy profile in which, on the equilibrium path, firms $i \in \{2, \ldots, n\}$ set p_i according to eq. (2), and the sniper sets $p_1 = v$ at the second stage. Those prices are undercut-proof, and so the sniper has no better reply at the on-path second stage.

Consider a first-stage deviation by $k \neq 1$. If this is a downward deviation then $p_1 = v$ remains a best reply for the sniper, and the undercut-proof prices means that there is no gain for k. An upward first-stage deviation prompts an undercut: the optimal "snipe" is either to match (undercut) the deviant price or that of a firm that was "jumped over" by the deviation.

If k prices higher but still less than $k-1$ so that $\tilde{p}_k \in (v(X_1/X_k), v(X_1/X_{k-1}))$, then the sniper undercuts firm k: $p_1 = p_k$. Firm k wins comparisons only against the $k-2$ firms $i \in \{2, \ldots, k-1\}$ and so makes sales of X_{k-1} and earns profit $\tilde{p}_k X_{k-1} < p_{k-1} X_{k-1} = p_k X_k$. If k matches p_{k-1} then it faces positive probability of losing comparisons against firm $k-1$ and so continues to earn strictly less than $p_{k-1} X_{k-1} = p_k X_k$.

If $k \in \{3, \ldots, n\}$ and k prices higher than $l \in \{2, \ldots, k-1\}$ so that $\tilde{p}_k\, (v(X_1/X_l), v(X_1/X_{l-1}))$, then the sniper undercuts k or one of those firms that k jumped over. Firm k now wins comparisons only against the $l-2$ firms $i \in \{2, \ldots, l-1\}$ and so earns profit $\tilde{p}_k X_{l-1} < p_{l-1} X_{l-1} = p_k X_k$. We can also readily deal with the case where $\tilde{p}_k = p_{l-1}$ just as before.

Firm k does not have a profitable deviation upward. For subgames following multi-player deviations off path, any equilibrium (any best reply of the sniper) can be played. □

[26] While the existence of a last-mover is crucial, the fact that the $n-1$ other firms move simultaneously is not. In Appendix C we show that any entirely distinct prices supported by the equilibrium play of pure strategies are also the (efficient, and undercut-proof) prices of eq. (2). We also specify such an equilibrium for a triopoly.

C.2. **A Stackelberg Game.** We can also consider a fully sequential-move *Stackelberg game* in which firms choose their prices according to a pre-determined sequence. Here we show how any distinct prices supported by the play of such a game correspond to our stable prices.

There are n firms and n periods. In each period $t \in \{1, \dots, n\}$, one firm i sets its price $p_i \in [0, v]$. We label firms in reverse order of play so that firm n moves first and firm 1 last. All firms observe a price once it is set. As usual, customers each purchase from the cheapest firm in their consideration set. Any ties are broken in favor of the lowest-indexed (latest-moving) of those tied firms.[27] For a concise exposition, we assume $m_1, m_2 > 0$.

Proposition C2 (Stable Prices in a Stackelberg Game). *Suppose $m_1, m_2 > 0$. In any subgame perfect equilibrium in which firms set entirely distinct prices on path, those prices are the stable prices of eq.* (2)*. Profits in any such equilibrium are vX_1 for each firm.*

Proof. Consider a subgame perfect equilibrium in which firms set distinct on-path prices.

The last mover observes the (distinct) prices of its predecessors. Its best reply is either v, or to match one of its predecessors (if not, it could do strictly better by slightly raising its price). The latter would violate a prediction of entirely distinct prices. And so, this price is v and all other firms price strictly below v in any such equilibrium.

The last mover must earn its captive-only profit, vX_1. If another firm made more than this, then the last mover would be able to match the price of that other firm, contradicting its choice of v at the final stage. And so all firms make vX_1 in equilibrium. The only way for firms to each make exactly vX_1 and have entirely distinct prices is for their prices to exactly satisfy the no-undercutting constraints of $vX_1 \geq p_i X_i$ for each i. □

Proposition C2 shows that any dispersion emerging from the sequential-move game coincides with our stable prices. Our next result, Proposition C3, shows that in the case of triopoly, those prices (set in an ascending order) are supported by the equilibrium play of pure strategies.

Proposition C3 (A Stackleberg Equilibrium: Triopoly). *Suppose $n = 3$ and $m_1, m_2 > 0$. There is a subgame perfect equilibrium of the Stackelberg game in which profits are vX_1 for each firm and on-path prices are $(p_1^\star, p_2^\star, p_3^\star) = (v, vX_1/X_2, vX_1/X_3)$. The strategies are:*

At $t = 1$, firm 3 sets $p_3 = vX_1/X_3$.

At $t = 2$, firm 2 plays the following strategy:

$$p_2 = \begin{cases} vX_1/X_2 & \text{if } p_3 \in [0, vX_1/X_3] \\ vX_1/X_3 & \text{if } p_3 \in (vX_1/X_3, vX_1/X_2] \\ p_3 X_2/X_3 & \text{if } p_3 \in (vX_1/X_2, v]. \end{cases} \tag{C1}$$

[27] This avoids uninteresting existence issues and does not bite in the on-path predictions we derive and report.

