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On the (ir)relevance of fee structures in certification*

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Abstract

Restrictions on certifiers' fee structures are irrelevant for maximizing their profits and trade efficiency, and for the implementability of (monotone) distributions of rents. The irrelevance results exploit that certification schemes involve two substitutable dimensions—the fee structure and the disclosure rule—and adaptations in the disclosure dimension can mitigate restrictions on the fee dimension. While restrictions on fee structures do affect market transparency, it has no impact on economic efficiency or rent distributions.

Keywords: certification, fee structures, disclosure rules, transparency
JEL classification: D82

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1 Introduction

Certifiers play a crucial role in many markets with asymmetric information: by their ability to make private information more public, they reduce informational asymmetries and thereby have the potential to enhance economic efficiency. However, most certifiers are guided by incentives other than directly promoting efficiency. Commercial certifiers for instance aim at maximizing profits and these goals may well be in conflict with enhancing aggregate efficiency or providing full and unbiased information to market participants. A typical example are credit rating agencies, who provide critical services to financial markets, and who, because of their questionable performance during the financial crisis of 2007–09, have much fallen under the scrutiny of regulators. Addressing a possible misalignment of incentives, regulators have pointed to the certifiers' fee structure as one of the main drivers of these problems.[1] Specifically, regulators blame fee structures that are increasing in certification outcomes—as has been common for credit rating agencies before the financial crisis—for inviting certification inflation, i.e., the practice of awarding better certificates than warranted. Regulatory proposals in the aftermath of the financial crisis, such as the Cuomo reform and the Franken amendment to the Dodd-Frank act, stipulate the use of flat fees that are paid upfront and irrespective of the certification outcome.[2]

Given the above interest in the fee structure of certifiers, this paper provides a systematic study of its economic relevance in adverse selection markets. Our main insight is that restrictions on the fee structure are economically irrelevant for the certifier's profits and trade efficiency, and for the implementability of (monotone) distributions of rents. The intuition for this irrelevance result is that certification schemes involve two dimensions — the fee structure and the disclosure rule. Our analysis shows that the two are substitutes in that restrictions on fee structures can be fully compensated by adapting the certifier's disclosure rule.[3] This explanation also reveals two limits of our irrelevance result: First, it breaks down if certifiers are unable to tailor their disclosure rules to the fee structure. Second and somewhat more subtle, certifiers' fee structures crucially affect the optimal disclosure rule and therefore the resulting transparency of markets. The subtlety is however that these differences in transparency have no impact on economic efficiency and profits.[4]

We derive these results by studying certification in a canonical model of adverse selection

[1]In addition to fee structures, regulators have also expressed concerns about the initiators of certification. By focusing on fee structures for seller-initiated certification, we fully abstract from this regulatory dimension and refer to other literature (e.g. Stahl and Strausz, 2017).

[2]E.g., WSJ (2008) explains the Cuomo-reform stipulates fixed, up-front fees so that "the firms would get paid for their review, even if they aren't hired to rate the deal." (`https://www.wsj.com/articles/SB121268203224348921` [last accessed Oct 26, 2022]). Such up-front payments were also part of the Franken amendment to the Dodd–Frank Act, which was eventually removed to allow for an extended review by the SEC (see Ozerturk, 2014).

[3]In the context of a moral hazard rather than a pure adverse selection market, Albano and Lizzeri (2001) note the substitutability between fees and disclosure rules, but apply this observation differently.

[4]Lizerri (1999)'s observation that the "intermediary reveals some information but only the minimal amount necessary to induce efficient trading behavior in the market" (p.224) is an expression of this subtlety.

in the spirit of Akerlof (1970). A seller has private information on the value of the object to be traded. Expressing the seller's private information as his type, the seller's type can affect both the seller's own consumption value and that of the buyer. The market interaction is such that any trade takes place at a price that equals the buyer's expected willingness to pay for the offered object. To this canonical setup, we introduce a profit-maximizing certifier, who can offer a certification scheme to the seller. The scheme prescribes both the fee structure and the disclosure rule by which the certification reveals the seller's private information.[5] Given this setup, we study the extent to which different types of fee structures affect economic outcomes. The most flexible fee structure is a "type-contingent fee", which allows the fee to condition directly on the seller's (initially private) type. An intermediate form is a "certificate-contingent fee", which allows the fee to condition on the specific certificate that results from the certification process but not on the seller's type directly. The least flexible fee structure is a "flat fee" that is fixed and unresponsive to the certification outcome or to the seller's type.

Our first irrelevance result shows that the fee structure has no impact on market efficiency and the certifier's maximum profit. Not only type- and certificate-contingent fees but also flat fees enable the certifier to extract all gains from trade in excess of the seller's outside option, and these profit-maximizing schemes induce efficient trade. The disclosure rules that support this efficient and profit-maximizing outcome are however markedly different. While with type– and certificate–contingent fees the certifier can maximize profits with a fully revealing disclosure rule, this is not so with flat fees. The profit-maximizing disclosure rule under flat fees is necessarily coarser as it involves a pooling of low and high seller types to level their different willingness to pay for certification. Hence, even though with respect to profits and trade efficiency, the fee structure does not matter, it does affect market transparency. In particular, restricting profit-maximizing certifiers to use only flat fees leads to less transparent market outcomes. Yet, this difference in market transparency does neither impact trade efficiency nor the seller's profits.[6]

Our second result goes beyond considering a profit-maximizing certifier and concerns the potential distribution of rents between the certifier and the seller given an efficient market outcome.[7] With a certificate contingent fee, any distribution of rents can be implemented, by charging the respective seller type an appropriate fee for the certificate that this type will be awarded. With flat fees, a distribution of rents is implementable for any prior if and only if the rents for *both* the seller and certifier are monotone increasing in the seller's type. These monotonicity requirements are necessary and sufficient for guaranteeing that the required pooling of types associated with the optimal flat-fee disclosure rule is feasible.

[5]We abstract from any moral hazard problems of certification and refer for such considerations to Ozerturk (2014) and Bizzotto and Vigier (2021).

[6]For papers that focus on the market transparency effect of certification see Pollrich and Wagner (2016), Stahl and Strausz (2017), Harbaugh and Rasmusen (2018), and Kattwinkel and Knoepfle (2022).

[7]Because in the canonical adverse selection setup, trade takes place at a price that equals the buyer's expected willingness to pay, buyer-rents are always zero.

Hence, fee schedules are also irrelevant for their ability to implement monotone distributions of rents.

