How Lotteries in School Choice Help to Level the Playing Field

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Discussion Paper No. 114

August 30, 2018

Collaborative Research Center Transregio 190 | www.rationality-and-competition.de
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How Lotteries in School Choice Help to Level the Playing Field*

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August 29, 2018

Abstract

The use of lotteries is advocated to desegregate schools. We study lottery quotas embedded in the two most common school choice mechanisms, namely deferred and immediate acceptance mechanisms. Some seats are allocated based on merit (e.g., grades) and some based on lottery draws. We focus on the effect of the lottery quota on truth-telling, the utility of students, and the student composition at schools, using theory and experiments. We find that the lottery quota strengthens truth-telling in equilibrium when the deferred acceptance mechanism is used while it has no clear effect on truth-telling in equilibrium for the immediate acceptance mechanism. This finds support in the experiment. Moreover, the lottery quota leads to more diverse school populations in the experiments, as predicted. Comparing the two mechanisms, students with the lowest grades profit more from the introduction of the lottery under immediate than under deferred acceptance.

Keywords: School choice, immediate acceptance mechanism, deferred acceptance mechanism, lotteries, experiment, market design.

JEL codes: C78, C91, D82, I24.

*The authors would like to thank Bistra Solakova for her contribution to this research in its early phase (see Solakova, 2011). We are grateful to Nina Bonge who helped us run the experiments and to Jennifer Rontganger for copy editing. We received valuable comments from participants of the Matching-in-Practice workshop in Cologne, the Market Design Workshop at ZEW Mannheim, and the German Society of Operations Research Conference at HU Berlin, as well as from Bob Hammond. Bettina Klaus and Dorothea Kübler gratefully acknowledge the hospitality of Stanford University where part of this paper was written. Financial support from Deutsche Forschungsgemeinschaft through CRC TRR 190 and from the Swiss National Science Foundation (SNSF) through Project 100018_162606 is gratefully acknowledged.

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1 Introduction

In urban areas in the US and in Europe, school choice has become common practice. Students are not automatically assigned to a school, but can choose a school that best fulfills their needs and matches their interests. School choice is supposed to permit fair access to good schools and to avoid unjust segregation with respect to socio-demographic characteristics that can arise if there is residential segregation and students are automatically assigned to their district school.

However, some of the high hopes in school choice have been dashed by reality. Allowing parents and students to choose among schools has not led to a desegregation of schools, as documented by Allen et al. (2014) for the UK. For Sweden, Bohlmark et al. (2016) document an *increase* in segregation following the introduction of school choice procedures. Three distinct reasons have been put forward:

(i) The persistence of segregation could be due to the preferences of students and parents. Homophily may lead parents to choose schools with students of a similar socio-economic background, or segregation of schools can result when there is residential segregation and parents prefer schools in their proximity.[1] Relatedly, school choice has not always led to schools with a strong academic record being in higher demand than poorly performing schools.[2]

(ii) School choice mechanisms themselves may inadvertently further segregation. If school choice procedures are complex and it is hard to find the optimal application strategies, then strategically sophisticated, better-informed parents have an advantage when trying to get their children into desirable schools.

(iii) The selection of students based on academic achievements can lead to a segregation of schools along this criterion, i.e., desirable schools admit the high-achieving students. This segregation along the selection criterion can lead to segregation according to socio-economic characteristics where they are correlated with academic achievement.

In this paper, we present mechanisms that deal with the second and third dimension. In particular, we vary the manipulability and hence the strategic complexity of the admissions procedure. Moreover, we study to what extent a lottery can reduce the segregation of schools along academic achievement. In particular, we take an existing

[1] Residential segregation may in turn be reinforced by prioritizing the geographic proximity of applicants which allows affluent parents to gain access to desired schools via the housing market.

[2] For the UK, Allen et al. (2014) write: "Social class differences in school preferences emerge: middle classes tend to value performance and peer group; lower SES [socio-economic status] groups may look for accessibility, friendliness of staff and support for those of lower ability." In a similar vein, school choices in the city of Amsterdam are not driven by the quality of the schools but by their geographical proximity and by the existence of friends who go to the same school, as documented by Ruijs and Oosterbeek (2017).

mechanism involving a lottery, examine it in light of its desegregation properties, and compare it to a natural alternative mechanism.

Segregation has been at the heart of a longstanding public debate in England; English grammar schools base their admission decision on students' exam scores. In regions where these selective grammar schools are numerous and attract the highest achieving students, their counterparts, the so-called comprehensive schools, are left with mostly lower achieving students.[3] To mitigate segregation, the official School Admissions Code 2007 in the UK proposed using lotteries alongside other admission criteria at oversubscribed schools, a proposal supported by Coldron et al. (2008) in their report on secondary school admissions in England. Noden et al. (2014) report that a small but growing number of English schools use lotteries as the main criterion to determine the student priorities. Similarly, in New York City, Educational Option schools use a combination of priorities based on academic performance and a lottery in an attempt to ensure a diverse student composition, but find it increasingly hard to attract high achieving applicants who instead flock to selective exam schools.[4]

To reduce segregation and equalize educational opportunities, the city of Berlin introduced a new admission procedure in the academic year 2010/2011. Schools were no longer allowed to use geographic proximity as an admission criterion, but had to use academic achievement instead. Moreover, in the case of more applicants than seats at a given school, it can assign at most 60% of the seats based on applicants' academic attainment and has to assign 30% via a lottery, with the remaining 10% reserved for cases of hardship.[5]

The introduction of the lottery quota was highly controversial. Left-leaning politicians who favored a less differentiated student composition across schools (and hence a more diverse student composition within schools) called for a larger lottery quota. Steffen Zillich (member of the Berlin parliament and the party Die Linke) argued that using a lottery opens up highly demanded schools to children from educationally deprived social groups and that lotteries counteract a further differentiation of schools.[6] Right-leaning politicians criticized a lottery as arbitrary and favored academic attainment as the principal determinant for priorities.[7]

[3]For the prevalence of selective grammar schools across English cities, see Coldron et al. (2008), Table 13.

[4]Nathanson et al. (2013) find that Educational Option programs are the modal (reported) first preference of low-achieving students while for the remaining students, more than half rank a selective program first and less than one fifth an Educational Option program.

[5]See Basteck et al. (2015) for details on the admission procedure.

[6]The protocol of the plenary session of the parliament of Berlin on June 11, 2009, reads as follows: "Das Los öffnet begehrte Schulen auch für Kinder aus bildungsfernen Schichten. Es ist das Element, das einer weiteren Ausdifferenzierung zwischen Schulen [...] entgegenwirkt.")

[7]Mieke Senftleben (member of the Berlin parliament for the market-liberal party FDP) said in the

Besides the use of a lottery, the Berlin mechanism is controversial in that it applies an immediate acceptance algorithm. This algorithm has been widely used in many cities, most notably in Boston where it first attracted the interest of economists. Following protests from parents and after the involvement of economists who helped design a new mechanism, Boston abandoned the immediate acceptance mechanism in 2005. The main criticism was that under the immediate acceptance mechanism parents have to manipulate their rank-order lists over schools to achieve a good outcome.[8] Such manipulations require strategic sophistication and information about the demand for the schools. Thus, the mechanism favors strategically sophisticated and better-informed parents over others.[9] Under the new mechanism in Boston that is based on the deferred acceptance algorithm, parents cannot gain from misrepresenting their true preferences. This property, called *strategy-proofness*, levels the playing field among the parents. Moreover, truthful reports can serve as a valuable feedback to school authorities on the quality of and the demand for particular schools.

In this paper, we use theory and experiments to investigate the existing mechanism in Berlin, and more generally, to understand the influence of a lottery quota on the two most frequently applied matching mechanisms, the immediate and the deferred acceptance mechanism. Specifically, following up on the controversies that accompanied the introduction of a lottery and taking into account the criticism of the immediate acceptance mechanism, we seek to understand whether the mechanism achieves the political goals of a more diverse student composition of schools and hence less segregation. In addition, we investigate an alternative mechanism with a lottery that is based on the deferred acceptance algorithm. We show how a lottery quota combined with the deferred acceptance mechanism levels the playing field in two of the three dimensions mentioned above: First, it gives students with lower academic achievements a chance to get a seat at their preferred school, thereby reducing segregation according to merit. Second, it reinforces the strategy-proofness of the deferred acceptance mechanism by making it a *strict* best response for more students to report their true preferences, thereby reducing the complexity of the choice task.

Ample evidence both from the field and the laboratory shows that participants in deferred acceptance mechanisms often fail to understand its incentive properties (Chen

plenary session on September 24, 2009, that a lottery undermines the principle of merit, since talent and effort become secondary. ("Eine Schülerlotterie untergräbt das Leistungsprinzip, denn Begabung, Anstrengung und Mühe werden zweitrangig.")

[8]For this reason, England outlawed the use of immediate acceptance in 2007 (Pathak and Sönmez, 2013).

[9]In support of this often voiced concern, Basteck and Mantovani (2018) find that subjects of lower cognitive ability fare worse than their high ability peers and that they are at a greater disadvantage under the manipulable immediate acceptance than under the strategy-proof deferred acceptance mechanism.

and Sönmez, 2006; Hassidim et al., 2016). Instead, they display systematic manipulations of their rank-order list (Echenique et al., 2016). The study of matching mechanisms is in part motivated by the question of how to help participants make the right choices (see for example Bó and Hakimov, 2018). Our study contributes to this discussion, showing that including a lottery quota in the deferred acceptance mechanism makes it strictly dominant to truthfully reveal the most preferred school[10] and in equilibrium can make it a *strict* (rather than a weak) best response to truthfully reveal the complete preferences.

We proceed as follows: First, we analyze the existing Berlin school choice mechanism and show that it satisfies a number of desirable properties that it inherits from the immediate acceptance mechanism without a lottery. We also describe a version of the deferred acceptance mechanism that incorporates a lottery. We show that this mechanism again preserves the desirable properties of the deferred acceptance algorithm, including strategy-proofness. Furthermore, we show that if students' priorities are the same across schools, then adding a lottery to the deferred acceptance mechanism always increases truth-telling in equilibrium (Theorem 1). For general priorities, adding a lottery to the deferred acceptance mechanism makes it a strictly dominant strategy to truthfully rank the most preferred school. While adding a lottery to the immediate acceptance mechanism gives some students an incentive to truthfully rank their most preferred school, some students might demote their most preferred school to avoid competition for a lottery seat.

To provide an empirical test of the predicted properties of the mechanisms involving lotteries, we conduct an experiment. We consider a setup where preferences over schools are correlated and academic achievement determines the students' priorities at schools. We compare the immediate acceptance mechanism with a lottery quota (which is a stylized version of the mechanism used in Berlin) to the immediate acceptance mechanism without a lottery and to the deferred acceptance mechanism with and without a lottery. This allows us to test whether, on average, the lottery quota leads to more truthful revelation of preferences under both mechanisms, as predicted by theory for the school choice problems we consider. Moreover, we investigate which students benefit most from the introduction of a lottery in the two mechanisms and what the effects of the lottery are on the distribution of payoffs across students and on the composition of schools.

The experimental findings support the main theoretical predictions concerning comparisons of the mechanisms. In particular, the results show that lotteries increase truth-telling and lead to a more diverse student body at schools with respect to academic achievement (lotteries harm good but not excellent students who are displaced by students with lower academic achievements but more luck in the lottery). While all of these findings hold for both school choice mechanisms, the immediate acceptance mechanism leads to more

[10] More precisely, misreporting the most preferred school is strictly dominated by truth-telling.

diverse schools than the deferred acceptance mechanism for the same lottery quota, at the cost of being manipulable. We conclude that where a lottery quota is used to strike a compromise between meritocratic and egalitarian principles, the size of the quota should take into account the mechanism used. In particular, it seems advisable to use deferred acceptance, though in combination with a larger quota than one would set under immediate acceptance.

The paper is organized as follows. Section 2 introduces four school choice mechanisms, namely Immediate Acceptance (IA), Deferred Acceptance (DA), Immediate Acceptance with a lottery quota for one third of the seats (IA33), and Deferred Acceptance with the same lottery quota (DA33). Apart from investigating normative trade-offs between these mechanisms (see Table 1) we also analyze the effect of a lottery on truth-telling under DA and IA in equilibrium. Section 3 presents the experimental design and the equilibrium analysis for the four mechanisms, including the hypotheses. Experimental results are found in section 4, and section 5 concludes.

2 School Choice Mechanisms and their Properties

A school choice problem is a many-to-one matching problem between a set of students and a set of schools that have a limited number of seats to allocate. Each student has strict preferences over schools and each school has strict priorities over students. Schools having priorities as opposed to preferences implies that they do not act strategically, e.g., because priorities are mandated by the local education authorities.

A solution to a school choice problem consists of a matching in which all students are matched to schools or remain unmatched (in our setup all students will be matched to schools in equilibrium). A school choice mechanism is a systematic way of selecting a matching for any given school choice problem, taking schools' priorities and students' (reported) preferences as inputs. The choice of a matching mechanism is a crucial decision that local educational authorities need to make. It is therefore important to understand the normative properties of these mechanisms to be able to provide policymakers with some arguments for their decisions. There are several mechanisms that have prevailed in practice, the two most popular being the **I**mmediate **A**cceptance (aka Boston) mechanism, **IA** for short, and the **D**eferred **A**cceptance (aka Gale-Shapley) mechanism, **DA** for short.

2.1 The Immediate Acceptance Mechanism (IA)

The *immediate acceptance mechanism* first requires all students to submit strict preferences over schools and all schools to submit strict priorities over students. Then, IA computes a matching via the following algorithm.

