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A Theory of Crowdfunding

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Roland Strausz (Humboldt University Berlin)

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Ludwig-Maximilians-Universität München | Humboldt-Universität zu Berlin
Spokesperson: Prof. Dr. Klaus M. Schmidt, University of Munich, 80539 Munich, Germany
+49 (89) 2180 3405 | info@rationality-and-competition.de

A Theory of Crowdfunding
- a mechanism design approach with demand uncertainty and moral hazard

Roland Strausz*

Abstract

Crowdfunding provides innovation in enabling entrepreneurs to contract with consumers before investment. Under aggregate demand uncertainty, this improves screening for valuable projects. Entrepreneurial moral hazard and private cost information threatens this benefit. Crowdfunding's after-markets enable consumers to actively implement deferred payments and thereby manage moral hazard. Popular crowdfunding platforms offer schemes that allow consumers to do so through conditional pledging behavior. Efficiency is sustainable only if expected returns exceed an agency cost associated with the entrepreneurial incentive problems. By reducing demand uncertainty, crowdfunding promotes welfare and complements traditional entrepreneurial financing, which focuses on controlling moral hazard.

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*Humboldt-Universität zu Berlin, Institute for Economic Theory 1, Spandauer Str. 1, D-10178 Berlin (Germany), strauszr@wiwi.hu-berlin.de. I thank Helmut Bester, Simon Board, Tilman Börgers, Peter Cramton, Francoise Forges, Willy Fuchs, Sebastian Schweighofer-Kodritsch, Daniel Krähmer, Nicolas Lambert, Matthias Lang, Johannes Maier, Moritz Meyer-ter-Vehn, Steven Tadelis, Colin von Negenborn, Georg Weizsäcker, and audiences at Ann Arbor (Michigan), Berkeley, EUI Florence, Harvard/MIT, Princeton, Stanford, Toulouse, Tutzing, and UCLA. research assistance. Financial support by Deutsche Forschungsgemeinschaft through CRC TRR 190 is gratefully acknowledged, as well as support by the 35 crowdfunders who pledged in my Kickstarter campaign "An Economic Theory of Crowdfunding" (`https://www.kickstarter.com/projects/682028702/an-economic-theory-of-crowdfunding/description`).

1 Introduction

Crowdfunding has attracted much attention in recent years as a new mode of financing entrepreneurs: through the internet, many individuals – the crowd – provide funds directly to the entrepreneur rather than through a financial intermediary to whom the task to oversee the investment is delegated.[1] Given the typical agency problems associated with entrepreneurial financing, the popularity of crowdfunding is surprising.[2] In particular, Diamond's (1984) seminal paper suggests that crowdfunding cannot handle agency problems well because, due to the large number of investors, the free-riding problem and duplication costs in monitoring are especially severe.

However, popular crowdfunding platforms like Kickstarter and Indiegogo not only dispense with the financial intermediary, they also change the returns to investment. Instead of promising a monetary return, they promise investors only the good which the entrepreneur intends to develop. Hence, with these so-called *reward-based* crowdfunding schemes the entrepreneur's consumers become her investors. Therefore, a crowdfunding platform, next to eliminating the financial intermediary, provides innovation in that it allows an entrepreneur to contract with her future consumers *before* investments are sunk.

The objective of this paper is to show that this latter innovation has an important efficiency effect that persists despite the presence of moral hazard and private cost information. The basic intuition behind the efficiency gain is straightforward.[3] By directly addressing consumers, the contract can elicit their demand and, thereby, obtain information about whether aggregate demand is large enough to cover the project's investment costs. Hence, by conditioning the investment decision on this information, crowdfunding has the potential to yield more efficient investment decisions.

In the presence of entrepreneurial moral hazard, it is, however, not clear whether the contracting parties can actually realize this potential efficiency gain. Due to private information, consumers have to be given incentives to honestly reveal their demand and, due to moral hazard, the entrepreneur has to be given incentives to properly invest. Private information about investment and production costs leads to additional complications. All these incentive problems may thwart the efficiency effect of crowdfunding to reduce demand uncertainty.

This leads us to our central research question. Defining crowdfunding covenants as contracts between an entrepreneur and her consumers before the project's investment, we

[1] Mollick (2014) explicitly defines crowdfunding as ventures "without standard financial intermediaries."

[2] The Economist (2012) reports: "talk of crowdfunding as a short-lived fad has largely ceased." Regulatory reforms such as SEC (2015) indicate that also regulators expect crowdfunding to persist.

[3] Agrawal et al. (2014) and Belleflamme et al. (2015) discuss this effect informally, while Chang (2015), Chemla and Tinn (2016), Ellman and Hurkens (2015), Gruener and Siemroth (2015), Schwienbacher (2015), and this paper model it explicitly.

investigate the potential of such contracts to implement efficient and profit-maximizing allocations. In particular, we characterize (constrained) efficient outcomes in the presence of entrepreneurial moral hazard, consumers' private information about demand, and the entrepreneur's private information about her cost structure. We moreover argue that popular crowdfunding schemes reflect their properties.

By modeling entrepreneurial moral hazard as the entrepreneur's ability to embezzle investment funds, we obtain our first two insights. First, deferred payments are the primary tool to control moral hazard. Second, the entrepreneur should be given as little as possible information on the size of these deferred payments. Intuitively, moral hazard is prevented if the entrepreneur expects the deferred payments for completing the project to exceed her payoff from embezzling funds, and providing information about their size makes it harder to control the entrepreneur's embezzlement.

Because the deferred payments contain the rents for the entrepreneur that are necessary to guarantee incentive-compatibility, they represent agency costs that augment the project's implementation costs. We show that they are strictly positive only in the presence of moral hazard, while private information about the project's cost can solely amplify them. This suggests the pecking order that, in crowdfunding, entrepreneurial moral hazard is a first-order problem, whereas private cost information is of second order.[4]

More precisely, we show that an investment policy is consistent with the entrepreneur's incentives if its expected returns exceed its augmented implementation costs. We call such investments *affluent* and identify such affluence as the crucial concept for both optimality and implementability. That is, optimal contracts implement efficient outcomes if and only if the efficient investment policy is affluent. More generally, the second-best investment policy maximizes aggregate surplus under the restriction that it is affluent. If first-best investment is not affluent then the second-best distorts investment decisions downwards to ensure affluency. These downward distortions raise the level of deferred payments and ensure that the investment policy is consistent with the entrepreneur's incentives.

In addition to characterizing optimal mechanisms in our theoretical benchmark, we argue that popular crowdfunding schemes indirectly implement their two main features – they induce deferred payments and limit the entrepreneur's information about their size. In order to identify the critical components by which crowdfunding schemes achieve this, we first describe the crowdfunding scheme as offered by Kickstarter, the most successful crowdfunding platform to date.

The entrepreneur is first asked to describe the following three elements of her project on Kickstarter's public webpage: 1) a description of the *reward* to the consumer, which is typically the entrepreneur's final product; 2) a *pledge level* p; and 3) a *target level* T. After describing these elements, the crowdfunding campaign starts and, for a fixed

[4]The amplification effect of private cost information may however be substantial.

period of time – usually 30 days – a consumer can pledge the amount p to support the project financially. During the campaign, Kickstarter provides accurate information on the aggregate level of pledges so that a consumer can, in principle, condition his decision to pledge on the contributions of previous consumers.

After the campaign ends, Kickstarter compares the target level T to the sum of pledges $P \equiv \tilde{n} \cdot p$, where $\tilde{n}$ is the number of pledging consumers. If aggregate pledges P fall short of the target level T, Kickstarter declares the crowdfunding campaign a failure and cancels the project. In this case, consumers do not pay their pledges and the entrepreneur has no obligation to invest. If aggregate pledges P exceed the target level T, Kickstarter declares the crowdfunding campaign a success. Only in this case Kickstarter collects the pledges from consumers and transfers them to the entrepreneur who, in return, develops the product and delivers the rewards. In the parlance of crowdfunding, Kickstarter uses an *all-or-nothing* reward-based crowdfunding scheme where the entrepreneur receives "all" if the campaign is successful and "nothing" in case of failure.

In line with our theoretical results, the all-or-nothing target is the crucial feature by which the scheme conditions investment on revealed demand.[5] When first-best investment is affluent then the optimal crowdfunding target corresponds to one that induces efficient investment. If first-best investment is not affluent, then the optimal target level has to be set inefficiently high so that underinvestment results.

Kickstarter's scheme itself does not, however, use deferred payments; as stated in its guidelines, Kickstarter transfers all pledges to the entrepreneur directly after the campaign. Yet, because entrepreneurs, following a successful crowdfunding campaign, sell their good also to non-crowdfunding consumers in an after-market, the overall crowdfunding mechanism should be viewed as a combination of the platform's scheme together with this after-market.

This combination allows an implicit but natural implementation of deferred payments as follows. Consumers who value the good start pledging, but as soon as they observe that the target has been reached, they stop and wait to buy the product in the after-market instead. Hence, in practice the deferred payments that are crucial for controlling moral hazard come from the consumers in the after-market. In addition, the consumers' conditional pledging behavior ensures that the entrepreneur learns only that demand is high enough to make the project profitable, but not by how much. Our formal results show that, in the presence of entrepreneurial moral hazard, this reflects an optimal degree

[5] All-or-nothing schemes are commonly used by platforms that focus on for-profit projects (e.g., Kickstarter, Sellaband, and PledgeMusic). Platforms that focus on non-profit projects (e.g., GoFundMe) often use the alternative "keep-what-you-raise" system, where pledges are triggered even if the target level is not reached. Indiegogo, which describes itself as both a for-profit and non-profit platform, actually gives the entrepreneur the choice between using the all-or-nothing (fixed funding) or the keep-what-you-raise (flexible funding) model.

of information revelation.[6]

A crucial feature of the Kickstarter scheme is, therefore, that the level of aggregate contributions is accurately reported while the campaign is active. Only this allows consumers to use a conditional pledging strategy by which they can actively mitigate threats of moral hazard. The conditional pledging behavior is also consistent with the empirical evidence on crowdfunding. For instance, Mollick (2014, p. 6) observes that "projects that succeed tend to do so by relatively small margins." Moreover, more specialized crowdfunding platforms such as PledgeMusic, whose after-markets are arguably small, explicitly use deferred payments.[7]

Finally, we note that as crowdfunding schemes themselves are, in the presence of moral hazard, unable to attain full efficiency in general, they complement rather than substitute traditional forms of venture capital – the strength of crowdfunding lies in learning about demand, whereas the advantage of venture capitalists (or banks) lies in controlling entrepreneurial moral hazard. Although we do not model it explicitly, we point out that this complementarity suggests a sequential financing strategy for entrepreneurs. The entrepreneur first approaches a venture capitalist (VC). The VC invests only if he is convinced that demand for the product is high. If not, the VC turns down the project and the entrepreneur starts a crowdfunding campaign with a target that, due to the large moral hazard problems associated with crowdfunding, has to be set at an inefficiently high level. After a successful campaign, the entrepreneur finances her project through crowdfunding. If, however, the crowdfunding campaign falls short of its target by a relatively small amount then this reveals that, in principle, demand is high enough to make the project profitable, but not high enough to also cover the high agency costs associated with crowdfunding. Given that the VC has better means to control moral hazard and, therefore, has smaller agency costs, the entrepreneur may then return to the VC to obtain funds. Indeed, Kickstarter reports that "78% of projects that raised more than 20% of their goal were successfully funded."[8]

The rest of the paper is organized as follows. The next section discusses the related literature. Section 3 introduces the setup and takes an intuitive approach that identifies the main trade-offs. Section 4 sets up the problem as one of mechanism design and characterizes (constrained) efficient mechanisms. Section 5 relates optimal mechanisms to real-life crowdfunding mechanisms and examines extensions. Section 6 concludes. All formal proofs are collected in the appendix.

[6] If efficient investment is affluent and marginal costs are zero, then a dynamic scheme with the efficient target actually simultaneously handles optimal deferral of payments and optimal information limitation. If efficient investment satisfies a slightly stricter affluency condition (i.e., the right hand side of (44) is to be divided by $1-c$), then it also does so for strictly positive marginal costs ($c > 0$).

[7] As we argue in the extensions, deferred payments are also obtained from consumers, who for some exogenous reason cannot participate in crowdfunding but can acquire the product only in the after-market.

[8] See `https://www.kickstarter.com/help/stats`, last retrieved Sep. 10, 2016.

2 Related literature

Being a relatively new phenomenon, the economic literature on crowdfunding is small but growing. Agrawal et al. (2014) and Belleflamme et al. (2015) discuss crowdfunding's economic underpinnings. Potential benefits stem from reducing demand uncertainty and using crowdfunding as a tool for price discrimination, whereas dealing with entrepreneurial moral hazard and informational asymmetries present crowdfunding's main challenges. Subsequent theoretical literature has studied these issues in closer detail by modeling them formally. In contrast to the current paper, most of this literature does not take a full-fledged mechanism design approach, but compares specific crowdfunding schemes as used in practice.[9]

Most closely related is Chemla and Tinn (2016), who likewise focus on the problem of entrepreneurial moral hazard. While these authors also analyze a model of demand uncertainty with binary consumer valuations, the analysis differs in two respects. First, rather than taking a general mechanism design approach, the authors compare two specific reward-based crowdfunding mechanisms: take-it-all vs. all-or-nothing schemes. Second, they assume that if consumers are indifferent between the option to crowdfund and the option to wait for the after market, they crowdfund with probability 1. At first sight, this deterministic tie-breaking rule seems innocuous, but its assumption effectively deprives consumers of an important tool for controlling moral hazard: conditional pledging. More precisely, our insight that the dynamic schemes which crowdfunding platforms use in practice allow consumers to use conditional pledging strategies can, in a static framework, not be represent by a deterministic tie-breaking rule. As a result, the tie-breaking rule affects economic results significantly. In particular, the conclusion in Chemla and Tinn (2016) that the share of consumers who have access to the campaign must be sufficiently low to prevent moral hazard does not hold with more general tie-breaking rules.

Similarly to Chemla and Tinn (2016), Chang (2015) also compares take-it-all and all-or-nothing reward-based crowdfunding schemes, but in a pure public good setup in which consumers have an identical but initially unknown valuation for the good. In line with Chemla and Tinn (2016), he shows that these crowdfunding schemes withstand moral hazard provided that only a small enough measure of consumers can acquire the good through crowdfunding.

Abstracting from moral hazard, Ellman and Hurkens (2015) study the benefits of crowdfunding as a tool for both price discrimination and for reducing demand uncertainty (market testing). They point out that the usual practice of crowdfunding to condition a campaign's success on the sum of pledges is generally suboptimal. This is in line with

[9] Most papers also model the entrepreneur as the principal, who offers a profit-maximizing crowdfunding scheme to consumers. This approach cannot handle well private cost information, because such private information renders the model in a signaling game (or informed principal problem).

earlier results of Cornelli (1996), who shows that profit-maximizing mechanisms condition the investment decision on the *composition* of aggregate contributions rather than the *sum* of aggregate contributions. While Cornelli (1996) obtains this result with a continuum of consumer types, Ellman and Hurkens (2015) refine this result by considering discrete types. Using mechanism design, they show that conditioning investment on the sum of pledges is generally only optimal with two types.

These results suggest that popular crowdfunding platforms provide entrepreneurs only with suboptimal tools for price discrimination and that their main efficiency effect lies in the reduction of demand uncertainty.[10] Gruener and Siemroth (2015) study this effect of crowdfunding with correlated signals and in the presence of wealth constraints. Hakenes and Schlegel (2015) investigate the incentives of potential consumers to actively acquire private information, which a firm subsequently elicits through a crowdfunding scheme.

Apart from the recent literature on crowdfunding, there is surprisingly little work in economics and finance that focuses on the firm's ability to learn about the value of its projects by addressing consumers directly. In contrast, the marketing literature explicitly addresses this issue in its subfield of market research, focusing on consumer surveys and product testing (e.g., Lauga and Ofek, 2009). Ding (2007), however, points out that market research mainly relies on voluntary, non-incentivized reporting by consumers. He emphasizes that consumers need to be given explicit incentives for revealing their information truthfully. In line with this view, we point out that crowdfunding schemes naturally provide explicit incentives for such truthtelling.

While most empirical crowdfunding studies aim at identifying the crucial features of successful crowdfunding campaigns, two studies explicitly address moral hazard. In particular, Mollick (2014) finds little evidence of fraud in reward-crowdfunding, indicating that, consistent with our results, this type of crowdfunding is able to handle potential moral hazard problems. In contrast, Hildebrand et al. (2016) identify an increased problem of moral hazard for investment-based crowdfunding. Moreover, Mollick and Kuppuswamy (2014) report in a survey of crowdfunding projects that "To see if there was demand for the project" was the most agreed-upon reason for entrepreneurs to use crowdfunding. Viotto da Cruz (2016) provides empirical support for the idea that entrepreneurs use reward-based crowdfunding to learn about market demand.

[10] Although suboptimal, popular schemes nevertheless enable firms to price-discriminate consumers to some degree. For example, Belleflamme et. al (2014) argue that crowdfunding allows a discrimination between consumers who obtain an additional benefit from participating in crowdfunding and those who do not.

3 A model of crowdfunding

In this section, we introduce the framework and develop some preliminary insights. The framework considers an entrepreneur who, prior to her investment decision, directly interacts with privately informed consumers about whether they value the product. In order to clearly demonstrate the potential of crowdfunding, we first model and discuss the role of demand uncertainty and, only in a second step, introduce entrepreneurial moral hazard and private information regarding the cost structure.

The entrepreneur. We consider a penniless entrepreneur who needs an upfront investment of $I > 0$ to develop her product. After developing it, the entrepreneur can produce the good at some marginal cost $c \in [0, 1)$. The entrepreneur is crucial for realizing the project and cannot sell her idea to outsiders. We normalize interest rates to zero and abstract from any uncertainty concerning the development of the product.

The crowd. We consider a total of n consumers and denote a specific consumer by the index $i \in \mathcal{N} \equiv \{1, \ldots, n\}$. A consumer i either values the good, $v_i = 1$, or not, $v_i = 0$.[11] Hence, the n-dimensional vector $v = (v_1, \ldots, v_n) \in \mathcal{V} \equiv \{0, 1\}^n$ represents the valuation profile of the consumers. We let $\pi(v)$ denote its corresponding probability. As a result, the probability that $\tilde{n}$ consumers value the product is

$$\Pr\{\tilde{n}\} \equiv \sum_{\{v : \sum_{i \in \mathcal{N}} v_i = \tilde{n}\}} \pi(v).$$

Since the marginal costs c are smaller than 1, we can take $\tilde{n}$ as the potential demand of the entrepreneur's good. Its randomness expresses the demand uncertainty.

Investing without demand uncertainty. Consider as a benchmark the case of perfect information, where the realized demand $\tilde{n}$ is observable so that the investment decision can directly condition on it. It is socially optimal that the entrepreneur invests if the project's revenue, $\tilde{n}$, covers the costs of production $I + \tilde{n}c$, i.e., if

$$\tilde{n} \geq n^* \equiv \frac{I}{1 - c}.$$

In this case, the project generates an ex ante expected aggregate surplus of[12]

$$S^* = \sum_{\tilde{n} = \lceil n^* \rceil}^{n} \Pr\{\tilde{n}\}[(1 - c)\tilde{n} - I].$$

We assume that S^* is strictly positive. Note that by investing for $\tilde{n} > n^*$ and subsequently selling the good at a price $p = 1$, the entrepreneur can appropriate the full surplus. Given

[11] The binary structure ensures that demand uncertainty expresses itself only concerning the question of whether the entrepreneur should invest without affecting actual pricing decisions. It clarifies that the model's driving force is *not* price discrimination. This is discussed in more detail in Section 5.

[12] Let $\lceil x \rceil$ denote the smallest integer larger than x.

that the entrepreneur obtains the funds, this behavior represents her optimal strategy. Anticipating the entrepreneur's optimal behavior, a competitive credit market is willing to lend the amount I at the normalized interest rate of zero. Unsurprisingly, perfect information combined with a competitive credit market yield an efficient outcome.

Investing with demand uncertainty. Next, consider the setup with demand uncertainty, i.e., the entrepreneur must decide to invest I without knowing $\tilde{n}$. If she does invest, it clearly remains optimal to set a price $p = 1$. Hence, expected profits from investing are

$$\bar{\Pi} = \left(\sum_{\tilde{n}=0}^{n} \Pr\{\tilde{n}\}(1-c)\tilde{n} \right) - I.$$

It is therefore profitable to invest only if $\bar{\Pi} \geq 0$. Even though the price $p = 1$ does not leave any consumer rents, the entrepreneur's decision to invest leads either to under- or overinvestment. For parameter constellations such that $\bar{\Pi} < 0$, the entrepreneur will not invest and, hence, underinvestment results (because the good is not produced for any $\tilde{n} > n^*$, where it would be efficient to produce). For the parameter constellation $\bar{\Pi} \geq 0$, the entrepreneur does invest I, but this implies overinvestment (because she produces the good even when it turns out that $\tilde{n} < n^*$).