At $t = 3$, firm 1 plays the following strategy:

$$p_1 = \begin{cases} v & \textit{if } \max\{p_2, p_3\} \in \left[0, v\frac{X_1}{X_3}\right] \\ v & \textit{if } \min\{p_2, p_3\} \in \left[0, v\frac{X_1}{X_3}\right], \max\{p_2, p_3\} \in \left(v\frac{X_1}{X_3}, v\frac{X_1}{X_2}\right] \\ \max\{p_2, p_3\} & \textit{if } \min\{p_2, p_3\} \in \left[0, v\frac{X_1}{X_3}\right], \max\{p_2, p_3\} \in \left(v\frac{X_1}{X_2}, v\right] \\ \min\{p_2, p_3\} & \textit{if } \min\{p_2, p_3\}, \max\{p_2, p_3\} \in \left(v\frac{X_1}{X_3}, v\frac{X_1}{X_2}\right] \\ \max\{p_2, p_3\} & \textit{if } \min\{p_2, p_3\} \in \left(v\frac{X_1}{X_3}, v\frac{X_1}{X_2}\right], \max\{p_2, p_3\} \in \left(v\frac{X_1}{X_2}, v\right], \frac{\min\{p_2,p_3\}}{\max\{p_2,p_3\}} \le \frac{X_2}{X_3} \\ \min\{p_2, p_3\} & \textit{if } \min\{p_2, p_3\} \in \left(v\frac{X_1}{X_3}, v\frac{X_1}{X_2}\right], \max\{p_2, p_3\} \in \left(v\frac{X_1}{X_2}, v\right], \frac{\min\{p_2,p_3\}}{\max\{p_2,p_3\}} > \frac{X_2}{X_3} \\ \max\{p_2, p_3\} & \textit{if } \min\{p_2, p_3\}, \max\{p_2, p_3\} \in \left(v\frac{X_1}{X_2}, v\right], \frac{\min\{p_2,p_3\}}{\max\{p_2,p_3\}} \le \frac{X_2}{X_3} \\ \min\{p_2, p_3\} & \textit{if } \min\{p_2, p_3\}, \max\{p_2, p_3\} \in \left(v\frac{X_1}{X_2}, v\right], \frac{\min\{p_2,p_3\}}{\max\{p_2,p_3\}} > \frac{X_2}{X_3}. \end{cases} \tag{C2}$$

Proof of Proposition C3. Firm 1 moves last. Its (complete) best response correspondence is:

If $\max\{p_2, p_3\} \in [0, v\frac{X_1}{X_3}]$, $\text{BR}_1(p_2, p_3) = v$.

If $\max\{p_2, p_3\} = v\frac{X_1}{X_3}$ and:

- $\min\{p_2, p_3\} \in [0, v\frac{X_1}{X_3})$, $\text{BR}_1(p_2, p_3) = v$.
- $\min\{p_2, p_3\} = v\frac{X_1}{X_3}$, $\text{BR}_1(p_2, p_3) = \{v\frac{X_1}{X_3}, v\}$.

If $\max\{p_2, p_3\} \in (v\frac{X_1}{X_3}, v\frac{X_1}{X_2})$ and:

- $\min\{p_2, p_3\} \in [0, v\frac{X_1}{X_3})$, $\text{BR}_1(p_2, p_3) = v$,
- $\min\{p_2, p_3\} = v\frac{X_1}{X_3}$, $\text{BR}_1(p_2, p_3) = \{v\frac{X_1}{X_3}, v\}$,
- $\min\{p_2, p_3\} \in (v\frac{X_1}{X_3}, v\frac{X_1}{X_2})$, $\text{BR}_1(p_2, p_3) = \min\{p_2, p_3\}$.

If $\max\{p_2, p_3\} = v\frac{X_1}{X_2}$ and:

- $\min\{p_2, p_3\} \in [0, v\frac{X_1}{X_3})$, $\text{BR}_1(p_2, p_3) = \{v\frac{X_1}{X_2}, v\}$,
- $\min\{p_2, p_3\} = v\frac{X_1}{X_3}$, $\text{BR}_1(p_2, p_3) = \{v\frac{X_1}{X_3}, v\frac{X_1}{X_2}, v\}$,
- $\min\{p_2, p_3\} \in (v\frac{X_1}{X_3}, v\frac{X_1}{X_2})$, $\text{BR}_1(p_2, p_3) = \min\{p_2, p_3\}$,
- $\min\{p_2, p_3\} = v\frac{X_1}{X_2}$, $\text{BR}_1(p_2, p_3) = v\frac{X_1}{X_2}$,

If $\max\{p_2, p_3\} \in (v\frac{X_1}{X_2}, v]$ and:

- $\min\{p_2, p_3\} \in [0, v\frac{X_1}{X_3}]$, $\text{BR}_1(p_2, p_3) = \max\{p_2, p_3\}$,
- $\min\{p_2, p_3\} \in (v\frac{X_1}{X_3}, v]$ and $\frac{\min\{p_2,p_3\}}{\max\{p_2,p_3\}} \le \frac{X_2}{X_3}$, $\text{BR}_1(p_2, p_3) = \max\{p_2, p_3\}$,
- $\min\{p_2, p_3\} \in (v\frac{X_1}{X_3}, v]$ and $\frac{\min\{p_2,p_3\}}{\max\{p_2,p_3\}} \ge \frac{X_2}{X_3}$, $\text{BR}_1(p_2, p_3) = \min\{p_2, p_3\}$.