The rest of the paper is organized as follows. In the next section, we present a canonical model of adverse selection, and discuss its relation to the existing literature. In Section 3, we derive equivalence results on profit-maximizing certification. Section 4 focuses on the distribution of rents. We conclude in Section 5.

2 Model

A canonical adverse selection market. A single buyer faces a single seller, who owns one unit of an indivisible good of a certain type. The good's type has $n \geq 1$ possible realizations.[8] Hence, we denote the good's type by a number i of the index set $I_n \equiv \{1, \ldots, n\}$, the set of possible types. Initially, type $i \in I_n$ is drawn from the commonly known prior distribution $\boldsymbol{\pi} = (\pi_1, \ldots, \pi_n)$, where π_i denotes the probability that the good's type is i. Before deciding whether to sell her good, the seller observes the realization of this draw privately.

The good's type determines the consumption value of the good to both the buyer and seller. In particular, the buyer's (consumption) value of a good of type i is q_i, whereas its (consumption) value to the seller is s_i. We assume that trade is always efficient, i.e. $q_i \geq s_i$ for all i.[9] Without loss of generality, we make the ordering assumption

$$q_1 \leq q_2 \leq \cdots \leq q_n. \tag{Q-MON}$$

After learning the good's type i, the seller decides whether to offer her good for sale. Hence, when deciding to sell the good, she already knows that consuming the good herself yields her the value s_i. This is the first defining feature of an adverse selection market.

If the seller actually decides to sell the good, we assume that the market interaction is such that she sells it at a price equal to the buyer's expected valuation for the good. Importantly, the buyer's expectation is fully rational and, for instance, takes into account information contained in the fact that the good is offered for sale. That the selling price equals the buyer's rational expectation is the second defining feature of an adverse selection market.[10] As a consequence, a sale results in a price that always (weakly) exceeds q_1. Hence, the seller's effective reservation value r_i for selling the good is given by $r_i = \max\{s_i, q_1\}$, which is the least utility a seller with a good of type i can ever get.

[8] We consider an adverse selection market with finite but arbitrarily many types, because, as explained in Remarks below, the proof of our result is based on an induction argument on the number of types. As the studies in the literature that use continuous type models are limits of an appropriate sequence of our canonical adverse selection model with finite types, our results also apply for these continuous type models.

[9] This assumption is inconsequential for our equivalence results, but simplifies the exposition and is standard in the literature.

[10] One common micro-foundation for this assumption is that there are, in fact, two identical buyers who bid for the good in a second-price auction.

Throughout we make the following assumptions.

$$r_1 \leq r_2 \leq \ldots \leq r_n \tag{R-MON}$$

$$q_1 - r_1 \leq q_2 - r_2 \leq \ldots \leq q_n - r_n \tag{S-MON}$$

Assumption (R-MON) states that the reservation values increase with type. This assumption is equivalent to assuming $s_i \leq s_{i+1}$ for all i such that $s_i > q_1$. Assumption (S-MON) asserts that the surplus from trade $q_i - r_i$ increases with the seller's type, and is non-negative for all types. (S-MON) implies that it is always efficient to trade the good. Note that, together, (R-MON) and (S-MON) imply (Q-MON).[11]

Canonical Certification. A certification technology $D = (\mathcal{C}, \mathbf{p})$ consists of a set of certificates $\mathcal{C} = \{C_1, \ldots, C_m\}$ and a family of distributions $\mathbf{p} = (p_1, \ldots, p_n)$. Each $p_i \in \Delta\mathcal{C}$ is a distribution over certificates, with the interpretation that when a good of type i is certified, it receives the certificate C_j with probability p_{ij}. Special cases of certification technologies are (i) *full disclosure*, where $\mathcal{C}^{FD} = \{C_1, \ldots, C_n\}$ and $p_i = e_i$ for all i,[12] and (ii) *cut-off diclosure*, where for some $1 \leq k \leq n$ we have $\mathcal{C}^k = \{\overline{C}, \underline{C}\}$ and $p_i = (1, 0)$ for $i \leq k$ and $p_i = (0, 1)$ otherwise.[13]

In addition to the certification technology, the certifier commands a certification fee. In general, a *fee schedule* $f : I_n \times \mathcal{C} \to \mathbb{R}_+$ determines the fee to be paid by the seller if her good is of type i and receives the certificate C_j. Fee schedules encompass the following special cases:

- Flat fee: $f(i, C_j) \equiv F \in \mathbb{R}_+$ for all $i \in I_n$ and $j \in I_m$.
- Type-contingent fee: for every $i \in I_n$ there is a number F_i^t such that $f(i, C_j) \equiv F_i^t$ for all $j \in I_m$.
- Certificate-contingent fee: for every $j \in I_m$ there is a number F_j^c such that $f(i, C_j) \equiv F_j^c$ for all $i \in I_n$.

The certification game. A certifier technology D together with a fee schedule f induce the following certification game.

1. Nature draws $i \in I_n$ according to $\boldsymbol{\pi}$ and reveals it to the seller;
2. Upon observing i, the seller decides whether to demand certification;
3. If the seller demands certification, the certifier draws a certificate C according to p, the seller makes the payment f;

[11] Note however that (Q-MON) together with either of the conditions (R-MON) or (S-MON) does not imply the respective other condition.

[12] Throughout we denote $e_i \in \mathbb{R}^n$ the i-th unit vector, i.e., $e_{ii} = 1$ and $e_{ij} = 0$ for all $j \neq i$.

[13] A no disclosure rule is the special case of cut-off disclosure with cut-off $k = 1$.

Fee structure	Disclosure	Papers
flat	endogenous	Doherty et al. (2012), Lizzeri (1999), Loerke & Niedermayer (2018), Opp et al. (2013), Pollrich & Wagner (2016), Viscusi (1978)
	exogenous	Bolton et al. (2012), Bouvard & Levy (2018), Durbin (1999), Farhi et al. (2013), Mathis et al. (2009), Skreta & Veldkamp (2009), Stahl & Strausz (2017), Strausz (2005)
certificate contingent	endogenous	Faure-Grimaud et al. (2009), Kovbasyuk (2018)
	exogenous	Fulghieri et al. (2014), Bar-Isaac & Shapiro (2013), Peyrache & Quesada (2011)
type contingent	endogenous	
	exogenous	Frenkel (2015)

Table 1: Classification of the literature on certification in markets with adverse selection. Not listed are Ozerturk (2014) and Bizzotto & Vigier (2021), who both explicitly compare flat to certificate-contingent fees.

4. The certificate is publicly disclosed, and the seller decides whether to offer the good for sale;

5. Buyers update their belief about the good, based on the published certificate and the seller's decision to offer the good for sale, and pay the price equal to the expected quality.

