An Experimental Test of the Global-Game Selection in Coordination Games with Asymmetric Players

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Abstract:

In symmetric binary-choice coordination games, the global-game selection (GGS) has been proven to predict a high proportion of observed choices correctly. In these games, the GGS is identical to the best response to Laplacian beliefs about the fraction of players choosing either action. This paper presents an experiment on asymmetric games in which the GGS differs from the best response to Laplacian beliefs. It shows that the best response to Laplacian beliefs is a better predictor of behavior in these games than the GGS. In the considered games, the GGS provides poor guidance and also fails to give the right qualitative comparative statics predictions. Simple cognitive hierarchy models yield better predictions. The best response to a Laplacian belief about the distribution of other players' actions yields the best prediction. Comparing maximum likelihood estimates for four probabilistic models shows that an estimated global-game equilibrium fits worse than a rather simple level-k or Laplacian-belief model combined with a standard error-response function.

JEL codes: C72, C91, D81

Keywords: coordination games, equilibrium selection, global game, Laplacian beliefs, private information, network effects

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1. Introduction

In this paper, I argue that in binary-action supermodular games, the best response to Laplacian beliefs may be a better predictor of individual behavior and for comparative-statics of the distribution of actions than the global-game selection.

The theory of global games was introduced by Carlsson and van Damme (1993) and advanced by Morris and Shin (2003) as a refinement concept for supermodular games with multiple Nash equilibria. A game is supermodular if strategies can be ordered such that each player has an incentive to switch to higher strategies, if other players choose higher strategies. Another term for supermodularity is strategic complementarity. Supermodular games are common in macroeconomics, investment and network theory, and in the description of financial markets. They have increasing best response functions and may have multiple Nash equilibria.

The global-game approach relaxes the assumption that the game is common knowledge among players. It imbeds the game to be analyzed in a larger class of games. The particular game is then assumed to be randomly drawn out of this world of possible games (which is expressed by the term *global* game). Players are not perfectly informed about the selected game, but instead receive private signals. They are, however, perfectly rational in analyzing their information and deducing the strategies of other players in the global game. The class of games and the distribution of signals are common knowledge, so that standard equilibrium concepts can be applied to the global game. The most important property of global games is that for a sufficiently small variance of private signals, a global game has a unique equilibrium. If the variance of private signals converges to zero, the global game converges to the original complete information game. Thus, the convergence point of global game equilibria for vanishing noise in private signals can be used as a refinement for the original game with multiple equilibria. This refinement is called "global-game selection" (GGS).

The theory of global games has been tested by laboratory experiments on games with symmetric players.[1] In those, the GGS is identical to the best response to Laplacian beliefs about the proportion of other agents who take the higher action (Morris and Shin, 2003). The experiments have shown that the distribution of actions observed in one-shot games with multiple equilibria can be described by the equilibrium of a global game with positive variance of private signals. Hence, the equilibrium of a global game with positive variance can be used as an "as if" approach. Subjects behave as if they had noisy private signals. In repeated games, subjects usually coordinate on one of the equilibria. While

[1] See, for example, Cabrales, Nagel, and Armenter (2007), Heinemann, Nagel, and Ockenfels (2004, 2009), and Duffy and Ochs (2012).

different groups of players may coordinate on different equilibria[2], the chosen equilibria are usually between the GGS and the payoff-dominant equilibrium. Furthermore, changes to the payoff function shift the actions in the direction predicted by the GGS. Thus, even though the GGS is not a perfect point predictor of behavior in supermodular complete information games, it indicates a lower bound to an interval of strategies that can be expected to be observed in these games, and it yields a prediction for qualitative comparative statics that is useful in theoretical analyzes of supermodular games[3]. Furthermore, Heinemann, Nagel and Ockenfels (2009, henceforth HNO) have shown that the GGS provides good advice for an individual player in a one-shot coordination game.