Step 1. Each student applies to the school he ranks first. Each school matches its school seats one by one with the highest priority applicants (it matches all applicants if there are fewer than the number of school seats). All matched students and school seats are removed. All remaining students are rejected and continue to the next step.

Step $k > 1$. Each rejected student applies to his best-ranked school that has not yet rejected him, i.e., his k^{th} ranked school. Each school matches its remaining school seats one by one with the highest priority applicants (it matches all applicants if there are fewer than the remaining number of school seats). All matched students and school seats are removed. All remaining students continue to the next step.

The IA algorithm terminates when all students are matched to school seats (in our setting, students find all schools acceptable and there are enough school seats available).

Doğan and Klaus (2018, Theorem 1) recently proved that the IA mechanism is the only mechanism satisfying non-wastefulness,[11] resource monotonicity,[12] consistency,[13] favoring-higher-ranks,[14] and either rank-respecting unavailable-type-invariance or weak uniform tie-breaking.[15,16] The first three properties are well known and applicable in many economic contexts. The last two properties specifically refer to the role of relative ranks of school seats in students' preferences for the resulting matching. For details, we refer the interested reader to Doğan and Klaus (2018).

One of the main shortcomings of IA is that it may give rise to justified envy: a student might prefer another school seat that is assigned to a lower priority student (*no justified envy* is a key element in the definition of *stability*).[17] Furthermore, IA is not *strategy-proof*; that is, a student may obtain a better match by misrepresenting his preferences. Apart from being a strategic robustness property, strategy-proofness in matching models represents a certain notion of fairness. Former Boston Public Schools superintendent Thomas Payzant, in a memo to the Boston School Committee on May 25th 2005, described the rationale for switching away from a manipulable school choice

[11] *Non-wastefulness* requires that no student prefers an unassigned school seat to his allotment.

[12] *Resource monotonicity* requires that the availability of more school seats has a (weakly) positive effect on all students.

[13] *Consistency* requires that if some students leave with their school seats, and if their school seats are removed accordingly, then the mechanism should allocate the remaining school seats among the remaining students in the same way as in the original problem.

[14] *Favoring-higher-ranks* requires that if there is a student who prefers another school seat to his allotment, then that school seat is not assigned to a student who ranks it lower.

[15] For a definition of these somewhat more technical properties see Doğan and Klaus (2018).

[16] Kojima and Ünver (2014), Afacan (2013), and Doğan and Klaus (2018, Theorem 2) provide further characterizations of the IA mechanism.

[17] A matching is *stable* if it is non-wasteful, no student finds his allotment unacceptable (individual rationality), and there is no justified envy (no pairwise blocking).

mechanism as follows: “A strategy-proof algorithm *levels the playing field* by diminishing the harm done to parents who do not strategize or do not strategize well.” Despite its various shortcomings (see Abdulkadiroğlu and Sönmez, 2003), the IA mechanism remains popular and is currently being used not only in Berlin, but in many school districts in the US (e.g., Minneapolis, Lee County of Florida, Denver, and Cambridge, Massachusetts) and in other countries (e.g., Spain, Calsamiglia and Güell, 2014).

2.2 The Deferred Acceptance Mechanism (DA)

The *deferred acceptance mechanism* first requires all students to submit strict preferences over schools and all schools to submit strict priorities over students. Then, DA computes a matching via the following algorithm (first proposed by Gale and Shapley, 1962).

Step 1. Each student applies to the school he ranks first. Each school tentatively matches its school seats one by one with the highest priority applicants (it tentatively matches all applicants if there are fewer than the number of school seats). All remaining students are rejected and continue to the next step.

Step $k > 1$. Each tentatively matched student applies again to the same school. Each rejected student applies to his best-ranked school that has not rejected him yet. Each school tentatively matches its school seats one by one with the highest priority applicants (it tentatively matches all applicants if there are fewer than the number of school seats). All remaining students are rejected and continue to the next step.

The DA algorithm terminates when all students are tentatively matched to school seats, at which point the current tentative matching becomes final (in our setting, students find all schools acceptable and there are enough school seats available).

Ehlers and Klaus (2016, Theorem 1) proved that the DA mechanism is the only mechanism satisfying non-wastefulness, individual rationality,[18] population monotonicity,[19] and strategy-proofness. Kojima and Manea (2010) and Ehlers and Klaus (2016, Theorem 2) provide further characterizations of the DA mechanism.

Interestingly, the axiomatic characterizations of allocation mechanisms in Ehlers and Klaus (2016), Kojima and Ünver (2014), and Doğan and Klaus (2018) together imply that the IA and the DA mechanisms have a joint normative basis, in that both satisfy the properties of non-wastefulness, individual rationality, resource monotonicity, and population monotonicity. What sets them apart is that the DA mechanism is strategy-

[18] *Individual rationality* requires that each student prefers his allotment over being unassigned.

[19] *Population monotonicity* requires that an increase in student numbers has a (weakly) negative effect on all students.

proof and stable but satisfies neither non-bossiness[20] nor favoring-higher-ranks, while the IA mechanism is non-bossy and favors higher ranks but satisfies neither strategy-proofness nor stability.

As mentioned before, several authors underline the policy importance of strategy-proofness for school choice mechanisms (see, e.g., Abdulkadiroğlu et al., 2006; Pathak and Sönmez, 2008). The argument is not simply that the truthful revelation of preferences is desirable per se, but that strategy-proofness *levels the playing field* for strategically unsophisticated and less informed students by making the simple strategy of reporting preferences truthfully weakly dominant. In fact, DA's strategy-proofness was one of the main reasons why the Boston and New York City public school systems, in collaboration with a team of economists (see, e.g., Abdulkadiroğlu, Pathak, and Roth, 2005; Abdulkadiroğlu, Pathak, Roth, and Sönmez, 2005; Pathak and Sönmez, 2008), decided to switch from an IA-based mechanism that was in place to a new DA-based mechanism.

2.3 The Immediate Acceptance Mechanism with a Lottery (IA33)

In Berlin, the academic year 2010/2011 saw the introduction of a new admission procedure for secondary schools. Geographical priorities were abolished. Instead, 60% of seats are allocated based on academic performance, and another 30% by lottery.[21] We approximate the Berlin secondary school mechanism by an IA mechanism in which one third (33%) of the seats are matched using a (single) lottery.[22] Note that the lottery quota is filled *after* allocating the seats based on academic merit, which increases the impact of the lottery on the final allocation (see Dur et al., 2017).

The *immediate acceptance mechanism with a 33% lottery* first requires all students to submit strict preferences over schools and all schools to submit strict priorities over students. Next, a *lottery ranking* of students is drawn randomly (from a uniform distribution). For simplicity, we assume that the number of seats at each school is a multiple of three. Then, based on the lottery ranking of students, IA33 computes a matching with the following algorithm.

Step 1. Each student applies to the school he ranks first. Each school matches the first two thirds of its school seats one by one with the highest priority applicants and the

[20] *Non-bossiness* requires that whenever a change in a student's preference relation does not bring about a change in his allotment, it does not bring about a change in anybody's allotment.

[21] The remaining 10% are reserved for cases of hardship. See Basteck et al. (2015) for details on the admission procedure.

[22] In practice, Berlin uses multiple school-specific lotteries. However, as the lottery is only relevant in the first round of the mechanism, running a single lottery is equivalent to running multiple lotteries.

remaining one third of its school seats one by one with the remaining highest lottery-priority applicants (it matches all applicants if there are fewer than the number of school seats). All matched students and school seats are removed. All remaining students are rejected and continue to the next step.

Step $k > 1$. Each rejected student applies to his best-ranked school that has not rejected him yet, i.e., his k^{th} ranked school. Each school matches its remaining school seats one by one with the highest priority applicants (it matches all applicants if there are fewer than the remaining number of school seats). All matched students and school seats are removed. All remaining students continue to the next step.

The IA33 algorithm terminates when all students are matched to school seats (in our setting, students find all schools acceptable and there are enough school seats available).

Finally, the IA33 *outcome* is a probabilistic matching, i.e., a convex combination over all deterministic matchings obtained via the above algorithm where each of the underlying lottery rankings occurs with equal probability.

Doğan and Klaus (2018, Theorem 2) characterized a larger set of IA mechanisms, the so-called *choice-based immediate acceptance mechanisms*, by weak non-wastefulness, resource monotonicity, non-bossiness, favoring-higher-ranks, and rank-respecting unavailable-type-invariance. These properties not only imply that in each step of the IA algorithm, students are chosen using a choice function, but also that the choice function satisfies certain properties (namely, acceptance, monotonicity, substitutability, sequence-monotonicity, and sequence-substitutability; see Doğan and Klaus, 2018, for details). At each step of the IA33 algorithm, we can express a school's admission decision via a choice function; in the first step two thirds of the school seats are assigned using priorities and the remaining one third of the school seats are assigned using the lottery. In all later steps students are chosen using only priorities. The corresponding choice function satisfies all the properties above. Hence, for any given realization of the lottery, the IA33 mechanism corresponds to a choice-based immediate acceptance mechanism.

Since the probabilistic matching produced by IA33 is the average of the deterministic matchings associated with each lottery realization, the normative properties discussed above extend to the IA33 mechanism in an ex-post sense. Moreover, as individual rationality, resource monotonicity, population monotonicity, and strategy-proofness are formulated as weak preference domination statements for each realization of the lottery, we can extend these notions to the probabilistic mechanism IA33 by extending preferences from deterministic matchings to probabilistic matchings using stochastic dominance or expected utility (both extensions will work). Thus, these properties also hold in an ex-ante sense (as does truncation invariance).

2.4 The Deferred Acceptance Mechanism with a Lottery (DA33)

The purpose of introducing a lottery in the Berlin public school system was to give students with lower academic achievements a shot at being placed at popular schools with positive probability and to limit the concentration of these students at unpopular schools.[23] It is well understood, especially since the redesign of the Boston and New York city public school assignment, that strategy-proofness and stability are key properties of a well-functioning school choice mechanism. Since IA33 lacks this property, we propose a lottery-based variant of the DA mechanism in which one third (33%) of school seats are matched using a lottery. As is the case in Berlin and our stylized IA33, we fix the precedence order so that in each step the lottery quota is filled *after* allocating the seats based on academic merit, which increases the impact of the lottery on the final allocation (see Dur et al., 2017).

The *deferred acceptance mechanism with a 33% lottery* first requires all students to submit strict preferences over schools and all schools to submit strict priorities over students. Next, a *lottery ranking* of students is drawn randomly (from a uniform distribution).[24] For simplicity, we assume that each school has a number of school seats that is a multiple of three. Then, based on the lottery ranking of students, DA33 computes a matching via the following algorithm.

Step 1. Each student applies to the school he ranks first. Each school tentatively matches the first two thirds of its school seats one by one with the highest priority applicants and the remaining one third of its school seats one by one with the remaining highest lottery-priority applicants (it tentatively matches all applicants if there are fewer than the number of school seats). All remaining students are rejected and continue to the next step.

Step $k > 1$. Each tentatively matched student applies again to the same school. Each rejected student applies to his best-ranked school that has not rejected him yet. Each school tentatively matches the first two thirds of its school seats one by one with the highest priority applicants and the remaining one third of its school seats one by one with the remaining highest lottery-priority applicants (it tentatively matches all applicants if there are fewer than the number of school seats). All remaining students are rejected and continue to the next step.

The DA33 algorithm terminates when all students are tentatively matched to school

[23]See Basteck et al. (2015) for details on the political objectives.

[24]For its superior welfare properties, we use a single rather than multiple lotteries (see Abdulkadiroğlu et al., 2009).

seats. Then, the current tentative matching becomes final. In our setting, students find all schools acceptable and there are enough school seats available.

The DA33 *outcome* is a probabilistic matching, i.e., a convex combination over all deterministic matchings obtained via the above algorithm where each of the underlying lottery rankings occurs with equal probability.

Ehlers and Klaus (2016, Theorem 3) characterized a larger set of DA mechanisms, the so-called *choice-based deferred acceptance mechanisms*, by unavailable-type invariance,[25] non-wastefulness, resource monotonicity, truncation invariance,[26] and strategy-proofness. These properties not only imply that at each step of the DA algorithm, students are chosen using a choice function, but the properties also induce that the choice function satisfies certain properties (namely, acceptance, monotonicity, and substitutability; see Ehlers and Klaus, 2016, for details). At each step of the DA33 algorithm, we can express a school's (tentative) admission decision via a choice function in which students are chosen first using priorities based on academic achievement and then using the lottery. Hence, for any given realization of the lottery, the DA33 mechanism corresponds to a choice-based deferred acceptance mechanism.

Moreover, as the probabilistic matching produced by DA33 is the average of the deterministic matchings associated with each lottery realization, the normative properties discussed above extend to the DA33 mechanism in an ex-post sense. Again, individual rationality, resource monotonicity, population monotonicity, and strategy-proofness are formulated as weak preference domination statements for each realization of the lottery. Therefore, it is possible to extend these notions to the probabilistic mechanism DA33 by generalizing preferences over deterministic matchings to probabilistic matchings using stochastic dominance or expected utility. As for IA33, these properties also hold in an ex-ante sense (as does truncation invariance).

2.5 Summary of Normative Trade-Offs

The previous sections have demonstrated that the four mechanisms share a number of properties, but they also differ with respect to some of them. Table 1 provides an overview of the main ex-post properties of the mechanisms. It is evident that none of the mechanisms dominates another in terms of these desirable properties.