Remarks. Assumptions (R-MON) and (S-MON) are satisfied in all models on certification in adverse selection markets that have been studied in the literature. In his seminal work, Akerlof (1970) assumes that $s_i = \alpha q_i$ for $\alpha \in (0, 1)$, which yields $r_i = \max\{\alpha q_i, q_1\}$ and $q_i - r_i = \min\{(1-\alpha)q_i, q_i - q_1\}$.[14] (R-MON) and (S-MON) are then implied by (Q-MON), i.e., that quality is increasing in type. Numerous authors assume $s_i = q_1$, for which our conditions are naturally satisfied. Lizzeri (1999) brings forward another specification with $s_i = q_i - \alpha$. In our terminology we thus get $r_i = \max\{q_i - \alpha, q_1\}$, and $q_i - r_i = \min\{\alpha, q_i - q_1\}$. Again, (R-MON) and (S-MON) are implied by (Q-MON).

Our timing implicitly assumes that the seller's decision whether to certify is observable. For our results this assumption is without loss. Unobservability of the certification decision only matters if the certification technology allows for non-certification as an outcome, i.e., the entries of the vectors p_i do not have to add up to one. We return to this issue in the discussion. Also, the seller cannot hide the certificate, in case she is not satisfied with the outcome. In our results, the seller never wishes to do so. Our model however does allow the seller not to sell the good after an (unlucky) certification outcome.[15]

The main driver of our irrelevance results is the assumption that certification schemes do

[14] Akerlof assumes $q_1 = 0$, hence $r_i = s_i$ in our terminology.

[15] Technically, it is this assumption that requires us to use an induction argument rather than build our proof on convexification (or rather majorization) arguments. This is so because the seller's final selling decision can depend on the actual realization of the certificate rather than its "expected" realization.

not only specify a fee structure but also a disclosure rule, because the disclosure rule then can act as a substitute for the fee structure. For this reason, Table 1 classifies the work on certification in adverse selection markets according to their underlying assumptions about the fee structure and whether the disclosure rule can be (at least in parts) endogenously chosen. It reveals that, except for Frenkel (2015), the literature either assumes a fee structure that is flat or certificate-contingent. About half the studies assume that the certification rule is endogenous, whereas the other half of the literature studies fixed certification rules and, thereby, mostly those that lead to full revelation.

3 Profit-Maximizing Certification

In this section, we focus on profit-maximizing certification. We first derive the maximal certifier profit under the most flexible fee structure. We next show that the certifier attains this profit also with less flexible fee structures and, in particular, with a flat fee.

To derive an upper bound on the certifier's profit note that trading a good of type i generates at most a surplus of q_i and does so if the buyer obtains the good i. Hence, an obvious upper bound on the certifier's profit is the ex ante expected consumption value $\sum_i \pi_i q_i$. However, each seller-type also has the option to sell the good without certification, which yields a price weakly above q_1, or to consume it by herself, which yields s_i. Taking into account the resulting outside option for the seller, the surplus the certifier can extract from a seller with a good of type i is at most $q_i - r_i$, which is non-negative as $q_i \geq s_i$. Hence, an upper bound on the certifier profit is given by

$$S^\star := \sum_{i=1}^{n} \pi_i (q_i - r_i). \tag{1}$$

Proposition 1. *The certifier obtains the profit $S^\star$ with a certification technology D of full disclosure and a type-contingent fee schedule f with $F_i^t = q_i - r_i$ for all $i \in I_n$.*

Proof. Let the certification technology be a full disclosure rule, i.e., $\mathcal{C} = \{C_1, \ldots, C_n\}$ and $p_i = e_i$. Set $f^\star(i, C_j) = q_i - r_i$ for all i, j.[16] We verify that there exists an equilibrium in which the seller always demands certification. To see this, choose the following off-path beliefs: buyers believe uncertified goods are of the lowest quality q_1 with probability one, hence an uncertified good sells at price q_1. If every seller-type certifies, then a good that is offered and carries certificate C_j sells at price q_j. Seller-type i's payoff from certification is $q_i - f^\star(i, C_i) = q_i - q_i + r_i = r_i$. Hence, type i prefers selling the good certified to either selling it uncertified (at a price q_1) or keeping the good for herself (and realizing value s_i). We have thus shown that it is an equilibrium that all seller types demand certification. The certifier's profit in this equilibrium equals $S^\star$. □

[16] Given that under full disclosure, a good i can only get certificate C_i, only the fees $f(i, C_i)$ play a role in verifying the equilibrium of the certification game.

Using a type-contingent fee schedule, Proposition 1 resembles classical results on first-degree price discrimination. The certifier charges each seller-type an amount equal to this type's willingness to pay (WTP) for certification. This WTP depends on the seller's outside option, which is either consuming the good herself or selling it uncertified. The maximum profit $S^\star$ is attained by making the latter option as unattractive as possible via inducing the off-path belief that an uncertified good has the lowest quality.

Our first equivalence result is that the certifier can also obtain the profit $S^\star$ with a certificate-contingent fee:

Proposition 2. *The certifier obtains the profit $S^\star$ with a certification technology D of full disclosure (i.e. $m = n$) and a certificate-contingent fee schedule f with $F^c_j = q_j - r_j$ for all $j \in I_n$.*

The result is a direct corollary of Proposition 1 because the certifier uses a disclosure rule that fully reveals the seller's type. Hence, the issued certificate equals the seller's type and charging the fee based on the certificate is equivalent to charging the fee based on the discovered type.

We finally show the main result of this section: the certifier can attain the profit $S^\star$ with a flat fee. Deriving this result is more subtle as the certifier cannot obtain $S^\star$ with a flat fee using a fully disclosing certification technology. Indeed, if she could do so, then necessarily the flat fee has to equal $F \equiv S^\star$, but, to low seller types, paying the fee $S^\star$ for a fully disclosing certificate is not worthwhile. Hence, if a flat fee is able to attain the profit $S^\star$, it has to do so with a certification technology D that is not fully revealing. In particular, the certification technology must ensure that all seller types are willing to pay the flat certification fee $S^\star$ and also induce them to sell their good independent of the specific certification outcome.

The main insight of the next proposition is that this is possible for any prior $\boldsymbol{\pi}$. Hence, our next equivalence result holds regardless of the prior $\boldsymbol{\pi}$, but the disclosure rule that sustains it depends on the prior $\boldsymbol{\pi}$ in a carefully calibrated way and does not yield a fully transparent market outcome, where all private information is fully disclosed to all market participants.