The condition special to the last set of cases above arises because firm 1 wishes to match (effectively undercut, given the tie-break rule) one of the two prices p_2, p_3 because they are both relatively high such that matching either gives firm 1 more than its captive-only profit vX_1. It prefers to match $\min\{p_2, p_3\}$ rather than $\max\{p_2, p_3\}$ if and only if the associated profit is greater: $\min\{p_2, p_3\}X_2 \ge \max\{p_2, p_3\}X_3 \Leftrightarrow \frac{\min\{p_2,p_3\}}{\max\{p_2,p_3\}} \ge \frac{X_2}{X_3}$.

At $t=2$, firm 2 moves. A best response correspondence for it is below. Where there are multiple best responses of firm 1 for given strategies of 2 and 3, we select the one that supports the equilibrium of the proposition (and those selections are reported in parentheses):

If $p_3 \in [0, v\frac{X_1}{X_3})$, $\mathrm{BR}_2(\mathrm{BR}_1(p_2,p_3),p_3) = \{v\frac{X_1}{X_2}, v\}$. (For $p_2 = v\frac{X_1}{X_2}$ let $\mathrm{BR}_1(p_2,p_3) = v$.)

If $p_3 = v\frac{X_1}{X_3}$, $\mathrm{BR}_2(\mathrm{BR}_1(p_2,p_3),p_3) = \{v\frac{X_1}{X_3}, v\frac{X_1}{X_2}, v\}$. (For $p_2 \in [v\frac{X_1}{X_3}, v]$ let $\mathrm{BR}_1(p_2,p_3) = v$.)

If $p_3 \in (v\frac{X_1}{X_3}, v\frac{X_1}{X_2})$, $\mathrm{BR}_2(\mathrm{BR}_1(p_2,p_3),p_3) = \{v\frac{X_1}{X_3}, v\}$. (For $p_2 = v\frac{X_1}{X_3}$ let $\mathrm{BR}_1(p_2,p_3) = v$.)

If $p_3 = v\frac{X_1}{X_2}$, $\mathrm{BR}_2(\mathrm{BR}_1(p_2,p_3),p_3) = \{v\frac{X_1}{X_3}, v\frac{X_1}{X_2}, v\}$. (For $p_2 = v\frac{X_1}{X_3}$ let $\mathrm{BR}_1(p_2,p_3) = \{v\frac{X_1}{X_2}, v\}$.)

If $p_3 \in (v\frac{X_1}{X_2}, v]$, $\mathrm{BR}_2(\mathrm{BR}_1(p_2,p_3),p_3) = \{\frac{p_3X_2}{X_3}, p_3\}$. (For $p_2 = \frac{p_3X_2}{X_3}$ let $\mathrm{BR}_1(p_2,p_3) = \max\{p_2,p_3\}$.)

At $t=1$, firm 3 moves. We consider its profits for a choice of p_3 given firms 1 and 2 will best respond as described above. Let this be π_3. Where there are multiple best replies (for concision call these BR_1 and BR_2), we select those that support the equilibrium we seek.

$$\pi_3 = \begin{cases} p_3X_3 & \text{if } p_3 \in \left[0, v\frac{X_1}{X_3}\right) \\ p_3X_3 & \text{if } p_3 = v\frac{X_1}{X_3} \qquad (\text{Letting } \mathrm{BR}_1 = v\frac{X_1}{X_2}; \mathrm{BR}_2 = v) \\ p_3X_2 & \text{if } p_3 \in \left(v\frac{X_1}{X_3}, v\frac{X_1}{X_2}\right) \quad (\text{Letting } \mathrm{BR}_2 = v\frac{X_1}{X_3}) \\ p_3X_2 & \text{if } p_3 = v\frac{X_1}{X_2} \qquad (\text{Letting } \mathrm{BR}_1 = v; \mathrm{BR}_2 = v\frac{X_1}{X_3}) \\ p_3X_1 & \text{if } p_3 \in \left(v\frac{X_1}{X_2}, v\right]. \end{cases} \tag{C3}$$

By inspection, firm 3 maximizes π_3 with any $p_3 \in \{v\frac{X_1}{X_3}, v\frac{X_1}{X_2}, v\}$. Let $p_3 = v\frac{X_1}{X_3}$. By the earlier derivations, we see a best response of firm 2 to that (given 1 will in turn give a best response) is $v\frac{X_1}{X_2}$. A best response of firm 1 to those choices of 2 and 3 is v. This gives the on-path prices. Resulting profits are vX_1 per firm. Selecting elements appropriately from the best response correspondences for firms 1 and 2 gives the best response functions of the proposition. □

Appendix D. Properties of the Expected Lowest Order Statistic

(Expected) order statistics are relevant to many calculations, especially those of an extreme value from a finite sample. The distribution from which draws are made is usually assumed to be continuous, as in auction theory (Krishna, 2013, Appendix C) or search theory (Janssen and Moraga-González, 2004). Watt (2022) derived conditions for concavity and convexity of the expected order statistics with respect to sample size. Here we study sampling without replacement from a distribution with finite support.[28] For concision we assume that each item is sampled with equal probability from the (remaining) population, however, we expect the results to readily extend to any distribution. In doing so, we confirm that these well-known results from the literature (with continuously distributed draws) also hold in this context.