This paper reports an experiment on one-shot supermodular games with complete information and *asymmetric* payoffs in which the GGS differs from the best response to Laplacian beliefs. It shows that the GGS fails to predict the observed responses of subjects to changes in the payoff function. The best response to Laplacian beliefs and Level-k models better describe observed behavior. Furthermore, the GGS gives a poor advice for an individual player. A player following this strategy would have achieved a payoff below the average realized payoff of our subjects. Level-1 and the best response to Laplacian beliefs yield higher expected payoffs. To my knowledge, this is the first experiment to test global games in asymmetric coordination games against alternative solution concepts.

Supermodular games with multiple equilibria are applied to a wide range of topics: currency and banking crises, government debt and twin crises, refinancing of short-term credit to firms, competition between trading venues, decisions to join a revolution, poverty trap models, marketing of network goods, antitrust regulation, and growth models with positive externalities of investment. Yet the applications have thus far been restricted to binary-choice games and most theory papers assume that all players share the same payoff function. By contrast, real-world players are typically asymmetric in many of the applications outlined above. For example, large financial institutions can take larger positions on the foreign-exchange market and exert a larger impact on the likelihood of a currency crisis than small ones. Banks that are highly interconnected via the inter-bank market, suffer more from runs on other banks than pure consumer banks. Firms that are highly dependent on using a network good (such as a particular software) gain more from its proliferation than other firms.

In the experiment, all subjects can decide between a safe and a risky action. The safe action yields a payoff that is independent of other players' actions. The payoff for the risky action increases in the number of other players who take the risky action. Subjects differ in the payoffs that they receive for a given number of risky choices. For some players, the risky

[2] See, for examples, VanHuyck, Battaglio, and Beil (1990), Heinemann, Nagel, and Ockenfels (2004) and Arifovic, Jiang, and Xu (2013).

[3] These properties were also demonstrated by Schmidt et al. (2003) who focus on the risk-dominant equilibrium that is closely related to the GGS.

action may be profitable if just a few others opt for the same action. For others, the profitability of the risky action requires that many other players select the risky action[4]. Subjects play 20 different one-shot games without feedback. Some of these games have a unique equilibrium, but most of the games have several equilibria in pure strategies.

Most of these games are designed in such a way that the GGS predicts that either all players choose risky or all choose safe. The best response to Laplacian beliefs, instead, predicts that types with high payoffs from network effects choose risky, while others choose safe. In the experiment, the proportion of risky choices differs significantly between subject types. Subjects who require only a few others to opt for the risky choice to render it profitable, are more inclined to choose the risky option than others. This asymmetry in behavior is consistent with Laplacian beliefs, but is not predicted by the GGS. The observed responses of subjects across games varies with changes in the payoff functions in an intuitive and predictable way: the higher the payoffs from risky choices, the more subjects take the risky option. These comparative statics are also predicted by Laplacian beliefs, but not by the GGS.

To compare the quality of different solutions concepts, their predictive power is measured by the probability that a subject's decision is in line with the respective solution concept. While only 60% of subjects' decisions in the experiment are in line with the GGS (not much more than a random prediction), 80% of observed decisions are in line with a best response to random behavior and with a best response to a uniform distribution on the proportion of players who choose risky. Note that the latter coincides with the GGS in symmetric binary-choice games.

Since the GGS does not predict different behavior for different types in our games, it also fails to give good advice to individual agents who play the game against some randomly selected players from the subject pool. The best response to Laplacian beliefs about other players' behavior or about the fraction of agents choosing the higher strategy yields higher expected payoffs.

For describing observed heterogeneity, we estimate a global game with positive variance of private signals and compare it with an estimated quantal-response equilibrium (QRE) and with noisy best responses to Laplacian and Level-1 beliefs. All four concepts capture the observed qualitative comparative statics properties and predict the observed asymmetry in behavior between different types of players. But, the fit of the estimated global game is not as good as it has been reported for symmetric coordination games in HNO. Noisy best responses to Laplacian or Level-1 beliefs provide a better fit. The estimated QRE does worst in this comparison. The best response to the estimated global game also yields a higher expected payoff than the GGS, however the resulting expected payoff is

[4] This is akin to firms who have a symmetric impact on the market of a network good (say, a business software), but differ by the payoffs that they receive from network effects.