Proposition 3. *For every prior $\boldsymbol{\pi}$ there exists a certification technology D with a flat fee schedule $f = S^\star$ such that the certification game induced by (D, f) exhibits an equilibrium in which the certifier's profit is $S^\star$.*

Before proving Proposition 3, we first state the following more technical sounding lemma, providing its proof in the appendix.

Lemma 1. *Suppose* $\mathbf{u} \in \mathbb{R}^n$ *satisfies*

$$u_1 \leq u_2 \leq \ldots \leq u_n, \tag{U-MON}$$

$$q_1 - u_1 \leq q_2 - u_2 \leq \ldots \leq q_n - u_n, \tag{S-MON'}$$

$$\sum_i \pi_i q_i = \sum_i \pi_i u_i, \tag{F-SUM}$$

$$r_i \leq u_i, \quad \forall i = 1, \ldots, n. \tag{F-IN}$$

Let $\mathcal{I}_- := \{i | q_i - u_i < 0\}, \mathcal{I}_0 := \{i | q_i - u_i = 0\}$ *and* $\mathcal{I}_+ := \{i | q_i - u_i > 0\}$. *There exists a certification technology* $D = (\mathcal{C}, \mathbf{p})$ *with* $\mathcal{C} = \{C_1, \ldots, C_n\}$ *and*

$$p_i = e_i \text{ for all } i \in \mathcal{I}_0 \cup \mathcal{I}_+, \tag{ID}$$

$$p_{ij} = 0 \text{ for all } i \in \mathcal{I}_- \text{ and } j \in (\mathcal{I}_- \cup \mathcal{I}_0) \setminus \{i\}, \tag{UP}$$

such that in the certification game induced by D (with zero fee) every type i demands certification, sells the certified good, and obtains expected utility u_i *from certification.*

While more technical, the lemma is informative about the structural properties of the associated disclosure rule. Moreover, it not only allows us to prove Proposition 3, but also extend our study of fee structures beyond profit-maximizing certifiers. In the next section, we use it to study the ability of different fee structures in supporting different non-profit-maximizing distributions of rents between the certifier and the seller. We however first end this section by using Lemma 1 to prove Proposition 3, and by providing two examples that highlight the role of properties (R-MON) and (S-MON) for the result.

Proof. To prove Proposition 3, we apply the lemma as follows. For all $i = 1, \ldots, n$ define $u_i = r_i + S^\star$. Equation (R-MON) implies (U-MON), and (S-MON) implies (S-MON'). By definition $S^\star > 0$, which yields (F-IN). Finally,

$$\sum_{i=1}^n \pi_i u_i = \sum_{i=1}^n \pi_i \big(r_i + S^\star\big) = \sum_{i=1}^n \pi_i r_i + \sum_{i=1}^n \pi_i (q_i - r_i) = \sum_{i=1}^n \pi_i q_i,$$

i.e., (F-SUM) holds. Lemma 1 implies there is a certification technology D with the described properties. Now add to D the flat certification fee $F^\star = S^\star$. We verify there is an equilibrium in which every seller-type demands certification, and offers the certified good for sale. As before, choose the out-off equilibrium beliefs that an uncertified good is of the lowest quality q_1. By construction of the certification technology from Lemma 1 every type sells the certified good, and this decision is independent of the (previously sunk) certification fee. Type i's expected utility from certifying at the certification fee $F^\star$ is $u_i - S^\star = r_i$. Hence, every type prefers certification over keeping the good or selling it uncertified. Because all seller-types demand certification, the certifier's profit equals $S^\star$. □

Remarks. A complication in proving Proposition 3 lies in the (ex post) possibility for the seller to retain the good after it is certified. More specifically, we require that the seller must be willing to sell his certified good for the *realized* rather than the *expected* certificate. While natural, the assumption prevents us to use popular majorization techniques in information design which are based on constructing disclosure rules that are a straightforward Blackwell garbling of full information. Instead, the proof of Lemma 1 develops an induction argument, guaranteeing structural properties of the disclosure rule so that the seller is indeed willing to sell ex post. In particular, types exceeding a certain threshold obtain a particular certificate with probability one, while lower types are either fully revealed or pooled together with these high types. This "upward pooling" reduces the value of the certificate obtained by high types but still ensures that the expected value from certification for low types is sufficiently high. In addition, pooling upwards ensures that low types sell their good for every *realized* certificate, because in the worst outcome their type is fully revealed.

Proposition 3 requires properties (R-MON) and (S-MON). We close this section by providing minimal examples to show that whenever one of these assumptions is violated, there exist priors such that the certifier cannot attain profit $S^\star$ with a flat certification fee.[17]

Example 1 (Necessity of (R-MON))**.** Consider a market with three seller types, $q_1 < q_2 < q_3$ and assume $r_2 > r_3 > r_1 = q_1$, violating (R-MON). Define

$$\bar{q}_{23} := \frac{\pi_2}{\pi_2 + \pi_3} q_2 + \frac{\pi_3}{\pi_2 + \pi_3} q_3$$

We have that $S^\star = \pi_2(q_2 - r_2) + \pi_3(q_3 - r_3) = \bar{q}_{23} - r_2 + \pi_3(r_2 - r_3) - \pi_1(\bar{q}_{23} - r_2)$. In particular, $S^\star > \bar{q}_{23} - r_2$ whenever $\pi_3/\pi_1 > \bar{q}_{23} - r_2 / r_2 - r_3 > 0$. For any disclosure rule type 2's expected payoff from certification is at most $\bar{q}_{23} - f$, where that bound results from pooling all q_3-types together with type q_2. That is, a necessary condition for type 2's participation is $\bar{q}_{23} - f \geq r_2$. But then, $f \leq \bar{q}_{23} - r_2 < S^\star$, whenever $\pi_3/\pi_1 > \bar{q}_{23} - r_2 / r_2 - r_3$.

Example 2 (Necessity of (S-MON))**.** Consider a market with three seller types, $q_1 < q_2 < q_3$ and assume $q_2 - r_2 > q_3 - r_3 > q_1 - r_1 = 0$. Again, we have $S^\star = \pi_2(q_2 - r_2) + \pi_3(q_3 - r_3)$. Note that $S^\star > q_3 - r_3$ whenever $\pi_2/1 - \pi_3 > q_3 - r_3 / q_2 - r_2$.[18] Type 3's payoff from certification is at most $q_3 - f$. To guarantee that type's participation requires $q_3 - f \geq r_3$. But we have$f \leq q_3 - r_3 < S^\star$, whenever $\pi_2/1 - \pi_3 > q_3 - r_3 / q_2 - r_2$.