[28] There is some work on order statistics and sampling from finite populations without replacement in the statistics literature. Nagaraja (1992, Section 7) and Wilks (1962, Section 8.8) covered aspects of this. However, the expected values of order statistics and their properties are not investigated.

D.1. **Setting.** There is a finite population of values $x_1 \leq x_2 \leq \cdots \leq x_n$, where (at least) one inequality strict so that $x_1 < x_n$. The population is sampled from randomly so that each remaining value has an equal probability of being drawn. Suppose $q \in \{1 \ldots, n\}$ draws are made. Let the random variable of the lowest value in the resulting sample be $X_{(1,q)}$. Denote its expected value by $\mathrm{E}[X_{(1,q)}]$, its cdf by F_q, and its pmf by f_q.

D.2. **Example.** Suppose $n = 3$. If there is $q = 1$ draw, then each of x_1, x_2, x_3 are drawn with equal probability and the expected lowest draw is the expected value, $(x_1 + x_2 + x_3)/3$.

With $q = 2$, the samples are either $\{x_1, x_2\}$, $\{x_1, x_3\}$, or $\{x_2, x_3\}$, each with probability 1/3. In the first two cases, x_1 is the lowest value, in the last, x_2 is. And so $\mathrm{E}[X_{(1,2)}] = (2x_1 + x_2)/3$.

With $q = 3$ draws, all values are drawn and the sample is certainly $\{x_1, x_2, x_3\}$ so $X_{(1,3)} = x_1$.

Because $x_1 < x_3$, $\mathrm{E}[X_{(1,1)}] > \mathrm{E}[X_{(1,2)}]$; and as $x_1 \leq x_2$, $\mathrm{E}[X_{(1,2)}] \geq \mathrm{E}[X_{(1,3)}]$. It is convex if

$$\mathrm{E}[X_{(1,1)}] - \mathrm{E}[X_{(1,2)}] \geq \mathrm{E}[X_{(1,2)}] - \mathrm{E}[X_{(1,3)}] \Leftrightarrow x_2 \geq x_3. \tag{D1}$$

D.3. **Distribution of $\mathbf{X_{(1,q)}}$.** Each sample of $q \in \{1, \ldots, n\}$ values is equally likely because the values are drawn with equal probability. The probability of any sample of size q is thus

$$\frac{1}{n}\frac{1}{n-1}\cdots\frac{1}{n-(q-1)}q! = \frac{1}{\binom{n}{q}}. \tag{D2}$$

To calculate probability with which a value is the lowest, we only need to multiply this by the number of distinct samples that can be drawn in which it is the smallest. For x_i such that $i \in \{1, \ldots, n-(q-1)\}$ to be the lowest value, all other $q-1$ values in the sample must be from the pool of $n-i$ larger values, and so there are $\binom{n-i}{q-1}$ samples in which x_i is lowest, giving

$$f_q(x_i) = \begin{cases} \frac{\binom{n-i}{q-1}}{\binom{n}{q}} & i \in \{1, \ldots, n-q+1\} \\ 0 & \text{otherwise.} \end{cases} \tag{D3}$$

And so the cdf (for now supposing entirely distinct population values) is, for $i \in \{1, \ldots, n-q\}$

$$F_q(x_i) = \sum_{j=1}^{i} \frac{\binom{n-j}{q-1}}{\binom{n}{q}} = \frac{1}{\binom{n}{q}}\left[\sum_{j=0}^{n}\binom{j}{q-1} - \sum_{j=i+1}^{n}\binom{n-j}{q-1} - \binom{n}{q-1}\right], \tag{D4}$$

$$= \frac{1}{\binom{n}{q}}\left[\binom{n+1}{q} - \binom{n-i}{q} - \binom{n}{q-1}\right] \tag{D5}$$

$$= \frac{1}{\binom{n}{q}}\left[\binom{n}{q-1}\left[\frac{n+1}{q} - 1\right] - \binom{n-i}{q}\right] = \frac{1}{\binom{n}{q}}\left[\binom{n}{q} - \binom{n-i}{q}\right] = 1 - \frac{\binom{n-i}{q}}{\binom{n}{q}}, \tag{D6}$$

and so in summary, $F_n(x_i) = 1$ for all $i \in \{1, \ldots, n\}$, while for $q \in \{1, \ldots, n-1\}$ we have

$$F_q(x_i) = \begin{cases} 1 - \frac{\binom{n-i}{q}}{\binom{n}{q}} & i \in \{1, \ldots, n-q\} \\ 1 & i \in \{n-q+1, \ldots, n\}. \end{cases} \tag{D7}$$

If there are $T \geq 1$ ties in the population's values at some y such that $x_t = x_{t+1} = x_{t+T} = y$, we can simply sum the probability masses corresponding to all the tied terms to give $f_q(y)$. We can continue to use the same cdf but only referring to the value y once, defining (D7) only for $i \in \{1, \ldots, n\} \setminus \{t, \ldots, t+T-1\}$, repeating the exclusion process for each value with a tie.[29]