Crowdfunding without moral hazard. We next consider the case of demand uncertainty but with an "all-or-nothing, reward-based crowdfunding scheme" (p, T) as introduced in the introduction. That is, the investment is now governed by a contract pair (p, T) with the interpretation that if $\hat{n}$ consumers pledge so that the total amount of pledges, $P = \hat{n}p$, exceeds T, then the entrepreneur obtains it "all": she receives the pledges P, invests, and produces a good for each consumer who pledged. If the total amount of pledges P falls short of T then the entrepreneur obtains "nothing": the pledges are not triggered, the entrepreneur does not receive any funding, and she does not invest.

It is straightforward to see that this crowdfunding scheme enables the entrepreneur to extract the maximum aggregate surplus S^* and thereby achieve an efficient outcome. Indeed, for any $p \in (0, 1]$, it is optimal for the consumer to pledge p if and only if $v = 1$. As a result, exactly $\tilde{n}$ consumers sign up so that the sum of pledges equals $P = \tilde{n}p$. Hence, the project is triggered if and only if $T \leq \tilde{n}p$. It follows that an all-or-nothing crowdfunding scheme (p, T) with $p \in (0, 1]$ yields the entrepreneur the expected profit

$$\Pi^c(p, T) = \sum_{\tilde{n}=\lceil T/p \rceil}^{n} \Pr\{\tilde{n}\}[(p-c)\tilde{n} - I].$$

Clearly, a pledge level $p = 1$ and target level $T = n^*$ maximize profits, enabling the entrepreneur to extract the surplus S^*, and yield an efficient outcome.

Apart from stressing the surprisingly simple way in which the crowdfunding pair (p, T) resolves the problem of demand uncertainty, it is worthwhile to point out three additional features. First, for any possible outcome of the crowdfunding scheme, the consumers and entrepreneurs obtain at least their outside option from non-participation. Hence, there is no regret over participation after a consumer learns whether the campaign has been a success or a failure. Second, despite the presence of a crowd, the crowdfunding scheme circumvents any potential coordination problems. This is because of the scheme's third feature: it eliminates any problems with strategic uncertainty concerning both the behavior and the private information of other consumers. In other words, the all-or-nothing crowdfunding scheme $(p, T) = (1, n^*)$ respects ex post participation constraints and implements the first best in dominant strategies.

Moral hazard. The setup until now abstracted from problems of moral hazard; consumers are guaranteed the promised good if their pledge is triggered. In practice, consumers may, however, be concerned about whether the entrepreneur will deliver a good that meets the initial specifications – or even deliver the good at all.

We capture the problem of moral hazard by assuming that after the entrepreneur has obtained the money from the crowdfunding platform, she can "make a run for it" and thereby keep a share $\alpha \in [0, 1]$. When the entrepreneur "runs" she does not incur any investment or production costs and consumers do not obtain their goods. The share $(1-\alpha)$ is lost and represents a cost for running off with the money. Hence, the parameter α measures the weakness of the institutional environment to prevent moral hazard. For the extreme $\alpha = 0$, there is effectively no moral hazard, whereas for the extreme $\alpha = 1$, the entrepreneur can keep all the pledges without incurring any costs.

It is important to stress that this modeling approach captures several types of moral hazard problems. First, we can take the running literally: The entrepreneur is able to flee with the money and thereby keep a share αP without being caught. Or, alternatively, run off with the amount P but with an expected fine of $(1-\alpha)P$.[13] Second, at a reduced cost of $(1-\alpha)P < I - \tilde{n}c$ the entrepreneur can provide the consumer with a product that matches the formal description but is still worthless to the consumer.[14] Third, by a (possibly expected) cost $(1-\alpha)P$, the entrepreneur can convincingly claim that the project failed so that, without fear of any legal repercussions, she does not need to deliver

[13]For example, the project "Code hero" raised $170,954 but never delivered any rewards, Polygon.com states "His critics believe he has run off with the money raised from the Kickstarter campaign" (`https://www.polygon.com/2012/12/18/3781782/code-hero-kickstarter-interview` , last retrieved Sep. 10, 2016), whereas the campaign "Asylum Playing Cards" resulted in legal fines "against a crowdfunded project that didn't follow through on its promise to backers" (`http://www.atg.wa.gov/news/news-releases/ag-makes-crowdfunded-company-pay-shady-deal` , last retrieved Sep. 10, 2016).

[14]For example, the crowdfunding project "Healbe GoBe" caused much controversy about whether the delivered product actually worked (see `http://blog.belgoat.com/24-hours-with-my-healbe-gobe/`, last retrieved Sep. 10, 2016).

the product and she can keep the pledges.[15]

In order to see that moral hazard undermines the simple crowdfunding scheme, note that, facing aggregated pledges P, the entrepreneur obtains a payoff αP from running and a profit $P - I - cP/p$ from investing. Hence, she runs if

$$\alpha P > P - I - cP/p. \tag{1}$$

The inequality not only holds for the extreme $\alpha = 1$ but also for any $\alpha \geq 1 - c/p$. For all these cases, consumers anticipate that the entrepreneur will not deliver the product and so are not willing to participate in the crowdfunding scheme.

In the remainder of this section, we introduce two intuitive but ad hoc changes to the crowdfunding scheme (p, T) that reduce entrepreneurial moral hazard. Using a mechanism design approach, the next section proves that the two changes lead to mechanisms that are indeed optimal in the class of all possible mechanisms; even if we also consider that the entrepreneur is privately informed about her investment and production costs.

Deferred payments. An intuitive way to mitigate the moral hazard problem is to transfer the consumers' pledges to the entrepreneur only *after* having produced the good. Since the penniless entrepreneur needs at least the amount I to develop the product, such a delay in payments is possible only up to the amount I.

Hence, a first, ad hoc step toward mitigating the moral hazard problem is to adjust the crowdfunding scheme (p, T) and introduce deferred payments as follows. As before, the variable p represents the pledge level of an individual consumer and T the target level which the sum of pledges, P, has to meet before the investment is triggered. In contrast to our previous interpretation, however, the entrepreneur, after learning P, initially obtains only the required amount I for investing in the development of the product and receives the remaining sum $P - I$ only after having developed the product.

In order to characterize crowdfunding schemes with deferred payments that prevent moral hazard, note that the entrepreneur now obtains only the payoff αI from a run and the payoff $P - I - cP/p$ from realizing the project. With a pledge level $p = 1$, she has no incentive to run if

$$\alpha I \leq P - I - cP \Rightarrow P \geq \bar{P} \equiv \frac{(1+\alpha)I}{1-c} = (1+\alpha)n^*. \tag{2}$$

In particular, the deferred crowdfunding scheme $(p, T) = (1, \bar{P})$ leads to an equilibrium outcome in which the entrepreneur never runs. Using this scheme, the project is triggered

[15] For example, Kickstarter explicitly refers to this possibility: "If a creator is making a good faith effort to complete their project and is transparent about it, backers should do their best to be patient and understanding while demanding continued accountability from the creator." (`https://www.kickstarter.com/help/faq/kickstarter%20basics#Acco`, last retrieved Sep. 10, 2016).

when at least $T = \bar{P}$ consumers pledge so that it induces the entrepreneur to diligently complete the project.

Even though crowdfunding schemes with a deferred payment enable a prevention of moral hazard, they only do so with an inefficiently high target level T. That is because by taking the money and running, the entrepreneur can ensure a rent of at least αI. To induce the entrepreneur not to run, the project must therefore yield her a surplus of at least αI. Yet, by definition, the efficient threshold n^* is such that when completing the project diligently, the project yields a surplus of exactly zero. Raising the target level from n^* to $(1 + \alpha)n^*$ ensures that the entrepreneur obtains a rent if the target level is triggered. Nonetheless, an inefficiently high target level implies that the scheme exhibits underinvestment.

The information trade-off. We showed that a crowdfunding scheme with deferred payments can circumvent the moral hazard problem while still revealing the aggregate demand from consumers. Since this deferred crowdfunding scheme does not yield an efficient outcome, the question arises as to whether we can reduce the inefficiency with more sophisticated schemes.

We next argue that by limiting the information which the entrepreneur learns about demand, this is indeed possible. Given the argument that the fundamental benefit of crowdfunding is to reduce the entrepreneur's uncertainty about consumer demand, this may sound somewhat paradoxical. Note, however, that with respect to implementing the efficient investment decision, the entrepreneur only needs to learn whether $\tilde{n}$ is above or below n^*. That is, the exact value of $\tilde{n}$ is not important.

In contrast, providing the entrepreneur with full information about $\tilde{n}$ intensifies moral hazard because, with full information, a prevention of moral hazard requires inequality (2) to hold for any possible realization of $P \geq T$. In particular, it has to hold for the most stringent case $P = T$. As a result, a crowdfunding scheme (p, T) with $p = 1$ prevents moral hazard if and only if the target T exceeds the threshold $\bar{P}$.

However, if the entrepreneur learned only that P exceeds T rather than the exact value of P itself, then, with the pledge level $p = 1$, she would rationally anticipate an expected payoff $E[P|P \geq T] - I - cE[P|P \geq T]$ from not running. Hence, a crowdfunding scheme that reveals only whether P exceeds T prevents moral hazard if

$$E[P|P \geq T] \geq \bar{P}. \tag{3}$$

Since the conditional expectation $E[P|P \geq T]$ is at least T, condition (3) is weaker than condition (2). This implies that a partially informative crowdfunding scheme deals with moral hazard more effectively. In other words, it withstands moral hazard with a target level T below $\bar{P}$. Reducing the informativeness of the crowdfunding scheme therefore allows us to reduce inefficiencies.

This shows that the extraction of demand information interacts with the moral hazard problem and, in the presence of both demand uncertainty and moral hazard, the information extraction problem is sophisticated; the entrepreneur should learn neither too much nor too little.

The after-market. With the help of deferred payments, we first solved the moral hazard problem in crowdfunding and, subsequently, improved the scheme's efficiency by reducing the entrepreneur's information concerning these deferred payments. The resulting crowdfunding scheme (p, T) is an *information-restricted, payout-deferred, all-or-nothing reward-based crowdfunding scheme.*

At first sight, this hypothetical scheme seems to contradict how crowdfunding works in practice. Actual crowdfunding platforms such as Kickstarter offer a dynamic scheme and provide accurate and up-to-date information during the campaign about cumulative pledges. Moreover, almost all platforms hand out the collected pledges to the entrepreneur immediately after a campaign has successfully ended. Hence, crowdfunding platforms do not appear to use deferred payment or actively hide information about pledges from the entrepreneur.

On closer inspection, however, the dynamic schemes that current crowdfunding platforms offer and the information they provide allow an indirect implementation of deferred payments. This is because, in practice, participation in crowdfunding is not the only way for consumers to obtain the product. Indeed, once entrepreneurs have successfully developed the product, they also offer them for sale to non-pledging consumers in an "after-market."

Together with a dynamic crowdfunding scheme, this after-market allows an indirect implementation of the opaque deferred payments that underlie our hypothetical scheme because it enables consumers to use a conditional pledging strategy.[16] In particular, consumers who value the product line up and start pledging $p = 1$ sequentially, but, as soon as the target level is reached, pledging stops. All remaining consumers wait and buy the good at the sequentially rational price of 1 in the after-market.[17] This strategy is optimal, implements deferred payments, and leaves the entrepreneur uninformed about their exact size.

Hence, it is the specific dynamic structure of crowdfunding schemes in combination with the after-market by which popular crowdfunding platforms induce opaque deferred

[16] Since an explicit implementation of deferred payments by the crowdfunding platform requires the delivery of the product to be verifiable, this conditional pledging behavior also solves a potential enforceability issue of deferred payments.

[17] This argument hinges on the idea that consumers are willing to pledge up to a level p that makes them indifferent between pledging or buying in the after-market. Therefore, it also holds if the investment's success probability is commonly known to be less than one or, similarly, if there is discounting but pledging and non-pledging consumers obtain the good at the same time.

payments. In particular, a crucial feature of these schemes is that crowdfunding platforms report up-to-date information about the cumulative pledge level so that consumers can condition their decision to pledge on the current level of pledges.

This conditional pledging behavior is also consistent with the empirical observation of Mollick (2014) that successful campaigns tend to overshoot their targets only by relatively small margins. Moreover, specialized crowdfunding platforms such as PledgeMusic, whose after-markets are non-existent, explicitly use deferred payments to address problems of moral hazard.[18]

Private cost information. Importantly, the reward-based crowdfunding scheme as derived above conditions on the entrepreneur's investment I and her marginal cost c. The scheme therefore implicitly assumes that the entrepreneur's cost structure (I, c) is observable. It seems, however, natural that entrepreneurs are better informed about their cost structure than consumers or the crowdfunding platform. Also in practice, consumers and crowdfunding platforms reportedly worry that crowdfunding will attract fraudulent entrepreneurs who falsely claim to be able to manufacture some highly attractive product at some very low cost – whereas the true costs to manufacture such products are prohibitively high.[19] Taking this informational asymmetry seriously, an implementation of our crowdfunding scheme would then require the entrepreneur to first truthfully report her cost structure. The need for truthful revelation creates an additional incentive problem.

Note, however, that false reports about the cost structure are only profitable if the entrepreneur is able to run off with the money without incurring the true cost of the project. In other words, if there is no moral hazard ($\alpha = 0$) then the entrepreneur's private cost information does not matter because the entrepreneur has nothing to gain from misrepresenting her costs. More precisely, the aforementioned concerns about fraudulent entrepreneurs is that they make false claims *together* with the intention to run off with the money. The presence of moral hazard is therefore a prerequisite for private cost information to cause problems. In other words, entrepreneurial moral hazard is a first-order problem in crowdfunding, while private cost information is of second order.

Although only of second order, we show in the next section that private cost information may nevertheless have strong effects. It intensifies incentive problems because it allows the entrepreneur to use more sophisticated deviations. In the presence of both moral hazard and private cost information, crowdfunding must deal with the *double* deviation that the entrepreneur can combine lies about the cost structure with the intention

[18]PledgeMusic explicitly mentions that it uses deferred payments to prevent fraud: `http://www.pledgemusic.com/blog/220-preventing-fraud`, last retrieved Sep. 10, 2016.

[19]To prevent such false claims, Kickstarter, for instance, requires the entrepreneurs of gadgets to show consumers an explicit prototype. As we discuss in more detail in footnote 28, the suspended crowdfunding campaign "Skarp" illustrates that Kickstarter implements these rules rigorously.

of taking the money and running. These double deviations are, however, rather intricate and we present their formal analysis in the next section.

To summarize, we argued in this section that our hypothetical *information-restricted, payout-deferred, all-or-nothing, reward-based crowdfunding schemes* can deal with moral hazard effectively. They are, moreover, consistent with and shed light on the specific features of popular crowdfunding schemes in practice. Yet, two open questions remain. First, in what sense are these schemes optimal? Second, do they also remain feasible in the presence of private cost information? The next section addresses these two questions by studying the crowdfunding problem as one of optimal mechanism design. Based on the model considered in this section, it analyzes an economic environment with both entrepreneurial moral hazard and private cost information, in which the hypothetical scheme that we derived in this section is generally optimal in terms of both efficiency and profits.

4 Crowdfunding and mechanism design

In this section we analyze the entrepreneur's problem as one of mechanism design and formalize the idea that payout-deferred, information-restricted, all-or-nothing reward-based crowdfunding schemes implement optimal allocations.

4.1 The mechanism design setup

In order to treat the entrepreneur's moral hazard, we use the framework of Myerson (1982), which handles both ex ante private information and moral hazard. This generalized framework introduces a mediator who coordinates the communication between economic agents and gives incentive-compatible recommendations concerning the unobservable actions that underly the moral hazard problem. One of the insights from this analysis is that crowdfunding platforms play the role of a mediator exactly in the sense of Myerson (1982).

Economic allocations. In order to cast the entrepreneur's investment problem in a framework of mechanism design, we first formalize the economic allocations. In particular, crowdfunding seeks to implement an allocation between one cash-constrained entrepreneur, player 0, and n consumers, players 1 to n. We denote by $i \in \mathcal{N} \equiv \{1, \ldots, n\}$ a generic consumer. An allocation involves monetary transfers and production decisions. Concerning monetary transfers, consumers can make transfers to the entrepreneur both before and after the entrepreneur's investment decision. We denote the ex ante transfer from consumer i to the entrepreneur by t_i^a and the ex post transfer by t_i^p. Concerning the production decisions, the allocation describes whether the entrepreneur invests, $x_0 = 1$, or not, $x_0 = 0$, and whether the entrepreneur produces a good for consumer i, $x_i = 1$, or not, $x_i = 0$. Consequently, *an economic allocation* is a collection $a = (t, x)$ of transfers $t = (t_1^a, \dots, t_n^a, t_1^p, \dots, t_n^p) \in \mathbb{R}^{2n}$ and outputs $x = (x_0, \dots, x_n) \in \mathcal{X} \equiv \{0, 1\}^{n+1}$.

Feasible allocations. A defining feature of the crowdfunding problem is that the entrepreneur does not have the resources to finance the required investment $I > 0$. The entrepreneur's financial constraints imply the following feasibility restrictions on the available allocations. First, if the entrepreneur invests, the transfers from consumers must cover investment costs I. Second, the entrepreneur cannot make any net positive ex ante transfers to consumers. Finally, aggregate payments over both periods must be sufficient to cover the entrepreneur's investment and production costs. To express these feasibility requirements, we say that the allocation $a = (t, x)$ is *budget feasible* if

$$\textstyle\sum_{i \in \mathcal{N}} t_i^a \geq x_0 I \text{ and } \sum_{i \in \mathcal{N}} t_i^a + t_i^p \geq x_0 I + c \sum_{i \in \mathcal{N}} x_i. \tag{4}$$

In addition, an entrepreneur can only produce a good for a consumer if she invested. To express this feasibility requirement, we say that the output schedule x is *development-feasible* when the entrepreneur invested in its development if the good is produced for at least one consumer:

$$\exists i \in \mathcal{N} : x_i = 1 \Rightarrow x_0 = 1. \tag{5}$$

This condition logically implies that if $x_0 = 0$ then $x_i = 0$ for all i.

An allocation $a \in \mathcal{A} \equiv \mathbb{R}^{2n} \times \mathcal{X}$ is *feasible* if it satisfies (4) and (5).

Payoffs. Let the n-dimensional vector $v = (v_1, \dots, v_n) \in \mathcal{V} \equiv \{0, 1\}^n$ represent the valuation profile of consumers. We denote the probability of $v \in \mathcal{V}$ by $\pi(v)$ and the conditional probability of $v_{\text{-}i} \in \mathcal{V}_{-1} \equiv \{0, 1\}^{n-1}$ given v_i as $\pi_i(v_{\text{-}i}|v_i)$. Assuming that individual types are drawn independently, it holds that $\pi_i(v_{\text{-}i}|0) = \pi_i(v_{\text{-}i}|1)$ so that we can express the conditional probability simply as $\pi_i(v_{\text{-}i})$.[20] Moreover, we assume that consumers are identical: $\pi_i(v_{\text{-}k}) = \pi_j(v_{\text{-}k})$ for any $v_{\text{-}k} \in \mathcal{V}_{\text{-}1}$ and i, j.

In addition to the valuation profile v, the entrepreneur's cost structure (I, c) is independently drawn from a finite set of possible cost structures $\mathcal{K} \subset \mathbb{R}_+ \times [0, 1)$. Let $\rho(I, c)$ represent the probability that the entrepreneur's project has the cost structure (I, c), which is private information to the entrepreneur. For the special case, where $\mathcal{K}$ is a singleton, the entrepreneur has no private information concerning (I, c). This is the case we studied in the previous section.

A feasible allocation $a \in \mathcal{A}$ yields a consumer i with value v_i the payoff

$$U_i(a|v_i) = v_i x_i - t_i^a - t_i^p; \tag{6}$$

[20]Although we introduce an independence assumption here to avoid possible complications due to correlated private information, we stress that all our results hold also with correlated values. This is because, even with independence, the efficient scheme does not leave any information rents.

and it yields the entrepreneur with costs (I,c) the payoff

$$\Pi(a|I,c) = \textstyle\sum_{i\in\mathcal{N}}[t_i^a + t_i^p - x_i c] - x_0 I. \tag{7}$$

Efficiency. An output schedule $x \in \mathcal{X}$ is *Pareto efficient* in state (I,c,v) if and only if it maximizes the *aggregate net surplus*

$$S(x|I,c,v) \equiv \textstyle\sum_{i\in\mathcal{N}}(v_i - c)x_i - Ix_0 = \Pi(a|I,c) + \sum_{i\in\mathcal{N}} U_i(a|v_i).$$

With respect to efficiency, two different types of production decisions matter: the overall investment decision x_0 and the individual production decisions x_i. Given $v_l = 0 \leq c < v_h = 1$, efficiency with respect to the individual allocations requires $x_i = v_i$. This yields a surplus of $\sum_{i\in\mathcal{N}} v_i(1-c) - I$.