[17] Assumptions (R-MON) and (S-MON) have no bite when there are only two seller-types. Assuming trade is efficient, i.e., $s_i < q_i$, we have that $r_1 = q_1 \leq r_2$ and $q_1 - r_1 = 0 \leq q_2 - r_2$, hence both assumptions are automatically satisfied.

[18] This condition holds for instance if $\pi_1 = \pi_3 = \varepsilon/2$ and $\varepsilon \to 0$. We have that $\pi_2/1 - \pi_3 = 2 - 2\varepsilon / 2 - \varepsilon \to 1$, and by assumption $q_3 - r_3 / q_2 - r_2 \in (0, 1)$.

4 Implementable Rent Distributions

In the previous section, we focused on a profit-maximizing certifier. In this section, we take a more general view and study the extent to which fee structures affect the ability to implement different distributions of rents among the certifier and different seller types. Such implementation concerns become relevant when regulators want, next to the fee structures, to influence also the certifier's objectives; or the certifier represents a self-regulated organization set up by the sellers themselves; or the certifier is a non-profit organization.[19] Characterizing the implementable distributions of rents under the fee structures of type-contingent, certificate-contingent, and flat fees, we focus on efficient allocations, i.e., where certification leads to efficient trade in equilibrium.

A rent vector $(S, u_1, \ldots, u_n)$ specifies the certifier's (expected) profit S and each seller type's utility $u_1, \ldots, u_n$. A rent vector is *feasible* if

$$S \geq 0, \tag{C-IR}$$

and

$$u_i \geq r_i, \quad \forall i. \tag{S-IR}$$

An efficient allocation gives rise to the aggregate surplus $\sum_i \pi_i q_i$. This aggregate surplus is distributed between the certifier and the seller types. A rent vector is exhaustive if

$$S + \sum_i \pi_i u_i = \sum_i \pi_i q_i. \tag{EX}$$

Our next result establishes that certification with type- or certificate-contingent fees allows for implementing any rent vector that is feasible and exhaustive.

Proposition 4. *Any rent vector $(S, u_1, \ldots, u_n)$ that is feasible and exhaustive can be implemented by certification with a type- or certificate-contingent fee.*

Proof. Let the certification technology be a full disclosure rule and take the fee structure $f(i, C_j) = q_i - u_i$ for all i, j (resp. $f(i, C_j) = q_j - u_j$ for a certificate-contingent fee). We verify there exists an equilibrium in which the seller always demands certification, and that implements the desired rents. If all seller types demand certification, a good with certificate C_i sells at a price q_i and in equilibrium sellers do sell their certified goods. Choose the out-of equilibrium belief that an uncertified good is of the lowest quality with probability one. Hence, an uncertified good sells at a price q_1. Seller type i obtains $q_i - f(i, C_i) = q_i - (q_i - u_i) = u_i$ from certification. Not certifying yields r_i. Feasibility then implies that every seller-type demands certification. Using (EX) and (C-IR) the certifier's profit is $\sum_i \pi_i f(i, C_i) = \sum_i \pi_i (q_i - u_i) = S \geq 0$. □

[19] See Zapechelnyuk (2020) or Vellodi (2021) for models where, rather than profits, the certifier maximizes welfare or consumer surplus.

Proposition 4 resembles once more classical results on first-degree price discrimination. In conjunction to revealing the seller's type publicly, the certifier charges each seller-type the difference between her contribution to surplus q_i and her designated utility level u_i. Via subsidies (i.e., negative fees) the certifier can shift rents across types. Re-distribution is limited by each individual's outside option, given by the above feasibility constraints, and by the fact that the certifier is the residual claimant of the surplus after re-distribution across seller types. As before, there is essentially no difference between type- and certificate-contingent fees, because the full disclosure rule allows for inferring the true type from the issued certificate.

When restricted to the use of flat fees, surplus can, by contrast, not be re-distributed via monetary payments, leaving the disclosure rule as the sole channel for redistribution. This introduces additional constraints on implementable rent distributions.

Proposition 5. *Any rent vector* $(S, u_1, \ldots, u_n)$ *that is feasible, exhaustive and satisfies* (U-MON) *and* (S-MON') *can be implemented by certification with a flat fee.*

Proof. We apply Lemma 1 to the vector of utilities $\mathbf{u}' = (u'_1, \ldots, u'_n)$ where $u'_i = u_i + S$. Conditions (U-MON) and (S-MON') are invariant upon adding constants, hence $\mathbf{u}'$ satisfies them, as by assumption $(u_1, \ldots, u_n)$ satisfies them. Similarly, $\mathbf{u}'$ satisfies (F-IN), because we have $u'_i = u_i + S \geq r_i$ due to (C-IR) and (S-IR). Finally, $\mathbf{u}'$ satisfies (F-SUM), because

$$\sum_i \pi_i u'_i = \sum_i \pi_i u_i + S \overset{\text{(EX)}}{=} \sum_i \pi_i q_i.$$

Hence, the vector $\mathbf{u}'$ satisfies all the conditions from Lemma 1. It is straightforward to verify that the resulting disclosure rule together with a (possibly non-zero) fee S implements rent vector $(S, u_1, \ldots, u_n)$. $\square$

The additional constraints (U-MON) and (S-MON') arise because redistribution via the disclosure rule is limited by Bayes' consistency. Hence, it is not possible to reduce the utility of some seller type below the average resulting from that type with the lowest type. Unlike monetary payments, probabilities cannot become negative. Though it is possible to find specific rent distributions that can be implemented with a flat certification fee while violating (U-MON) or (S-MON'), the next two examples show that this is not possible in general.

Example 3 (Necessity of (U-MON))**.** Suppose $n = 2$ and assume (S, u_1, u_2) satisfies (C-IR), (S-IR) and (EX). In addition assume $u_1 > u_2$. Let $(u'_1, u'_2) = (u_1 + S, u_2 + S)$. A certification technology with flat fee S implements utilities (u_1, u_2) if and only if a certification technology with flat fee 0 implements utilities (u'_1, u'_2). Note that $u_1 > u_2$ implies $u'_1 > u'_2$. Then (EX) implies that $u'_1 > \bar{q} > u'_2$. But the utility of type 1 cannot be increased above $\bar{q}$ by certification alone.