[25] *Unavailable-type invariance* requires that the chosen allocation only depends on preferences over the set of available school seats.

[26] *Truncation invariance* requires that if a student truncates his preference relation in such a way that his allotment remains acceptable under the truncated preference relation, then the allocation is the same under both profiles.

Table 1: Ex-Post Properties of the Mechanisms

	NW	IR	MON	TI	NB	NJ	SP
IA	✓	✓	✓	✓	✓		
IA33	✓	✓	✓	✓	✓		
DA	✓	✓	✓	✓		✓	✓
DA33	✓	✓	✓	✓		(✓)	✓

Notation:
NW stands for non-wastefulness (Footnote 11),
IR stands for individual rationality (Footnote 18),
MON stands for resource monotonicity and population monotonicity (Footnotes 12 & 19),
TI stands for truncation invariance (Footnote 26),
NB stands for non-bossiness (Footnote 20),
NJ stands for no-justified-envy and stability (Footnote 17), and
SP stands for strategy-proofness.

The Table 1 entry in parentheses for DA33 with respect to the no-justified envy property (NJ) stands for a modified version of this property. While a student may envy another student of lower academic achievement who is admitted based on the lottery, no justified envy (NJ) holds within each quota. Thus, no student of higher priority with respect to academic achievement will envy another student who is admitted based on this priority, and no student with higher lottery priority will envy another student who is admitted based on the lottery.

It is also interesting to note that the stronger strategic robustness property *group strategy-proofness*, which requires that no group of students together can obtain better matches by misrepresenting their preferences, is equivalent to strategy-proofness and non-bossiness. We see in Table 1 that DA and DA33 pick up one part of group strategy-proofness (strategy-proofness) while IA and IA33 pick up the other part (non-bossiness).

One important difference between the mechanisms concerns the incentives of participants to state the truth, that is, strategy-proofness (SP). Given the central importance of strategy-proofness in real applications also due to its fairness aspect, we investigate it in more detail. First, we study the effect of the lottery quota from a theoretical perspective. Then, with the help of a lab experiment, we ask how the manipulability of IA and IA33, in contrast to the strategy-proofness of DA and DA33, affects students' reporting strategies.

2.6 The Effect of a Lottery on Truth-Telling

Under DA, truthful reporting is a weakly dominant strategy. There may, however, exist non-truth-telling equilibria, for example where applicants demote schools in their ranking at which they stand no chance of being admitted. Introducing a lottery gives every applying student a non-zero chance to be admitted to a school and thus rules out some of these non-truth-telling equilibria. As a result, the introduction of a lottery can lead to more truth-telling in equilibrium. For the case where students' priorities are the same across schools, for example when they are induced by exam scores or other measures of academic achievement, we are able to state this formally in Theorem 1 below.[27]

For Theorem 1, we may choose between two different assumptions on how students compare probability distributions over schools. For one, we may assume that any student strictly prefers a probability distribution over another if and only if the former yields a *higher expected utility*. Alternatively, we may assume that any student strictly prefers a probability distribution over another if and only if the former *stochastically dominates* the latter.[28]

Finally, a school choice game under DA33 requires students to submit rank-order lists of schools that may be interpreted as students' reported strict preferences over schools. For given preferences regarding probability distributions over schools (based on expected utility or stochastic dominance), a profile of student's reports is a *pure strategy Nash equilibrium* in the school choice game under DA33, if no student can achieve a strictly preferred probability distribution over schools by changing his preference report. Note that when comparing distributions using stochastic dominance rather than expected utility, fewer probability distributions are comparable, which implies that in any strategy profile there are (weakly) fewer profitable deviations and hence the set of equilibria is (weakly) larger.

Theorem 1 (**For Aligned Priorities, an Equilibrium under DA33 is an Equilibrium under DA but not Vice Versa**).
Assume students' priorities are the same across schools and assume either that students compare probability distributions over schools based on expected utility or that they compare them by stochastic dominance. Then,

(a) *any pure strategy Nash equilibrium in the school choice game under DA33 is a pure strategy Nash equilibrium in the school choice game under DA, and*

[27] We establish the result for DA33, but it holds more generally for any positive lottery quota.

[28] In our school choice model, probability distribution $\mathbb{P}_1$ stochastically dominates probability distribution $\mathbb{P}_2$ if the following holds: the probability to be matched to the most preferred school under $\mathbb{P}_1$ is at least as high as under $\mathbb{P}_2$, the probability to be matched to the two most preferred schools under $\mathbb{P}_1$ is at least as high as under $\mathbb{P}_2$, the probability to be matched to the three most preferred schools under $\mathbb{P}_1$ is at least as high as under $\mathbb{P}_2$, etc., with one of these probabilities being strictly higher for $\mathbb{P}_1$.

(b) *some pure strategy Nash equilibria in the school choice game under DA are not pure strategy Nash equilibria in the school choice game under DA33 (and hence violate truth-telling in ways that cannot arise in an equilibrium under DA33).*

Theorem 1 implies that if students' priorities are the same across schools then the set of equilibria in the school choice game under DA33 is a strict subset of the set of equilibria under DA. Since both sets of equilibria include truth-telling as an equilibrium, all of the additional equilibria that occur under DA involve some students not telling the truth – in this sense *there is relatively more truth-telling in equilibrium under DA33 than under DA.*

Proof of Theorem 1. **(a)** Let P be a profile of students' preference reports over schools that is a pure strategy Nash equilibrium in the school choice game under DA33. We show that P is also a pure strategy Nash equilibrium in the school choice game under DA.

Assume, toward a contradiction, that P is not a pure strategy Nash equilibrium in the school choice game under DA. Then, at profile P, some student i can get a preferred school by reporting different preferences. Since truth-telling is a weakly dominant strategy under DA, student i can also get a preferred school by reporting his true preferences P_i^t.

Now, recall that the DA33 produces a probability distribution by using the deterministic DA33 algorithm for each possible lottery ranking and by taking the average over the obtained outcomes. Furthermore, in the deterministic DA33 algorithm for any lottery ranking, it is a weakly dominant strategy for student i to report his true preferences P_i^t. Hence, for any lottery ranking, student i, when telling the truth P_i^t instead of P_i, will get the same or a (strictly) preferred school. Moreover for the particular lottery ranking that coincides with the priorities of schools over students, the associated deterministic DA33 and the DA coincide – thus, with our previous observation that student i is assigned a (strictly) preferred school under DA when he reports the truth P_i^t instead of P_i, we can conclude that the DA33 probability distribution when student i tells the truth P_i^t stochastically dominates (and hence yields a strictly higher expected utility than) the DA33 probability distribution based on P_i; a contradiction to P being a pure strategy Nash equilibrium in the school choice game under DA33.

(b) The following example shows that the set of pure strategy Nash equilibria in the school choice game under DA33 can be a strict subset of the set of pure strategy Nash equilibria in the school choice game under DA.

We assume that there are two schools, A and B, each with three seats and a common priority ranking over students $1, 2, \ldots, 6$ such that students are labeled by their rank. All students consider A to be a better school than B. Under DA, students 1, 2, and 3 are assigned to A and students 4, 5, and 6 are assigned to B. It is a pure strategy Nash equilibrium in the school choice game under DA that students 1, 2, and 3 truthfully report school A as their best school while students 4, 5, and 6 falsely report school B as their best school. This is not a pure strategy Nash equilibrium in the school choice game under DA33 because each of the students 4, 5, and 6 has a positive probability of being assigned to school A if he ranks it first and hence would have an incentive to unilaterally deviate to truth-telling. $\square$

Next, we demonstrate that if the schools' priorities over students are sufficiently heterogeneous, as for instance when they are given by walkzones, then the introduction of a lottery can create new non-truth-telling equilibria.

Example 1 (**For General Priorities, an Equilibrium under DA33 might not be an Equilibrium under DA**). Consider three schools, A, B, and C, with three seats each, one at each school reserved for the lottery quota, and a common priority ranking over students $1, 2, \ldots, 6$ such that students are labeled by their rank. Both schools A and B rank students $1, 2, \ldots, 6$ higher than the remaining three students 7, 8, and 9. Schools' priorities and students' preferences are as follows (entries marked with $\diamond$ are not relevant).

schools' priorities	1st	2nd	3rd	4th	5th	6th	7th	8th	9th
A	1	2	3	4	5	6	7	8	9
B	1	2	3	4	5	6	9	7	8
C	$\diamond$	$\diamond$	$\diamond$	$\diamond$	$\diamond$	$\diamond$	$\diamond$	$\diamond$	$\diamond$

students' preferences	1st	2nd	3rd
1 & 2	A	$\diamond$	$\diamond$
3 & 4	B	$\diamond$	$\diamond$
5 & 6	C	$\diamond$	$\diamond$
7	B	A	C
8 & 9	A	B	C

In the truth-telling equilibrium under DA33, students 1 and 2 report school A as their best and are admitted (based on their rank), while students 3 and 4 as well as 5 and 6 are likewise admitted to schools B and C. Students 7, 8, and 9 compete for the lottery seats at A and B (one per school). It is easy to see that given any lottery ranking, student 7 will never be assigned to school A: if student 7 is first in the lottery ranking he will be assigned to school B while if either student 8 or 9 are first in the lottery ranking, one of them will be assigned to school A. Hence, a unilateral deviation from truth-telling by student 7 where he demotes A and reports B as his best and C as his second best school yields the same outcome and is still an equilibrium under DA33.

We show that the above non-truth-telling equilibrium under DA33 is not an equilibrium under DA. Under truth-telling, students 1–6 are admitted to the same school as before. For student 7, we now find that he is admitted to A (his second-most preferred school) while student 9 is admitted to B. If student 7 was to unilaterally deviate by demoting A in his reported ranking – reporting B as his best and C as his second best school – he would still not be admitted to B but would instead be admitted to C, making him strictly worse off than under truth-telling. □

Remark 1 (**For General Priorities, in Equilibrium under DA33 First-Ranked Schools Must be Reported First**)**.** Since truth-telling is weakly dominant and since with a lottery, students are admitted to their first-ranked school whenever they are ranked first by the lottery, it is a *strictly* dominated strategy (dominated by truth-telling) not to rank the most preferred school first. Hence, in all pure strategy Nash equilibria in the school choice game under DA33, students will truthfully reveal their most preferred school, independently of schools' priorities. □

Next, consider the effect of a lottery on truth-telling under IA. In any pure strategy equilibrium of the school choice game induced by the IA algorithm, we can classify players according to the following three types.

(i) Students who in equilibrium receive their most preferred school (and may truthfully rank it first).

(ii) Students who in equilibrium see their k most preferred schools filled by applicants of higher priority and who rank their $(k+1)^{th}$ most preferred school first in order to be admitted there.

(iii) Students who in equilibrium see their k most preferred schools filled by applicants of higher priority and who are admitted to their $(k+1)^{th}$ most preferred school even without ranking it first.

Students of type (ii) have a strict incentive to misreport their preferences, and students of type (iii) are indifferent between several (mis)reports, including ranking any of their $(k+1)$ most preferred schools first. Introducing a lottery gives applicants of type (ii) and (iii) a positive probability to obtain their most preferred school, and hence increases their incentives to apply truthfully – in particular to report their most preferred school first. However, some students of type (i) may lose their priority as we move from IA to IA33 and hence may misreport their most preferred school, opting for a safe seat at a less preferred school instead of competing for a lottery seat. Hence, while the introduction of a lottery increases the incentives to report preferences truthfully for some students, its overall effect on truth-telling is ambiguous.

3 Design of the Experiment

The experiment investigates behavior under the two most important school choice mechanisms, IA and DA, in combination with a lottery quota. It is the first experiment to study the effect of a lottery under IA and DA on truth-telling, the utility of students, and the student composition at schools. We also use the experiment to understand the differences of the effect of the lottery on the properties of the two mechanisms.

3.1 The Experimental School Choice Problems

The participants in the experiment are faced with a school choice problem. Each of them is part of a group of 12 students who are competing for seats at three schools. The three schools differ in size and popularity. Schools A and B have three seats each while school C has six seats. School A is the most popular school in that it is most preferred by nine out of the 12 students. School B is the second-most popular school in that it is most preferred by three out of 12 students. School C is the unanimously least preferred school and all students prefer any school over being unmatched. Thus, our setting features two preference types – a majority of students whose preferences are $A \succ B \succ C \succ \emptyset$ (A-types) and a minority whose preferences are $B \succ A \succ C \succ \emptyset$ (B-types) – and aims to capture both a high correlation of preferences (A is the most and C the least preferred school) as well as some degree of heterogeneity.

Schools have the same strict priorities over students, which we interpret as a ranking of students by academic achievement. To simplify notation, we identify students' names and their relative position in this ranking – student 1 is the strongest student enjoying the highest priority, student 2 the second strongest, etc., and student 12 is the weakest student.

Table 2 indicates how we construct the 20 different school choice problems, namely the preference profiles for the 12 students identified by their rank i (the complete profiles can be found in Appendix B.1, Table 10).

Table 2: Preference Profiles

i	$P1$	$P2$	$\ldots$	$P20$
1–6	A	A	A	A
7	A	A	$\ldots$	B
8	A	A	$\ldots$	B
9	A	B	$\ldots$	B
10	B	A	$\ldots$	A
11	B	B	$\ldots$	A
12	B	B	$\ldots$	A

Notes: Preference types (most-preferred schools) of students 1–12 in profiles $P1$–$P20$.