Proposition D1 (Decreasing). *If the lowest population value is distinct, i.e., $x_1 < x_2$, then*

$$\mathrm{E}[X_{(1,q)}] > \mathrm{E}[X_{(1,q+1)}] \qquad q \in \{1, \ldots, n-1\}. \tag{D8}$$

If there are $T \in \{2, \ldots, n-1\}$ lowest population values, such that $x_1 = \cdots = x_T < x_{T+1}$, then

$$\mathrm{E}[X_{(1,q)}] > \mathrm{E}[X_{(1,q+1)}] \qquad q \in \{1, \ldots, n-T\}, \tag{D9}$$

$$\mathrm{E}[X_{(1,q)}] = \mathrm{E}[X_{(1,q+1)}] = x_1 \qquad q \in \{n-T+1, \ldots, n-1\}. \tag{D10}$$

Proof of Proposition D1. Let $T \in \{1, \ldots, n-1\}$ items have value x_1, so $x_1 = \cdots = x_T < x_{T+1}$.

A sample of $q \in \{n-T+1, \ldots, n\}$ draws is guaranteed to contain the lowest value, x_1, so further sampling does not change the expectation of $X_{(1,q)}$.

Now consider a sample of size $q \in \{1, \ldots, n-T\}$. We show that the cdf of $X_{(1,q)}$ is first-order stochastic dominance ordered, by showing $F_q(x_i) < F_{q+1}(x_i)$ for each $i \in \{T, \ldots, n-q\}$ (recall that $F_q(x_i) = F_{q+1}(x_i) = 1$ for $i \in \{n-q+1, n\}$). For $i = n-q$, it can seen from (D7) that $F_q(x_{n-q}) < 1 = F_{q+1}(x_{n-q})$. For $i \in \{T, \ldots, n-q-1\}$

$$F_q(x_i) < F_{q+1}(x_i) \Leftrightarrow \binom{n-i}{q+1}\binom{n}{q} < \binom{n-i}{q}\binom{n}{q+1} \Leftrightarrow i > 0, \tag{D11}$$

where the last step is seen by substituting the binomial terms with their factorial definitions. This also covers the case in which there are any ties in (non-lowest) population values, for which only the highest-indexed terms at each shared value need to be checked as per (D11). □

D.4. **Convexity of $\mathbf{X_{(1,q)}}$.** Using the pmf from (D3) we now write $\mathrm{E}[X_{(1,q)}]$ explicitly,

$$\mathrm{E}[X_{(1,q)}] = \frac{x_1\binom{n-1}{q-1} + x_2\binom{n-2}{q-1} + \cdots + x_{n-(q-1)}\binom{q-1}{q-1} + 0 + \cdots + 0}{\binom{n}{q}} = \frac{\sum_{j=1}^{n-q+1} x_j \binom{n-j}{q-1}}{\binom{n}{q}}. \tag{D12}$$

Assume $n \geq 3$. The discrete notion of convexity we use is $\Delta_q > 0$, where, for $q \in \{1, \ldots, n-2\}$

$$\Delta_q \equiv \Big[\mathrm{E}[X_{(1,q)}] - \mathrm{E}[X_{(1,q+1)}]\Big] - \Big[\mathrm{E}[X_{(1,q+1)}] - \mathrm{E}[X_{(1,q+2)}]\Big], \tag{D13}$$

$$= \sum_{j=1}^{n-q} x_j \left[\frac{\binom{n-j}{q-1}}{\binom{n}{q}} - \frac{\binom{n-j}{q}}{\binom{n}{q+1}}\right] + x_{n-q+1}\frac{1}{\binom{n}{q}} - \sum_{j=1}^{n-q-1} x_j \left[\frac{\binom{n-j}{q}}{\binom{n}{q+1}} - \frac{\binom{n-j}{q+1}}{\binom{n}{q+2}}\right] - x_{n-q}\frac{1}{\binom{n}{q+1}}, \tag{D14}$$

$$= \sum_{j=1}^{n-q-1} x_j \left[\frac{\binom{n-j}{q-1}}{\binom{n}{q}} - 2\frac{\binom{n-j}{q}}{\binom{n}{q+1}} + \frac{\binom{n-j}{q+1}}{\binom{n}{q+2}}\right] + x_{n-q}\left[\frac{\binom{q}{q-1}}{\binom{n}{q}} - \frac{2}{\binom{n}{q+1}}\right] + x_{n-q+1}\frac{1}{\binom{n}{q}}. \tag{D15}$$

This is a weighted sum of $x_1, \ldots, x_{n-q+1}$. To prove convexity we will use the following lemma.

[29]The pmf and cdf here are special cases of those in Nagaraja (1992, Equation 7.1), which are for any order statistic (not only the lowest). That work assumes that population values are completely ordered and distinct.