Example 4 (Necessity of (S-MON')). Suppose $n = 2$ and assume (S, u_1, u_2) satisfies (C-IR), (S-IR) and (EX). In addition assume $q_1 - u_1 > q_2 - u_2$. Again, add $S \geq 0$ to the utilities to get (u_1', u_2') which then satisfies $\pi_1 u_1' + \pi_2 u_2' = \pi_1 q_1 + \pi_2 q_2$ and $q_1 - u_1' > q_2 - u_2'$. Together, these imply $q_1 - u_1' > 0 > q_2 - u_2'$, and thus $u_2' > q_2$. It is clearly impossible to implement such a utility vector with certification, because any certified good is worth less than q_2.

5 Conclusion

We have proven our results abstracting from costs of certification. In practice, there are however (at least) two natural sources for certification costs. First, certification may require cost intensive investigations. Second, it is costly to generate and publicly reveal the respective certificate. For the former case it is natural to assume that costs are type-specific. Our results readily extend if we assume that it costs c_i to certify a type i seller. With a type-contingent fee certification of type i is optimal whenever $q_i - r_i \geq c_i$ and the certifier's maximal profit becomes

$$S_c^\star = \sum_i \pi_i \max\left\{q_i - r_i - c_i, 0\right\}.$$[20]

To see that Proposition 3 continues to hold, observe that Lemma 1 is independent of certification costs. After dropping all seller-types that are not certified in the above procedure (i.e., all i such that $q_i - r_i < c_i$) we can apply the lemma to the resulting smaller market.

Matters are more delicate when the certification costs directly depend on the amount of information that the certifier discovers. For instance, it seems natural to assume this cost is increasing in the precision of certification.[21] A corollary of our results is then that the imprecise disclosure associated with a flat fee leads to less costs than the full disclosure associated with a type- or certificate-contingent fee structure and is socially superior.

While focusing on a single, monopolistic certifier and not explicitly modelling competition, our analysis on the distribution of rents is indicative of competitive pressures that lower monopolistic rents to the certifier. It is an exciting avenue for future research to study whether competition between certifiers yields additional insights about the economic effect of limitations on the fee structures in certification.

Appendix: Proof of Lemma 1.

In this appendix we prove Lemma 1. The proof proceeds by induction over n. In all cases the equilibrium entails full certification, and we assume the following off-path beliefs: the

[20] As before, the certifier uses full disclosure and charges the fee $f(i, C_j) = q_i - r_i$ for all i, j s.t. $q_i - r_i \geq c_i$.

[21] See Pomatto et al. (2022) for an axiomatic foundation of such costs, or Ozerturk (2014) and Bizzotto and Vigier (2021) for specific examples of such costs with binary certification outcomes, or Ball and Kattwinkel (2022) for a mechanism design foundation of such costs based on probabilistic verification.

buyer believes an uncertified good is of the lowest quality, hence it sells at a price q_1.

We start by proving the claim for $n = 2$. Note that by (F-SUM) we have that $1 \in \mathcal{I}_0$ if and only if $2 \in \mathcal{I}_0$. Hence, if $\mathcal{I}_0 \neq \emptyset$ a full disclosure rule with $p_i = e_i$ yields the desired result.

If $\mathcal{I}_0 = \emptyset$ conditions (U-MON), (F-SUM) and (S-MON') imply that $q_1 < u_1 \leq u_2 < q_2$. Hence, $1 \in \mathcal{I}_-$ and $2 \in \mathcal{I}_+$. Consider a certification technology with $\mathcal{C} = \{C_1, C_2\}$, and $p_1 = ({}^{u_2 - u_1}/_{u_2 - q_1}, {}^{u_1 - q_1}/_{u_2 - q_1})$ as well as $p_2 = (0, 1)$. Clearly, this rule satisfies (ID) and (UP). Suppose both seller types demand certification. We have $v_1 = q_1$, and

$$\begin{aligned}
v_2 &= \frac{\pi_1 p_{12}}{\pi_1 p_{12} + \pi_2} q_1 + \frac{\pi_2}{\pi_1 p_{12} + \pi_2} q_2 \\
&= \frac{\pi_1 (u_1 - q_1)}{\pi_1 (u_1 - q_1) + \pi_2 (u_2 - q_1)} q_1 + \frac{\pi_2 (u_2 - q_1)}{\pi_1 (u_1 - q_1) + \pi_2 (u_2 - q_1)} q_2 \\
&\overset{\text{(F-SUM)}}{=} \frac{\pi_1 (u_1 - q_1)}{\pi_2 (q_2 - q_1)} q_1 + \frac{\pi_2 (u_2 - q_1)}{\pi_2 (q_2 - q_1)} q_2 \\
&\overset{\text{(F-SUM)}}{=} \frac{\pi_2 (q_2 - u_2) q_1 + \pi_2 (u_2 - q_1) q_2}{\pi_2 (q_2 - q_1)} \\
&= u_2
\end{aligned}$$

Because $u_2 \geq r_2$, the type-2 seller indeed demands certification and sells the certified good. For the type-1 seller we have that $u_2 = v_2 > q_1 \geq r_1$. Hence, this type sells the certified good for either of the two certificates. That type's expected value from certification is $p_{11} q_1 + p_{12} u_2 = q_1 + p_{12}(q_2 - q_1) = q_1 + u_1 - q_1 = u_1$. Hence, both types demand certification. This completes the proof for $n = 2$.

Now suppose the claim is proven for every $\tilde{n} \leq n - 1$. We prove it also holds for n.

<u>Case 1:</u> $|\mathcal{I}_0| = k \geq 1$. From (S-MON') we get that there are $1 \leq \underline{k} < \overline{k} \leq n$ such that $\mathcal{I}_- = \{1, \ldots, \underline{k}\}$ and $\mathcal{I}_+ = \{\overline{k}, \ldots, n\}$. We construct a sub-market of size $n - k$ with

$$(\tilde{q}_i, \tilde{r}_i, \tilde{u}_i) = \begin{cases} (q_i, r_i, u_i), & i = 1, \ldots, \underline{k}, \\ (q_{i+k}, r_{i+k}, u_{i+k}), & i = \underline{k} + 1, \ldots, n - k. \end{cases}$$

and the prior belief $\tilde{\pi}$ given by

$$\tilde{\pi}_i = \frac{\pi_i}{\sum_{j \in \mathcal{I}_- \cup \mathcal{I}_+} \pi_j}.$$

Using the respective conditions for the original market, the sub-market satisfies conditions (U-MON), (S-MON') and (F-IN). Because $q_i = u_i$ for all $i \in \mathcal{I}_0$ it also satisfies (F-SUM). For the submarket we have that $\tilde{\mathcal{I}}_- = \mathcal{I}_-$, $\tilde{\mathcal{I}}_+ = \{\underline{k} + 1, \ldots, n - k\}$, and $\tilde{\mathcal{I}}_0 = \emptyset$. By induction hypothesis there exists a disclosure rule $\tilde{D}$ with $\tilde{\mathcal{C}} = \{C_1, \ldots, C_{n-k}\}$, $\tilde{p}_i = e_i$ for all $i = \underline{k} + 1, \ldots, n - k$, and $\tilde{p}_{ij} = 0$ whenever $i \in \{1, \ldots, \underline{k}\}$ and $j \in \{1, \ldots, \underline{k}\} \setminus \{i\}$.