We assume that the six strongest students 1–6 prefer school A, while the three students who prefer B are ranked between 7 and 12.[29] Permuting the rank of the three students who most prefer B, we arrive at 20 different preference profiles, and hence 20 different experimental school choice problems used in our treatments. We will denote the set of A-type students in rank 7–12 by 7–12(A) and the set of B-type students in rank 7–12 by 7–12(B).

To induce the preference profiles in the experiment, students receive a payoff that depends on both the school he is matched to and his own type. A-type students receive 22€ if admitted to A and 16€ if admitted to B while the payoffs for B-type students are reversed – they receive 22€ if admitted to B and 16€ if admitted to A. All students receive 10€ if admitted to C and 0€ if not admitted to any school. While it is possible for a student to be unassigned if he lists too few schools, he can always guarantee a school seat for himself by ranking enough schools, so that school C is effectively the safety school in our experiment.

3.2 Equilibrium Predictions

We analyze the corresponding school choice games for our mechanisms (IA, IA33, DA, and DA33) assuming complete information and players maximizing their expected payoffs.

[29] Hence, we focus on cases in which there is a positive correlation between academic rank and a preference for the top school. The converse case where stronger applicants prefer the less popular school would tend to yield a trivial and straightforward match.

Equilibria under IA. *In every (Nash) equilibrium of the school choice game induced by the IA mechanism and any of the preference profiles $P1$–$P20$:*

- *Students* 1–3 *rank A first (not ranking A first is strictly dominated) and are admitted.*
- *Students* 4–6 *rank B first (strict best response) and are admitted.*
- *Students* 7–12 *are admitted to C (they can report any ranking).*

students	1–3	4–6	7–12(A)	7–12(B)
match probability at A	1	0	0	0
match probability at B	0	1	0	0
match probability at C	0	0	1	1

In equilibrium, at least three students truthfully report their most preferred school (students 1–3) and at least three students (students 4–6) misrepresent their preferences. All six remaining students (students 7–12) will always be matched to school C no matter which preferences they report. We derive the set of equilibria in detail in Appendix A.1.

Equilibria under IA33. *In every (Nash) equilibrium of the school choice game induced by the IA33 mechanism and any of the preference profiles $P1$–$P20$:*

- *Students* 1–2 *rank A first (not ranking A first is strictly dominated) and are admitted.*
- *Students* 3–4 *rank B first (strict best response) and are admitted.*
- *All other A-type students with ranks between* 5–12 *rank A first (strict best response). They are admitted to A with probability $1/5 = 0.2$ and are otherwise admitted to C.*
- *All three B-type students with ranks between* 7–12 *rank B first (strict best response). They are admitted to B with probability $1/3 \approx 0.333$ and are otherwise admitted to C.*

students	1–2	3–4	5–12(A)	7–12(B)
match probability at A	1	0	0.2	0
match probability at B	0	1	0	0.333
match probability at C	0	0	0.8	0.667

In equilibrium, apart from students 3 and 4, all students truthfully report their most preferred school. Intuitively, the two top students (students 1–2) can always obtain their most preferred school A. The two next best students (students 3–4) would have to enter a lottery for the one remaining seat at school A, where they have no priority over lower-ranked applicants, or they can apply to school B and be sure to be matched to their second-most preferred school – in equilibrium, they opt for the latter. With both top schools almost filled, the decision of lower-ranked students (students 5–12) at which school to enter the lottery for the remaining seat is governed by their type. We derive the set of equilibria in detail in Appendix A.2.

Equilibria under DA. *In every (Nash) equilibrium of the school choice game induced by the DA mechanism and any of the preference profiles $P1$–$P20$:*

- *Students 1–3 rank A first (not ranking A first is strictly dominated) and are admitted.*
- *Students 4–6 rank B above C (they may rank A or B first) and are admitted to B.*
- *Students 7–12 are admitted to C (they can report any ranking).*

students	1–3	4–6	7–12(A)	7–12(B)
match probability at A	1	0	0	0
match probability at B	0	1	0	0
match probability at C	0	0	1	1

In equilibrium, at least three students truthfully report their most preferred school (students 1–3). For all students it is a weakly dominant strategy to report their true preferences, but three students (4–6) may misreport B to be most preferred and the remaining students (7–12) will be matched to C independently of their reported preferences. We derive the set of equilibria in detail in Appendix A.3.

Equilibria under DA33. *In every (Nash) equilibrium of the school choice game induced by the DA33 mechanism and any of the preference profiles $P1$–$P20$:*

- *Students 1–2 rank A first (not ranking A first is strictly dominated) and are admitted.*
- *Students 3–12 report their true preferences (not ranking the most preferred school first is strictly dominated and truthfully ranking all schools is a strict best response).*
- *Students 3–4 are admitted to A with probability $2/15 \approx 0.133$ and are otherwise admitted to B.*

- *Student* 5 *is admitted to* A *with probability* $2/15 \approx 0.133$*, to* B *with probability* $34/105 \approx 0.324$*, and is otherwise admitted to* C*.*

- *All other* A*-type students with ranks between* 6–12 *are admitted to* A *with probability* $2/15 \approx 0.133$*, to* B *with probability* $3/35 \approx 0.086$*, and are otherwise admitted to* C*.*

- *All three* B*-type students with ranks between* 7–12 *are admitted to* A *with probability* $1/45 \approx 0.022$*, to* B *with probability* $1/5 = 0.2$*, and are otherwise admitted to* C*.*

students	1–2	3–4	5	6–12(A)	7–12(B)
match probability at A	1	0.133	0.133	0.133	0.022
match probability at B	0	0.867	0.324	0.086	0.2
match probability at C	0	0	0.543	0.781	0.778

In contrast to DA, the lottery ensures that in the DA33 mechanism every student has a chance to be admitted to his first-ranked school. Hence, all students rank their most preferred school first and all but students 1 and 2 have an incentive to truthfully report their complete preferences. We derive the set of equilibria in detail in Appendix A.4.

Remark 2 (IA and DA have the same Equilibrium Outcome). Observe that the unique equilibrium outcomes for our experimental school choice problems under IA and DA are the same (although the underlying strategy profiles can differ). This design feature simplifies the comparison of the results of the two mechanisms with a lottery.

3.3 Experimental Design and Procedures

Treatments

We employ a between-subjects design where each treatment is devoted to one of the four mechanisms (IA, IA33, DA, and DA33). Treatment IA33 is based on the mechanism that is used in Berlin for secondary schools. The 20 school choice problems described in Table 2 were used in every treatment in the order of their presentation in Appendix B.1. For each school choice problem, the students' ranks were randomly assigned anew to the participants, and each participant also received a new lottery ranking in treatments IA33 and DA33. Thus, each participant played 20 school choice games with changing roles and with slightly different preference profiles. While students were aware of their priority ranking based on academic achievement when submitting their preferences over schools, the lottery ranking was not disclosed to them until the end of the round. This reflects the fact that students have a good idea of their grade point averages when applying to secondary schools, but that the lottery ranking is drawn only after the applications have been submitted.

Experimental Procedures

The computerized experiment was conducted at the TU-WZB Experimental Lab in Berlin.[30] Overall, we conducted 16 sessions, with four sessions for each of the treatments IA, IA33, DA, and DA33, respectively. There were 24 subjects in a session, divided into two matching groups of 12 participants. Subjects stayed together in this group of 12 for the entire 20 rounds of the experiment. Thus, we end up with eight independent observations and data from 4 x 24 = 96 participants per treatment, yielding a total number of 384 subjects. The sessions lasted an average of 90 minutes and subjects earned around 20€The experiment was programmed with z-Tree (Fischbacher, 2007). Each subject participated in only one session.

Table 3: Experimental Treatments

Treatment	Lottery	# of subjects
IA	no	96
IA33	yes	96
DA	no	96
DA33	yes	96

In each session, subjects were randomly assigned to the computer terminals in the lab. They were given written instructions. To ensure that everybody understood the tasks, we conducted a quiz before starting with the experiment. For the quiz, the subjects had to work through examples of allocating seats in a school choice problem. They had to apply the same algorithm as the one used in their respective treatment. We checked their written answers and clarified all remaining questions in private.

An English translation of the experimental instructions and the quiz as well as screenshots (in German) can be found in Appendix B.

[30] The first sessions were run in 2011 and formed part of a master's thesis (Solakova, 2011). Additional sessions were run in 2015 and 2017.

3.4 Hypotheses

Truth-Telling

Table 4 shows which students are predicted to truthfully reveal their full preferences, truthfully report their most preferred school, etc.

Table 4: Truthful Revelation in Equilibrium

i	IA	DA	IA33	DA33
1	$\bar{\boldsymbol{t}}$	$\bar{\boldsymbol{t}}$	$\bar{\boldsymbol{t}}$	$\bar{\boldsymbol{t}}$
2	$\bar{\boldsymbol{t}}$	$\bar{\boldsymbol{t}}$	$\bar{\boldsymbol{t}}$	$\bar{\boldsymbol{t}}$
3	$\bar{\boldsymbol{t}}$	$\bar{\boldsymbol{t}}$	$\boldsymbol{\ell}$	$\boldsymbol{t}$
4	$\boldsymbol{\ell}$	$\underline{\boldsymbol{t}}$	$\boldsymbol{\ell}$	$\boldsymbol{t}$
5	$\boldsymbol{\ell}$	$\underline{\boldsymbol{t}}$	$\bar{\boldsymbol{t}}$	$\boldsymbol{t}$
6	$\boldsymbol{\ell}$	$\underline{\boldsymbol{t}}$	$\bar{\boldsymbol{t}}$	$\boldsymbol{t}$
7–12(A)	$\sim$	$\sim$	$\bar{\boldsymbol{t}}$	$\boldsymbol{t}$
7–12(B)	$\sim$	$\sim$	$\bar{\boldsymbol{t}}$	$\boldsymbol{t}$

Notation:
$\boldsymbol{t}$ indicates that applicants reveal their full preferences truthfully,
$\bar{\boldsymbol{t}}$ indicates that applicants rank their most preferred school first,
$\underline{\boldsymbol{t}}$ indicates that applicants rank their least preferred below their second-most preferred school,
$\boldsymbol{\ell}$ indicates that applicants misrepresent their most preferred school and rank their second-most preferred school first, and
$\sim$ indicates that applicants are indifferent as reports are ineffectual.

As we move from IA to DA or from IA33 to DA33, truth-telling becomes a (weakly) dominant strategy and hence we expect truth-telling rates to increase. Moving from IA to DA, this effect should be strongest for the good but not excellent students who are not good enough to ensure themselves a seat at their most preferred school, but would, in equilibrium, be good enough to be admitted to their second-most preferred school under immediate acceptance, provided they misreport their preferences and rank it first. This yields our first hypothesis.

Hypothesis 1. *On average, truth-telling increases as we move from IA to DA and from IA33 to DA33.*

Next, we consider the effect of a lottery on truth-telling under immediate acceptance. Moving from IA to IA33 increases the chances for students 5–12 to be admitted to their most preferred school when ranking it first. Hence, we expect truth-telling to increase for these students. The overall increase in truth-telling is tempered by the fact that student 3 is no longer guaranteed a seat at school A and may refrain from truth-telling to avoid the lottery at school A (see section 2.6 and Table 4).

Hypothesis 2. *On average, truth-telling increases as we move from IA to IA33.*

For deferred acceptance, truth-telling is an equilibrium under both DA and DA33 but the introduction of the lottery eliminates most of the non-truth-telling equilibria that exist besides truth-telling under DA (see Theorem 1 and Table 4). Moreover, the lottery quota makes it a dominated strategy not to report the most preferred school truthfully and makes reporting the true preferences over all three schools a strict best response for all but the two top-ranked students. Without the lottery, truth-telling is never a strict best response in equilibrium. Truthfully revealing the most preferred school is a strict best response only for the three best students in the equilibrium under DA, and truthfully ranking the second-most over the least preferred school is a strict best response only for students 4–6.

Hypothesis 3. *On average, truth-telling increases as we move from DA to DA33.*

Student Composition of Schools and Payoff Distribution

Given the political reasons that motivated the introduction of a lottery, we investigate whether it leads to a more equitable distribution of payoffs among students and to smaller differences in the student compositions across schools. Moreover, we analyze whether the lottery is more effective in reaching these goals when used in combination with IA or DA.

Table 5 shows the expected payoffs in equilibrium for students in each rank across treatments.

The following hypothesis summarizes the effects of introducing a lottery quota both for IA and DA.

Hypothesis 4. *Moving from IA to IA33 and from DA to DA33,*

(i) *increases the average payoffs of the lower-ranked students, but decreases the average payoffs of the good but not excellent students and*

(ii) *leads to a more equitable distribution of expected payoffs across ranks, as measured by the Gini index.*

Table 5: Expected Equilibrium Payoffs

i	IA/DA	IA33	DA33
1	22.00	22.00	22.00
2	22.00	22.00	22.00
3	22.00	16.00	16.80
4	16.00	16.00	16.80
5	16.00	12.40	13.54
6	16.00	12.40	12.11
7–12(A)	10.00	12.40	12.11
7–12(B)	10.00	14.00	12.53
Sum of payoffs	174.00	180.00	177.20
Average payoff	14.50	15.00	14.77
Gini index	0.181	0.111	0.121

Notes: Expected equilibrium payoffs for students 1–12 in each treatment. Students in rank 7–12 may be of preference type A or B.

As the importance of academic rank decreases with the introduction of a lottery, the higher-ranked students $3, 5$, and 6 lose while the lower-ranked students 7–12 gain. No reduction in payoff is predicted for student 4 because in equilibrium his rank ensures him a seat at school B under both IA33 and DA33.