Lemma D1. *Take two vectors with $m \geq 2$ elements, $\mathbf{x} \in \mathbb{R}_+^m$ and $\mathbf{a} \in \mathbb{R}^m$, where elements are such that $0 \leq x_1 \leq \cdots \leq x_m$ and $a_1 \leq \cdots \leq a_m$ with $a_1 < a_m$.*

If (i) $\sum_{i=1}^m a_i = 0$; and (ii) $\exists\, i^\star \in \{1, \ldots, m-1\}$ such that $a_i < 0$ if $i \leq i^\star$ and $a_i > 0$ if $i > i^\star$, then $\sum_{i=1}^m a_i x_i \geq 0$. If, in addition, the elements in $\mathbf{x}$ are entirely distinct, then $\sum_{i=1}^m a_i x_i > 0$.

Proof of Lemma D1. We proceed by noting a lower bound of the weighted sum of interest

$$\sum_{i=1}^{m} a_i x_i \geq \sum_{i=1}^{i^\star} a_i x_{i^\star} + \sum_{i=i^\star+1}^{m} a_i x_{i^\star+1} = x_{i^\star} \sum_{i=1}^{i^\star} a_i + x_{i^\star+1} \sum_{i=i^\star+1}^{m} a_i = (x_{i^\star+1} - x_{i^\star}) \sum_{i=i^\star+1}^{m} a_i \geq 0, \quad \text{(D16)}$$

where the last inequality is strict if $x_{i^\star} < x_{i^\star+1}$, which it is if all elements of $\mathbf{x}$ are distinct. $\square$

We now apply Lemma D1 to prove Proposition D2.

Proposition D2 (Convexity). *For $n \geq 3$ non-negative population values, $0 \leq x_1 \leq \cdots \leq x_n$,*

$$\Delta_q \equiv \Big[\mathrm{E}[X_{(1,q)}] - \mathrm{E}[X_{(1,q+1)}]\Big] - \Big[\mathrm{E}[X_{(1,q+1)}] - \mathrm{E}[X_{(1,q+2)}]\Big] \geq 0, \quad \text{(D17)}$$

for $q \in \{1, \ldots, n-2\}$: the expected lowest order statistic is weakly convex in the sample size.

If also the population values are entirely distinct, so that $0 \leq x_1 < \cdots < x_n$, then $\Delta_q > 0$ for $q \in \{1, \ldots, n-2\}$: the expected lowest order statistic is strictly convex in the sample size.

Proof of Proposition D2. From equation eq. (D15) we can see that Δ_q is a weighted sum of the terms x_i with $i \in \{1, \ldots, n-q+1\}$. We will show that the weights, a_i, satisfy requirements (i) and (ii) of Lemma D1, and then apply the lemma directly.

Let a_i denote the coefficient on x_i in (D15) for each $i \in \{1, \ldots, n-q+1\}$, so that

$$\sum_{j=1}^{n-q+1} a_i \equiv \sum_{j=1}^{n-q-1} \left[\frac{\binom{n-j}{q-1}}{\binom{n}{q}} - 2\frac{\binom{n-j}{q}}{\binom{n}{q+1}} + \frac{\binom{n-j}{q+1}}{\binom{n}{q+2}} \right] + \left[\frac{\binom{q}{q-1}}{\binom{n}{q}} - \frac{2}{\binom{n}{q+1}} \right] + \frac{1}{\binom{n}{q}}, \quad \text{(D18)}$$

$$= \left[\frac{\binom{n+1}{q} - \binom{n}{q-1} - \binom{q+1}{q}}{\binom{n}{q}} - 2\frac{\binom{n+1}{q+1} - \binom{n}{q} - \binom{q+1}{q+1}}{\binom{n}{q+1}} + \frac{\binom{n+1}{q+2} - \binom{n}{q+1}}{\binom{n}{n+2}} \right] \quad \text{(D19)}$$

$$+ \left[\frac{\binom{q}{q-1}}{\binom{n}{q}} - \frac{2}{\binom{n}{q+1}} \right] + \frac{1}{\binom{n}{q}}, \quad \text{(D20)}$$

$$= \left[1 - \frac{q+1}{\binom{n}{q}} - 2\left[1 - \frac{1}{\binom{n}{q+1}}\right] + 1 \right] + \left[\frac{\binom{q}{q-1}}{\binom{n}{q}} - \frac{2}{\binom{n}{q+1}} \right] + \frac{1}{\binom{n}{q}}, \quad \text{(D21)}$$

$$= \frac{1}{\binom{n}{q}} \left[\frac{(q+1)(2-n+q)}{n-q} + q - \frac{2(q+1)}{n-q} + 1 \right] = 0, \quad \text{(D22)}$$

which is property (i) of Lemma D1. Before showing that property (ii) holds, we simplify

$$a_i = \left[\frac{\binom{n-i}{q-1}}{\binom{n}{q}} - 2\frac{\binom{n-i}{q}}{\binom{n}{q+1}} + \frac{\binom{n-i}{q+1}}{\binom{n}{q+2}} \right] = \frac{q(n-q)}{n-i-q+1} - \frac{2(q+1)}{n-i} + \frac{(q+2)(n-i-q)}{(n-q)(n-q-1)}, \quad \text{(D23)}$$

for $i \in \{1, \ldots, n-q-1\}$. The RHS of (D23) also calculates a_{n-q} correctly. One can confirm $a_1 = 0$, $a_2 = -2/(n(n-1)) < 0$ and, recalling (D17), $a_{n-q+1} = 1/\binom{n}{q} > 0$. Because a_1 does not affect a sum of a_i or $a_i x_i$ terms, we ignore it and proceed with a_i for $i \in \{2, \ldots, n-q+1\}$. We now show $\exists i^\star \in \{2, \ldots, n-q\}$ such that $a_i < 0$ if $i \in \{2, \ldots, i^\star\}$ and $a_i > 0$ if $i \in \{i^\star+1, \ldots, n-q+1\}$.