Construct a disclosure rule for the grand market as follows. Let $\mathcal{C} = \{C_1, \ldots, C_n\}$, and set $p_i = e_i$ for $i \in \mathcal{I}_0$. $p_i = \widetilde{p}_{i-k}$ for $i \in \mathcal{I}_+$, and for $i \in \mathcal{I}_-$ set

$$p_{ij} = \begin{cases} \widetilde{p}_{ij}, & \text{if } j \in \mathcal{I}_- \\ 0, & \text{if } j \in \mathcal{I}_0, \\ \widetilde{p}_{ij-k}, & \text{if } j \in \mathcal{I}_+ \end{cases}$$

By construction and from the induction hypothesis this disclosure rule satisfies (ID) and (UP). Now suppose every seller-type demands certification in equilibrium. For any $i \in \mathcal{I}_- \cup \mathcal{I}_0$ the certificate C_i is solely awarded to type i, hence $v_i = q_i$. Furthermore, for any $j \in \mathcal{I}_+$ the respective certificate C_j has the same composition as in the sub-market described above, hence $v_j = u_j$. This implies that every seller-type indeed offers the certified good for sale. Also, for every type $j \in \mathcal{I}_o \cup \mathcal{I}_+$ we have that $p_j = e_j$ and $v_j = u_j \geq r_j$, hence any such type demands certification. Any type $i \in \mathcal{I}_-$ has the same expected payoff from certification as in the sub-market, namely $\widetilde{u}_i$. By construction we have that $\widetilde{u}_i = u_i \geq r_i$ and also these types demand certification.

Case 2: $\mathcal{I}_0 = \emptyset$. In this case (S-MON') together with (F-SUM) implies existence of a *unique* $k \in \{1, \ldots, n-1\}$ such that $\mathcal{I}_- = \{1, \ldots, k\}$ and $\mathcal{I}_+ = \{k+1 \ldots, n\}$. Define $\widehat{\boldsymbol{\alpha}} = (\widehat{\alpha}_1, \ldots, \widehat{\alpha}_k)$ via

$$\widehat{\alpha}_i = \frac{u_i - q_i}{u_{k+1} - q_i}, \qquad i = 1, \ldots, k.$$

Because $u_{k+1} - q_i \geq u_i - q_i > 0$ we have that $\widehat{\alpha}_i \in (0, 1]$ for all $i = 1, \ldots, k$. Further, define the set $\mathcal{A} \subset [0,1]^k$ as follows. $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_k) \in \mathcal{A}$ if and only if for all $i = 1, \ldots, k-1$

$$u_{i+1} - \frac{\alpha_{i+1}}{1 - \alpha_{i+1}}(u_{k+1} - u_{i+1}) \geq u_i - \frac{\alpha_i}{1 - \alpha_i}(u_{k+1} - u_i) \tag{2}$$

$$q_{i+1} - u_{i+1} + \frac{\alpha_{i+1}}{1 - \alpha_{i+1}}(u_{k+1} - u_{i+1}) \geq q_i - u_i + \frac{\alpha_i}{1 - \alpha_i}(u_{k+1} - u_i) \tag{3}$$

and $\alpha_i \leq \widehat{\alpha}_i < 1$ for all $i = 1, \ldots, k$. Because (2) and (3) are continuous in $\boldsymbol{\alpha}$, the set $\mathcal{A}$ is path-connected.

Consider the mapping $f : \mathcal{A} \to \mathbb{R}$ given by

$$f(\alpha_1, \ldots, \alpha_k) = \sum_{i=1}^{k} \alpha_i \pi_i (q_i - u_{k+1}) + \pi_{k+1}(q_{k+1} - u_{k+1}). \tag{4}$$

We next argue that there exists $\boldsymbol{\alpha}^\star \in \mathcal{A}$ such that $f(\boldsymbol{\alpha}^\star) = 0$. By assumptions (U-MON) and (S-MON) we have $(0, \ldots, 0) \in \mathcal{A}$, and $f(0, \ldots, 0) = \pi_{k+1}(q_{k+1} - u_{k+1}) > 0$. Considering $\widehat{\boldsymbol{\alpha}} = (\widehat{\alpha}_1, \ldots, \widehat{\alpha}_k)$ we have that

$$\frac{\widehat{\alpha}_i}{1 - \widehat{\alpha}_i}(u_{k+1} - u_i) = u_i - q_i.$$

Therefore, condition (2) follows from (Q-MON), while condition (3) is trivially satisfied because $0 \geq 0$. We have

$$\begin{aligned} f(\widehat{\boldsymbol{\alpha}}) &= \sum_{i=1}^{k} \frac{u_i - q_i}{u_{k+1} - q_i} \pi_i (q_i - u_{k+1}) + \pi_{k+1}(q_{k+1} - u_{k+1}) \\ &= \sum_{i=1}^{k+1} \pi_i (q_i - u_i) \overset{\text{(F-SUM)}}{=} - \sum_{i=k+2}^{n} \pi_i (q_i - u_i) < 0, \end{aligned}$$

where the strict inequality follows from $\{k+2, \ldots, n\} \subset \mathcal{I}_+$. Applying the intermediate value theorem, we get existence of $\boldsymbol{\alpha}^\star = (\alpha_1^\star, \ldots, \alpha_k^\star) \in \mathcal{A}$ such that $f(\boldsymbol{\alpha}^\star) = 0$, i.e.,

$$\sum_{i=1}^{k} \alpha_i^\star \pi_i q_i + \pi_{k+1} q_{k+1} = \left(\sum_{i=1}^{k} \alpha_i^\star \pi_i + \pi_{k+1} \right) u_{k+1}. \tag{5}$$