Defining

$$n^*(I,c) \equiv \frac{I}{1-c};\ \mathcal{V}^*(I,c) \equiv \{v : \sum_{i\in\mathcal{N}} v_i > n^*(I,c)\};\ \text{and}\ \pi^*(I,c) \equiv \sum_{v\in\mathcal{V}^*(I,c)} \pi(v),$$

we can characterize the Pareto *efficient output schedule* $x^* : \mathcal{K}\times\mathcal{V} \to \mathcal{X}$ as follows. For $v \in \mathcal{V}^*(I,c)$, it exhibits $x_0^* = 1$ and $x_i^* = v_i$ for all i. For $v \in \mathcal{V}\backslash\mathcal{V}^*(I,c)$, it exhibits $x_0^* = x_i^* = 0$ for all i.[21] Under an efficient output schedule, the entrepreneur invests only if $v \in \mathcal{V}^*(I,c)$, implying that $\pi^*(I,c)$ expresses the probability that the project is executed with cost structure (I,c).

Although transfers are immaterial for Pareto efficiency, we must nevertheless ensure that the efficient output schedule $x^*(I,c,v)$ can indeed be made part of some feasible allocation $a \in \mathcal{A}$. In order to specify one such feasible allocation, we define the *first-best* allocation $a^*(I,c,v) = (t^*(I,c,v), x^*(I,c,v))$ as follows. For $v \in \mathcal{V}^*(I,c)$, it exhibits $t_i^{a*}(I,c,v) = v_i$ and $t_i^{p*}(I,c,v) = 0$. For $v \in \mathcal{V}\backslash\mathcal{V}^*(I,c)$, $a^*(I,c,v)$ is defined by $t_i^{a*}(I,c,v) = t_i^{p*}(I,c,v) = 0$. By construction $a^*(I,c,v)$ is feasible and yields an ex ante expected gross surplus (gross of investment costs) of $W^* \equiv \sum_{(I,c)\in\mathcal{K}} \rho(I,c) W^*(I,c)$, where

$$W^*(I,c) \equiv \sum_{v\in\mathcal{V}^*(I,c)} \sum_{i\in\mathcal{N}} \pi(v) v_i (1-c). \tag{8}$$

For future reference, we say that an output schedule $x : \mathcal{K}\times V \to \mathcal{X}$ is *development efficient* if for all $(I,c,v) \in \mathcal{K}\times\mathcal{V}$,

$$x_0(I,c,v) = 1 \Rightarrow \exists i \in \mathcal{N} : x_i(I,c,v) = 1. \tag{9}$$

This condition is the converse of development feasibility (5). If it does not hold, it implies the inefficiency that there is a state (I,c,v) in which the entrepreneur invests I but

[21]For $\sum_{i\in\mathcal{N}} v_i = n^*(I,c)$, the output schedule $x_0^* = 1$, $x_i^* = v_i$ is also efficient, but this is immaterial (and can only arise for the non-generic case that I is a multiple of $1-c$).

no consumer consumes the good. Although technically feasible, a schedule that is not development efficient wastes the investment $I > 0$ and is not Pareto efficient.

For future reference, the following lemma summarizes these considerations.

Lemma 1 *The first-best allocation* $\{a^*(I,c,v) = (t^*(I,c,v), x^*(I,c,v))\}_{(I,c,v)\in\mathcal{K}\times\mathcal{V}}$ *is feasible and exhibits an output schedule that is development efficient. It yields an expected net surplus of* S^**, where*

$$S^* \equiv \sum_{(I,c)\in\mathcal{K}} \rho(I,c)\left[W^*(I,c) - \pi^*(I,c)I\right].$$

Mechanisms. We next turn to mechanisms. A *mechanism* Γ is a set of rules between the entrepreneur and the n consumers that induces a game. Its outcome is an allocation $a \in \mathcal{A}$ with payoffs $\Pi(a|I,c)$ and $U_i(a|v_i)$. In line with Myerson (1982), we interpret the crowdfunding platform as the mediator who runs the mechanism. It coordinates the communication between participants and enforces the rules the mechanism specifies for the game.

A *deterministic direct mechanism* is a function $\gamma : \mathcal{K}\times\mathcal{V} \to \mathcal{A}$, which induces the following game. First, the entrepreneur and consumers simultaneously and independently send a (confidential) report of their private information to the platform. Based on the collected reports (I^r, c^r, v^r) and in line with the rules γ, the platform collects the funds $T = \sum_{i\in\mathcal{N}} t_i^a(I^r, c^r, v^r)$ from the consumers and transfers them to the entrepreneur together with the recommendation $x_0(I^r, c^r, v^r)$ of whether to invest.

To capture the moral hazard problem, we explicitly assume that the platform cannot coerce the entrepreneur into following the recommendation $x_0 = 1$. That is, the entrepreneur is free to follow or reject it. If, however, the entrepreneur follows the recommendation, the platform enforces the production schedule $x(I^r, c^r, v^r)$ and the transfers $t_i^p(I^r, c^r, v^r)$. If the entrepreneur does not follow the recommendation to invest, but runs, then individual production schedules are 0, and no ex post transfers flow, i.e., $x_i = t_i^p = 0$. Moreover, consumers forfeit their ex ante transfers t_i^a, whereas the entrepreneur retains only the amount αT so that the amount $(1-\alpha)T$ is lost.

We can express a payout-deferred, information-restricted, all-or-nothing, reward-based crowdfunding scheme, as introduced in Section 3, by a direct mechanism $\gamma = (t, x)$ with the following structure. For each reported cost structure (I, c) there is a threshold $T(I,c) > 0$ such that for each reported $(i, v) \in \mathcal{N}\times\mathcal{V}$ it holds

$$x_0(I,c,v) = \begin{cases} 1 & \text{if } T(I,c) < n(v); \\ 0 & \text{if } T(I,c) > n(v); \end{cases} \quad x_i(I,c,v) = \begin{cases} v_i & \text{if } T(I,c) < n(v); \\ 0 & \text{if } T(I,c) > n(v); \end{cases} \tag{10}$$

and

$$(t_i^a(I,c,v), t_i^p(I,c,v)) = \begin{cases} (v_i I/n(v), v_i[1 - I/n(v)]) & \text{if } T(I,c) < n(v); \\ (0,0) & \text{if } T(I,c) > n(v), \end{cases} \tag{11}$$

where $n(v) \equiv \sum_{i \in \mathcal{N}} v_i$.

Hence, we say that a mechanism γ is a *crowdfunding mechanism* if for some threshold function $T : \mathcal{K} \to \mathbb{R}_+$ it satisfies (10) and (11), and in case $T(I,c) = \sum_{i \in \mathcal{N}} v_i$, it satisfies for this knife-edge either

$$(x_0(I,c,v), x_i(I,c,v), t_i^a(I,c,v), t_i^p(I,c,v)) = (0,0,0,0) \tag{12}$$

or

$$(x_0(I,c,v), x_i(I,c,v), t_i^a(I,c,v), t_i^p(I,c,v)) = (1, v_i, v_i I/n(v), v_i[1 - I/n(v)]) \,. \tag{13}$$

In order to also address stochastic direct mechanisms, we write direct mechanisms as distributions over deterministic direct mechanisms. That is, we define a (stochastic) *direct mechanism* as a collection $\Gamma = \{(p_l, \gamma_l)\}_{l \in \mathcal{L}} = \{(p_l, t_l, x_l)\}_{l \in \mathcal{L}}$ with $\mathcal{L} \equiv \{1, \ldots, L\}$, $\sum_{l \in \mathcal{L}} p_l = 1$, and for all $l \in \mathcal{L}$ it holds $p_l > 0$ and that γ_l is a deterministic direct mechanism. The interpretation of Γ is that, first, the platform draws the direct mechanism $\gamma_l = (t_l, x_l)$ with probability p_l and, subsequently, executes it as explained above. A direct mechanism is deterministic if $L = 1$.

In the following we show that efficient mechanisms do not require randomization. Moreover, it is also straightforward to argue that deterministic production decisions x_i and transfers t are optimal. Yet, as we explain in more detail below, the entrepreneur's private cost information may result in constrained efficient mechanisms that necessarily exhibit some minor randomization. These schemes randomize between (at most) two crowdfunding mechanisms with identical target functions $T(I,c)$ but with one that satisfies (12) and the other (13). This randomization is due to the discrete number of consumers.

Feasible mechanisms. We next introduce the concept of feasible mechanisms, which, in line with standard mechanism design, are direct mechanisms that are incentive-compatible and individual rational.

A direct mechanism γ is *incentive-compatible* if its induced game has a perfect Bayesian equilibrium in which 1) consumers are *truthful*; they reveal their values honestly, i.e., $v_i^r = v_i$, and 2) the entrepreneur is *truthful* and *(on path) obedient*; she reveals her costs (I,c) honestly and follows the recommendation upon honestly revealing her costs.[22]

To formalize the notion of truthful revelation by consumers for a (possibly random) mechanism Γ, it is helpful to define the conditional expected utilities given a reported value v_i^r

$$U_i^{\Gamma}(v_i^r|v_i) \equiv \sum_{l \in \mathcal{L}} p_l U_i^{\gamma_l}(v_i^r|v_i),$$

[22]In line with Myerson (1982) and the applicability of the revelation principle, obedience is imposed only on the equilibrium path.

where

$$U_i^\gamma(v_i^r|v_i) \equiv \sum_{(I,c)\in\mathcal{K}} \rho(I,c) U_i^\gamma(v_i^r|I,c,v_i) \text{ and } U_i^\gamma(v_i^r|I,c,v_i) \equiv \sum_{v_{\text{-}i}\in\mathcal{V}_{\text{-}i}} \pi_i(v_{\text{-}i}) U_i(\gamma(I,c,v_i^r,v_{\text{-}i})|v_i).$$

Consequently, we say that a mechanism $\Gamma = \{p_l, \gamma_l\}_{l\in\mathcal{L}}$ is *C-truthful* if

$$U_i^\Gamma(0|0) \geq U_i^\Gamma(1|0) \text{ and } U_i^\Gamma(1|1) \geq U_i^\Gamma(0|1), \forall i \in \mathcal{N}. \tag{14}$$

To formalize the notion of the entrepreneur's truthfulness and obedience, we define for a deterministic direct mechanism $\gamma = (t, x)$ the conditional profit

$$\Pi^\gamma(I^r, c^r|I,c,v) \equiv \Pi(\gamma(I^r,c^r,v)|I,c);$$

and the set $\mathcal{T}^\gamma(I,c)$ as the set of aggregate ex ante transfers which γ induces conditional on a recommendation of investment and the entrepreneur reporting the cost structure (I,c):

$$\mathcal{T}^\gamma(I,c) \equiv \{T|\exists v \in \mathcal{V} : \textstyle\sum_{i\in\mathcal{N}} t_i^a(I,c,v) = T \wedge x_0(I,c,v) = 1\}.$$

Given this set we define, for any $T \in \mathcal{T}^\gamma(I,c)$, the set $\mathcal{V}^\gamma(T|I,c)$ of valuation profiles for which the mechanism γ induces the recommendation to invest together with ex ante transfers T:

$$\mathcal{V}^\gamma(T|I,c) \equiv \{v \in \mathcal{V}|x_0(I,c,v) = 1 \wedge \textstyle\sum_{i\in\mathcal{N}} t_i^a(I,c,v) = T\}.$$

Hence, given a mechanism $\Gamma = \{(p_l, \gamma_l)\}_{l\in\mathcal{L}}$, the probability that the platform will send a recommendation to invest together with ex ante transfers T conditional on the entrepreneur reporting (I,c) is

$$P^\Gamma(T|I,c) \equiv \sum_{l\in\mathcal{L}} \sum_{v\in\mathcal{V}^{\gamma_l}(T|I,c)} p_l \pi(v).$$

After reporting the cost structure (I,c) and receiving a recommendation to invest together with an ex ante transfer $T \in \mathcal{T}^\Gamma(I,c) \equiv \cup_l \mathcal{T}^{\gamma_l}(I,c)$, the entrepreneur has some belief $\eta^\Gamma(v,l|T,I,c)$ that the consumers' valuation is v and the platform has picked the deterministic direct mechanism γ_l. This belief is Bayes-consistent if

$$\eta^\Gamma(v,l|T,I,c) = \begin{cases} \dfrac{p_l\pi(v)}{P^\Gamma(T|I,c)} & \text{if } (v,l) \in \mathcal{V}^{\gamma_l}(T|I,c) \times \mathcal{L}; \\ 0 & \text{otherwise.} \end{cases} \tag{15}$$

After reporting the cost structure (I^r, c^r), obtaining the recommendation to invest and an ex ante transfer T, an entrepreneur with cost structure (I,c) anticipates obtaining a profit of

$$\Pi_o^\Gamma(T|I,c,I^r,c^r) \equiv \sum_{l\in\mathcal{L}} \sum_{v\in\mathcal{V}} \eta^\Gamma(v,l|T,I^r,c^r) \Pi^{\gamma_l}(I^r,c^r|I,c,v)$$

from obediently following the recommendation to invest.

Finally, we can express the (maximum) expected profit of an entrepreneur with cost structure (I, c) who reports (I^r, c^r) as

$$\begin{aligned}\Pi^{\Gamma}(I^r, c^r|I, c) \;\equiv\; & \sum_{T \in \mathcal{T}^{\Gamma}(I^r, c^r)} P^{\Gamma}(T|I^r, c^r) \max\{\Pi_o^{\Gamma}(T|I, c, I^r, c^r), \alpha T\} \\ & + \sum_{l \in \mathcal{L}} \sum_{\{v: x_{l0}(I^r, c^r, v) = 0\}} p_l \pi(v) \Pi^{\gamma_l}(I^r, c^r|I, c, v). \qquad (16)\end{aligned}$$

The first term in this expression collects the events that after reporting the cost structure (I^r, c^r), the entrepreneur receives the recommendation to invest, together with some transfer T. In this case, she can decide whether to follow the recommendation or take the money and run. The maximum-operator reflects the entrepreneur's optimal decision given her updated belief after receiving the transfer T. The second term collects the events of the entrepreneur receiving the recommendation not to invest.

With this notation, we say that a mechanism $\Gamma = \{(p_l, \gamma_l)\}_{l \in \mathcal{L}}$ is *(on path) obedient* if an entrepreneur, who reveals her cost structure (I, c) honestly, is better off investing than she would be if she took the money and ran:

$$\Pi_o^{\Gamma}(T|I, c, I, c) \geq \alpha T, \text{ for all } T \in \mathcal{T}^{\Gamma}(I, c) \text{ and } (I, c) \in \mathcal{K}. \qquad (17)$$

Moreover, we say that a mechanism $\Gamma = \{(p_l, \gamma_l)\}_{l \in \mathcal{L}}$ is *E-truthful* if

$$\Pi^{\Gamma}(I, c) \geq \Pi^{\Gamma}(I^r, c^r|I, c), \text{ for all } (I, c, I^r, c^r) \in \mathcal{K} \times \mathcal{K}, \qquad (18)$$

with $\Pi^{\Gamma}(I, c) \equiv \Pi^{\Gamma}(I, c|I, c)$.

We say that a direct mechanism is *incentive-compatible* if and only if it is C-truthful, E-truthful, and on path obedient.

As participation is voluntary it must yield the consumers and the entrepreneur at least their outside option. Taking these outside options as 0, the entrepreneur's participation is not an issue because any feasible allocation yields the entrepreneur a non-negative payoff. Hence, her participation constraint is satisfied for every outcome and therefore even in an ex post sense.

In contrast, a feasible allocation does not guarantee that a consumer will obtain his outside option of zero. As noted in the previous section, all-or-nothing crowdfunding schemes have, in the absence of moral hazard, the attractive feature that they respect participation constraints even after a consumer learns whether the crowdfunding campaign has been a success or a failure. Consumers, therefore, do not regret their participation. Yet, rather than imposing ex post participation constraints by assumption, we will assume a less restrictive form of participation. This allows us to show the extent to which ex post individual rationality of the optimal mechanism is a result rather than an assumption.[23] In particular, we assume that the consumer has to receive his outside option conditional on his own type and the project's cost structure. Formally, we say that an incentive-compatible direct mechanism is *individual rational* if for all $(i, I, c) \in \mathcal{N} \times \mathcal{K}$ it holds

$$U_i^\Gamma(I, c|0) \equiv \textstyle\sum_{l \in \mathcal{L}} p_l U_i^{\gamma_l}(0|I, c, 0) \geq 0; \tag{19}$$

and

$$U_i^\Gamma(I, c|1) \equiv \textstyle\sum_{l \in \mathcal{L}} p_l U_i^{\gamma_l}(1|I, c, 1) \geq 0. \tag{20}$$

To summarize, we say that a mechanism Γ is (strictly) *feasible*, if it is incentive-compatible and individual rational and its induced allocations $a(I, c, v)$ are feasible. Following the previous definitions, $\Gamma = \{(p_l, \gamma_l)\}_{l \in \mathcal{L}}$ with $\gamma_l = (t_l, x_l)$ is feasible if and only if it satisfies the constraints (21)–(29):

$$\textstyle\sum_{i \in \mathcal{N}} t_{li}^a(I, c, v) \geq x_{l0}(I, c, v)I, \ \forall (l, I, c, v); \tag{21}$$

$$\textstyle\sum_{i \in \mathcal{N}} t_{li}^a(I, c, v) + t_{li}^p(I, c, v) \geq x_{l0}(I, c, v)I + c \sum_{i \in \mathcal{N}} x_{li}(I, c, v), \ \forall (l, I, c, v); \tag{22}$$

$$\exists i \in N : x_{li}(I, c, v) = 1 \Rightarrow x_{l0}(I, c, v) = 1, \ \forall (l, I, c, v); \tag{23}$$

$$U_i^\Gamma(0|0) \geq U_i^\Gamma(1|0), \ \forall i; \tag{24}$$

$$U_i^\Gamma(1|1) \geq U_i^\Gamma(0|1), \ \forall i; \tag{25}$$

$$\Pi_o^\Gamma(T|I, c, I, c) \geq \alpha T, \ \forall (I, c), \forall T \in \mathcal{T}^\Gamma(I, c); \tag{26}$$

$$\Pi^\Gamma(I, c) \geq \Pi^\Gamma(I', c'|I, c), \ \forall (I, c, I', c'); \tag{27}$$

$$U_i^\Gamma(I, c|0) \geq 0, \ \forall (i, I, c); \tag{28}$$

$$U_i^\Gamma(I, c|1) \geq 0, \ \forall (i, I, c). \tag{29}$$

A feasible mechanism Γ yields an ex ante net aggregate surplus of

$$S^\Gamma \equiv \sum_{l \in \mathcal{L}} \sum_{(I, c) \in \mathcal{K}} p_l \rho(I, c) S^{x_l}(I, c),$$

with

$$S^x(I, c) \equiv \sum_{v \in \mathcal{V}} \pi(v) S(x(I, c, v)|I, c, v) = \sum_{v \in \mathcal{V}} \pi(v) \left[\sum_{i \in \mathcal{N}}^{n} (v_i - c) x_i(I, c, v) - I x_0(I, c, v) \right].$$

Similarly, a feasible mechanism Γ yields the entrepreneur with costs (I, c) an expected payoff $\Pi^\Gamma(I, c) = \sum_{l \in \mathcal{L}} p_l \Pi^{\gamma_l}(I, c)$ with

$$\Pi^{\gamma_l}(I, c) \equiv \textstyle\sum_{v \in \mathcal{V}} \pi(v) \Pi(\gamma_l(I, c, v)|I, c).$$

[23] This approach also mitigates the problem that the extent to which ex post individual rationality constrains crowdfunding platforms is unclear. As it turns out, in practice it is possible for consumers to not honor their pledges after a successful campaign. For instance, Kickstarter lists such "dropped pledges" in its receipt to entrepreneurs and deducts them from the entrepreneur's transfer.

Finally, we say that two feasible mechanisms Γ and Γ' are *payoff-equivalent* if they lead to identical payoffs for each consumer type v_i in each cost state:

$$U_i^{\Gamma}(I, c|v_i) = U_i^{\Gamma'}(I, c|v_i), \text{ for all } (i, I, c, v_i) \in \mathcal{N}\times\mathcal{K}\times\{0, 1\};$$

and for each entrepreneur type (I, c):

$$\Pi^{\Gamma}(I, c) = \Pi^{\Gamma'}(I, c), \text{ for all } (I, c) \in \mathcal{K}.$$

Implementability. A (stochastic) *allocation function* $f : \mathcal{K}\times V \to \Delta\mathcal{A}$ specifies for any cost structure (I, c) and any value profile v a distribution over the feasible allocations $a \in \mathcal{A}$. It is *implementable* if there exists some (not necessarily direct) mechanism such that the induced game has a perfect Bayesian equilibrium outcome in which, for each $(I, c, v) \in \mathcal{K}\times\mathcal{V}$, the allocation coincides with $f(I, c, v)$. In this case, we say that the mechanism *implements* the allocation function f.