Firstly, suppose $q = n-2$. We know $a_2 < 0$ and $a_{n-q+1} = a_3 > 0$ and so $i^\star = 2$, as required.

Now suppose $q < n-2$ (which implies $n > 3$). For $i \in \{2, \ldots, n-q\}$, we use (D23) and show

$$a_i = 0 \Leftrightarrow i = 2\frac{n+1}{q+2} \in (2, n-q+1), \tag{D24}$$

which implies there is the desired threshold term, $i^\star \in \{2, \ldots, n-q\}$.

We can now apply Lemma D1: if the population values are non-negative, then $\Delta_q \geq 0$; and if, in addition, the population values are entirely distinct, then $\Delta_q > 0$. $\square$

References

Anderson, S. P., and A. De Palma (2005): "Price Dispersion and Consumer Reservation Prices," *Journal of Economics and Management Strategy*, 14(1), 61–91.

Anderson, S. P., A. De Palma, and J.-F. Thisse (1992): *Discrete Choice Theory of Product Differentiation*. The MIT Press, Cambridge.

Anderson, S. P., and R. Renault (1999): "Pricing, Product Diversity, and Search Costs: A Bertrand-Chamberlin-Diamond Model," *RAND Journal of Economics*, 30(4), 719–735.

——— (2018): "Firm Pricing with Consumer Search," in *Handbook of Game Theory and Industrial Organization, Volume II*, ed. by L. C. Corchón, and M. A. Marini, pp. 177–224. Edward Elgar Publishing.

Armstrong, M., and J. S. Vickers (2001): "Competitive Price Discrimination," *RAND Journal of Economics*, 32(4), 579–605.

——— (2010): "Competitive Non-Linear Pricing and Bundling," *Review of Economic Studies*, 77(1), 30–60.

——— (2022): "Patterns of Competitive Interaction," *Econometrica*, 90(1), 153–191.

Armstrong, M., J. S. Vickers, and J. Zhou (2009a): "Consumer Protection and the Incentive to Become Informed," *Journal of the European Economic Association*, 7(2-3), 399–410.

——— (2009b): "Prominence and Consumer Search," *RAND Journal of Economics*, 40(2), 209–233.

Baye, M. R., D. Kovenock, and C. G. de Vries (1992): "It Takes Two to Tango: Equilibria in a Model of Sales," *Games and Economic Behavior*, 4(4), 493–510.

Burdett, K., and K. L. Judd (1983): "Equilibrium Price Dispersion," *Econometrica*, 51(4), 955–969.

Chen, Y., and C. He (2011): "Paid Placement: Advertising and Search on the Internet," *Economic Journal*, 121(556), 309–328.

Chen, Y., Z. Li, and T. Zhang (2022): "Experience Goods and Consumer Search," *American Economic Journal: Microeconomics*, 14(3), 591–621.

CHEN, Y., AND M. H. RIORDAN (2008): "Price-Increasing Competition," *RAND Journal of Economics*, 39(4), 1042–1058.

CHEN, Y., AND R. W. ROSENTHAL (1996a): "Asking Prices as Commitment Devices," *International Economic Review*, 37(1), 129–155.

——— (1996b): "On the Use of Ceiling-Price Commitments by Monopolists," *RAND Journal of Economics*, 27(2), 207–220.

CHEN, Y., AND T. ZHANG (2011): "Equilibrium Price Dispersion with Heterogeneous Searchers," *International Journal of Industrial Organization*, 29(6), 645–654.

——— (2018a): "Entry and Welfare in Search Markets," *Economic Journal*, 128(608), 55–80.

——— (2018b): "Intermediaries and Consumer Search," *International Journal of Industrial Organization*, 57, 255–277.

DASGUPTA, P., AND E. MASKIN (1986): "The Existence of Equilibrium in Discontinuous Economic Games, I: Theory," *The Review of Economic Studies*, 53(1), 1–26.

DE CORNIÈRE, A. (2016): "Search Advertising," *American Economic Journal: Microeconomics*, 8(3), 156–188.

DE CORNIÈRE, A., AND G. TAYLOR (2019): "A model of biased intermediation," *RAND Journal of Economics*, 50(4), 854–882.

DE LOS SANTOS, B., A. HORTAÇSU, AND M. R. WILDENBEEST (2012): "Testing Models of Consumer Search Using Data on Web Browsing and Purchasing Behavior," *American Economic Review*, 102(6), 2955–80.

DENECKERE, R., D. KOVENOCK, AND R. LEE (1992): "A Model of Price Leadership Based on Consumer Loyalty," *Journal of Industrial Economics*, 40(2), 147–156.

DIAMOND, P. A. (1971): "A Model of Price Adjustment," *Journal of Economic Theory*, 3(2), 156–168.