We use $\boldsymbol{\alpha}^\star$ to construct a sub-market of size $n-1$, by dropping type $k+1$ and adjusting the target utilities u for types $1, \ldots, k$. Formally, let

$$(\widetilde{q}_i, \widetilde{r}_i) = \begin{cases} (q_i, r_i), & i = 1 \ldots, k \\ (q_{i+1}, r_{i+1}), & i = k+1, \ldots, n-1 \end{cases}$$

and

$$\widetilde{u}_i = \begin{cases} u_i - \frac{\alpha_i^\star}{1-\alpha_i^\star}(u_{k+1} - u_i), & i = 1 \ldots, k \\ u_{i+1}, & i = k+1, \ldots, n-1 \end{cases}$$

and

$$\widetilde{\pi}_i = \begin{cases} \frac{(1-\alpha_i^\star)\pi_i}{\sum_{j=1}^{k}(1-\alpha_j^\star)\pi_j + \sum_{l=k+2}^{n} \pi_l}, & i = 1 \ldots, k \\ \frac{\pi_{i+1}}{\sum_{j=1}^{k}(1-\alpha_j^\star)\pi_j + \sum_{l=k+2}^{n} \pi_l}, & i = k+1, \ldots, n-1. \end{cases}$$

We verify that $(\widetilde{q}, \widetilde{r}, \widetilde{u})$ satisfies the conditions of the lemma. We have $\widetilde{u}_i \leq u_i$ for all $i = 1, \ldots, k$, hence (2) together with (U-MON) for the original market imply that the sub-market satisfies (U-MON). By definition we have that $\alpha_i^\star \leq \widehat{\alpha}_i$, and thus $q_i - \widetilde{u}_i \leq 0$ for all $i = 1 \ldots, k$. Hence, (3) together with (S-MON') for the original market implies (S-MON') for the sub-market. (F-IN) holds for the sub-market because it holds for the original market and we have $\widetilde{u}_i \geq q_i \geq r_i$ for all $i = 1, \ldots, k$. To see that (F-SUM) holds, observe that

$$\begin{aligned}
\sum_{j=1}^{n-1} \widetilde{\pi}_j \big(\widetilde{q}_j - \widetilde{u}_j\big) &= \frac{\sum_{i=1}^{k}(1-\alpha_i^\star)\pi_i \left(q_i - u_i + \frac{\alpha_i^\star}{1-\alpha_i^\star}(u_{k+1} - u_i)\right) + \sum_{l=k+2}^{n} \pi_l (q_l - u_l)}{\sum_{j=1}^{k}(1-\alpha_j^\star)\pi_j + \sum_{l=k+2}^{n}\pi_l} \\
&= \frac{\sum_{i=1}^{k} \pi_i \left((1-\alpha_i^\star)(q_i - u_i) + \alpha_i^\star(u_{k+1} - u_i)\right) + \sum_{l=k+2}^{n} \pi_l (q_l - u_l)}{\sum_{j=1}^{k}(1-\alpha_j^\star)\pi_j + \sum_{l=k+2}^{n}\pi_l} \\
&= \frac{\sum_{i=1}^{k} \pi_i \alpha_i^\star (u_{k+1} - q_i) + \sum_{l \neq k+1} \pi_l (q_l - u_l)}{\sum_{j=1}^{k}(1-\alpha_j^\star)\pi_j + \sum_{l=k+2}^{n}\pi_l} \\
&= \frac{\pi_{k+1}(q_{k+1} - u_{k+1}) + \sum_{l \neq k+1} \pi_l (q_l - u_l)}{\sum_{j=1}^{k}(1-\alpha_j^\star)\pi_j + \sum_{l=k+2}^{n}\pi_l} \\
&= 0,
\end{aligned}$$

where the penultimate equality uses $f(\boldsymbol{\alpha}^\star) = 0$, and the last equality follows from (F-SUM) for the original market. Hence, the sub-market satisfies all conditions from the lemma. Note that in the sub-market we have $\widetilde{\mathcal{I}}_+ = \{k+1, \ldots, n-1\}$. By induction hypothesis there exists $(\widetilde{p}_1, \ldots, \widetilde{p}_{n-1})$, such that the associated disclosure game has the desired properties. In particular $\widetilde{p}_j = e_j$ for all $j > k$, and $\widetilde{p}_{ij} = 0$ if $i, j \leq k$ and $i \neq j$. Note that in the sub-market the value of certificate C_j is $\widetilde{v}_j = q_j$ if $j = 1, \ldots, k$ and $\widetilde{v}_j = \widetilde{u}_j = u_{j+1}$ if $j = k+1, \ldots, n-1$

To complete the proof we construct a disclosure rule for the grand market as follows. Let $\mathcal{C} = \{C_1, \ldots, C_n\}$. Set $p_i = \widetilde{p}_{i-1}$ for any $i = k+2, \ldots, n$, $p_{k+1} = e_{k+1}$, and for $i = 1, \ldots, k$ set

$$p_{ij} = \begin{cases} \widetilde{p}_{ij}(1-\alpha_i^\star), & j \neq k+1, \\ \alpha_i^\star & j = k+1 \end{cases}$$

The properties of $\widetilde{p}$ imply that the disclosure rule satisfies 1.–3.

It remains to verify that the certification game induced by D has the desired equilibrium. To this end, assume that all types demand certification in equilibrium. Then by construction $v_i = q_i$ for all $i = 1, \ldots, k$. We have $v_{k+1} = u_{k+1}$ by construction of $\boldsymbol{\alpha}^\star$. The properties of the sub-market yield $v_i = u_i$ for all $i = k+2, \ldots, n$. Properties 2. and 3. thus imply that each seller-type offers the certified good for sale for any certificate she obtains with positive probability. It is then in fact the case that types $i = k+1, \ldots, n$ demand certification, because their expected value from certification is $u_i \geq r_i$ by (F-IN). The expected value from certification for a type $i \in \mathcal{I}_-$ is

$$\begin{aligned}\sum_{j=1}^{n} p_{ij} v_j &= \sum_{j=1}^{k} \widetilde{p}_{ij}(1-\alpha_i^\star)v_j + \alpha_i^\star v_{k+1} + \sum_{j=k+2}^{n} \widetilde{p}_{ij-1}(1-\alpha_i^\star)v_j \\ &= (1-\alpha_i^\star)\sum_{j=1}^{n-1} \widetilde{p}_{ij}\widetilde{v}_j + \alpha_i^\star u_{k+1} \\ &= (1-\alpha_i^\star)\widetilde{u}_i + \alpha_i^\star u_{k+1} \\ &= (1-\alpha_i^\star)\left(u_i - \frac{\alpha_i^\star}{1-\alpha_i^\star}(u_{k+1}-u_i)\right) + \alpha_i^\star u_{k+1} \\ &= u_i,\end{aligned}$$

where the third inequality uses the induction hypothesis. Hence, also all types $i \in \mathcal{I}_-$ demand certification. We have thus verified existence of the desired equilibrium in the original market. $\square$

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