Given that in general the lower-ranked students expect lower payoffs in equilibrium, the redistribution of payoffs toward these students reduces the inequality in expected payoffs as measured by the Gini index, presented in the last row of Table 5.

Next, we investigate the effect of the lottery on the student composition of the schools. Harmonizing the student composition was one of the main motivations for adopting the lottery in Berlin. Table 6 provides the expected average ranks of students at the schools under the four mechanisms.

For both IA and DA, providing lower-ranked students with access to more popular schools, should reduce differences in student composition.

Hypothesis 5. *Moving from IA to IA33 and from DA to DA33 decreases the difference between the average ranks of students at the three schools.*

Table 6: Expected Average Student Rank at Schools

i	IA/DA	IA33	DA33
A	2	3.63	3.27
B	5	5.5	5.45
C	9.5	8.43	8.64
Δ_{AB}	3	1.87	2.18
Δ_{BC}	4.5	2.93	3.19
Δ_{AC}	7.5	4.8	5.37

Notes: Averages of the equilibrium outcomes for all 20 preference profiles $P1$–$P20$. Δ_{ij} denotes the difference in average ranks between schools i and j.

Finally, we compare DA33 and IA33. Notice that good but not excellent students – those who are just below the cutoff to be admitted to the most competitive school A based on their academic rank (in particular 3, 4, and 5) – have an advantage under DA33 compared to IA33. Under DA33, they can first enter a lottery at their most preferred school and, if they are rejected, they can still apply to their second-most preferred school where they are likely to be admitted based on their rank. In contrast, under IA33 they have to decide whether to apply for the lottery seat at A or whether to rely on their high rank and apply to B where they are most likely admitted in Step 1. However, the fact that these good but not excellent students fare better under DA33 than under IA33 comes at the expense of lower-ranked students who are less likely to be admitted to their most preferred school (see section 3.2).

Hypothesis 6. *Comparing IA33 to DA33,*

(i) *the good but not excellent students fare better under DA33 while the lower-ranked students fare better under IA33;*

(ii) *inequality, as measured by the Gini index, is lower under IA33 than under DA33; and*

(iii) *schools are more similar with respect to the average rank of their students under IA33 than under DA33.*

We note in passing that we expect no differences in the distribution of payoffs and the composition of schools when comparing IA and DA without a lottery, since the matching outcomes are identical in equilibrium (see Remark 2).

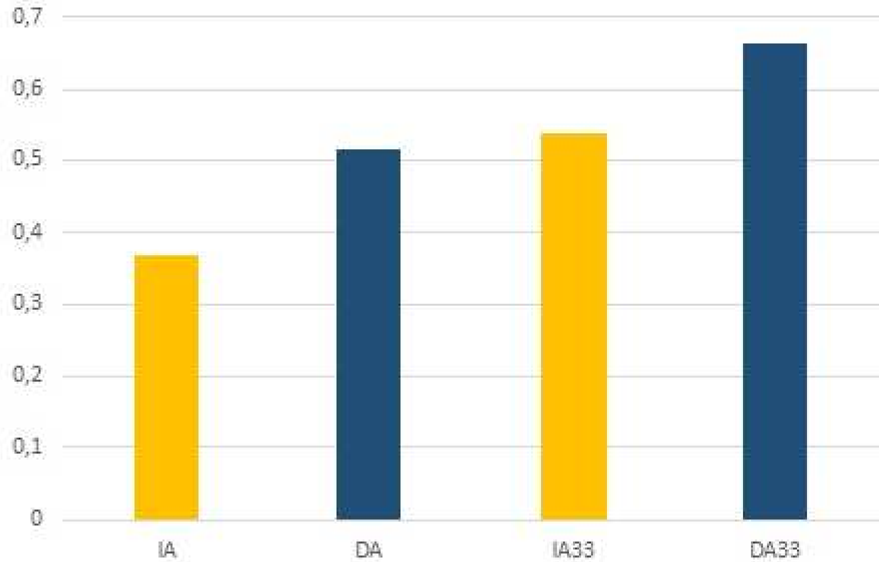
4 Experimental Results

We first analyze the observed preference manipulations across the four mechanisms (IA, IA33, DA, and DA33). Then, we investigate the payoffs of students, the distributional effects of the lottery quota as well as the composition of schools.

Truth-telling

We say that a participant is truth-telling if he states his full preferences truthfully. In the following, we report our findings with respect to this measure, but the results remain qualitatively unchanged if we consider truthful reports of the first preference only (analyses available upon request from the authors). Figure 1 reports the average truth-telling rates across mechanisms.

Figure 1: Average truth-telling rates



Notes: The figure displays the proportion of participants per treatment that submit their full lists truthfully.

The left panel of Table 7 presents the average truth-telling rates at each rank separately.

In line with Hypothesis 1, we find that, on average, truth-telling rates are significantly higher under DA than under IA, as indicated by the MWU test with $p = .002$ in the last row of the right panel. Moreover, the effect is strongest for the good but not excellent students 4–6, significant at $p = .001$. When comparing IA33 and DA33, average truth-telling rates are significantly higher under DA33 ($p = .005$). We find the strongest increase in truth-telling for students 3 and 4 and a smaller significant increase for students 2 and 5. Hence, as hypothesized, the increase in truth-telling due to a switch from immediate to deferred acceptance is most pronounced for good but not excellent students.

Table 7: Observed Truth-telling Rates

i	Truth-telling rates (treatment averages) IA	DA	IA33	DA33	Mann-Whitney U test (p-values) IA vs. DA	IA33 vs. DA33	IA vs. IA33	DA vs. DA33
1	.756	.794	.813	.85	.561	.363	.492	.262
2	.8	.813	.781	.894	.705	**.017**	.706	.071
3	.794	.8	.288	.631	.872	**.001**	**.001**	**.014**
4	.15	.519	.219	.581	**.001**	**.001**	.096	.397
5	.081	.45	.406	.669	**.001**	**.008**	**.001**	**.006**
6	.069	.444	.431	.594	**.001**	.071	**.001**	**.038**
7–12(A)	.162	.377	.567	.644	**.003**	.225	**.001**	**.002**
7–12(B)	.429	.406	.615	.608	.489	.958	**.002**	**.001**
1–12	.369	.514	.54	.665	**.002**	**.005**	**.002**	**.005**

Notes: The left panel presents the average truth-telling rates, i.e., the proportion of subjects submitting the full list of three schools truthfully. In the right panel, the Mann-Whitney U test is based on eight matching group averages per treatment. All p-values with $p \leq 0.05$ are in bold.

Result 1. *The average truth-telling rate increases as we move from IA to DA and from IA33 to DA33.*

Next, consider the effect of a lottery quota on truth-telling. Moving from IA to IA33 increases overall truth-telling, as predicted. In particular, students 4–6 (for student 4 significant only at the 10% level) as well as students ranked 7–12 truthfully reveal their preferences more often. Note that these are precisely the students for which the lottery increases the chances of being admitted to their most preferred school if they rank it first. At the same time, the introduction of the lottery induces student 3 to misrepresent his preferences significantly more often, since he loses his priority seat at school A. These observations are in line with Hypothesis 2.

Result 2. *The average truth-telling rate increases as we move from IA to IA33.*

Finally, we consider the effect of a lottery on truth-telling under DA. Moving from DA to DA33, we find that the truth-telling rates increase significantly for students in rank 5 and below. Only student 3, who under DA33 loses his priority seat at his most preferred school A, misrepresents his preferences more often under DA33 than under DA. Hence, in line with Hypothesis 3 we find:

Result 3. *The average truth-telling rate increases as we move from DA to DA33.*

Student Composition of Schools and Payoff Distribution

We now turn to the experimental results on students' payoffs. It turns out that almost all students were matched to a school. Out of 1,920 observations per treatment (eight groups of 12 students over 20 rounds), a student remained unmatched in four cases in IA (0.2%), in 10 cases in IA33 (0.5%), in one case in DA (0.05%), and in three cases in DA33 (0.15%).

The left panel of Table 8 displays the average payoffs for each student rank.

Note that the lowest feasible average payoff for the group of 12 participants is 13, while the highest is 16.[31] In the right panel, we compare the average payoffs per rank between treatments using MWU tests.

[31] In the worst case, school A is filled with B-types and school B with A-types, while in the best case, both schools A and B are filled only with students of their respective type.

Table 8: Individual Payoffs

	Payoffs (treatment averages)				Mann-Whitney U test (p-values)			
i	IA	DA	IA33	DA33	IA vs. DA	IA33 vs. DA33	IA vs. IA33	DA vs. DA33
1	21.738	21.925	22.000	21.850	.441	.144	.144	.538
2	21.888	21.888	21.713	21.775	.644	.952	.522	.334
3	21.775	21.700	14.550	16.863	.637	**.001**	**.001**	**.001**
4	15.113	16.075	15.363	16.750	**.003**	**.001**	.336	**.001**
5	15.513	15.963	13.763	13.525	**.026**	.672	**.002**	**.001**
6	15.413	15.963	12.675	12.525	**.012**	.916	**.001**	**.001**
7–12(A)	10.480	10.004	12.592	11.804	**.001**	**.005**	**.001**	**.001**
7–12(B)	10.492	10.275	12.871	12.154	.239	**.010**	**.001**	**.001**
1–12	14.529	14.529	14.704	14.597	.915	.052	**.010**	.064
Gini index	.174	.179	.116	.136	.114	**.001**	**.001**	**.001**

Notes: The left panel displays the average realized payoffs and the Gini index. Averages are taken for the eight matching groups where for each group we compute the average payoff at a given rank over the 20 periods. In the right panel, the Mann-Whitney U test is based on the eight matching group averages per treatment. All p-values with $p \leq 0.05$ in bold.

Introducing a lottery under IA and DA redistributes the payoffs between the students as predicted. For both IA and DA, introducing the lottery significantly reduces payoffs for students 3, 5, and 6, while it increases payoffs for lower-ranked students. For student 4, whose rank should in equilibrium ensure him (at least) a seat at B both with and without the lottery, there is no significant difference between IA and IA33 and a small significant increase in DA33 relative to DA. Hence, in line with Hypothesis 4 we find:

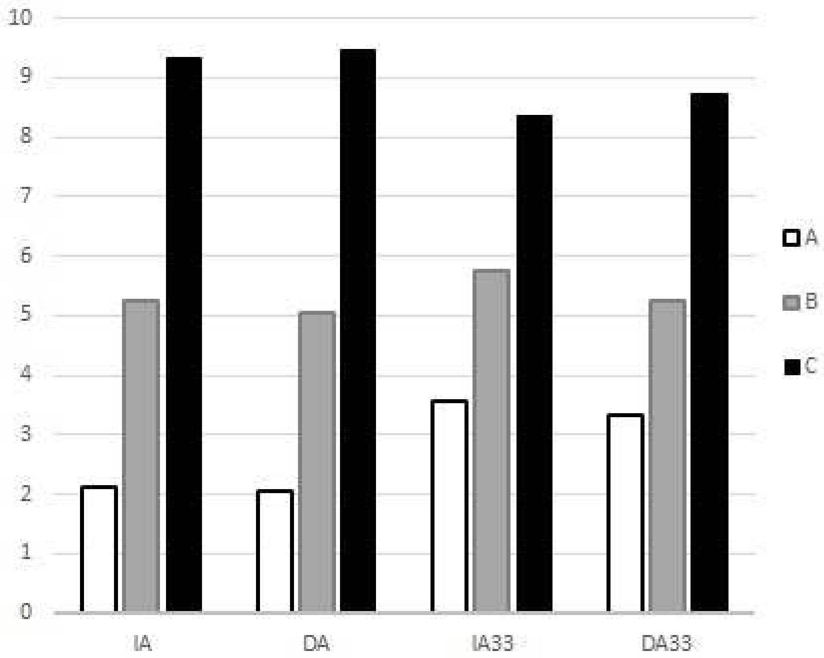
Result 4. *Moving from IA to IA33 and from DA to DA33*

(i) increases the average payoffs of the lower-ranked students, but decreases the average payoffs of the good but not excellent students and

(ii) leads to a more equitable distribution of average payoffs across students, as measured by the Gini index.

Beyond its effect on individual payoffs, we also find that the introduction of a lottery leads to a more equal student composition across schools with respect to average ranks. Figure 2 and Table 9 present the average student ranks at the three schools.

Figure 2: Composition of schools



Notes: The bars indicate the average rank of students assigned to each school.

Table 9: Composition of schools

	Average student rank (treatment averages)				Mann-Whitney U test (p-values)			
	IA	DA	IA33	DA33	IA vs. DA	IA33 vs. DA33	IA vs. IA33	DA vs. DA33
A	2.11	2.04	3.57	3.31				
B	5.25	5.04	5.76	5.25				
C	9.33	9.46	8.36	8.72				
Δ_{AB}	3.14	3.00	2.19	1.94	**0.015**	0.142	**0.001**	**0.001**
Δ_{BC}	4.01	4.42	2.60	3.47	**0.003**	**0.001**	**0.001**	**0.001**
Δ_{AC}	7.21	7.42	4.79	5.41	**0.027**	**0.002**	**0.001**	**0.001**

Notes: Δ_{ij} denotes the difference in average ranks between schools i and j. The Mann-Whitney U tests are based on the 8 matching group averages per treatment. All p-values with $p \leq 0.05$ are in bold.