(slightly) smaller than for a best response to Laplacian beliefs, although the global game is already fitted to observations.

These results indicate that the best response to Laplacian beliefs may be more useful than the global-game approach for predicting or describing aggregate behavior in asymmetric binary-choice coordination games. This is remarkable also because the best response to Laplacian beliefs is much simpler to derive. In the concluding section, this is discussed in the light of various goals that decision makers may have in mind when they want to predict the expected outcome of a one-shot supermodular game. For many research questions and also applied problems, the simple calculation of the best response to Laplacian beliefs may be more useful than solving a global game.

In the literature, the theory of global games has almost never[5] been applied to asymmetric games for two reasons: (i) deriving the GGS is, in general, challenging and a simple way for deriving it exists only for symmetric binary-choice games; (ii) The GGS of an asymmetric game with more than two players is, in general, not noise independent. A GGS is called noise independent if it does not depend on the assumed distribution of private signals. If noise independence fails, then multiple equilibria are replaced by multiple global-game selections.[6] Basteck, Daniëls, and Heinemann (2013) show how the GGS of a supermodular game with more than 2 strategies or asymmetric players can be derived by splitting the game into smaller games, deriving the GGS for each of these smaller games, and patching the selected strategy profiles together. If a supermodular game can be decomposed into binary-action games with symmetric players and the global-game selections for these games (that are straightforward to derive) yield a unique GGS for the larger game, then the GGS of the larger game is also noise independent. The theory part of this paper explains this procedure and demonstrates how it can be applied to solve for the GGS in the asymmetric games underlying the experiment.

The remainder of this paper is structured as follows: Section 2 gives a short overview of some empirical tests on the theory of global games. Section 3 formally introduces global games and explains the decomposition result by Basteck, Daniëls, and Heinemann (2013) that is applied in Section 4 to derive the GGS of the asymmetric games underlying the experiment. Section 5 describes the experimental design and Section 6 derives the results of the experiment. Section 7 concludes by discussing whether and how global games and Laplacian beliefs can be used as descriptive theories for coordination games.

[5] A notable exception is Corsetti et al. (2004)

[6] This may still be useful, because the GGSs are closer to each other than the extreme Nash equilibria and also share some comparative statics properties.

2. Empirical tests of the theory of global games

Van Huyck, Battaglio, and Beil (1990) show that in a minimum-effort game, groups of 14 to 16 players converge to the risk-dominant equilibrium, while groups with 2 players often converge to the payoff dominant equilibrium. Heinemann, Nagel, and Ockenfels (2004) show that convergence points of repeated coordination games with groups of 15 are between the GGS and the payoff-dominant equilibrium. Global-game equilibria predict the comparative statics with respect to parameters of the payoff-function, but not with respect to the precision of signals. They also show that behavior in global games (with noisy signals) tends to converge to strategies that are more efficient than the unique equilibrium. Cabrales, Nagel, and Armenter (2007), on the other hand, find convergence to the unique Nash equilibrium in a different global game. Szkup and Trevino (2020) test behavior in 2-player global games with different noise levels. They corroborate that comparative statics of global-game equilibria with respect to the precision of private signals is reversed to observations. For precision approaching infinity, subjects' behaviour approximates the payoff-dominant equilibrium. Observations can be explained by a model that allows for "sentiments" in the sense of optimism in achieving the efficient outcome with high precision of signals and pessimism for low precision. They also find deviations from the unique equilibrium of the global game in direction of more efficient payoffs. Using coordination games with perfect information, Arifovic, Jiang, and Xu (2013) show that there is a path-dependency of the outcome if the threshold to success of the risky action varies over time. Arifovic and Jiang (2019) show that extrinsic signals ("sunspots") may affect behavior in the neighborhood of the GGS.