Likewise, a (stochastic) *output schedule* $x : \mathcal{K}\times\mathcal{V} \to \Delta\mathcal{X}$ specifies for any $(I, c, v) \in \mathcal{K}\times\mathcal{V}$ a probability distribution over output schedules $x \in \mathcal{X}$.[24] It is *implementable* if there exists some mechanism such that the induced game has a perfect Bayesian equilibrium outcome in which, for each $(I, c, v) \in \mathcal{K}\times\mathcal{V}$, the induced output coincides with $x(I, c, v)$. In this case, we say the mechanism *implements* output schedule $x(\cdot)$.

Appealing to the (mediated) revelation principle in Myerson (1982), an allocation function $f(\cdot)$ is implementable if and only if there exists a feasible mechanism Γ such that it implements f. Likewise, an output schedule $x(\cdot)$ is implementable if and only if there exists a feasible mechanism Γ such that it implements x. Hence, as usual, the revelation principle motivates incentive-compatibility as a defining feature of feasibility.

One question that initially arises is whether the efficient output schedule x^* is always implementable. Our first proposition confirms that this is not the case.

Proposition 1 *The efficient output schedule x^* is not always implementable.*

Intuitively, the inefficiency results from a tension between the entrepreneur's budget constraint and the moral hazard problem. So consumers can ensure that the entrepreneur realizes her project, simply giving her the required amount I does not suffice. Due to the moral hazard problem, she must also be given an explicit incentive to invest this amount properly and not run off with it. The proposition shows that for the efficient output schedule x^* this is, in general, not possible.

[24] In contrast, deterministic output schedules are functions $x : \mathcal{K}\times\mathcal{V} \to \mathcal{X}$.

4.2 Optimal allocations and mechanisms

A (possibly constrained) efficient mechanism $\breve{\Gamma} = \{(\breve{p}_l, \breve{t}_l, \breve{x}_l)\}_{l\in\mathcal{L}}$ maximizes S^Γ subject to constraints (21)–(29). In order to solve this maximization problem, we follow the usual approach in mechanism design to focus first on a relaxed maximization problem that considers only a subset – albeit the relevant subset – of the overall incentive and individual rationality constraints. In particular, we disregard the individual rationality constraint of the high valuation type (29) and replace the entrepreneur's truthtelling constraint (27) by

$$\Pi^\Gamma(I, c) \geq \sum_{T\in\mathcal{T}^\Gamma(I',c')} P^\Gamma(T|I', c')\alpha T, \ \forall(I, c, I', c'). \tag{30}$$

The constraint is weaker than (27), because its right-hand side is larger than the right-hand side of (30), whereas their left-hand sides are identical.[25]

Formally, we say that Γ is *weakly feasible* if it satisfies constraints (21)–(26), (28), and (30) and an output schedule $\breve{x} : \mathcal{K}\times\mathcal{V} \to \Delta\mathcal{X}$ is *weakly-implementable* if there exists a weakly feasible mechanism $\breve{\Gamma}$. A weakly feasible mechanism $\breve{\Gamma}$ is *optimal* if it maximizes S^Γ over all weakly feasible mechanisms.

In the following, we derive an optimal weakly feasible mechanism $\breve{\Gamma}$ with the feature that it is also (strictly) feasible. Hence, it also represents a constrained efficient mechanism $\breve{\Gamma}$. In particular, we show that such a mechanism is a crowdfunding mechanism, i.e., there is a threshold function $T(I, c)$ so that all the deterministic mechanisms γ_l in $\breve{\Gamma}$ satisfy (10)–(13).

We first derive a series of lemmas that allow us to simplify the maximization problem. The first lemma establishes the relatively intuitive result that development-efficiency is a necessary feature of optimal weakly feasible mechanisms.

Lemma 2 *A weakly feasible mechanism* $\breve{\Gamma} = \{(\breve{p}_l, \breve{t}_l, \breve{x}_l)\}_{l\in\mathcal{L}}$ *is optimal only if each* $\breve{x}_l$ *is development-efficient.*

The next lemma validates the suggestion of the previous section that, in order to optimally control entrepreneurial moral hazard, a mechanism uses deferred payments and limits the entrepreneur's information. In particular, it shows that development-efficiency is a sufficient condition under which it is optimal to initially provide the entrepreneur only with the investment amount I and, hence, minimize the information which she gleans from receiving a recommendation to invest. The result is an illustration of Myerson's general observation that, accompanying a recommendation, mediators should give agents only

[25] Referring to (16), it considers only one element within the maximum operator and for $x_0(I', c', v) = 0$, constraint (22) implies $\Pi^{\gamma_l}(I', c'|I, c, v) = \sum_{i\in\mathcal{N}}[t^a_{li}(I', c', v) + t^p_{li}(I', c', v)] \geq 0$.

the minimum information possible, as more information only makes it harder to satisfy incentive-compatibility.

Lemma 3 *Suppose* $\check{\Gamma} = \{(\check{p}_l, \check{t}_l, \check{x}_l)\}_{l\in\mathcal{L}}$ *is weakly feasible and* $\{\check{x}_l\}_{l\in\mathcal{L}}$ *are development-efficient. Then there are transfer schedules* $\{\hat{t}_l\}_{l\in\mathcal{L}}$ *such that (21) binds and the direct mechanism* $\hat{\Gamma} = \{(\check{p}_l, \hat{t}_l, \check{x}_l)\}_{l\in\mathcal{L}}$ *is weakly feasible and payoff equivalent, and (22) simplifies to*

$$\textstyle\sum_{i\in\mathcal{N}} t^p_{li}(I,c,v) \geq c\sum_{i\in\mathcal{N}} x_{li}(I,c,v), \forall(l,I,c,v) \in \mathcal{L}\times\mathcal{K}\times\mathcal{V}. \tag{31}$$

Because Lemma 2 shows that an optimal weakly feasible mechanism is development-efficient, there is no loss of generality in restricting attention to weakly feasible direct mechanisms that give the entrepreneur exactly the amount I if the entrepreneur is to develop the product.

Combining Lemmas 2 and 3 allows us to considerably simplify the optimization problem. Indeed, if the feasibility constraint (21) binds then $\mathcal{T}^\Gamma(I,c) = \{I\}$ so that the obedience constraint (26) has to hold only with regard to $T = I$. By defining, for an output schedule $x \in \mathbb{R}^{n+1}$, the set and probability

$$\mathcal{V}^x(I,c) \equiv \{v|x_0(I,c,v) = 1\} \text{ and } \pi^x(I,c) = \sum_{v\in\mathcal{V}^x(I,c)} \pi(v),$$

the obedience constraint (26) simplifies to

$$\sum_{l\in\mathcal{L}} \sum_{v\in\mathcal{V}^{x_l}(I,c)} \sum_{i\in\mathcal{N}} p_l\pi(v)(t^p_{li}(I,c,v) - cx_{li}(I,c,v)) \geq \sum_{l\in\mathcal{L}} p_l\pi^{x_l}(I,c)\alpha I, \forall(I,c)\in\mathcal{K}; \tag{32}$$

and the relaxed truthfulness constraint (30) to

$$\Pi^\Gamma(I,c) \geq \pi^\Gamma(I',c')\alpha I',\ \forall(I,c,I',c') \in \mathcal{K}\times\mathcal{K}, \tag{33}$$

where $\pi^\Gamma(I,c) \equiv \sum_{l\in\mathcal{L}} p_l\pi^{x_l}(I,c)$.

Following the previous two lemmas, there is no loss of generality to focus on weakly feasible mechanisms $\check{\gamma} = (\check{t}, \check{x})$ that satisfy (23), (24), (25), (28), (31), (32), and (33), and (21) in equality. Given this observation, we next prove that optimal weakly feasible mechanisms do not produce a product for consumers who do not value them.

Lemma 4 *A weakly feasible mechanism* $\check{\Gamma} = \{(\check{p}_l, \check{t}_l, \check{x}_l)\}_{l\in\mathcal{L}}$ *is optimal only if it holds that*

$$x_{il}(I,c,0,v_{\text{-}i}) = 0, \forall(l,i,I,c,v_{\text{-}i}) \in \mathcal{L}\times\mathcal{N}\times\mathcal{K}\times\mathcal{V}_{\text{-}i}. \tag{34}$$

The result sounds intuitive, since it implies that an optimal weakly feasible mechanism does not display any form of artificial inefficiency. It is, however, not immediate because, in general, artificial inefficiencies may help to relax incentive constraints. The next lemma shows that it also implies that there is no loss of generality in assuming that an optimal weakly feasible mechanism leaves no rents to consumers.

Lemma 5 *Suppose* $\check{\Gamma} = \{(\check{p}_l, \check{t}_l, \check{x}_l)\}_{l\in\mathcal{L}}$ *is weakly feasible and* $\check{x}_l$ *satisfies (34). Then there exists a weakly feasible mechanism* $\hat{\Gamma} = \{(\check{p}_l, \hat{t}_l, \check{x}_l)\}_{l\in\mathcal{L}}$ *which yields the same aggregate surplus* $S^{\check{\Gamma}}$ *and exhibits*

$$U_i^{\hat{\gamma}_l}(0|I,c,0) = U_i^{\hat{\gamma}_l}(1|I,c,1) = 0, \forall(l,i,I,c) \in \mathcal{L}\times\mathcal{N}\times\mathcal{K}. \tag{35}$$

The lemma provides the insight that optimal weakly feasible mechanisms extract all rents from consumers and assign them as revenues to the entrepreneur. The intuition as to why this rent extraction is optimal follows directly from the moral hazard problem: by giving all rents in the form of deferred payments to the entrepreneur, she has the least incentives to run with the money.

As we show in the next lemma, the rent extraction result implies that there is no conflict between maximizing the aggregate surplus and maximizing the entrepreneur's ex ante expected profits. In order to make this statement explicit, define for a mechanism $\Gamma = \{(p_l, \gamma_l)\}_{l\in\mathcal{L}} = \{(p_l, t_l, x_l)\}_{l\in\mathcal{L}}$ the entrepreneur's ex ante expected profits as

$$\Pi^\Gamma \equiv \sum_{l\in\mathcal{L}} \sum_{(I,c)\in\mathcal{K}} p_l \rho(I,c)\Pi^\gamma(I,c);$$

and the aggregate surplus in the cost state (I,c) as

$$S^\Gamma(I,c) \equiv \sum_{l\in\mathcal{L}} p_l S^{x_l}(I,c).$$

Lemma 6 *It is without loss of generality to assume that both an optimal weakly feasible mechanism* $\check{\Gamma} = \{(\check{p}_l, \check{t}_l, \check{x}_l)\}_{l\in\mathcal{L}}$ *maximizes the entrepreneur's ex ante expected profits* Π^Γ*, and exhibits* $S^{\check{x}_l}(I,c) = \Pi^{\check{\gamma}_l}(I,c)$ *for all* (l,I,c) *so that for all* (I,c) *it also holds that* $\Pi^{\check{\Gamma}}(I,c) = S^{\check{\Gamma}}(I,c)$*.*

To summarize, Lemmas 2 to 6 imply that, with respect to the optimal weakly feasible mechanism, it is without loss of generality to replace the constraints (21)–(29) with the following constraints

$$\sum_{i\in\mathcal{N}} t^a_{li}(I,c,v) = x_{l0}(I,c,v)I, \ \forall(l,I,c,v); \tag{36}$$

$$\sum_{i\in\mathcal{N}} t^p_{li}(I,c,v) \geq c\sum_{i\in\mathcal{N}} x_{li}(I,c,v), \forall(l,I,c,v); \tag{37}$$

$$\exists i\in\mathcal{N} : x_{li}(I,c,v) = 1 \Rightarrow x_{l0}(I,c,v) = 1, \ \forall(l,I,c,v); \tag{38}$$

$$U_i^{\gamma_l}(1|I,c,1) = 0, \ \forall(l,i,I,c); \tag{39}$$

$$U_i^{\gamma_l}(0|I,c,0) = 0, \ \forall(l,i,I,c); \tag{40}$$

$$\sum_{l\in\mathcal{L}} \sum_{v\in\mathcal{V}^{x_l}(I,c)} \sum_{i\in\mathcal{N}} p_l\pi(v)(t^p_{li}(I,c,v) - cx_{li}(I,c,v)) \geq \pi^\Gamma(I,c)\alpha I, \forall(I,c); \tag{41}$$

$$x_{li}(I,c,0,v_{\text{-}i}) = 0, \forall(l,I,c,v_{\text{-}i}); \tag{42}$$

$$S^\Gamma(I,c) \geq \pi^\Gamma(I',c')\alpha I', \ \forall(I,c,I',c'). \tag{43}$$

Constraint (43) effectively represents the entrepreneur's incentive constraint (18). The insight that the mechanism leaves all rents to the entrepreneur in order to optimally deal with the entrepreneur's moral hazard problem, enables us to rewrite this constraint as depending only on output schedules and not on transfers.

Since the deterministic version of this constraint turns out to play a key role for implementability, we say that an output schedule $x \in \mathbb{R}^{n+1}$ is *affluent* if for all $(I, c) \in \mathcal{K}$ it holds

$$S^x(I, c) \geq \Phi(x) \equiv \max_{(\tilde{I}, \tilde{c}) \in \mathcal{K}} \alpha \pi^x(\tilde{I}, \tilde{c})\tilde{I}. \tag{44}$$

We moreover denote by $(\bar{I}(x), \bar{c}(x))$ a maximizer of the right-hand side of (44). Note that for a deterministic mechanism $\Gamma = (1, \gamma_1) = (1, x_1, t_1)$, constraint (43) amounts to the requirement that x_1 is affluent. This leads to the following result.

Proposition 2 *The efficient output schedule x^* is implementable if and only if it is affluent. If implementable, a crowdfunding mechanism implements it and thereby maximizes both aggregate surplus and the entrepreneur's ex ante expected profits.*

The proposition identifies affluency as the crucial condition: it is both necessary and sufficient for the implementability of the efficient output schedule. The intuition behind this result is that the entrepreneur needs to receive a rent of at least $\Phi(x^*)$ to induce her to invest properly rather than employing the combined strategy of misreporting her cost structure and, subsequently, taking the money and running. Since the consumers ultimately pay this rent, the project then has to generate a surplus of at least $\Phi(x^*)$ so that the consumers' participation is still individual rational. The efficient output schedule x^*, however, only guarantees such a surplus if it is affluent.

More generally, we can interpret the required rent $\Phi(x)$ as the *agency costs* of implementing some output schedule x. To obtain more insights concerning the extent to which moral hazard and private cost information are responsible for these agency costs, note that if the entrepreneur cannot falsify her cost structure, the output schedule x induces the entrepreneur to invest if

$$S^x(I, c) \geq \Phi^m(x) \equiv \alpha \cdot \pi^x(I, c)I.$$

This suggests interpreting $\Phi^m(x)$ as the agency cost associated with moral hazard and the remaining part

$$\Phi^i(x) \equiv \Phi(x) - \Phi^m(x) = \alpha \cdot \left[\pi^x(\bar{I}(x), \bar{c}(x))\bar{I}(x) - \pi^x(I, c)I\right] \geq 0$$

as the agency cost associated with private information about the cost structure.

The proposition further shows that if there is no moral hazard problem ($\alpha = 0$), the efficient output schedule is implementable even if the entrepreneur has private information about the cost structure. In this case, agency costs $\Phi^m(x)$ and $\Phi^i(x)$ are both zero. Hence, private cost information alone does not lead to distortions in crowdfunding. This observation formalizes the insight of Section 3 that entrepreneurial moral hazard is a first-order problem in crowdfunding while private cost information is of second order.

It also demonstrates that the presence of private cost information does not alter the intuition behind the inefficiency result of Proposition 1. Effectively, the existence of a tension between the entrepreneur's budget constraint and the moral hazard problem remains solely responsible for the inefficiencies, and prevents the implementability of the efficient output.

Yet, even though private cost information by itself cannot lead to an inefficiency, it does, however, intensify the moral hazard problem. This is because with private cost information, consumers have to grant enough rents to prevent the *double* deviation of the entrepreneur combining lies about the cost structure with the intent to take the money and run. In the extreme, this multiplier effect destroys all potential benefits from crowdfunding. In particular, if there is a cost structure (I, c) in $\mathcal{K}$ for which $S^{x^*}(I, c) = 0$, then an affluent output schedule necessarily exhibits $\pi^x(\tilde{I}, \tilde{c}) = 0$ for all $(\tilde{I}, \tilde{c}) \in \mathcal{K}$. This means that crowdfunding is ineffective: for any demand realization and any cost structure, implementability implies $x_0 = 0$.

We next address the question of which constrained efficient output schedule is optimal when the efficient output schedule is not affluent. Note that affluency is a necessary condition for an implementable output schedule x. Hence, an intuitive approach toward finding the constrained efficient output level is to start with the efficient output x^* and adapt it to make it affluent. Because the efficient output x^* maximizes $S^x(\cdot)$ and, hence, the left-hand side of (44), such an adaptation requires a change in x that lowers its right-hand side. That is, the output schedule should decrease $\pi^x(\cdot)$. Effectively, this means lowering the likelihood that the entrepreneur will receive a recommendation to invest when reporting the cost structure $(\bar{I}(x), \bar{c}(x))$. Intuitively, this change reduces the profitability of the double deviation to misreport the cost structure as $(\bar{I}(x), \bar{c}(x))$ and subsequently take the money and run.

The required adaptation of x^* implies a downward distortion of the output schedule: the constrained efficient mechanism has to recommend the entrepreneur not to invest for some demand revelations that yield a positive surplus. Hence, lowering π^x comes at the cost of underinvestment. These costs are minimized when the mechanism makes the inefficient recommendation not to invest for those demand realizations that yield the least surplus. In terms of crowdfunding, this means that the crowdfunding target T is raised above the efficient one, as the demand realizations closest to target yield the least.

The reasoning to adapt x^* toward some affluent output schedule suggests that also the constrained efficient mechanism is a crowdfunding mechanism, but with an inefficiently high target T. Since the adaptation away from x^* comes at a cost, the crowdfunding target should be raised such that the affluency constraint (44) is just met. Due to the discreteness of the problem, this is generally not possible with deterministic output schedules. As a consequence, we cannot exclude the possibility that the optimal mechanism is stochastic and displays the minor form of randomness in that it randomizes between two crowdfunding schemes such that the affluency constraint is satisfied with equality.

In two steps, we formally confirm that the heuristic arguments presented above are correct. In a first lemma, we show that optimal weakly feasible mechanisms necessarily exhibit a single cutoff T for each cost structure (I, c). This implies that crowdfunding mechanisms implement them. Proposition 3 then shows that these weakly feasible mechanisms are actually (strictly) feasible.

Lemma 7 *A weakly feasible mechanism $\check{\Gamma} = \{(\check{p}_l, \check{t}_l, \check{x}_l)\}_{l \in \mathcal{L}}$ that satisfies (36)–(43) is optimal only if for each $(I, c) \in \mathcal{K}$ there exists some $T \in \mathcal{N}$ such that for all $(l, i, v) \in \mathcal{L} \times \mathcal{N} \times \mathcal{V}$ it holds*

$$\check{x}_{l0}(I, c, v) = \begin{cases} 1 & \text{if } n(v) > T; \\ 0 & \text{if } n(v) < T; \end{cases} \quad \text{and } \check{x}_{li}(I, c, v) = \begin{cases} v_i & \text{if } n(v) > T; \\ 0 & \text{if } n(v) < T. \end{cases} \tag{45}$$

The next proposition shows that any output schedule that satisfies (45), is actually implementable by a (strictly) feasible mechanism that, in addition to (36)–(43), also satisfies properties (11)–(13).

Proposition 3 *If the efficient output x^* is not affluent, the optimal allocation is constrained efficient. A crowdfunding mechanism implements it and thereby also maximizes the entrepreneur's ex ante expected profits.*

5 Interpretation and extensions

In this section, we relate our formal analysis and results to crowdfunding in practice. Moreover, we discuss the extent to which additional economic forces strengthen or weaken our results.

5.1 Interpretations

Our first observation concerns the role of the crowdfunding platform itself. In our formal analysis the platform structures the communication between entrepreneur and consumers, and executes the mechanism. This is consistent with the role that crowdfunding platforms play in practice. Platforms such as Kickstarter emphasize that they themselves are not directly involved in the development of the product and take no responsibility for the entrepreneur's project. Wikipedia therefore refers to these internet platforms as "internet-mediated registries" and see them as "a moderating organization."[26] Tellingly, the technical term of the platform's role in the theory of mechanism design is "mediator" (e.g., Myerson, 1982). Although it seems the platform's role is only minor, it is nevertheless crucial. Due to commitment and communication problems, neither the entrepreneur nor the consumers can perform this role.