The difference in average ranks between schools A and C is larger under IA and DA than under the two mechanisms with a lottery, IA33 and DA33. Moreover, the same is true for the difference between A and B as well as B and C (all significant at $p = .001$). Thus, lotteries have the desired effect of making schools more similar in terms of admitted students' academic achievement, both under IA and under DA. This is in line with Hypothesis 5.

Result 5. *Moving from IA to IA33 and from DA to DA33 decreases the difference between the average ranks of students at the three schools.*

With respect to Hypothesis 6, we compare the distribution of payoffs and the composition of schools between IA33 and DA33.

The data confirm our hypothesis that the beneficial effect of a lottery for lower-ranked students is more pronounced under IA33 than under DA33. Students in rank 7–12 earn significantly more under IA33, both A-types ($p = .005$) and B-types ($p = .010$), see Table 8. Correspondingly, the Gini-index is significantly lower for IA33 than for DA33. Moreover, IA33 leads to more similar schools than DA33 with respect to the average rank of the students. While there is no significant change in the difference in average ranks between the two most popular schools A and B, the differences between them and the least preferred school are significantly lower under IA33. The difference in average ranks between the most preferred and the least preferred school, Δ_{AC}, is 5.41 under DA33 and only 4.79 under IA33. Hence, in line with Hypothesis 6 we find:

Result 6. *Comparing IA33 with DA33,*

(i) the good but not excellent students fare significantly better under DA33 than under IA33, while the lower-ranked students fare better under IA33 than under DA33;

(ii) inequality as measured by the Gini index is lower under IA33 than under DA33; and

(iii) the difference between the average ranks at schools is smaller under IA33 than under DA33.

Finally, comparing IA and DA without a lottery, there is no difference in the Gini index. However, the individual payoffs of students 4–6 and 7–12(A) differ under these two mechanisms, in spite of the equilibrium prediction of exactly equal payoffs. This is due to out-of-equilibrium behavior that harms students 4–6 and gives an advantage to students 7–12 under IA relative to DA.

5 Conclusions

The paper is the first to study how to embed lotteries in school choice mechanisms with the aim to reduce disparities in the student composition between schools. We show that there are important differences between the effect of a lottery quota on the immediate acceptance (IA33) and the deferred acceptance mechanism (DA33).

Our experiment replicates previous evidence that truth-telling in the strategy-proof DA mechanism is significantly lower than predicted, see the survey by Hakimov and Kübler (2018). More importantly, the experiment shows that lottery quotas increase truth-telling rates under the DA mechanism. This is in line with the theoretical result that a lottery quota reduces the set of equilibrium strategies under DA.

The experiment also provides evidence that the same lottery leads to more diverse schools under IA than under DA. In particular, the lottery quota provides a larger benefit for low priority students under IA than under DA, at the expense of the good but not excellent students who prefer DA33 over IA33.

The DA mechanism is strategically simple and thereby levels the playing field. It leads to high truth-telling rates when combined with a lottery. This ensures that the number of applications represent valuable feedback for school authorities. We consider it advisable to use DA in combination with a lottery instead of IA, but to choose a higher lottery quota relative to what would be an appropriate choice under IA.

Appendix

A Proofs of Equilibrium Predictions

A.1 Equilibria under IA

students	1–3	4–6	7–12(A)	7–12(B)
match probability at A	1	0	0	0
match probability at B	0	1	0	0
match probability at C	0	0	1	1

Proof (Equilibria under IA). Since each school has at least three seats, the top three students 1–3 will be admitted wherever they apply first. Hence, for them not ranking A first is strictly dominated. Given that 1–3 will be matched to school A in equilibrium, the three next best students 4–6 will be admitted either to B or C, depending on where they apply first. Hence, for them it is a strict best response (to students 1–3) to rank B first. All remaining students 7–12 will then be admitted to C independent of the order in which they rank the three schools, as there are no other seats available. □

A.2 Equilibria under IA33

students	1–2	3–4	5–12(A)	7–12(B)
match probability at A	1	0	0.2	0
match probability at B	0	1	0	0.333
match probability at C	0	0	0.8	0.667

Proof (Equilibria under IA33). In equilibrium, school A allocates all its three seats in Step 1 of the IA33 algorithm, otherwise one of the nine A-type students could change his strategy, report his type truthfully and obtain a seat at school A. Similarly, school B allocates all its three seats in Step 1 of the IA33 algorithm, otherwise one of the three B-type students could change his strategy, report his most preferred school truthfully and obtain a seat at school B. Note that since schools A and B allocate all their seats in Step 1, it is payoff irrelevant for students which school they rank second.

This leaves school C with at most six applicants in Step 1, so that each of them would be admitted and receive the lowest possible payoff of 10. However, given the positive probability of receiving a more preferred school seat via the lottery, no-one initially applies to school C in equilibrium. All that remains to show is who initially applies to school A and who initially applies to school B.

Given that students 1 and 2 will be admitted wherever they apply, not ranking A first is strictly dominated. The two next best students 3 and 4, are guaranteed a seat at school B if they apply there, which yields a payoff of 16. If they were to apply to school A, they would enter a lottery for the one remaining seat, knowing that they would be admitted to C if they lose. Hence, for students 3 or 4 to be willing to apply to school A there would need to be at least a 50% chance of being admitted to A (which yields a payoff of 22 – the expected payoff in the lottery with 50% is $1/2 \cdot 22 + 1/2 \cdot 10 = 16$) or at most four applicants at A and thus at least eight applicants at B. However, then one of the worse-ranked A-type students applying to B would deviate and apply to A instead (receiving an expected payoff of $1/3 \cdot 22 + 2/3 \cdot 10 = 14$ instead of $1/6 \cdot 16 + 5/6 \cdot 10 = 11$). We conclude that in equilibrium, students 3 and 4 initially apply to B and it is a strict best response for them to do so.

Next, we show that there are five applicants for the remaining lottery seat at school A (so seven applicants at A, including students 1 and 2). Assume there were at most four applicants in the lottery at school A and hence at least four in the lottery at school B. Then, there is at least one A-type student applying to B where he receives an expected payoff of at most $1/4 \cdot 16 + 3/4 \cdot 10 = 11.5$. However, by applying to school A instead, that A-type student would receive at least $1/5 \cdot 22 + 4/5 10 = 12.4$ and hence deviate. Assume, on the other hand, there were at least six applicants for the lottery at school A and hence at most two in the lottery at school B. Then, there is at least one B-type student applying to A who could profitably deviate by applying to his more preferred and, at the same time, less competitive school.

Finally, knowing that there are five applicants in the lottery at school A and three at school B, we also know that no B-type student is applying to A because he would receive an expected payoff of $1/5 \cdot 16 + 4/5 \cdot 10 = 11.2$ while deviating and applying to B would then yield $1/4 \cdot 22 + 3/4 \cdot 10 = 13$. Hence, all B-type students apply to school B and all remaining A-type students (all A-type students ranked 5 and worse) apply to school A. Finally, by the payoff comparisons in the preceding paragraph, these strategies are strict best responses. $\square$

A.3 Equilibria under DA

students	1–3	4–6	7–12(A)	7–12(B)
match probability at A	1	0	0	0
match probability at B	0	1	0	0
match probability at C	0	0	1	1

Proof (Equilibria under DA). Since each school has at least three seats, the top three students 1–3 will be admitted wherever they apply first. Hence, for them not ranking A first is strictly dominated. Given that 1–3 will be matched to school A in equilibrium, the three next best students, 4–6, will be admitted either to B or C, depending on which one they rank higher. Hence, for them it is a strict best response (to students 1–3) to rank B above C. All remaining students,7–12, will then be admitted to C independent of the order in which they rank the three schools, as there are no other seats available. □

A.4 Equilibria under DA33

students	1–2	3–4	5	6–12(A)	7–12(B)
match probability at A	1	0.133	0.133	0.133	0.022
match probability at B	0	0.867	0.324	0.086	0.2
match probability at C	0	0	0.543	0.781	0.778

Proof (Equilibria under DA33). Under the DA33 mechanism, it is a weakly dominant strategy for all students to report their true preferences (each student receives either a regular school seat based on his rank or a lottery seat based on his lottery rank independently of the step in the DA-algorithm – since matches are tentative until the very end of the algorithm, a student can safely report schools he might not match with without influencing later match chances).

By Remark 1, not ranking school A first is strictly dominated for each student. and for all but two students (students 1–2 are the exception) it even becomes a strict best response to report their true preferences.

Next, given that everybody reports their most preferred school truthfully, students 1 and 2 are admitted to school A. Next, we explain why all remaining A-type students 3–12(A) report their preferences truthfully in equilibrium: if a 3–12(A) student is ranked second by the lottery while some other 3–12(A) student is ranked first, he will be admitted to his second-ranked school – hence, it is strictly better to rank B second and thereby truthfully reveal his preferences, than to rank C second (and not obtain B with positive probability).

Hence, all A-type students report their preferences truthfully and thus the two merit seats at school A will be assigned to 3–6(A) students. Next, we explain why all B-type students 7–12(B) report their preferences truthfully in equilibrium: if a 7–12(B) student is ranked second by the lottery while some other 7–12(B) student is ranked first, he will be admitted to his second-ranked school – hence, it is strictly better to rank A second and thereby truthfully reveal his preferences, than to rank C second (and not obtain A with positive probability).

When computing the resulting match probabilities, we need to take into account that the DA33 algorithm uses the same lottery ranking in all its steps. We can neglect the lottery ranking of students 1 and 2 who are admitted to A based on their rank and focus only on the lottery ranking of the remaining students $3-12$. In the following, we consider five cases and calculate the associated conditional match probabilities.

Case 1: With probability $2/10 = 1/5$ student 3 or student 4 has the best lottery rank. Conditional on this event, each of them is admitted to A with probability $1/2$, while the other (3 or 4) and student 5 are admitted to B based on rank. The remaining seven students are admitted to B based on their lottery rank with equal probability $1/7$.

students	$1-2$	$3-4$	5	$6-12(A)$	$7-12(B)$
match probability at A	1	$1/2$	0	0	0
match probability at B	0	$1/2$	1	$1/7$	$1/7$
match probability at C	0	0	0	$6/7$	$6/7$

Case 2: With probability $5/10 = 1/2$ an A-type student ranked $5-12$ (there are five such students) has the best lottery rank. Conditional on this event, each of them is admitted to A with probability $1/5$, while students 3 and 4 are admitted to B based on rank. The remaining seven students are admitted to B based on their lottery rank with equal probability $1/7$.

students	$1-2$	$3-4$	5	$6-12(A)$	$7-12(B)$
match probability at A	1	0	$1/5$	$1/5$	0
match probability at B	0	1	$4/5 \cdot 1/7 = 4/35$	$4/5 \cdot 1/7 = 4/35$	$1/7$
match probability at C	0	0	$24/35$	$24/35$	$6/7$

Case 3: With probability $3/10 \cdot 2/9 = 1/15$ a B-type student has the best lottery rank (there are three such students) and either student 3 or student 4 has the second best lottery rank. Conditional on this event, each of the B-type students is admitted to B with probability $1/3$ and students 3 and 4 are each admitted to A with probability $1/2$, while the other (3 or 4) and student 5 are admitted to B based on rank. The remaining six students are admitted to C.

students	$1-2$	$3-4$	5	$6-12(A)$	$7-12(B)$
match probability at A	1	$1/2$	0	0	0
match probability at B	0	$1/2$	1	0	$1/3$
match probability at C	0	0	0	1	$2/3$

Case 4: With probability $3/10 \cdot 5/9 = 1/6$ a B-type student (there are three such students) has the best lottery rank and an A-type student ranked $5-12$ has the second best lottery rank (there are five such students). Conditional on this event, each of the B-type students is admitted to B with probability $1/3$ and each of the A-type student ranked $5-12$ is admitted to A with probability $1/5$, while students 3 and 4 are admitted to B based on rank. The remaining six students are admitted to C.

students	$1-2$	$3-4$	5	$6-12(A)$	$7-12(B)$
match probability at A	1	0	$1/5$	$1/5$	0
match probability at B	0	1	0	0	$1/3$
match probability at C	0	0	$4/5$	$4/5$	$2/3$

Case 5: With probability $3/10 \cdot 2/9 = 1/15$ a B-type student (there are three such students) has the best lottery rank and another B-type student has the second best lottery rank (there are two other such students). Conditional on this event, each of the B-type students is admitted to B with probability $1/3$ and to A with probability $1/3$, while students 3 and 4 are admitted to B based on rank. The remaining six students are admitted to C.

students	$1-2$	$3-4$	5	$6-12(A)$	$7-12(B)$
match probability at A	1	0	0	0	$1/3$
match probability at B	0	1	0	0	$1/3$
match probability at C	0	0	1	1	$1/3$

Hence, adding up all probabilities, we obtain

students	$1-2$	$3-4$
match prob. at A	$1/5 + 1/2 + 1/15 + 1/6 + 1/15 = 1$	$1/10 + 1/30 = 2/15 \approx .133$
match prob. at B	0	$1/10 + 1/2 + 1/30 + 1/6 + 1/15 = 13/15 \approx .867$
match prob. at C	0	0

students	5	$6-12(A)$
match prob. at A	$1/10 + 1/30 = 2/15 \approx .133$	$1/10 + 1/30 = 2/15 \approx .133$
match prob. at B	$1/5 + 2/35 + 1/15 = 34/105 \approx .324$	$1/35 + 2/35 = 3/35 \approx .086$
match prob. at C	$12/35 + 2/15 + 1/15 = 57/105 \approx .543$	$6/35 + 12/35 + 1/15 + 2/15 + 1/15 = 82/105 \approx .781$

students	$7-12(B)$	all students
match prob. at A	$1/45 \approx .022$	$2 + 4/15 + 2/15 + 8/15 + 1/15 = 3$
match prob. at B	$1/35 + 1/14 + 1/45 + 1/18 + 1/45 = 1/5 = .2$	$26/15 + 34/105 + 12/35 + 3/5 = 3$
match prob. at C	$6/35 + 3/7 + 2/45 + 1/9 + 1/45 = 7/9 \approx .778$	$57/105 + 328/105 + 21/9 = 6$

$\square$

B Documentation of the Experiment

B.1 Preference Profiles

Table 10 displays the 20 preference profiles used for all groups in the order indicated in the table.