ELLISON, G. (2005): "A Model of Add-On Pricing," *Quarterly Journal of Economics*, 120(2), 585–637.

FERSHTMAN, C., AND A. FISHMAN (1992): "Price Cycles and Booms: Dynamic Search Equilibrium," *American Economic Review*, 82(5), 1221–1233.

FERSHTMAN, C., A. FISHMAN, AND J. ZHOU (2018): "Search and Categorization," *International Journal of Industrial Organization*, 57, 225–254.

HONKA, E., AND P. CHINTAGUNTA (2016): "Simultaneous or Sequential? Search Strategies in the US Auto Insurance Industry," *Marketing Science*, 36(1), 21–42.

JANSSEN, M., P. PICHLER, AND S. WEIDENHOLZER (2011): "Oligopolistic Markets with Sequential Search and Production Cost Uncertainty," *RAND Journal of Economics*, 42(3), 444–470.

JANSSEN, M. C., J. L. MORAGA-GONZÁLEZ, AND M. R. WILDENBEEST (2005): "Truly Costly Sequential Search and Oligopolistic Pricing," *International Journal of Industrial Organization*, 23(5-6), 451–466.

JANSSEN, M. C. W., AND J. L. MORAGA-GONZÁLEZ (2004): "Strategic Pricing, Consumer Search and the Number of Firms," *Review of Economic Studies*, 71(4), 1089–1118.

JOHNEN, J., AND D. RONAYNE (2021): "The Only Dance in Town: Unique Equilibrium in a Generalized Model of Sales," *Journal of Industrial Economics*, 69(3), 595–614.

JOHNSON, J. P. (2017): "Unplanned Purchases and Retail Competition," *American Economic Review*, 107(3), 931–965.

Kalai, E. (2004): "Large Robust Games," *Econometrica*, 72(6), 1631–1665.

Krishna, V. (2013): *Auction Theory.* Academic Press, MA, USA.

McAfee, R. P. (1994): "Endogenous Availability, Cartels, and Merger in an Equilibrium Price Dispersion," *Journal of Economic Theory*, 62(1), 24–47.

Moraga-González, J. L., and V. Petrikaitė (2013): "Search Costs, Demand-Side Economies, and the Incentives to Merge under Bertrand Competition," *RAND Journal of Economics*, 44(3), 391–424.

Moraga-González, J. L., Z. Sándor, and M. R. Wildenbeest (2017): "Non-Sequential Search Equilibrium with Search Cost Heterogeneity," *International Journal of Industrial Organization*, 50, 392–414.

——— (2021): "Simultaneous Search for Differentiated Products: the Impact of Search Costs and Firm Prominence," *Economic Journal*, 131(635), 1308–1330.

Morgan, P., and R. Manning (1985): "Optimal Search," *Econometrica*, 53(4), 923–944.

Myatt, D. P., and D. Ronayne (2024a): "Asymmetric Models of Sales," *Working Paper.*

——— (2024b): "Stable Price Dispersion," *Working Paper.*

Nagaraja, H. (1992): "Order Statistics from Discrete Distributions," *Statistics*, 23(3), 189–216.

Rhodes, A. (2015): "Multiproduct Retailing," *Review of Economic Studies*, 82(1), 360–390.

Rhodes, A., M. Watanabe, and J. Zhou (2021): "Multiproduct Intermediaries," *Journal of Political Economy*, 129(2), 421–464.

Rhodes, A., and J. Zhou (2019): "Consumer Search and Retail Market Structure," *Management Science*, 65(6), 2607–2623.

Ronayne, D. (2021): "Price Comparison Websites," *International Economic Review*, 62(3), 1081–1110.

Rosenthal, R. W. (1980): "A Model in which an Increase in the Number of Sellers Leads to a Higher Price," *Econometrica*, 48(6), 1575–1579.

Stahl, D. O. (1989): "Oligopolistic Pricing with Sequential Consumer Search," *American Economic Review*, 79(4), 700–712.

Teh, T.-H., and J. Wright (2022): "Intermediation and Steering: Competition in Prices and Commissions," *American Economic Journal: Microeconomics*, 14(2), 281–321.

Varian, H. R. (1980): "A Model of Sales," *American Economic Review*, 70(4), 651–659.

Wang, C., and J. Wright (2020): "Search Platforms: Showrooming and Price Parity Clauses," *RAND Journal of Economics*, 51(1), 32–58.

Watt, M. (2022): "Concavity and Convexity of Order Statistics in Sample Size," *Working Paper.*

Wilks, S. S. (1962): *Mathematical Statistics.* New York: John Wiley.

Wolinsky, A. (1983): "Retail Trade Concentration Due to Consumers' Imperfect Information," *Bell Journal of Economics*, 14(1), 275–282.

——— (1986): "True Monopolistic Competition as a Result of Imperfect Information," *Quarterly Journal of Economics*, 101(3), 493–511.

Zhou, J. (2011): "Ordered Search in Differentiated Markets," *International Journal of Industrial Organization*, 29(2), 253–262.

——— (2014): "Multiproduct Search and the Joint Search Effect," *American Economic Review*, 104(9), 2918–2939.