A further notable feature of optimal mechanisms is that they do not exhibit negative transfers. Hence, consumers do not receive any money from the entrepreneur – meaning the entrepreneur does not share any of her revenues. As a result, the optimal crowdfunding scheme is reward-based instead of investment-based; it does not turn consumers into real investors. This feature is consistent with popular reward-crowdfunding platforms such as Kickstarter, which explicitly prohibit any monetary transfers to crowdfunders.[27]

In line with the many all-or-nothing crowdfunding platforms such as Kickstarter, the direct mechanisms that are optimal in our framework condition the investment decision on the sum of reported valuations rather than each individual consumer report. Clearly, the conditional investment is crucial for exploiting crowdfunding's fundamental benefit of extracting demand information directly from consumers. Our results show that moral hazard and private cost information do not undermine this fundamental benefit of crowdfunding.

The analysis further reveals that deferred payments are crucial for controlling moral hazard. And, optimally, the entrepreneur should not learn the exact amount of these deferred payments. In the mechanism design problem, we derived an optimal mechanism with symmetric transfer schedules. In case of an investment, all consumers who value the product, equally share the investment cost upfront and make identical deferred payments later. The formal analysis shows, however, that the optimum determines only the aggregate payments of consumers and not the individual ones. Hence, instead of sharing the initial investment by all consumers, an optimal mechanism can just as well ask for some consumers to pay in full upfront, while other consumers only pay later. In Section 3, we exploited this feature when arguing that current crowdfunding platforms allow an indirect implementation of opaque deferred payments via the after-market and by offering a dynamic scheme which allows consumers to use conditional pledging strategies. In terms of a direct mechanism, these conditional strategies induce the asymmetric payment schedules $(t_i^a, t_i^p) = (1, 0)$ and $(t_i^a, t_i^p) = (0, 1)$.

[26] See `https://en.wikipedia.org/wiki/Crowdfunding` and `https://www.kickstarter.com/help/faq/kickstarter%20basics#Acco` for explicit statements concerning the accountability of projects, last retrieved Sep. 10, 2016.

[27] See `https://www.kickstarter.com/rules?ref=footer`, last retrieved Sep. 10, 2016.

Our analysis shows, moreover, that in the presence of private cost information the set of possible cost structures, $\mathcal{K}$, affects the efficiency of crowdfunding in two ways. First, the efficient output is implementable only if it is affluent, which means that for *all* cost structures in $\mathcal{K}$, the project yields enough rents. Second, the lower the surpluses associated with the least favorable cost structure, the more the constrained efficient mechanism has to distort investment downwards for more favorable cost structures. Hence, even though our results demonstrate that, in direct comparison to moral hazard, private cost information is only a second-order problem, it may nevertheless substantially amplify the moral hazard problem. In particular, the more expensive cost structures exert a negative externality on projects with a more favorable cost structure. This negative externality implies that, for controlling moral hazard, the set of possible cost structures plays a crucial role. This is consistent with the observation that, in practice, crowdfunding platforms have strict rules concerning the projects that they allow on their platforms. For instance, for manufacturing products, Kickstarter requires a working prototype and bans the use of photorealistic renderings.[28] The platform explains that these rules are to ensure that entrepreneurs offer only serious projects, generating genuine benefits. Since our results clarify that only in the presence of moral hazard does excluding such non-serious projects make sense, these rules indicate that platforms do view moral hazard as a potential problem.

5.2 Extensions

The starting point of our analysis was the idea that aggregate demand uncertainty provides an economic rationale for reward-crowdfunding schemes. We subsequently presented an economic model for which such crowdfunding schemes are indeed fully optimal – even in the presence of entrepreneurial moral hazard and private cost information. In the remainder of this section, we discuss the extent to which additional economic forces may strengthen or weaken our results.

Limited consumer reach. Motivated by the observation that crowdfunding allows entrepreneurs to contract with consumers before they make an investment, our formal analysis took this idea to the extreme. It implicitly assumed that the entrepreneur could

[28]See `https://www.kickstarter.com/rules/prototypes`, last retrieved Sep. 10, 2016. The crowdfunding campaign "Skarp" illustrates that Kickstarter takes these rules very seriously. This campaign raised more than 4 million dollars, making it one of Kickstarter's largest campaigns ever. Yet, after Kickstarter discovered that the project did not have a working prototype, it cancelled the campaign. See `https://www.kickstarter.com/projects/skarp/the-skarp-laser-razor-21st-century-shaving` last retrieved Sep. 10, 2016. (The fact that most crowdfunding campaigns fail to reach their target level, makes it efficient for Kickstarter to check up only on those projects, whose campaign ends successfully.)

contract with *every* potential consumer. Given this extreme position, the revelation principle implies that there is no loss of generality in assuming that mechanisms allow consumers to acquire the product only through the mechanism.

Yet, in practice, not all consumers are able to participate in the mechanism. A share of consumers may, for instance, fail to notice the crowdfunding scheme, not have access to the internet or only arrive in the market after the product has been developed. Hence, a relevant extension of our framework is to consider mechanisms which, for some exogenous reason, reach only a limited number of consumers.

In order to make this more concrete, consider an extension of the model in which it is known that only a share of $\beta \in (0, 1)$ can partake in the mechanism. This purely proportional case, that a consumer's ability to participate is independent of his valuation, already yields new insights.

Note first that the crowdfunding scheme is still able to reduce demand uncertainty: a pledge by $\tilde{n}$ consumers means that, in expectation, $\tilde{n}(\beta) \equiv \tilde{n}/\beta$ consumer will like the product. It follows that the previous analysis still applies when we factor in the parameter β. That is, investment is socially efficient if

$$\tilde{n}(\beta) \geq I/(1-c) \Rightarrow \tilde{n} \geq n^*(\beta) \equiv \beta I/(1-c).$$

A first new insight of this extension is, however, that, with limited consumer reach, deferred payments may not be needed explicitly for the reward-based crowdfunding scheme (p, T) to withstand moral hazard, even for the extreme case $\alpha = 1$. To see this, note that if the scheme can reach only a share β of potential consumers then inequality (1), which describes the condition under which the entrepreneur has a strict incentive to run, changes to

$$\alpha P > P/\beta - I - cP/(p\beta) \Rightarrow \beta > \bar{\beta} \equiv \frac{1 - c/p}{\alpha + I/P}. \tag{46}$$

Hence, whereas under full consumer reach ($\beta = 1$), a reward-based crowdfunding scheme (p, T) without deferred payments is unable to withstand moral hazard if $\alpha > 1 - c/p - I/P$, it does withstand moral hazard when its consumer reach is limited to $\beta < \bar{\beta}$. The reason for this follows the logic behind deferred payments: the limited consumer reach effectively implies that a pledge level P constitutes a deferred payment of $(1 - \beta)P/\beta > 0$.[29]

A second new insight is that, when the share of crowdfunding consumers is small, consumers necessarily become real investors. To see this, note that because the entrepreneur needs the amount I to develop the product, the (average) ex ante transfer of a pledging consumer needs to be at least $I/\tilde{n}$. When β is small so that $n^*(\beta)$ is smaller than 1, it follows that for $\tilde{n}$ close to $n^*(\beta)$, the consumer's ex ante transfer *exceeds* his willingness to pay. Individual rationality then implies that the ex post transfer to the consumer is

[29] It is precisely by appealing to limited consumer reach that Chang (2015) and Chemla and Tinn (2016) argue that the specific crowdfunding schemes they consider can withstand moral hazard.

negative, meaning that after the investment the entrepreneur refunds consumers part of their money. Hence, the optimal mechanism turns consumers into real investors; they finance the entrepreneur's investment *and* share in her revenues.

As noted, reward crowdfunding schemes such as Kickstarter explicitly prohibit monetary transfers to crowdfunders. Our formal analysis confirms that this is indeed not needed if the investment I is small compared to the number of potential consumers that the platform can reach. For relatively large investments, however, such restrictions may matter.[30]

Finally, note that all-or-nothing crowdfunding projects also give consumers an incentive to participate in the crowdfunding scheme if the target level has not been met. That is because a consumer may be pivotal for the decision to invest and produce the good. Hence, facing a crowdfunding scheme (p, T) a consumer is strictly better off participating (provided he expects the price not to be lowered in the after-market, which in our setup would indeed not be the case). Hence, next to eliciting the consumer's valuation in an incentive-compatible manner, crowdfunding schemes also exhibit features that make participation incentive-compatible.[31]

Price discrimination. We assumed that consumers either do not value the good or value it at the same positive amount. This assumption allows us to focus on the problem of aggregate demand uncertainty and sidestep issues of price discrimination.

Indeed, economic theory has shown that, with respect to price discrimination, it is generally suboptimal for the entrepreneur to condition the execution of the project on the sum of pledges. Cornelli (1996) makes this observation in a model in which consumers' valuations are drawn from a continuum. She shows that, to achieve optimal price discrimination, the actual *composition* rather than the sum itself matters. More recently, Ellman and Hurkens (2015) extended this result to discrete valuations and show that conditioning the project's execution on the sum of pledges is generally profit-maximizing only when, as in our context, the support contains only two buyer valuations.

Our general insight that entrepreneurial moral hazard does not destroy the potential

[30] Ordanini et al. (2011) report the case of Cameesa, a Chicago-based clothing company which in 2008 introduced a reward-based crowdfunding model by which it also shared its revenue with its crowdfunders. Supporters of a successful project not only obtained the shirt, but also shared in some of the revenue of its future sales. (see `http://www.cnet.com/news/cameesa-a-threadless-where-customers-are-also-investors/`, last retrieved Sep. 10, 2016).

[31] Next to the probability to be pivotal and the consumer's expectation of the price in the after-market, a consumer's specific incentives to participate will also depend on other factors from which our model abstracts: time-preferences, the probability that the project will succeed, and the possibility that the consumer can better judge the product after it has been successfully produced. Yet, given that there is no private information about these factors, we can, without affecting qualitative results, integrate these factors in the analysis as a discount factor between the consumer's value before and after investment.

benefits of crowdfunding extends, however, to models with non-trivial concerns for price discrimination and, in particular, with more than two valuations. To see this, note that the new issue that arises in such models is that eliciting the consumers' private information requires them giving a strictly positive information rent. As a result, incentive-compatible mechanisms cannot assign the entire surplus to the entrepreneur. As our results show, a prevention of moral hazard does, however, not require that the entrepreneur extracts the full surplus; she just needs a large enough share to make the investment affluent. It then depends on the exact distributions and parameter constellations, whether crowdfunding schemes that condition on the sum of pledges can achieve full efficiency. In general however, consumers' information rents reduce the set of affluent investment profiles and, therefore, tends to lead to more distorted outcomes.

Hence, for the literature that restricts attention to comparing specific crowdfunding schemes but abstracts from moral hazard (e.g. Belleflamme et al., 2014 and Ellman and Hurkens, 2015), our results imply that the presence of moral hazard does not destroy the potential benefits which this literature identifies. In particular, our insight that dynamic crowdfunding schemes which condition on the sum of pledges allow consumers to use conditional pledging strategies that implement opaque deferred payments and thereby mitigate moral hazard, extends to such models.

Alternative funding. By enabling direct interaction with consumers prior to the investment, crowdfunding leads to a transformation of the entrepreneurial business model. Ordanini et al. (2011) emphasize that this transformation takes place at a fundamental level, blurring the traditional separation of finance and marketing.[32]

Although this fundamental perspective is correct if one views reward crowdfunding as an exclusive alternative to specialized venture capitalists, we emphasize that crowdfunding and venture capital financing are not mutually exclusive. On the contrary, we view the two forms as highly complementary. In line with Diamond (1984), we see the advantage of venture capitalists (or banks) in reducing the moral hazard problem, which in terms of the paper's model implies a reduction in α. In contrast, the strength of crowdfunding lies in learning about consumer demand for the project.

Because the analysis of a full-fledged model which combines venture capitalists and crowdfunding lies outside the scope of the current paper, we simply mention that we see no reason why a venture capitalist may not use crowdfunding to learn about demand or why after a successful crowdfunding campaign an entrepreneur may not approach a venture capitalist. Indeed, Dingman (2013) reports that exactly this occurred in the case of the Pebble Smart Watch. Venture capitalists decided to support the entrepreneur's project only after a successful crowdfunding campaign on Kickstarter. Quoting a managing part-

[32]In contrast, "investment-based crowdfunding" upholds the traditional separation between finance and marketing, because the consumers and the crowd-investors typically do not coincide.

ner of a venture capitalist firm: "What venture capital always wants is to get validation, and with Kickstarter, he [i.e., the entrepreneur] could prove there was a market."

When the crowdfunding mechanism is constrained efficient, it may – as we already pointed out in the introduction – even be profitable for venture capitalists to invest in projects whose crowdfunding campaign have failed. Constrained efficient campaigns set an inefficiently high target. Hence, campaigns that fall short of the target by a relatively small amount, reveal the information that, while there is enough demand for the project to be profitable – i.e., have a positive net present value – there is not enough demand to also control the excessive moral hazard problem associated with crowdfunding. For a venture capitalist, who can better control the moral hazard problem than the crowd, an investment in a project with a failed crowdfunding campaign may therefore still be profitable. This is consistent with Kickstarter's own observation that unsuccessful campaigns which raise a substantial part of their goal often get alternative funding.

6 Conclusion

Crowdfunding provides innovation in that, prior to the product's development, an entrepreneur contracts with consumers. Under aggregate demand uncertainty, this enables entrepreneurs to use crowdfunding as a tool to screen for valuable projects and thereby improve investment decisions. Our formal analysis confirms that optimal mechanisms do indeed take on this role of screening, even in the presence of moral hazard and private information. All-or-nothing reward crowdfunding schemes such as those used by Kickstarter and other crowdfunding platforms implement the crucial features of these mechanisms. In particular, they are consistent with the idea that crowdfunding improves the identification of valuable entrepreneurial projects. This promotes social welfare.

Our analysis further shows that the susceptibility of crowdfunding to entrepreneurial moral hazard can prevent the implementation of fully efficient outcomes. Private cost information may substantially exacerbate these inefficiencies. In particular, crowdfunding attains fully efficient outcomes only if they are affluent, meaning that the project's ex ante expected return exceeds the agency costs associated with moral hazard and private information. Constrained efficient mechanisms exhibit underinvestment, resulting in crowdfunding schemes with inefficiently high target levels.

As crowdfunding schemes by themselves are, in the presence of moral hazard and private cost information, unable to attain efficiency in general, we see them as complements rather than substitutes for traditional venture capital. We therefore expect a convergence of the two financing forms so that venture capitalists can provide their expertise in reducing moral hazard, while crowdfunding platforms enable a better screening for project value. Current policy measures such as the US JOBS Act and its implementation in SEC

(2015) will make such mixed forms easier to develop and will enable them to take advantage of their respective strengths. The website of the crowdfunding platform Rockethub already explicitly mentions this possible effect of the JOBS Act.[33]

In order to focus on the trade-off between demand uncertainty and entrepreneurial moral hazard – which we view as two fundamental first-order problems in crowdfunding – our analysis necessarily abstracts from many other relevant aspects and makes a number of simplifying assumptions. For instance, we do not address the role of crowdfunders in promoting the product. We further model the entrepreneur's investment technology as a deterministic one, leading to a well-defined private good without any network effects or any other form of externalities.[34] We also restricted attention to a model in which price discrimination is not an issue. As shown by Ellman and Hurkens (2015), price discrimination may, even in the absence of moral hazard, lead to a constrained efficient "second-best" outcome. We, however, expect a model with both moral hazard and price discrimination to yield similar insights concerning the role of deferred payments and of restricting the entrepreneur's information. A proper analysis is more involved, because with price discrimination, welfare and profit maximization no longer coincide.

Apart from pointing out that crowdfunding and external capital provision in the form of venture capital are complements, we also do not provide a formal analysis of the interaction between external financing and reward crowdfunding. We moreover leave aside possible issues concerning the platform's commitment to enforce the mechanism honestly. Since the platform is a long-term player we conjecture that it can uphold its honesty by reputational arguments of repeated games (see Strausz, 2005). A proper analysis would, however, require an explicit modeling of the platforms' objectives, but these objectives seem somewhat ambiguous.[35] Although we consider all these issues to be important, they lie outside the scope of the current investigation, which is to shed light on the salient features of popular crowdfunding platforms to prevent entrepreneurial moral hazard.

Finally, an appealing practical feature of these popular crowdfunding schemes is their simplicity; consumers seem to accept and understand their rules. Thus, we can view our model as providing a benchmark in which these simple crowdfunding contracts are fully optimal, suggesting that crowdfunding platforms gain little from using more sophisticated schemes in slightly more complex environments.

[33]See `http://www.rockethub.com/education/faq#jobs-act-index`, last retrieved Sep. 10, 2016.

[34]Yet, our results readily extend to investments with a commonly known stochastic outcome, e.g., projects that are successful only with a commonly known probability.

[35]In particular, modeling crowdfunding platforms as profit-maximizing firms does not seem in line with their self-stated missions. For example, in 2015 Kickstarter reincorporated as a Benefit Corporation, which is a legal corporate entity that, in addition to profit, explicitly includes positive impact on society, workers, the community, and the environment as its legally defined goals (see `https://www.kickstarter.com/charter`, last retrieved Sep. 10, 2016).

Appendix

This appendix collects the formal proofs.

Proof of Lemma 1: Follows directly from the text. Q.E.D.

Proof of Proposition 1: Note first that the efficient output schedule x^* is deterministic. Hence, if it is implementable, there exists some Γ that randomizes only over direct mechanisms $\gamma_l = (t_l, x_l)$ that exhibit $x_l = x^*$. Since consumers and the entrepreneur are risk-neutral, it is without loss of generality to restrict attention to deterministic transfers as well. Hence, the efficient output schedule x^* is implementable if there exists a feasible deterministic mechanism $\Gamma = (1, \gamma^*)$ that implements x^*. We show, by contradiction, that such a direct mechanism γ^* does not exist.

Moreover, because K is a singleton, the entrepreneur has no private information about her cost structure and, hence, the E-truthful condition (18) is vacuous. We can therefore disregard it and the direct mechanism γ^* effectively only request reports from consumers. To safe on notation, we therefore suppress the dependence of variables and mechanisms on the publicly observable cost structure (I, c). Let $\mathbf{1}^n$ denote the vector $(1, \ldots, 1) \in \mathbb{R}^n$. Since $n^* = I/(1-c) = n - 1/2$, it follows $\mathcal{V}^* = \{\mathbf{1}^n\}$ so that the efficient output schedule $x^*(v)$ exhibits $x_0^*(v) = x_i^*(v) = 0$ for $v \neq \mathbf{1}^n$, and $x_0^*(v) = x_i^*(v) = 1$ for $v = \mathbf{1}^n$.

Suppose to the contrary that a direct mechanism γ^* that implements x^* does exist. Then there exists a transfer schedule t so that the mechanism $\Gamma^* = (1, \gamma^*)$ with $\gamma^* = (x^*, t)$ is feasible, i.e., satisfies (21)–(29). Since $x_0^*(v) = 1$ implies $v = 1^n$, it follows that $\mathcal{T}^{\gamma^*} = \{\sum_{i\in\mathcal{N}} t_i^a(\mathbf{1}^n)\}$ is a singleton and $\mathcal{V}^{\gamma^*} = \{\mathbf{1}^n\}$. Consequently, $\pi^\Gamma(1^n|T) = 1$ and $\pi^\Gamma(v|T) = 0$ for all $v \neq \mathbf{1}^n$. Using $\alpha = 1$, $c = 0$, and $T = \sum_{i\in\mathcal{N}} t_i^a(\mathbf{1}^n)$, (26) rewrites after multiplying by $\pi(\mathbf{1}^n)$ as

$$\sum_{i\in\mathcal{N}} t_i^p(\mathbf{1}^n)\pi(\mathbf{1}^n) \geq I\pi(\mathbf{1}^n). \tag{47}$$

Since $x_0(\mathbf{1}^n) = 1$, the constraint (21) implies after multiplying with $\pi(\mathbf{1}^n)$

$$\sum_{i\in\mathcal{N}} t_i^a(\mathbf{1}^n)\pi(\mathbf{1}^n) \geq I\pi(\mathbf{1}^n). \tag{48}$$

Note further that (22) for each $v \neq \mathbf{1}^n$ implies

$$\sum_{i\in\mathcal{N}} t_i^a(v) + t_i^p(v) \geq 0.$$

Multiplying with $\pi(v)$ and adding over all $v \neq \mathbf{1}^n$ yields

$$\sum_{v\neq\mathbf{1}^n}\sum_{i\in\mathcal{N}} (t_i^a(v) + t_i^p(v))\pi(v) \geq 0. \tag{49}$$

Combining (47), (48), and (49), and using $I = n - 1/2$ yields

$$\sum_{i\in\mathcal{N}}\sum_{v\in\mathcal{V}} (t_i^a(v) + t_i^p(v))\pi(v) \geq 2I\pi(\mathbf{1}^n) = (2n-1)\pi(\mathbf{1}^n). \tag{50}$$

We now show that (50) contradicts (28) and (29). First note that (28) implies after a multiplication by $\pi_i(0)$ for each i

$$\sum_{v_{\text{-}i}\in\mathcal{V}_{\text{-}i}} (t_i^a(0, v_{\text{-}i}) + t_i^p(0, v_{\text{-}i}))\pi(0, v_{\text{-}i}) \leq 0.$$

Summing over i, we have

$$\sum_{i\in\mathcal{N}} \sum_{v_{\text{-}i}\in\mathcal{V}_{\text{-}i}} (t_i^a(0, v_{\text{-}i}) + t_i^p(0, v_{\text{-}i}))\pi(0, v_{\text{-}i}) \leq 0. \tag{51}$$

Because $\sum_{v_{\text{-}i}} \pi_i(v_{\text{-}i})x_i^*(1, v_{\text{-}i}) = \pi_i(\mathbf{1}^{n-1})$, constraint (29) implies, after a multiplication with $\pi_i(1)$, that for each i

$$\sum_{v_{\text{-}i}\in\mathcal{V}_{\text{-}i}} (t_i^a(1, v_{\text{-}i}) + t_i^p(1, v_{\text{-}i}))\pi(1, v_{\text{-}i}) \leq \pi(\mathbf{1}^n).$$

Summing over i yields

$$\sum_{i} \sum_{v_{\text{-}i}\in\mathcal{V}_{\text{-}i}} (t_i^a(1, v_{\text{-}i}) + t_i^p(1, v_{\text{-}i}))\pi(1, v_{\text{-}i}) \leq \pi(\mathbf{1}^n)n. \tag{52}$$

Combining (51) and (52) yields

$$\sum_{i\in\mathcal{N}} \sum_{v\in\mathcal{V}} (t_i^a(v) + t_i^p(v))\pi(v) \leq \pi(\mathbf{1}^n)n.$$

But since $2n - 1 > n$, this contradicts (50). Q.E.D.