If subjects behave as if they have noisy private signals, the distribution of actions can be described by the equilibrium of a global game, where the variance of private signals is a parameter that can be fitted to maximize the likelihood of observations. HNO estimate a global game for a one-shot binary-choice coordination game with perfect information.

In the experiment by HNO, N subjects simultaneously had to choose between two options A and B. The payoff for A was a fixed amount $X \leq 15$ Euros, that varied between the different games. The payoff for B was 15 Euros, provided that at least K group members chose B, and zero otherwise. The hurdle K was also varied between games. Group size N varied from 4 to 10 between sessions, so that the total experiment spanned a range of 90 different coordination games. The GGS in these games selects A if and only if

$$X > 15 \cdot \left(1 - \frac{K-1}{N}\right) .$$

HNO estimate a global game with positive variance of private signals to describe the distribution of choices observed in the experiment. Subjects were modelled *as if* they had different signals about an underlying state parameter, while effectively they possessed perfect information about the games' payoffs. Fitting the variance of private signals to observations, HNO found a surprisingly good fit of actual observations, and the fitted global games also yielded acceptable out-of-sample predictions. Furthermore, HNO find that the GGS is very close to the best response of what subjects actually do. Thus, it can be taken as a recommendation for individually optimal behavior. From experiments on repeated coordination games, we know that they tend to follow best-response dynamics: that is, subjects coordinate on the strategy that is a best response to the first round(s).[7] In this sense, the GGS can be used as a descriptive theory for repeated coordination games, at least, if they fall into the class tested by HNO.

To the best of my knowledge, there are no experiments on supermodular coordination games with *asymmetric* payoff functions. Thus, we do not know whether the GGS can also serve as a descriptive theory for these games. To begin filling this gap, this paper presents an experiment designed to test the GGS as a selection theory for coordination games with asymmetric payoff functions.

3. Definition of global game and global-game selection

Before we turn to the experiment, this section introduces some theory needed to solve for the GGS of the asymmetric coordination games that are employed in the experiment. Readers primarily interested in the experiment may wish to skip this section.

Let us start by introducing some notation borrowed from Basteck, Daniëls, and Heinemann (2013, henceforth BDH). We denote the set of players by I. Each player i has an ordered finite action set $A_i = \{0,1,2, \dots ,m_i\}$. Actions are denoted by $a_i \in A_i$, an action profile by $a \in A = \prod_{i \in I} A_i$. The lowest and highest action profiles are then given by 0 and m.

A complete information game Γ is specified by payoff functions $g_i : A \rightarrow \mathbf{R}$. Game Γ is **supermodular** (actions are strategic complements), if for all i and for all $a_i \leq a'_i$ and $a_{-i} \leq a'_{-i}$:

$$g_i(a'_i, a_{-i}) - g_i(a_i, a_{-i}) \leq g_i(a'_i, a'_{-i}) - g_i(a_i, a'_{-i}).$$

In words, supermodularity implies that best-response functions are non-decreasing. Supermodular games often have multiple equilibria[8].

Following Frankel, Morris, and Pauzner (2003), a **global game** $G^v(u,\phi,f)$ is defined by

- payoff functions $u_i(a_i, a_{-i}, \theta)$, where $\theta \in \mathbf{R}$ is called state parameter, such that

[7] See, for example Van Huyck, Battaglio, and Beil (1990) or Heinemann, Nagel and Ockenfels (2004).

[8] This has been phrased as a case of "strong strategic complementarities" by Angeletos and Pavan (2004), who argue that in this case, constructive ambiguity is better than full transparency, because full transparency bears the danger of coordination on an inefficient equilibrium, while ambiguity can be modelled as a global game with a unique equilibrium.