Table 10: Preference Profiles of the Experimental School Choice Problems

i	$P1$	$P2$	$P3$	$P4$	$P5$	$P6$	$P7$	$P8$	$P9$	$P10$	$P11$	$P12$	$P13$	$P14$	$P15$	$P16$	$P17$	$P18$	$P19$	$P20$
1	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A
2	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A
3	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A
4	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A
5	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A
6	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A
7	A	A	A	A	A	A	A	A	A	A	B	B	B	B	B	B	B	B	B	B
8	A	A	A	A	B	B	B	B	B	B	A	A	A	A	A	A	B	B	B	B
9	A	B	B	B	A	A	A	B	B	B	A	A	A	B	B	B	A	A	A	B
10	B	A	B	B	A	B	B	A	A	B	A	B	B	A	A	B	A	A	B	A
11	B	B	A	B	B	A	B	A	B	A	B	A	B	A	B	A	A	B	A	A
12	B	B	B	A	B	B	A	B	A	A	B	B	A	B	A	A	B	A	A	A

Notes: Preference types (most preferred schools) of students 1–12 in profiles $P1$–$P20$.

B.2 Instructions and Quizzes

We only provide the instructions and the quiz for treatments IA33 and DA33. The instructions and quiz for IA and DA are available from the authors.

Instructions for Treatment IA33 (translated from German)

The experiment you are about to participate in is part of a project funded by the Deutsche Forschungsgemeinschaft (DFG) to analyze decision-making processes. In this experiment, you can earn a considerable amount of money, depending on your decisions and the decisions of the other participants. It is therefore essential that you read the instructions carefully.

Please do not speak to the other participants. If you have a question, please raise your hand. We will then come to you and answer your question. Please do not ask your question(s) in public.

Decision Situation

In this experiment, we simulate a procedure that matches places at schools to students. In this procedure, students apply to a central clearinghouse that allocates the students to the schools. You and the other participants take on the role of the students.

Procedure during the Experiment

At the beginning of the experiment, you and the other participants will be divided into groups of 12 students each.

After you and the other participants have read the instructions, everyone will be asked to answer five short control questions. With these questions, we will test your understanding of the procedure and of the decisions you have to make. One of the experimenters will come to you and will go through all the answers with you. This will give you the opportunity to ask any remaining questions.

You and the other 11 students are applying to the central clearinghouse for a school seat. Schools A and B each have three seats to offer, school C is offering 6 seats. In total, there are 12 school seats available to you and the other 11 participants. A computer program, which is explained in further detail below, determines who gains a seat at which school depending on the applications sent by you and the other participants. You will then be told whether your application was successful and which school seats you and the other participants of your group received.

The experiment consists of 24 rounds. In every round, the application procedure starts anew, and each round is independent from the others. This means that whether you get a seat or not and at which school you are admitted only depends on your application in this round. Your chances in the current round are therefore independent of the decisions that you and the other participants took in previous rounds.

Payoffs

At the end of the experiment, one round will be picked at random to determine your payoff. This payoff depends on the school you were admitted to in that particular round. Additionally, you will receive 5€ for your participation in the experiment.

At the beginning of each round, a rank (indicating your ranking in terms of school grades in relation to the other students) is assigned to you and a payoff table is shown. The table shows which amount every participant gets for a seat at a certain school. Every participant knows the payoff table for the round. In every round, the amount of money you get for being admitted to a certain school is determined anew, and you are assigned a new rank. The rank is shown in the upper-left corner of your screen.

To help you familiarize yourself with the structure of the payoff table, we will now show you an example.

Table 11: Payoff Table

	School A	School B	School C
Payoff of the student in rank 1	22 Euro	16 Euro	10 Euro
Payoff of the student in rank 2	22 Euro	16 Euro	10 Euro
Payoff of the student in rank 3	22 Euro	16 Euro	10 Euro
Payoff of the student in rank 4	22 Euro	16 Euro	10 Euro
Payoff of the student in rank 5	22 Euro	16 Euro	10 Euro
Payoff of the student in rank 6	22 Euro	16 Euro	10 Euro
Payoff of the student in rank 7	22 Euro	16 Euro	10 Euro
Payoff of the student in rank 8	22 Euro	16 Euro	10 Euro
Payoff of the student in rank 9	22 Euro	16 Euro	10 Euro
Payoff of the student in rank 10	16 Euro	22 Euro	10 Euro
Payoff of the student in rank 11	16 Euro	22 Euro	10 Euro
Payoff of the student in rank 12	16 Euro	22 Euro	10 Euro

Notes: Example of a payoff table for all participants in your group. [The German instructions show the screenshot of this page.]

This table can be read as follows: Assume that you are the student ranked 6th. This means that you get 10€ if you are admitted to school C and this round is chosen for the payoff. Accordingly, you get 16€ if you are admitted to school B or 22€ if you are admitted to school A. If you are not admitted to any school, you will receive 0€İn either case, you will receive an additional 5€ for your participation in the experiment.

Important: The payoff table shown above only serves as an example. It is not connected in any way to your situation in the actual experiment.

Possible Choices

In each round, your seat at one of the schools will be determined according to the following procedure: You apply to the schools by creating a rank-order list of the schools according to your preference. On that list, you can state which school is your first, your second, and your third preference. You will see the following boxes to list your choices in each round:

First preference		Second preference		Third preference	
School A	☐	School B	☐	School C	☐
School A	☐	School B	☐	School C	☐
School A	☐	School B	☐	School C	☐

Notes: Boxes to indicate your preference. [The German instructions show the screenshot of this page.]

You are free to choose which school you will list as your first, second, and third preference. You are also free to decide whether to list one, two, all three, or none of the schools. For each choice, you can only indicate one school, and each school can only be listed once. You can only be admitted to the schools which you have explicitly indicated in your preference list.

Please make a choice for each preference and then press the button "My choice."

After all participants have entered their preference lists, the computer program will determine the school seats that you and the other participants in your group will receive. You will then see at which school you are admitted or whether you are not admitted at all. You will also see which schools the other participants are admitted to. The allocation procedure that determines your rank is explained in detail below. Please read the rules of the allocation procedure carefully!

The Allocation Procedure

In each round, you and the other students in your group will be randomly assigned a rank. The rank varies between 1 and 12, with each of the students occupying exactly one rank. Rank 1 means that the student has the best grades, rank 12 means that the student has the worst grades. Schools prefer students with better grades over students with worse grades.

The allocation of the participants to the school seats works as follows:

Round 1:

- Each student applies to the school he listed as his first preference.
- If fewer or exactly as many students as there are seats at a school have listed that school as their first preference, all of them get a seat at the school. If more students have listed a school as their first preference than there are seats at the school, 66% of the seats at the school (i.e., two seats at schools A and B, respectively, and four seats at school C) will be given to the students with the highest ranks. The remaining

33% (i.e., one seat at schools A and B, respectively, and two seats at school C) will be assigned through a lottery. The lottery gives everyone participating the same chance and is therefore entirely independent of the ranks. For the students who were assigned a seat, the allocation procedure ends.

Round 2:

- Each student who did not receive a seat in the first round applies to the school he listed as his second preference. If he only listed one school, the allocation procedure ends, and he will receive no seat.
- If fewer or exactly as many students as there are remaining seats at a school have listed that school as their second preference, all of them get a seat at that school. If more students have listed a school as their first preference than there are seats still available at that school, the seats will be given to the students with the highest ranks. For the students who were assigned a seat, the allocation procedure ends.

Round 3:

- Each student who did not receive a seat in the first or second round applies to the school he listed as his third preference. If he only named two preferences, the allocation procedure ends, and he will receive no seat.
- In the last round, there are fewer or exactly as many students as there are seats still available at a school who have listed the school as their third preference. All of them receive a seat at that school and the allocation procedure ends.

The allocation procedure ends when either none of the students are rejected by a school or when all rejected applicants have already applied to all schools on their preference list. A participant who has not received a seat at a school at this point is left without a seat.

It is irrelevant for the payoff whether you receive a seat at a school through your rank or through the lottery. Your payoff is exclusively determined by the school you are assigned to.

Important Information for your Decision

A computer executes the allocation procedure. The description above explains the procedure applied. After each round, you will be shown at which schools you and the other 11 participants have received a seat. Afterwards, the next round starts. The experiment ends after 24 rounds.

Example of the Allocation Procedure

To illustrate the allocation procedure, we will now show you an example. In this example, there are 12 students and three schools (Schools A and B have 3 seats to fill and school C has 6 seats to fill). The students were assigned the ranks 1 to 12. The student in rank 1 has the best grades, the student in rank 2 has the second-best grades etc. The following table shows the preference lists of all 12 participants.

Student	First Choice	Second Choice	Third Choice
Student in rank 1	A	B	C
Student in rank 2	A	B	C
Student in rank 3	B	A	C
Student in rank 4	A	B	C
Student in rank 5	C	A	C
Student in rank 6	A	B	C
Student in rank 7	C	B	A
Student in rank 8	B	A	C
Student in rank 9	C	A	B
Student in rank 10	A	B	C
Student in rank 11	A	B	C
Student in rank 12	A	C	B

Notes: Preference lists of the students (example).

Important: The preference lists provided here are merely for illustrative purposes and are unrelated to your own decision situation.

We will now go through each round of the allocation procedure to illustrate how the procedure works.

Round 1:

- With their first preferences, the students with ranks 1, 2, 4, 6, 10, 11, and 12 apply to school A which only offers three seats. The students with the two highest ranks (rank 1 and 2) are admitted to school A. One additional student (one of those ranked 4, 6, 10, 11, or 12) is admitted to school A through the lottery. Let's say that this is the student in rank 11. Overall, three students will be admitted (rank 1 and 2 for the first two seats and rank 11 for the third seat because of the lottery).

The other students in the ranks 4, 6, 10, and 12 do not receive a seat in the first round.

- Given their first preference, the students in ranks 3 and 8 apply to school B which also has three seats to fill. Since there are fewer applicants than seats at the school, the students ranked 3 and 8 are admitted to school B.
- Given their first preferences, the students ranked 5th, 7th, and 9th apply to school C which has six seats to fill. Since there are fewer applicants than seats at the school, the students ranked 5th, 7th, and 9th are admitted to school C.
- The allocation procedure ends for the students in ranks 1, 2, 3, 5, 7, 8, 9, and 11 who have all received a seat. The students in ranks 4, 6, 10, and 12 did not receive a seat this round and go to the second round.

Round 2:

- The students in ranks 4, 6, 10, and 12 have not yet received a seat. They apply to their second-most preferred school. Given their second preference, the students in ranks 4, 6, and 10 apply to school B which still has one seat available. The student with the highest rank (rank 4) is admitted to school B. The students ranked 6th and 10th do not receive a seat at school B.
- Given his second preference, the student ranked 12th applies to school C which has three available seats left, and is admitted.
- The allocation procedure ends for the students in ranks 4 and 12 who have each received a seat. The students in ranks 6 and 10 have not received a seat yet and go into the third round.

Round 3:

- The students ranked 6th and 10th apply to their third-most preferred school, school C and are admitted because school C still has exactly two available seats. The allocation procedure ends.

The following assignments result from the allocation procedure:

Rank	1	2	3	4	5	6	7	8	9	10	11	12
School	A	A	B	B	C	C	C	B	C	C	A	C

Just to remind you again: the situation described above only illustrates the allocation procedure.

Do you have any questions?

Thank you for participating in the experiment!

Quiz for Treatment IA33

Please answer the following questions and raise your hand when you are done! One of the experimenters will then come to you.

There are six students and two schools: A and B. School A and B each have three seats to fill. The students are ranked by their grades, the best student being in rank 1 and the worst student being in rank 6.

	First Preference	Second Preference
Student in rank 1	A	B
Student in rank 2	A	B
Student in rank 3	B	A
Student in rank 4	B	A
Student in rank 5	B	A
Student in rank 6	B	A

Notes: The lottery for school B draws the student in rank 6.
At which school is the student in rank 4 admitted? (A or B)

	First Preference	Second Preference
Student in rank 1	A	B
Student in rank 2	A	B
Student in rank 3	A	B
Student in rank 4	A	B
Student in rank 5	B	A
Student in rank 6	B	A

Notes: The lottery for school A draws the student in rank 3.
At which school is the student in rank 2 admitted? (A or B)

	First Preference	Second Preference
Student in rank 1	A	B
Student in rank 2	A	B
Student in rank 3	B	A
Student in rank 4	A	B
Student in rank 5	A	B
Student in rank 6	B	A

Notes: The lottery for school A draws the student in rank 5. At which school is the student in rank 5 admitted? (A or B)

	First Preference	Second Preference
Student in rank 1	A	B
Student in rank 2	A	B
Student in rank 3	A	B
Student in rank 4	A	B
Student in rank 5	A	B
Student in rank 6	A	B

Notes: The lottery for school A draws the student in rank 4. At which school is the student in rank 3 admitted? (A or B)

	First Preference	Second Preference
Student in rank 1	A	B
Student in rank 2	A	B
Student in rank 3	B	A
Student in rank 4	B	A
Student in rank 5	A	B
Student in rank 6	A	B

Notes: The lottery for school A draws the student in rank 6. At which school is the student in rank 1 admitted? (A or B)

Instructions for Treatment DA33 (translated from German)

[See instructions for IA33 up to:]

The Allocation Procedure

In each round, you and the other students in your group will be randomly assigned a rank. The rank varies between 1 and 12, with each of the students occupying exactly one rank. Rank 1 means that the student has the best grades, rank 12 means that the student has the worst grades. Schools prefer students with better grades over students with worse grades.