Proof of Lemma 2: Consider a weakly feasible mechanism $\check{\Gamma} = \{(\check{p}_l, \check{t}_l, \check{x}_l)\}_{l\in\mathcal{L}}$ with some $\check{x}_l$ that is not development-efficient. That is, $\check{\Gamma}$ satisfies (21)-(26), (28), and (30) and there exists a combination $(\tilde{I}, \tilde{c}, \bar{v})$ such that $\check{x}_{l0}(\tilde{I}, \tilde{c}, \bar{v}) = 1$ and $\check{x}_{li}(\tilde{I}, \tilde{c}, \bar{v}) = 0$ for all $i \in N$. Lowering $\check{x}_{l0}(\tilde{I}, \tilde{c}, \bar{v})$ to zero raises the objective S^Γ by $p_l\rho(\tilde{I}, \tilde{c})\pi(\bar{v})\tilde{I}$. We show that this change yields a weakly feasible Γ', and as a result $\check{\Gamma}$ is not optimal. To show that Γ' is weakly feasible, we show that it satisfies (21)–(26), (28), and (30), given that $\check{\Gamma}$ satisfies these constraints. Note first that the change does not affect any of the constraints (24), (25), and (28), while it affects (21) and (22) only for $(l, \tilde{I}, \tilde{c}, \bar{v})$ by lowering the right-hand side by $\tilde{I}$. Hence, these constraints remain satisfied. Note further that because $\check{x}_{li}(\tilde{I}, \tilde{c}, \bar{v}) = 0$ for all $i \in N$, (23) is vacuous for $(l, \tilde{I}, \tilde{c}, \bar{v})$ so that the change does not affect it. Moreover, the change only affects (26) for $(\tilde{I}, \tilde{c}, \bar{v})$ by raising the left-hand side and, hence, it remains satisfied. Finally, the change also keeps (30) satisfied, because it raises $\Pi^{\check{\Gamma}}(I, c)$, i.e., the left-hand side, while it lowers $P^{\check{\Gamma}}(T|\tilde{I}, \tilde{c})$, i.e., the right-hand side. Q.E.D.

Proof of Lemma 3: Fix a weakly feasible $\check{\Gamma} = \{(\check{p}_l, \check{t}_l, \check{x}_l)\}_{l\in\mathcal{L}}$ with $\check{x}_1, \ldots, \check{x}_L$ development-efficient. Define for each (l, I, c, v),

$$K_l(I, c, v) \equiv \sum_{i\in\mathcal{N}} \check{t}_{li}^a(I, c, v) - I\check{x}_{l0}(I, c, v).$$

Since $\check{\Gamma}$ is weakly feasible, (21) implies that $K_l(I,c,v) \geq 0$ for all (l,I,c,v). For any (l,I,c,v), let $n_l(I,c,v) \equiv \sum_{i\in\mathcal{N}} \check{x}_{li}(I,c,v)$ represent the total number of consumers with $x_i = 1$. For any (l,I,c,v) with $\check{x}_{l0}(I,c,v) = 0$, define $\hat{t}^a_{li}(I,c,v) \equiv 0$ and $\hat{t}^p_{li}(I,c,v) \equiv \check{t}^a_{li}(I,c,v) + \check{t}^p_{li}(I,c,v)$. Similarly, for $\check{x}_{l0}(I,c,v) = 1$ define $\hat{t}^a_{li}(I,c,v) \equiv \check{t}^a_{li}(I,c,v) - \check{x}_{li}(I,c,v)K_l(I,c,v)/n_l(I,c,v)$ and $\hat{t}^p_{li}(I,c,v) \equiv \check{t}^p_{li}(I,c,v) + \check{x}_{li}(I,c,v)K_l(I,c,v)/n_l(I,c,v)$. Since $\check{\Gamma}$ is weakly feasible and $\check{x}_l$ is development-efficient, it holds $n_l(I,c,v) > 0$ if and only if $\check{x}_{l0}(I,c,v) = 1$. Hence, the transformed transfer schedule $\hat{t}$ is well defined.

By construction, $\sum_{i\in\mathcal{N}} \hat{t}^a_{li}(I,c,v) = 0$ for any (l,I,c,v) with $\check{x}_{l0}(I,c,v) = 0$, and $\sum_{i\in\mathcal{N}} \hat{t}^a_{li}(I,c,v) = \sum_{i\in\mathcal{N}} \check{t}^a_{li}(I,c,v) - \check{x}_{li}(I,c,v)K_l(I,c,v)/n_l(I,c,v) = \sum_{i\in\mathcal{N}} \check{t}^a_{li}(I,c,v) - K_l(I,c,v) = I$ for any (l,I,c,v) with $\check{x}_{l0}(I,c,v) = 1$. Hence, $(\hat{t}, \check{x}_l)$ satisfies (21) in equality. We show that, because $\check{\Gamma}$ is weakly feasible, $\hat{\Gamma} = \{(\check{p}_l, \hat{t}_l, \check{x}_l)\}$ is weakly feasible. To see this, note first that – because $\hat{t}^a_{li}(I,c,v) + \hat{t}^p_{li}(I,c,v) = \check{t}^a_{li}(I,c,v) + \check{t}^p_{li}(I,c,v)$ for all (l,I,c,v) – the change from $\check{\Gamma}$ to $\hat{\Gamma}$ leaves all constraints (22)–(25) and (28) unaffected. We therefore only have to check that $\hat{\Gamma}$ remains to satisfy (26) and (30).

In order to show that $\hat{\Gamma}$ satisfies (26), first note that, by construction of $\hat{t}_l$, for all (l,I,c) we have

$$v \in \mathcal{V}^{\hat{\gamma}_l}(I|I,c) \Leftrightarrow \exists T \in \mathcal{T}^{\check{\Gamma}}(I,c) \colon v \in \mathcal{V}^{\check{\gamma}_l}(T|I,c).$$

Hence, for all (l,I,c) we have

$$\{(v,l)|v \in \mathcal{V}^{\hat{\gamma}_l}(I|I,c)\} = \{(v,l)|\exists T \in \mathcal{T}^{\check{\Gamma}}(I,c) \colon v \in \mathcal{V}^{\check{\gamma}_l}(T|I,c)\}, \tag{53}$$

which for all (l,I,c) implies

$$\sum_{v\in\mathcal{V}^{\hat{\gamma}_l}(I|I,c)} \pi(v) = \sum_{T\in\mathcal{T}^{\check{\Gamma}}(I,c)} \sum_{v\in\mathcal{V}^{\check{\gamma}_l}(T|I,c)} \pi(v).$$

Multiplying by p_l, summing over l, and rearranging terms yields

$$P^{\hat{\Gamma}}(I|I,c) = \sum_{l\in\mathcal{L}} \sum_{v\in\mathcal{V}^{\hat{\gamma}_l}(I|I,c)} p_l\pi(v) = \sum_{T\in\mathcal{T}^{\check{\Gamma}}(I,c)} \sum_{l\in\mathcal{L}} \sum_{v\in\mathcal{V}^{\check{\gamma}_l}(T|I,c)} p_l\pi(v) = \sum_{T\in\mathcal{T}^{\check{\Gamma}}(I,c)} P^{\check{\Gamma}}(T|I,c). \tag{54}$$

Note that, by definition of Π^{Γ}_o,

$$P^{\check{\Gamma}}(T|I,c)\Pi^{\check{\Gamma}}_o(T|I,c,I,c) = \sum_{l\in\mathcal{L}} \sum_{v\in\mathcal{V}^{\check{\gamma}_l}(T|I,c)} p_l\pi(v)\Pi^{\check{\gamma}_l}(I,c|I,c,v).$$

Because $\check{\Gamma}$ satisfies (26), a multiplication of (26) by $P^{\check{\Gamma}}(T|I,c)$ yields

$$\sum_{l\in\mathcal{L}} \sum_{v\in\mathcal{V}^{\check{\gamma}_l}(T|I,c)} p_l\pi(v)\Pi^{\check{\gamma}_l}(I,c|I,c,v) \geq P^{\check{\Gamma}}(T|I,c)\alpha T, \forall T \in \mathcal{T}^{\check{\Gamma}}(I,c).$$

Summing over $T \in \mathcal{T}^{\check{\Gamma}}(I,c)$ and noting that $T \geq I$ yields after an exchange of sums,

$$\sum_{l\in\mathcal{L}} \sum_{T\in\mathcal{T}^{\check{\Gamma}}(I,c)} \sum_{v\in\mathcal{V}^{\check{\gamma}_l}(T|I,c)} p_l\pi(v)\Pi^{\check{\gamma}_l}(I,c|I,c,v) \geq \sum_{T\in\mathcal{T}^{\check{\Gamma}}(I,c)} P^{\check{\Gamma}}(T|I,c)\alpha I.$$

Using (53), $\Pi^{\hat{\gamma}_l}(I,c|I,c,v) = \Pi^{\check{\gamma}_l}(I,c|I,c,v)$, and (54) yields

$$\sum_{l\in\mathcal{L}} \sum_{v\in\mathcal{V}^{\hat{\gamma}_l}(I|I,c)} p_l \pi(v) \Pi^{\hat{\gamma}_l}(I,c|I,c,v) \geq P^{\hat{\Gamma}}(I|I,c)\alpha I.$$

Dividing both sides by $P^{\hat{\Gamma}}(I|I,c)$ shows that $\hat{\Gamma}$ satisfies (26), since $\mathcal{T}^{\hat{\Gamma}}(I,c) = \{I\}$.

Moreover, since $\check{\Gamma}$ satisfies (30) and, for any $T \in \mathcal{T}^{\check{\Gamma}}(I,c)$, we have $T \geq I$ and $\mathcal{T}^{\hat{\Gamma}}(I,c) = \{I\}$, it follows for all $(I,c,I',c') \in \mathcal{K}\times\mathcal{K}$ that, by (54),

$$\Pi^{\hat{\Gamma}}(I,c) = \Pi^{\check{\Gamma}}(I,c) \geq \sum_{T\in\mathcal{T}^{\check{\Gamma}}(I',c')} P^{\check{\Gamma}}(T|I',c')\alpha T \geq \sum_{T\in\mathcal{T}^{\check{\Gamma}}(I',c')} P^{\hat{\Gamma}}(T|I',c')\alpha I' = P^{\hat{\Gamma}}(I'|I',c')\alpha I',$$

which shows that $\hat{\Gamma}$ satisfies (30).

We conclude that $\hat{\Gamma}$ is weakly feasible. Because for all (l,I,c,v) we have $\hat{x}_{l0}(I,c,v) = \check{x}_{l0}(I,c,v)$, $\hat{x}_{li}(I,c,v) = \check{x}_{li}(I,c,v)$, and $\hat{t}^a_{li}(I,c,v) + \hat{t}^p_{li}(I,c,v) = \check{t}^a_{li}(I,c,v) + \check{t}^p_{li}(I,c,v)$, $\hat{\Gamma}$ is payoff equivalent to $\check{\Gamma}$. Finally, because (21) holds in equality for $\hat{\Gamma}$, (22) reduces to (31). Q.E.D.

Proof of Lemma 4: To see that any maximizer $\check{\Gamma} = \{(\check{p}_l, \check{t}_l, \check{x}_l\}_{l\in\mathcal{L}}$ of S^Γ subject to the constraints (23), (24), (25), (28), (31), (32), and (33), and (21) in equality, exhibits (34), suppose to the contrary that it is violated for some $(I,c,0,v_{\text{-}i}) \in \mathcal{K}\times\mathcal{V}$, i.e., for some l, we have $\check{x}_{li}(I,c,0,v_{\text{-}i}) = 1$. But then lowering it to 0 and lowering $\check{t}^p_{li}(I,c,0,v_{\text{-}i})$ by c raises the objective by $\check{p}_l\rho(I,c)\pi(0,v_{\text{-}i})c$ so that $\check{\Gamma}$ is not optimal if the changed mechanism respects all the constraints. To see that it does so, first note that the change does not affect (21) and (23). The combined reduction in $\check{x}_i(I,c,0,v_{\text{-}i})$ and $\check{t}^p_i(I,c,0,v_{\text{-}i})$ also implies that (31) and (32) remain satisfied, while also $\Pi(\gamma(I',c',v)|I,c)$ remains unaffected for any $(I,c,I',c') \in K^2$. Hence, $\Pi^\gamma(I,c)$ remains unaffected and, therefore (33) remains satisfied. The change further relaxes (24) and (28), since it raises the left-hand side. Finally, the change also keeps (25) satisfied, because it does not affect its left-hand side, while it lowers the right-hand side by $\check{p}_l\rho(I,c)\pi_i(v_{\text{-}i})(1-c)$. Q.E.D.

Proof of Lemma 5: We first prove that if $\check{\Gamma} = \{(\check{p}_l, \check{t}_l, \check{x}_l)\}_{l\in\mathcal{L}}$ is weakly feasible, then we find $\hat{t}$ such that the mechanism $\hat{\Gamma} = \{(\check{p}_l, \hat{t}_l, \check{x}_l)\}_{l\in\mathcal{L}}$ exhibits $U_i^{\check{\gamma}_l}(0|I,c,0) = 0$ for any (l,I,c) and is weakly feasible. If $U_i^{\check{\gamma}_l}(0|I,c,0) = 0$ for all (l,i,I,c), the result is immediate by taking $\hat{\Gamma} = \check{\Gamma}$. Hence, suppose $U_i^{\check{\gamma}_l}(0|I,c,0) > 0$ for some $(l,i,\tilde{I},\tilde{c})$. Fix $(l,i,\tilde{I},\tilde{c})$ and define $U \equiv U_i^{\check{\gamma}_l}(0|I,c,0) > 0$. Consider the transfer schedule $\hat{t}_l(.) = (\check{t}^a_l(.), \hat{t}^p_l(.))$ with $\hat{t}^p_l(.)$ defined as follows. For all $v_{\text{-}i} \in \mathcal{V}_{\text{-}i}$ set $\hat{t}^p_{li}(\tilde{I},\tilde{c},0,v_{\text{-}i}) = \check{t}^p_{li}(\tilde{I},\tilde{c},0,v_{\text{-}i}) + U$ and $\hat{t}^p_{li}(\tilde{I},\tilde{c},1,v_{\text{-}i}) = \check{t}^p_{li}(\tilde{I},\tilde{c},1,v_{\text{-}i}) + U$. By construction the mechanism $\hat{\gamma}_l = (\check{t}^a_l, \hat{t}^p_l, \check{x}_l)$ exhibits $U_i^{\hat{\gamma}_l}(0|\tilde{I},\tilde{c},0) = 0$.

$$U_i^{\hat{\gamma}_l}(0|\tilde{I},\tilde{c},0) = \sum_{v_{\text{-}i}\in\mathcal{V}_{\text{-}i}} \pi_i(v_{\text{-}i}) U_i(\hat{\gamma}_l(\tilde{I},\tilde{c},0,v_{\text{-}i})|0)$$

$$= \sum_{v_{\text{-}i} \in \mathcal{V}_{\text{-}i}} \pi_i(v_{\text{-}i})[-\check{t}^a_{li}(\tilde{I}, \tilde{c}, 0, v_{\text{-}i}) - \hat{t}^p_{li}(\tilde{I}, \tilde{c}, 0, v_{\text{-}i})]$$

$$= \sum_{v_{\text{-}i} \in \mathcal{V}_{\text{-}i}} \pi_i(v_{\text{-}i})[-\check{t}^a_{li}(\tilde{I}, \tilde{c}, 0, v_{\text{-}i}) - \check{t}^p_{li}(\tilde{I}, \tilde{c}, 0, v_{\text{-}i}) - U]$$

$$= U_i^{\check{\gamma}_l}(0|\tilde{I}, \tilde{c}, 0) - U = 0$$

Because $\hat{\gamma}_l$ and $\check{\gamma}_l$ exhibit the same output schedule $\check{x}_l$, they generate the same surplus $S^{\check{x}_l}(I, c)$ for all (I, c). Hence, if we define the mechanism $\hat{\Gamma}$ as identical to $\check{\Gamma}$ but with $\check{\gamma}_l$ exchanged for $\hat{\gamma}_l$ for any (l, i, I, c) such that $U_i^{\check{\gamma}_l}(0|I, c, 0) > 0$, then $S^{\hat{\Gamma}} = S^{\check{\Gamma}}$. We next show that, because $\check{\Gamma}$ is weakly feasible, so is the constructed $\hat{\Gamma}$, i.e., it satisfies (23), (24), (25), (28), (31), (32), and (33), and (21) in equality. To see this, note first that the change from $\check{\Gamma}$ to $\hat{\Gamma}$ affects only the transfers $t_i^p(.)$ so that (21) and (23) remain unaffected and, therefore, satisfied for $\hat{\Gamma}$. Because $\check{t}_i^p(I, c, 0, v_{\text{-}i})$ and $\check{t}_i^p(I, c, 1, v_{\text{-}i})$ are changed by the same amount, the change lowers the left- and right-hand side of (24) and (25) also by the same amount so that they remain satisfied. To see (28), note $U_i^{\hat{\Gamma}}(I, c|0) = \sum_{l \in \mathcal{L}} p_l U_i^{\hat{\gamma}_l}(0|I, c, 0) = 0$. Moreover, the change from $\check{\Gamma}$ to $\hat{\Gamma}$ only raises the transfers, i.e., $\hat{t}_i^p(I, c, v) \geq \check{t}_i^p(I, c, v)$, the constraints (31), (32), and (33) are relaxed so that $\hat{\Gamma}$ remains to satisfy them.

To see the second statement, consider a weakly feasible $\check{\Gamma}$ with $\check{x}_l$ satisfying (34), we construct a weakly feasible $\hat{\Gamma}$ that exhibits $U_i^{\hat{\gamma}_l}(1|I, c, 1) = 0$ and yields the same aggregate surplus $S^{\check{\Gamma}}$. By the first two statements of the lemma, we can adapt $\check{\Gamma}$ to $\tilde{\Gamma}$ so that $U_i^{\tilde{\gamma}_l}(0|I, c, 0) = U_i^{\tilde{\Gamma}}(0|0) = 0$ and $\tilde{\Gamma}$ satisfies (28), (21), (23), (24), (25), (31), (32), (33) and (21) in equality. Consider the transfer schedule $\hat{t}_l(.) = (\tilde{t}_l^a(.), \hat{t}_l^p(.))$ with $\hat{t}_l^p(I, c, v)$ defined by $\hat{t}^p_{li}(I, c, 0, v_{\text{-}i}) = \tilde{t}^p_{li}(I, c, 0, v_{\text{-}i})$ and $\hat{t}^p_{li}(I, c, 1, v_{\text{-}i}) = \tilde{t}^p_{li}(I, c, 1, v_{\text{-}i}) + U_i^{\tilde{\gamma}_l}(1|I, c, 1)$ for all $v_{\text{-}i} \in \mathcal{V}_{\text{-}i}$. Now consider the mechanism $\hat{\Gamma} = \{(\tilde{p}_l, \tilde{t}_l^a, \hat{t}_l^p, \tilde{x})\}_{l \in \mathcal{L}}$ so that $\hat{\Gamma}$ differs from $\tilde{\Gamma}$ only concerning $t^p_{li}(I, c, 1, v_{\text{-}i})$ so that, by construction, $U_i^{\hat{\gamma}_l}(1|I, c, 1) = 0$.

$$\begin{aligned}
U_i^{\hat{\gamma}_l}(1|I, c, 1) &= \sum_{v_{\text{-}i} \in \mathcal{V}_{\text{-}i}} \pi_i(v_{\text{-}i}) U_i(\hat{\gamma}_l(I, c, 1, v_{\text{-}i})|1) \\
&= \sum_{v_{\text{-}i} \in \mathcal{V}_{\text{-}i}} \pi_i(v_{\text{-}i})[\tilde{x}^a_{li}(I, c, 1, v_{\text{-}i}) - \tilde{t}^a_{li}(I, c, 1, v_{\text{-}i}) - \hat{t}^p_{li}(I, c, 1, v_{\text{-}i})] \\
&= \sum_{v_{\text{-}i} \in \mathcal{V}_{\text{-}i}} \pi_i(v_{\text{-}i})[\tilde{x}^a_{li}(I, c, 1, v_{\text{-}i}) - \tilde{t}^a_{li}(I, c, 1, v_{\text{-}i}) - \tilde{t}^p_{li}(I, c, 1, v_{\text{-}i}) - U_i^{\tilde{\gamma}_l}(1|I, c, 1)] \\
&= U_i^{\tilde{\gamma}_l}(1|I, c, 1) - U_i^{\tilde{\gamma}_l}(1|I, c, 1) = 0.
\end{aligned}$$

Note that because $\hat{\Gamma}$ and $\tilde{\Gamma}$ and $\check{\Gamma}$ exhibit identical output schedules $\check{x}_l$, they generate the same surplus $S^{\hat{\Gamma}} = S^{\tilde{\Gamma}} = S^{\check{\Gamma}}$. Hence, it remains to show that $\hat{\Gamma}$ is weakly feasible. This follows since $\tilde{\Gamma}$ satisfies (23), (24), (25), (28), (31), (32), and (33), and (21) in equality, so does $\hat{\Gamma}$. To see this, note first that the change from $\tilde{\Gamma}$ to $\hat{\Gamma}$ only affects the transfers $t^p_{li}(I, c, 1, v_{\text{-}i})$ by (weakly) raising them. Hence, (21), (23), and (28) remain unaffected. Moreover, since the change only raises transfers $t_i^p(I, c, 1, v_{\text{-}i})$, it relaxes the constraints (24), (31), (32), and (33). It remains to show that $\hat{\Gamma}$ respects (25). In order to see this, note that because $\check{\Gamma}$ satisfies, by assumption of the lemma, (34) for each $\check{x}_l$ also $\hat{\Gamma}$ satisfies (34). Hence, $U_i^{\hat{\Gamma}}(0|1) = U_i^{\hat{\Gamma}}(0|0) = 0 = U_i^{\hat{\Gamma}}(1|1)$ so that $\hat{\Gamma}$ satisfies (25) in equality. Q.E.D.

Proof of Lemma 6: Following Lemma 5, we may assume without loss of generality that an optimal weakly feasible mechanism $\check{\Gamma}$ satisfies $U_i^{\hat{\gamma}_l}(0|I,c,0) = U_i^{\hat{\gamma}_l}(1|I,c,1) = 0$ for all (l,i,I,c). It then follows for any (l,I,c) that

$$\begin{aligned}
\Pi^{\check{\gamma}_l}(I,c) &= \sum_{v\in\mathcal{V}} \pi(v)\Pi(\check{\gamma}_l(I,c,v)|I,c) \\
&= \sum_{v\in\mathcal{V}} \pi(v)\left[\sum_{i\in\mathcal{N}}[\check{t}^a_{li}(I,c,v) + \check{t}^p_{li}(I,c,v) - \check{x}_{li}(I,c,v)c] - I\check{x}_{l0}(I,c,v)\right] \\
&= \left[\sum_{i\in\mathcal{N}}\sum_{v\in\mathcal{V}} \pi(v)\check{x}_{li}(I,c,v)(1-c)\right] - \sum_{v\in\mathcal{V}}\pi(v)I\check{x}_{l0}(I,c,v) = S^{\check{x}}(I,c),
\end{aligned}$$

where, using $U_i^{\check{\gamma}_l}(0|I,c,0) = U_i^{\check{\gamma}_l}(1|I,c,1) = 0$, the third equality follows from

$$\begin{aligned}
&\sum_{v\in\mathcal{V}}\sum_{i\in\mathcal{N}} \pi(v)[\check{t}^a_{li}(I,c,v) + \check{t}^p_{li}(I,c,v) - \check{x}_{li}(I,c,v)c] \\
&= \sum_{i\in\mathcal{N}}\sum_{(v_i,v_{-i})\in\mathcal{V}} \pi(v_i,v_{-i})[\check{t}^a_{li}(I,c,v_i,v_{-i}) + \check{t}^p_{li}(I,c,v_i,v_{-i}) - \check{x}_{li}(I,c,v_i,v_{-i})c] \\
&= \sum_{i\in\mathcal{N}}\Bigg[\sum_{(0,v_{-i})\in\mathcal{V}} \pi(0,v_{-i})[\check{t}^a_{li}(I,c,0,v_{-i}) + \check{t}^p_{li}(I,c,0,v_{-i}) - \check{x}_{li}(I,c,0,v_{-i})c] \\
&\qquad + \sum_{(1,v_{-i})\in\mathcal{V}} \pi(1,v_{-i})[\check{t}^a_{li}(I,c,1,v_{-i}) + \check{t}^p_{li}(I,c,1,v_{-i}) - \check{x}_{li}(I,c,1,v_{-i})c]\Bigg] \\
&= \sum_{i\in\mathcal{N}}\Bigg[\pi_i(0)\sum_{v_{-i}\in\mathcal{V}_{-i}} \pi_i(v_{-i})[\check{t}^a_{li}(I,c,0,v_{-i}) + \check{t}^p_{li}(I,c,0,v_{-i}) - \check{x}_{li}(I,c,0,v_{-i})c] \\
&\qquad + \pi_i(1)\sum_{v_{-i}\in\mathcal{V}_{-i}} \pi_i(v_{-i})[\check{t}^a_{li}(I,c,1,v_{-i}) + \check{t}^p_{li}(I,c,1,v_{-i}) - \check{x}_{li}(I,c,1,v_{-i})c]\Bigg] \\
&= \sum_{i\in\mathcal{N}}\Bigg[\pi_i(0)[-U_i^{\check{\gamma}_l}(0|I,c,0) + \sum_{v_{-i}\in\mathcal{V}_{-i}} \pi_i(v_{-i})\check{x}_{li}(I,c,0,v_{-i})(1-c)] \\
&\qquad + \pi_i(1)[-U_i^{\check{\gamma}_l}(1|I,c,1) + \sum_{v_{-i}\in\mathcal{V}_{-i}} \pi_i(v_{-i})\check{x}_{li}(I,c,1,v_{-i})(1-c)]\Bigg] \\
&= \sum_{i\in\mathcal{N}}\Bigg[\pi_i(0)\sum_{v_{-i}\in\mathcal{V}_{-i}} \pi_i(v_{-i})\check{x}_{li}(I,c,0,v_{-i})(1-c) + \pi_i(1)\sum_{v_{-i}\in\mathcal{V}_{-i}} \pi_i(v_{-i})\check{x}_{li}(I,c,1,v_{-i})(1-c)\Bigg] \\
&= \sum_{i\in\mathcal{N}}\sum_{v\in\mathcal{V}} \pi(v)\check{x}_{li}(I,c,v)(1-c).
\end{aligned}$$

Q.E.D.

Proof of Proposition 2: If the efficient output schedule x^* is implementable, then the optimal feasible mechanism $\check{\Gamma}$ must implement it, because, by definition, no other output schedule yields a larger surplus. Moreover, the proof of Proposition 1 already noted that, because x^* is deterministic, it is implementable if and only if there exists a transfer schedule $\check{t}$ such that the deterministic mechanism $\check{\Gamma} = (1, \check{\gamma}) = (1, \check{t}, x^*)$ is feasible.

Note that for deterministic mechanisms, constraint (43) simplifies to

$$S^{x^*}(I, c) \geq \pi^{x^*}(I', c')\alpha I', \ \forall (I, c, I', c') \in \mathcal{K} \times \mathcal{K}.$$

It is therefore immediate that affluency is a necessary condition for the implementability of x^* by a weakly feasible mechanism $\check{\Gamma}$ and, hence, also for the implementability by a (fully) feasible mechanism $\check{\Gamma}$.

It remains to prove that affluency is also a sufficient condition for the implementability of x^*. We will do so constructively and, under the assumption that x^* is affluent, construct an explicit crowdfunding mechanism that implements it.

Because x^* is development-efficient, it holds $n(v) = \sum_{i \in \mathcal{N}} v_i > 0$ for any $x_0^*(I, c, v) = 1$ so that defining $\check{t} = (\check{t}^a, \check{t}^p)$ as

$$(\check{t}_i^a(I, c, v), \check{t}_i^p(I, c, v)) \equiv \begin{cases} (v_i I/n(v), v_i[1 - I/n(v)]) & \text{if } x_0^*(I, c, v) = 1, \\ (0, 0) & \text{otherwise,} \end{cases}$$

yields a well-defined $\check{t}$. For $T(I, c) = I/(1 - c)$, the output schedule x^* and transfers $\check{t}$ satisfy (10)–(13) and the deterministic mechanism $\check{\Gamma} = (1, \check{\gamma}) = (1, \check{t}, x^*)$ is, therefore, a crowdfunding mechanism.

As we next show, given that x^* is affluent, the crowdfunding mechanism $\check{\Gamma}$ satisfies constraints (36)–(43) so that it is weakly feasible and, moreover, (27) and (29) so that it is also feasible.

To see (36), note for $x_0^*(I, c, v) = 0$, it follows $\sum_{i \in \mathcal{N}} \check{t}_i^a(I, c, v) = 0 = x_0^*(I, c, v)I$. Moreover, because x^* is development-efficient it follows for $x_0^*(I, c, v) = 1$ that

$$\textstyle\sum_{i \in \mathcal{N}} \check{t}_i^a(I, c, v) = \sum_{i \in \mathcal{N}} v_i I/n(v) = \left[\sum_{i \in \mathcal{N}} v_i\right] I / \sum_j v_j = I = x_0^*(I, c, v)I.$$

Note that (38) holds, because x^* is development-feasible.

To see (39) and (40), note that, because x^* is development-efficient,

$$U_i^{\check{\gamma}}(v_i | I, c, v_i) = \sum_{v_{\text{-}i} \in \mathcal{V}_{\text{-}i}} \pi_i(v_{\text{-}i})[v_i x_i^*(I, c, 1) - \check{t}_i^a(I, c, 1) - \check{t}_i^p(I, c, 1)] = 0.$$

To see (37), note that, since x^* is development-efficient, for $x_0^*(I, c, v) = 0$ we have $\sum_{i \in \mathcal{N}} \check{t}_i^p(I, c, v) = 0 = \sum_{i \in \mathcal{N}} x_i^*(I, c, v)c$. Moreover, because x^* is development-efficient and $x_0^*(I, c, v) = 1$ implies $n(v) \geq I/(1 - c)$, for $x_0^*(I, c, v) = 1$ it follows that

$$\textstyle\sum_{i \in \mathcal{N}} \check{t}_i^p(I, c, v) = \sum_{i \in \mathcal{N}} v_i[1 - I/n(v)] = n(v) - I \geq cn(v) = c \sum_{i \in \mathcal{N}} x_i^*(I, c, v).$$

To see (41), note that since x^* is development efficient and affluent, we have

$$\sum_{v \in \mathcal{V}^{x^*}(I,c)} \sum_{i \in \mathcal{N}} \pi(v)(\check{t}_i^p(I,c,v) - c x_i^*(I,c,v))$$

$$= \sum_{v \in \mathcal{V}^{x^*}(I,c)} \pi(v) \left[\sum_{i: x_i^*(I,c,v)=1} (1 - I/n(v) - c) \right] = \sum_{v \in \mathcal{V}^{x^*}(I,c)} \pi(v)[n(v)(1-c) - I]$$

$$= \sum_{v \in \mathcal{V}} \pi(v) \left[\sum_{i \in \mathcal{N}}^{n} (v_i - c) x_i^*(I,c,v) - I x_0^*(I,c,v) \right] = S^{x^*}(I,c) \geq \pi^{x^*}(I,c)\alpha I.$$

Finally, (43) follows because x^* is affluent and x^* satisfies (42) by definition. Hence, $\check{\gamma}$ is weakly feasible,

We next show that $\check{\gamma}$ also satisfies the constraints (27) and (29).

To see (27), note that, because $x_0^*(I',c',v) = 0$ implies $\Pi^{\check{\gamma}}(I',c'|I,c,v) = 0$, (27) holds if

$$\Pi^{\check{\Gamma}}(I,c) \geq \pi^{x^*}(I',c') \max\{\Pi_o^{\check{\Gamma}}(T|I,c,I',c'), \alpha I'\}.$$

That is, it holds if

$$\Pi^{\check{\Gamma}}(I,c) \geq \pi^{x^*}(I',c')\Pi_o^{\check{\Gamma}}(I'|I,c,I',c') \text{ and } \Pi^{\check{\Gamma}}(I,c) \geq \pi^{x^*}(I',c')\alpha I'.$$

The latter follows, since, by Lemma 6, $\Pi^{\check{\Gamma}}(I,c) = S^{x^*}(I,c)$ and x^* is affluent. To see also the former inequality, note, because $x_0^*(I',c',v) = 0$ implies $\Pi^{\check{\gamma}}(I',c'|I,c,v) = 0$, we have

$$\pi^{x^*}(I',c')\Pi_o^{\check{\Gamma}}(I'|I,c,I',c') = \pi^{x^*}(I',c') \sum_{v \in \mathcal{V}} \eta^{\check{\Gamma}}(v,1|I',I',c')\Pi^{\check{\gamma}}(I',c'|I,c,v)$$

$$= \sum_{v \in \mathcal{V}^{x^*}(I',c')} \pi(v)\Pi^{\check{\gamma}}(I',c'|I,c,v) = \sum_{v \in \mathcal{V}} \pi(v)\Pi^{\check{\gamma}}(I',c'|I,c,v) = \sum_{v \in \mathcal{V}} \pi(v)\Pi(\check{\gamma}(I',c',v)|I,c)$$

$$= \sum_{v \in \mathcal{V}} \pi(v) \left\{ \sum_{i \in \mathcal{N}} \left[\check{t}_i^a(I',c',v) + \check{t}_i^p(I',c',v) - x_i^*(I',c',v)c \right] - I x_0^*(I',c',v) \right\}$$

$$= \sum_{v \in \mathcal{V}} \pi(v) \left\{ x_0^*(I',c',v)[n(v)(1-c) - I] \right\}$$

$$\leq \sum_{v \in \mathcal{V}} \pi(v) \left\{ x_0^*(I,c,v)[n(v)(1-c) - I] \right\} = S^{\check{\gamma}}(I,c) = \Pi^{\check{\gamma}}(I,c) = \Pi^{\check{\Gamma}}(I,c),$$

where the inequality follows because x^* is efficient.

Finally, to see (29), note

$$\begin{aligned} U_i^{\check{\Gamma}}(I,c|1) &= U_i^{\check{\gamma}}(1|I,c,1) \\ &= \sum_{v_{-i} \in \mathcal{V}_{-i}} \pi_i(v_{-i}) U_i(\check{\gamma}(I,c,1,v_{-i})|1) \\ &= \sum_{v_{-i} \in \mathcal{V}_{-i}} \pi_i(v_{-i})[x_i^*(I,c,1,v_{-i}) - \check{t}_i^a(I,c,1,v_{-i}) - \check{t}_i^p(I,c,1,v_{-i})] \\ &= \sum_{v_{-i}: x_0^*(I,c,1,v_{-i})=1} \pi_i(v_{-i}) x_i^*(I,c,1,v_{-i})[1-1] = 0. \end{aligned}$$

We conclude that the crowdfunding mechanism $\check{\Gamma}$ is feasible and, therefore, implements x^* and yields surplus and ex ante profits of S^{x^*}. As a feasible mechanism cannot yield more than S^{x^*}, it must be optimal and maximize ex ante profits. Q.E.D.

Proof of Lemma 7: The proof consists of 3 steps. We first prove that, for an optimal $\check{\Gamma} = \{(\check{p}_l, \check{t}_l, \check{x}_l)\}_{l\in\mathcal{L}}$ satisfying (36)–(43), for all $(l, I, c, v) \in \mathcal{L}\times\mathcal{K}\times\mathcal{V}$, it holds

$$\check{x}_{l0}(I, c, v) = 1 \Rightarrow \check{x}_{li}(I, c, v) = v_i. \tag{55}$$

Second, we prove that if $\check{\Gamma}$ is optimal, then for each $(l, I, c) \in \mathcal{L}\times\mathcal{K}$ there exists a $T \in \mathcal{N}$ such that (10) holds. In a final step, we prove that T is independent of l so that for each $(I, c) \in \mathcal{K}$ there exists a $T \in \mathcal{N}$ such that (10) holds for any $l \in \mathcal{L}$.

Step 1: Consider a $\check{\Gamma} = \{(\check{p}_l, \check{t}_l, \check{x}_l)\}_{l\in\mathcal{L}}$ that satisfies (36)–(43), but for which condition (55) is not satisfied. Hence, it holds that for some $(l, I, c, v) \in \mathcal{L}\times K \times\mathcal{V}$ that $\check{x}_{l0}(I, c, v) = 1$ but $\check{x}_{li}(I, c, v) \neq v_i \in \{0, 1\}$. Constraint (42) then implies $v_i = 1$ so that $\check{x}_{li}(I, c, v) = 0$. It then follows that by raising $\check{x}_{li}(I, c, v)$ to 1, the objective $S^{\check{\Gamma}}$ is increased by $\check{p}_l\rho(I, c)\pi(v)(1 - c)$. By accompanying the raise in $\check{x}_{li}(I, c, v)$ by a raise in $\check{t}^p_{li}(I, c, v)$ of 1 a changed mechanism obtains that remains to respect all constraints (36)–(43). It is therefore also weakly feasible, and hence $\check{\Gamma}$ is not optimal.

Step 2: Next we show that if $\check{\Gamma}$ is optimal then i) $\check{x}_{l0}(I, c, \hat{v}) = 1$ implies $x_{l0}(I, c, \bar{v}) = 1$ for any $\bar{v}$ such that $n(\bar{v}) > n(\hat{v})$, and ii) $\check{x}_{l0}(I, c, \hat{v}) = 0$ implies $x_{l0}(I, c, \bar{v}) = 0$ for any $n(\bar{v}) < n(\hat{v})$. From this it then directly follows that, for any $(l, I, c) \in \mathcal{L}\times\mathcal{K}$, there is a $T \in \mathcal{N}$ such that, for all $v \in \mathcal{V}$, it holds $x_{0l}(I, c, v) = 1$ if $n(v) > T$ and $x_{0l}(I, c, v) = 0$ if $n(v) < T$.

To see i) and ii), assume to the contrary that one of the two conditions does not hold, meaning there exists an $(\bar{l}, \tilde{I}, \tilde{c}) \in \mathcal{L}\times\mathcal{K}$ and $\bar{v}, \hat{v} \in \mathcal{V}$ with $n(\bar{v}) < n(\hat{v})$ such that $\check{x}_{\bar{l}0}(\tilde{I}, \tilde{c}, \bar{v}) = 1$ and $\check{x}_{\bar{l}0}(\tilde{I}, \tilde{c}, \hat{v}) = 0$. Since $n(\bar{v}) < n(\hat{v})$ there exists a bijection $j : \mathcal{N} \to \mathcal{N}$ such that $\bar{v}_i = 1$ implies $\hat{v}_{j(i)} = 1$. To show that $\check{\Gamma}$ is not optimal, we distinguish three cases: 1. $\pi(\bar{v}) = \pi(\hat{v})$; 2. $\pi(\bar{v}) < \pi(\hat{v})$, and 3. $\pi(\bar{v}) > \pi(\hat{v})$.