In addition, a second list is created in which each student randomly receives a lottery number between 1 and 12. Each lottery number is assigned exactly once. The lottery numbers are determined anew in each round and are independent of the grades.

The allocation of the participants to the school seats works as follows: Two thirds of the seats are reserved for the students with the best grades. One third of the seats are allocated based on lottery numbers. Applicants with a lower lottery number have priority over applicants with a higher lottery number. Admission by means of grades and lottery numbers takes place in several rounds. In the first round, each student applies to the school that he listed as his first preference. If a school rejects the application of a student, he applies in later rounds to the school he listed as his second preference. If he is also rejected there in a later round, he applies to the school he listed as his third preference. The allocation of seats is temporary in each round. This means that a temporary seat in a later round can be awarded to a better placed applicant.

Each round of the procedure consists of two parts: in Part 1, two thirds of the seats are awarded to the candidates with the highest ranks. There are two places at schools A and B, and 4 places at school C. In Part 2, the remaining third of the seats (one seat at schools A and B and two seats at school C) are allocated based on the lottery numbers.

Round 1:

- Each student applies to the school he listed as his first preference. Schools A and B temporarily assign

  - two seats to the two applicants with the best grades
  - one seat among the remaining applicants with the lowest lottery number.

  School C temporarily assigns

  - four seats to the four applicants with the best grades

- two seats among the remaining applicants with the two lowest lottery numbers.

- Applicants who do not get a seat are finally rejected at the respective school. If there are not enough applicants at a school, the surplus seats remain temporarily free.

Round 2:

- Each student who was finally rejected at his most preferred school applies to the school he listed as his second preference. Each school compares the new applicants with the applicants from the previous round, which they have temporarily accepted.

- Schools A and B temporarily assign

  - two seats to the two applicants with the best grades
  - one seat among the remaining applicants with the lowest lottery number.

  School C temporarily assigns

  - four seats to the four applicants with the best grades
  - two seats among the remaining applicants with the two lowest lottery numbers.

- Applicants who do not get a seat are finally rejected at the respective school. If there are not enough applicants at a school, the surplus seats remain temporarily free.

(...)

The procedure continues according to these rules. The procedure ends when a round is reached in which either one of the participants is no longer finally rejected by a school or each of the finally rejected candidates has applied to all the schools indicated on his preference list. At the end of the procedure, all temporary admissions become final: each student is given a seat at that school which has temporarily admitted him at that time. A student who was not admitted by any school now gets no seat.

In summary, the admission procedure works as follows:

1. In each round, the seats at the schools are only temporarily given. For example, a student who is rejected in a round by his preferred school will still have a chance to get a place at his second-most favourite school further in the course of the procedure, even if the latter has already assigned all its seats temporarily.

2. All students apply twice if required: Once in Part 1 for one of the seats reserved for the students with the best grades and, if this application was not successful, once again for one of the seats in Part 2 awarded according to the lottery numbers.

3. A school which has finally rejected a student does not take him into account for the further course of the procedure. So, it does not help the student if he mentions the same school twice on his preference list. If a student is rejected by all the schools mentioned in his preference list, he will leave the procedure and will not be given any seat.

Important Information for your Decision

- If more than one participant applies to a school, in Part 1 of a round only the school grades determine which candidate is temporarily admitted. In Part 2 of a round, only the lottery numbers are used. For your payoffs, it is irrelevant whether you have received a seat at the school due to your grade or due to your lottery number.
- A computer executes the allocation procedure. The description above explains the procedure applied. After each round, you will be shown at which schools you and the other 11 participants have received a seat. Afterwards, the next round starts. The experiment ends after 24 rounds.

Example of the Allocation Procedure

To illustrate the allocation procedure, we will now show you an example. In this example, there are 12 students and three schools (Schools A and B have 3 seats to fill and school C has 6 seats to fill). The students are assigned the ranks 1 to 12. The student in rank 1 has the best grades, the student in rank 2 has the second-best grades, etc. The following table shows the preference lists of all 12 participants. In addition, the last column of the table shows the lottery number of each student, assigned by a single lottery.

Student	First Choice	Second Choice	Third Choice	Lottery Number
Student in rank 1	A	B	C	7
Student in rank 2	A	B	C	3
Student in rank 3	B	A	C	1
Student in rank 4	A	B	C	8
Student in rank 5	C	A	C	12
Student in rank 6	A	B	C	11
Student in rank 7	C	B	A	1
Student in rank 8	B	A	C	2
Student in rank 9	C	A	B	6
Student in rank 10	A	B	C	9
Student in rank 11	A	B	C	4
Student in rank 12	A	C	B	5

Notes: Preference lists of the students and lottery numbers (example).

Important: The preference lists provided here are merely for illustrative purposes and are unrelated to your own decision situation.

We will now go through each round of the allocation procedure to illustrate how the procedure works.

Round 1:

- **Part 1:**
  The students with ranks 1, 2, 4, 6, 10, 11, and 12 apply to school A, which offers two seats based on the ranks. The students at the two highest ranks (ranks 1 and 2) are temporarily admitted to school A. The students with the ranks 3 and 8 apply at their most preferred school B, which also awards two seats according to ranks. Since there are fewer applicants than seats available, the students with ranks 3 and 8 are temporarily admitted by school B. The students with ranks 5, 7, and 9 apply to school C, which awards four seats according to ranks. Since there are fewer applicants than seats available, the students with ranks 5, 7, and 9 are temporarily admitted by school C.

- **Part 2:**
  School A assigns a further seat among the remaining applicants (ranks 4, 6, 10, 11,

and 12) based on the lottery number. This seat goes to the student with rank 11 who has the lowest lottery number of the remaining candidates (4). A total of three students are admitted to school A (ranks 1 and 2 for the first two seats and rank 11 through the lottery for the third seat). The students with ranks 4, 6, 10, and 12 do not get a seat in this round and are rejected by school A. At schools B and C, no seats are allocated through the lottery, since there were enough seats for all applicants.

Round 2:

The students with ranks 4, 6, 10, and 12 have not yet received a seat. They apply to their second-most preferred school.

- **Part 1:**
  The students with ranks 4, 6, and 10 apply to their second choice, school B. The student with rank 4 is temporarily admitted by school B, thus replacing the student with rank 8, who temporarily has no seat at school B. The students with ranks 6 and 10 do not receive a seat at school B. The student with rank 12 applies to his second choice, school C, which still has two available seats awarded according to the grades, and he is temporarily admitted.

- **Part 2:**
  The students with ranks 6, 8, and 10 have not received any seat in Part 1. They compete for the lottery seat at school B. Student 8 receives this seat because he has drawn the lowest lottery number (2).

Round 3:

- **Part 1:**
  The students with ranks 6 and 10 do not have any seat after the second round and apply to their third school, school C. They are admitted, since school C still has two available seats left. The allocation procedure ends.

The following assignments result from the allocation procedure:

Rank	1	2	3	4	5	6	7	8	9	10	11	12
School	A	A	B	B	C	C	C	B	C	C	A	C

Just to remind you again: the situation described above only illustrates the allocation procedure.

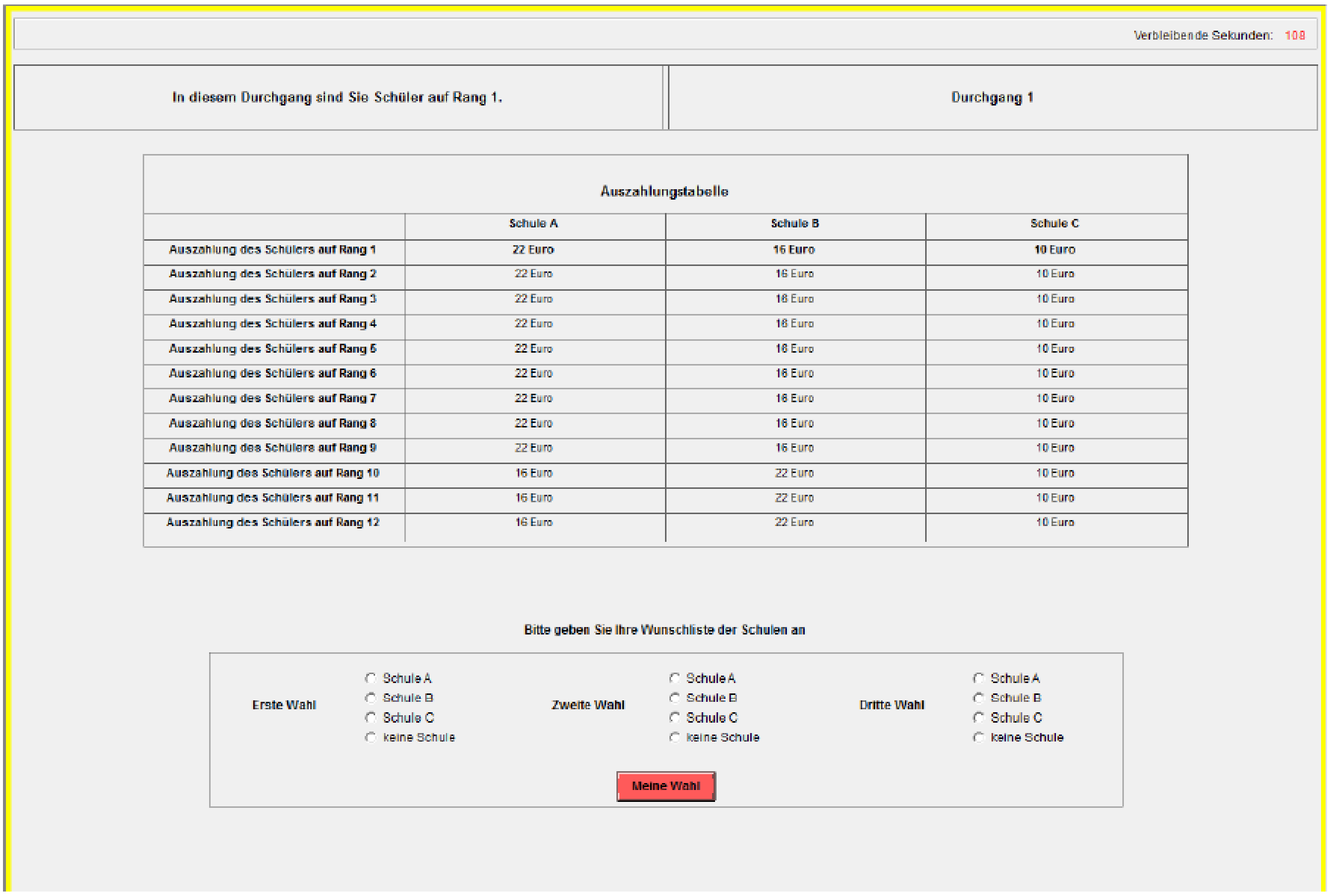
Do you have any questions?

Thank you for participating in the experiment!

B.3 Quiz for Treatment DA33

The quiz was the same as for IA33, the only difference being that the matching outcome had to be determined under DA33 instead of IA33.

B.4 Screenshots



In diesem Durchgang sind Sie Schüler auf Rang 1 mit der Losnummer 4.	Durchgang 1

Auszahlungstabelle

	Schule A	Schule B	Schule C
Auszahlung des Schülers auf Rang 1	**22 Euro**	**16 Euro**	**10 Euro**
Auszahlung des Schülers auf Rang 2	22 Euro	16 Euro	10 Euro
Auszahlung des Schülers auf Rang 3	22 Euro	16 Euro	10 Euro
Auszahlung des Schülers auf Rang 4	22 Euro	16 Euro	10 Euro
Auszahlung des Schülers auf Rang 5	22 Euro	16 Euro	10 Euro
Auszahlung des Schülers auf Rang 6	22 Euro	16 Euro	10 Euro
Auszahlung des Schülers auf Rang 7	22 Euro	16 Euro	10 Euro
Auszahlung des Schülers auf Rang 8	22 Euro	16 Euro	10 Euro
Auszahlung des Schülers auf Rang 9	22 Euro	16 Euro	10 Euro
Auszahlung des Schülers auf Rang 10	16 Euro	22 Euro	10 Euro
Auszahlung des Schülers auf Rang 11	16 Euro	22 Euro	10 Euro
Auszahlung des Schülers auf Rang 12	16 Euro	22 Euro	10 Euro

Sie sind Schüler auf Rang 1.	Sie haben einen Platz an der Schule A erhalten.	Ihr Gewinn in diesem Durchgang beträgt: 22 Euro.

Zuordnungstabelle

Durchgang	Schüler 1	Schüler 2	Schüler 3	Schüler 4	Schüler 5	Schüler 6	Schüler 7	Schüler 8	Schüler 9	Schüler 10	Schüler 11	Schüler 12
1	Schule A	Schule A	Schule B	Schule B	Schule C	Schule A	Schule C	Schule C	Schule C	Schule C	Schule C	Schule B

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