Case 1: Adapt the mechanism $\check{\Gamma}$ to the mechanism $\hat{\Gamma}$ by only replacing $\check{\gamma}_{\bar{l}}$ by the mechanism $\hat{\gamma} = (\hat{t}, \hat{x})$, which is identical to $\check{\gamma}_{\bar{l}}$ for all $(I, c, v) \in \mathcal{K}\times\mathcal{V}$ except for $(\tilde{I}, \tilde{c}, \bar{v})$ and $(\tilde{I}, \tilde{c}, \hat{v})$. Hence, for all $(I, c, v) \in (\mathcal{K}\times\mathcal{V})\backslash\{(\tilde{I}, \tilde{c}, \bar{v}), (\tilde{I}, \tilde{c}, \hat{v})\}$, it holds $\hat{t}(I, c, v) = \check{t}_{\bar{l}}(I, c, v) \in \mathbb{R}^{2n}$ and $\hat{x}(I, c, v) = \check{x}_{\bar{l}}(I, c, v) \in \{0, 1\}^{n+1}$. For all $i \in \mathcal{N}$, let $\hat{x}_0(\tilde{I}, \tilde{c}, \bar{v}) = \hat{x}_i(\tilde{I}, \tilde{c}, \bar{v}) = 0$, $\hat{t}^a_i(\tilde{I}, \tilde{c}, \bar{v}) = \check{t}^a_{\bar{l}i}(\tilde{I}, \tilde{c}, \bar{v}) - \check{x}_{\bar{l}i}(\tilde{I}, \tilde{c}, \bar{v})\tilde{I}/n(\bar{v})$, and $\hat{t}^p_i(\tilde{I}, \tilde{c}, \bar{v}) = \check{t}^p_{\bar{l}i}(\tilde{I}, \tilde{c}, \bar{v}) - \check{x}_{\bar{l}i}(\tilde{I}, \tilde{c}, \bar{v})[1 - \tilde{I}/n(\bar{v})]$. Moreover, for all $i \in \mathcal{N}$, let $\hat{x}_0(\tilde{I}, \tilde{c}, \hat{v}) = 1$ and $\hat{x}_{j(i)}(\tilde{I}, \tilde{c}, \hat{v}) = \check{x}_{\bar{l}i}(\tilde{I}, \tilde{c}, \bar{v})$, $\hat{t}^a_{j(i)}(\tilde{I}, \tilde{c}, \hat{v}) = \check{t}^a_{\bar{l}j(i)}(\tilde{I}, \tilde{c}, \hat{v}) + \hat{x}_{j(i)}(\tilde{I}, \tilde{c}, \hat{v})\tilde{I}/n(\bar{v})$, and $\hat{t}^p_{j(i)}(\tilde{I}, \tilde{c}, \hat{v}) = \check{t}^p_{\bar{l}j(i)}(\tilde{I}, \tilde{c}, \hat{v}) + \hat{x}_{j(i)}(\tilde{I}, \tilde{c}, \hat{v})[1 - \tilde{I}/n(\bar{v})]$. Because $\pi(\bar{v}) = \pi(\hat{v})$, it holds $\pi^{\check{x}_{\bar{l}}}(\tilde{I}, \tilde{c}) = \pi^{\hat{x}}(\tilde{I}, \tilde{c})$ and, therefore, $\pi^{\check{\Gamma}}(\tilde{I}, \tilde{c}) = \pi^{\hat{\Gamma}}(\tilde{I}, \tilde{c})$.

Case 2: Consider the mechanism $\hat{\Gamma} = \{(\hat{p}_l, \check{\gamma}_l)\}_{l\in\{0,\ldots,L\}}$, which, in addition to the same collection of deterministic mechanisms $\check{\gamma}_l$ as $\check{\Gamma}$ but with $\check{\gamma}_{\bar{l}}$ exchanged by the deterministic mechanism $\hat{\gamma}$ as defined in Case 1, also contains the deterministic mechanism

$\check{\gamma}_0 = (\check{t}_0, \check{x}_0)$. This deterministic mechanism is identical to $\check{\gamma}_{\bar{l}}$ for all $(I, c, v) \in \mathcal{K}\times\mathcal{V}$ except for $(\tilde{I}, \tilde{c}, \bar{v})$. Hence, for all $(I, c, v) \in \mathcal{K}\times\mathcal{V}\backslash\{(\tilde{I}, \tilde{c}, \bar{v})\}$, let $\check{t}_0(I, c, v) = \check{t}_{\bar{l}}(I, c, v) \in \mathbb{R}^{2n}$ and $\check{x}_0(I, c, v) = \check{x}_{\bar{l}}(I, c, v) \in \{0, 1\}^{n+1}$. For all $i \in \mathcal{N}$, let $\check{x}_{00}(\tilde{I}, \tilde{c}, \bar{v}) = \check{x}_{0i}(\tilde{I}, \tilde{c}, \bar{v}) = 0$, $\check{t}^a_{0i}(\tilde{I}, \tilde{c}, \bar{v}) = \check{t}^a_{\bar{l}i}(\tilde{I}, \tilde{c}, \bar{v}) - \check{x}_{\bar{l}i}(\tilde{I}, \tilde{c}, \bar{v})\tilde{I}/n(\bar{v})$, and $\check{t}^p_{0i}(\tilde{I}, \tilde{c}, \bar{v}) = \check{t}^p_{\bar{l}i}(\tilde{I}, \tilde{c}, \bar{v}) - \check{x}_{\bar{l}i}(\tilde{I}, \tilde{c}, \bar{v})[1 - \tilde{I}/n(\bar{v})]$. For $\hat{\Gamma}$ we further set $\hat{p}_l = \check{p}_l$ for all $l \in \mathcal{L}\backslash\{\bar{l}\}$, $\hat{p}_{\bar{l}} = \check{p}_{\bar{l}}\pi(\bar{v})/\pi(\hat{v}) < \check{p}_{\bar{l}}$ and $\hat{p}_0 = \check{p}_{\bar{l}}[\pi(\hat{v}) - \pi(\bar{v})]/\pi(\hat{v}) \in (0, 1)$. Hence, $\sum_{l=0}^{L} \hat{p}_l = 1$. Note that $\pi^{\check{\Gamma}}(\tilde{I}, \tilde{c}) = \sum_{l\in\mathcal{L}} \check{p}_l\pi^{\check{x}_l}(\tilde{I}, \tilde{c}) = \sum_{l\in\{0,...,L\}} \hat{p}_l\pi^{\hat{x}_l}(\tilde{I}, \tilde{c}) = \pi^{\hat{\Gamma}}(\tilde{I}, \tilde{c})$.

Case 3: Consider the mechanism $\hat{\Gamma} = \{(\hat{p}_l, \check{\gamma}_l)\}_{l\in\{0,...,L\}}$, which, in addition to the same collection of deterministic mechanisms $\check{\gamma}_l$ as $\check{\Gamma}$ but with $\check{\gamma}_{\bar{l}}$ exchanged by the deterministic mechanism $\hat{\gamma}$ as defined in Case 1, also contains the deterministic mechanism $\check{\gamma}_0 = (\check{t}_0, \check{x}_0)$. This deterministic mechanism is identical to $\check{\gamma}_{\bar{l}}$ for all $(I, c, v) \in \mathcal{K}\times\mathcal{V}$ except for $(\tilde{I}, \tilde{c}, \hat{v})$. Hence, for all $(I, c, v) \in (\mathcal{K}\times\mathcal{V})\backslash\{(\tilde{I}, \tilde{c}, \hat{v})\}$, let $\check{t}_0(I, c, v) = \check{t}_{\bar{l}}(I, c, v) \in \mathbb{R}^{2n}$ and $\check{x}_0(I, c, v) = \check{x}_{\bar{l}}(I, c, v) \in \{0, 1\}^{n+1}$. For all $i \in \mathcal{N}$, let $\check{x}_{00}(\tilde{I}, \tilde{c}, \hat{v}) = 1$ and $\check{x}_{0j(i)}(\tilde{I}, \tilde{c}, \hat{v}) = \check{x}_{\bar{l}i}(\tilde{I}, \tilde{c}, \bar{v})$, $\check{t}^a_{0j(i)}(\tilde{I}, \tilde{c}, \hat{v}) = \check{t}^a_{\bar{l}j(i)}(\tilde{I}, \tilde{c}, \hat{v}) + \check{x}_{0j(i)}(\tilde{I}, \tilde{c}, \hat{v})\tilde{I}/n(\bar{v})$, and $\check{t}^p_{0j(i)}(\tilde{I}, \tilde{c}, \hat{v}) = \check{t}^p_{\bar{l}j(i)}(\tilde{I}, \tilde{c}, \hat{v}) + \check{x}_{0j(i)}(\tilde{I}, \tilde{c}, \hat{v})[1 - \tilde{I}/n(\bar{v})]$. For $\hat{\Gamma}$, we further set $\hat{p}_l = \check{p}_l$ for all $l \in \mathcal{L}\backslash\{\bar{l}\}$, $\hat{p}_{\bar{l}} = \check{p}_{\bar{l}}\pi(\hat{v})/\pi(\bar{v}) < \check{p}_{\bar{l}}$ and $\hat{p}_0 = \check{p}_{\bar{l}}[\pi(\bar{v}) - \pi(\hat{v})]/\pi(\bar{v}) \in (0, 1)$. Hence, $\sum_{l=0}^{L} \hat{p}_l = 1$. Note that $\pi^{\check{\Gamma}}(\tilde{I}, \tilde{c}) = \sum_{l\in\mathcal{L}} \check{p}_l\pi^{\check{x}_l}(\tilde{I}, \tilde{c}) = \sum_{l=0}^{L} \hat{p}_l\pi^{\hat{x}_l}(\tilde{I}, \tilde{c}) = \pi^{\hat{\Gamma}}(\tilde{I}, \tilde{c})$.

In all 3 cases, we obtain an adapted mechanism $\hat{\Gamma}$ that satisfies (36)–(43), but, because $\sum_{i\in\mathcal{N}} x_i(\tilde{I}, \tilde{c}, \hat{v}) = n(\bar{v}) < n(\hat{v})$, it does not satisfy (55). According to step 1, the mechanism $\hat{\Gamma}$ is not optimal. Since $S^{\check{\Gamma}} = S^{\hat{\Gamma}}$, this means that also $\check{\Gamma}$ is not optimal.

Step 3: Due to step 2, if $\check{\Gamma}$ is optimal, then, for any $(l, I, c) \in \mathcal{L}\times\mathcal{K}$, there exists an integer $T_l(I, c) \in \mathcal{N}$ such that if $x_{l0}(I, c, v_1) \neq x_{l0}(I, c, v_2)$ and $n(v_1) = n(v_2)$, then $n(v_1) = n(v_2) = T_l(I, c)$. Moreover, $T_l(I, c)$ is a cutoff in the sense that $x_{l0}(I, c, v) = 0$ for all $v \in \mathcal{V}$ such that $n(v) < T_l(I, c)$, and $x_{l0}(I, c, v) = 1$ for all $v \in \mathcal{V}$ such that $n(v) > T_l(I, c)$.

We next show that for an optimal $\check{\Gamma}$ there is a cutoff $T_l(I, c)$ that is independent of l. That is, we show that if $x_{\bar{l}0}(\tilde{I}, \tilde{c}, \bar{v}_1) \neq x_{\bar{l}0}(\tilde{I}, \tilde{c}, \bar{v}_2)$, $n(\bar{v}_1) = n(\bar{v}_2) = n(\bar{v})$, $x_{\hat{l}0}(\tilde{I}, \tilde{c}, \hat{v}_1) \neq x_{\hat{l}0}(\tilde{I}, \tilde{c}, \hat{v}_2)$ and $n(\hat{v}_1) = n(\hat{v}_2) = n(\hat{v})$, then $n(\bar{v}) = n(\hat{v})$. By step 2 it then follows that $T(\tilde{I}, \tilde{c}) = n(\bar{v}) = n(\hat{v})$ is such an l-independent cutoff.

To see this, suppose to the contrary that $n(\bar{v}) \neq n(\hat{v})$ and, without of loss of generality, assume $n(\bar{v}) < n(\hat{v})$. This implies a bijection $j : \mathcal{N} \to \mathcal{N}$ such that $\bar{v}_i = 1$ implies $\hat{v}_{j(i)} = 1$. By step 1, optimality of $\check{\Gamma}$ implies $\check{x}_{\bar{l}i}(\tilde{I}, \tilde{c}, \bar{v}) = v_i$, and $\check{x}_{\hat{l}0}(\tilde{I}, \tilde{c}, \hat{v}) = 0$ implies $\check{x}_{\hat{l}i}(\tilde{I}, \tilde{c}, \hat{v}) = 0$.

Consider the (deterministic) direct mechanism $\check{\gamma}_{\bar{l}'}$ that is identical to $\check{\gamma}_{\bar{l}}$ except for $(\tilde{I}, \tilde{c}, \bar{v})$ in that $\check{x}_{\bar{l}'0}(\tilde{I}, \tilde{c}, \bar{v}) = 0$ and, for all $i \in \mathcal{N}$, it holds $\check{x}_{\bar{l}'i}(\tilde{I}, \tilde{c}, \bar{v}) = 0$, $\check{t}^a_{\bar{l}'i}(\tilde{I}, \tilde{c}, \bar{v}) = \check{t}^a_{\bar{l}i}(\tilde{I}, \tilde{c}, \bar{v}) - \check{x}_{\bar{l}i}(\tilde{I}, \tilde{c}, \bar{v})\tilde{I}/n(\bar{v})$, and $\check{t}^p_{\bar{l}'i}(\tilde{I}, \tilde{c}, \bar{v}) = \check{t}^p_{\bar{l}i}(\tilde{I}, \tilde{c}, \bar{v}) - \check{x}_{\bar{l}i}(\tilde{I}, \tilde{c}, \bar{v})[1 - \tilde{I}/n(\bar{v})]$.

Consider the (deterministic) direct mechanism $\check{\gamma}_{\hat{l}'}$ which is identical to $\check{\gamma}_{\hat{l}}$ except for $(\tilde{I}, \tilde{c}, \hat{v})$ in that $\check{x}_{\hat{l}'0}(\tilde{I}, \tilde{c}, \hat{v}) = 1$ and, for all $i \in \mathcal{N}$, it holds $\check{x}_{\hat{l}'j(i)}(\tilde{I}, \tilde{c}, \hat{v}) = \check{x}_{\bar{l}i}(\tilde{I}, \tilde{c}, \bar{v})$,

$\check{t}^a_{\bar{l}'j(i)}(\tilde{I},\tilde{c},\hat{v}) = \check{t}^a_{\hat{l}j(i)}(\tilde{I},\tilde{c},\hat{v}) + \check{x}_{\bar{l}'j(i)}(\tilde{I},\tilde{c},\bar{v})\tilde{I}/n(\bar{v})$, and, similarly, $\check{t}^p_{\hat{l}'j(i)}(\tilde{I},\tilde{c},\hat{v}) = \check{t}^p_{\hat{l}j(i)}(\tilde{I},\tilde{c},\hat{v}) + \check{x}_{\bar{l}'j(i)}(\tilde{I},\tilde{c},\bar{v})[1-\tilde{I}/n(\bar{v})]$.

Once more, we distinguish three cases: 1. $\pi(\bar{v}) = \pi(\hat{v})$; 2. $\pi(\bar{v}) < \pi(\hat{v})$, and 3. $\pi(\bar{v}) > \pi(\hat{v})$.

Case 1: We adapt the mechanism $\check{\Gamma}$ to $\hat{\Gamma}$ by exchanging $\check{\gamma}_{\bar{l}}$ by $\check{\gamma}_{\bar{l}'}$ and $\check{\gamma}_{\hat{l}}$ by $\check{\gamma}_{\hat{l}'}$. It then follows that, because $\pi(\bar{v}) = \pi(\hat{v})$, we have $\pi^{\check{\Gamma}}(\tilde{I},\tilde{c}) = \sum_{l\in\mathcal{L}} \pi^{\check{x}_l}(\tilde{I},\tilde{c}) = \sum_{l\in\mathcal{L}} \pi^{\hat{x}_l}(\tilde{I},\tilde{c}) = \pi^{\hat{\Gamma}}(\tilde{I},\tilde{c})$.

Case 2: We adapt the mechanism $\check{\Gamma}$ to $\hat{\Gamma}$ by exchanging $\check{\gamma}_{\bar{l}}$ by $\check{\gamma}_{\bar{l}'}$ and $\check{\gamma}_{\hat{l}}$ by $\check{\gamma}_{\hat{l}'}$. In addition, we add to the collection $\hat{\Gamma}$ the mechanism $\check{\gamma}_0 = (\check{t}_0, \check{x}_0)$ as defined in Case 2 above. For $\hat{\Gamma}$ we further set $\hat{p}_l = \check{p}_l$ for all $l \in \mathcal{L}\backslash\{\hat{l}\}$, $\hat{p}_{\hat{l}} = \check{p}_{\hat{l}}\pi(\bar{v})/\pi(\hat{v}) < \check{p}_{\hat{l}}$ and $\hat{p}_0 = \check{p}_{\hat{l}}[\pi(\hat{v}) - \pi(\bar{v})]/\pi(\hat{v}) \in (0,1)$. Hence, $\sum_{l=0}^{L} \hat{p}_l = 1$. Note that $\pi^{\check{\Gamma}}(\tilde{I},\tilde{c}) = \sum_{l\in\mathcal{L}} \check{p}_l \pi^{\check{x}_l}(\tilde{I},\tilde{c}) = \sum_{l\in\{0,\ldots,L\}} \hat{p}_l \pi^{\hat{x}_l}(\tilde{I},\tilde{c}) = \pi^{\hat{\Gamma}}(\tilde{I},\tilde{c})$.

Case 3: We adapt the mechanism $\check{\Gamma}$ to $\hat{\Gamma}$ by exchanging $\check{\gamma}_{\bar{l}}$ by $\check{\gamma}_{\bar{l}'}$ and $\check{\gamma}_{\hat{l}}$ by $\check{\gamma}_{\hat{l}'}$. In addition, we add to the collection $\hat{\Gamma}$ the mechanism $\check{\gamma}_0 = (\check{t}_0, \check{x}_0)$ as defined in Case 2 above. For $\hat{\Gamma}$ we further set $\hat{p}_l = \check{p}_l$ for all $l \in \mathcal{L}\backslash\{\hat{l}\}$, $\hat{p}_{\hat{l}} = \check{p}_{\hat{l}}\pi(\bar{v})/\pi(\hat{v}) < \check{p}_{\hat{l}}$ and $\hat{p}_0 = \check{p}_{\hat{l}}[\pi(\hat{v}) - \pi(\bar{v})]/\pi(\hat{v}) \in (0,1)$. Hence, $\sum_{l=0}^{L} \hat{p}_l = 1$. Note that $\pi^{\check{\Gamma}}(\tilde{I},\tilde{c}) = \sum_{l\in\mathcal{L}} \check{p}_l \pi^{\check{x}_l}(\tilde{I},\tilde{c}) = \sum_{l\in\{0,\ldots,L\}} \hat{p}_l \pi^{\hat{x}_l}(\tilde{I},\tilde{c}) = \pi^{\hat{\Gamma}}(\tilde{I},\tilde{c})$.

In all 3 cases, we obtain an adapted mechanism $\hat{\Gamma}$ that satisfies (36)–(43), but, because $\sum_{i\in\mathcal{N}} x_{\hat{l}i}(\tilde{I},\tilde{c},\hat{v}) = n(\bar{v}) < n(\hat{v})$, it does not satisfy (55). According to Lemma 7, the mechanism $\hat{\Gamma}$ is not optimal. Since $S^{\check{\Gamma}} = S^{\hat{\Gamma}}$, also $\check{\Gamma}$ is not optimal. Q.E.D.

Proof of Proposition 3: By Lemmas 2–6, we can assume that the optimal weakly feasible mechanism $\check{\Gamma} = \{(\check{p}_l, \check{t}_l, \check{x}_l)\}_{l\in\mathcal{L}}$ satisfies (36)–(43). By Lemma 7, we can moreover assume that for an optimal weakly feasible mechanism, there is a function $T : \mathcal{K} \to \mathbb{N}$ that satisfies (10). Lemma 7 implies that for any $(l,i,I,c,v) \in \mathcal{L}\times\mathcal{N}\times\mathcal{K}\times\mathcal{V}$ such that $n(v) = T(I,c)$, we have $(\check{x}_0(I,c,v), \check{x}_{li}(I,c,v)) = (0,0)$ or $(\check{x}_0(I,c,v), \check{x}_{li}(I,c,v)) = (1, v_i)$. Hence, the optimal weakly feasible mechanism specifies a unique output schedule $x(I,c,v) \in \{0,1\}^{n+1}$ for any (I,c,v) such that $n(v) \neq T(I,c)$, and it mixes between at most two output schedules when $n(v) = T(I,c)$.

With these observations, the proposition then follows by noting that we can complete any collection $\{(\hat{p}_l, \hat{x}_l)\}_{l\in\mathcal{L}}$ that satisfies the above conditions by a transfers schedule $\{\hat{t}_l\}_{l\in\mathcal{L}}$ as defined by (11)–(13). The resulting mechanism $\hat{\Gamma} = \{(\hat{p}_l, \hat{t}_l, \hat{x}_l)\}_{l\in\mathcal{L}}$ then satisfies (36)–(43) and the constraints (27) and (29). It is therefore not only weakly feasible but also (strictly) feasible. We conclude that any constrained efficient allocation is implementable by a crowdfunding mechanism and maximizes the entrepreneur's ex ante profits. Q.E.D.

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