(A1) for each state θ, the complete information game given by $u_i\ (\cdot,\theta)$ is a supermodular game,

(A2) there exist states $\underline{\theta}$ and $\overline{\theta}$, such that the lowest and highest action are strictly dominant in the games given by $u_i(\cdot,\underline{\theta})$ and $u_i(\cdot,\overline{\theta})$, respectively,

(A3) each u_i satisfies weak state monotonicity, which means that for all i and $a_i < \tilde{a}_i$, the payoff difference $u_i(\tilde{a}_i, a_{-i}, \theta) - u_i(a_i, a_{-i}, \theta)$ is weakly increasing in θ. This implies that higher states make higher actions more appealing.

- a distribution for the state parameter with continuous density ϕ, and
- a tuple of density functions f_i for each i with finite support and a scale parameter $\nu \in (0,1]$. In the global game, players do not observe state θ. Instead, each player i receives a private signal $x_i = \theta + \nu\eta_i$, where the idiosyncratic noise term η_i is distributed according to density function f_i.

A global game G embeds a complete information game Γ at state θ^*, if $g_i(a) = u_i(a, \theta^*)$ for all players i and for all action profiles a.

Theorem (analogue to Frankel, Morris, and Pauzner (2003):
As the scale parameter v goes to zero, the global game $G^v(u,\phi,f)$ has an essentially unique limit equilibrium.

More precisely, denote a pure strategy of the global game by $s_i : \mathbf{R} \rightarrow A_i$, such that player i chooses action $s_i(x_i)$ when receiving signal x_i. There is a strategy combination s, such that for $v \rightarrow 0$, any equilibrium $s^v(x)$ of $G^v(\cdot)$ converges to $s(x)$ for all x except possibly at the finitely many discontinuities of s.

If the global game's limit-equilibrium strategy profile is continuous at state θ^*, its value at this state determines a particular Nash equilibrium of the complete information game, called **global-game selection** (GGS).

The first question that arises when defining a selection for a class of games with multiple equilibria is whether the selection is actually unique. A complete-information game can be extended to many different global games distinguished by the extended payoff function u, the prior distribution of the state variable ϕ, and the tuple of noise distributions for private signals f. Hence, we would like to know under which conditions the GGS is independent of u, ϕ, and f ? If it is not independent, then multiple Nash equilibria of the underlying complete-information game are replaced by potentially different limit equilibria of the different global games. Frankel, Morris, and Pauzner (2003) show that the GGS is independent of ϕ. BDH show that the GGS is independent of u. The combination of these two results implies that one may use without loss of generality a particular global-game embedding, such as $u_i(a,\theta) = g_i(a) + \theta a_i$ (BDH).

Proof: For a sufficiently wide support of ϕ, u_i satisfies the global-game assumptions (A1) to (A3). Obviously, u_i embeds g at $\theta^*=0$.

Unfortunately, the GGS may depend on f. This is known since Frankel, Morris, and Pauzner (2003) and Morris and Shin (2003), who constructed the first examples of global games in which the limit equilibria for $v \to 0$ depend on the distribution f. The GGS is called **noise independent**, if the GGS is independent of the particular density function of private signals f. Carlsson and Van Damme (1993) had already shown that for any two-player-two-action game, the GGS is independent of f. In symmetric 2-player-2-action games, the GGS is actually identical to the risk-dominant equilibrium defined by Harsanyi and Selten (1988). Table 1, taken from BDH gives an overview of the games, for which noise independence can be established simply by counting the number of players and actions. It shows that symmetric complete-information games with two actions for each player, symmetric 2-player games with 3 actions for each player and asymmetric 2-player games, in which at least one of the players can only choose between two possible actions are noise independent. In these games, the GGS can be calculated by solving the simplest possible global-game. Larger games, however, may not be noise independent. For these games, noise independence can be established by using *p*-dominance or potential maximizers, arguably complicated concepts that most applied researchers do not want to go into.

Table 1. Noise (In)dependence in Supermodular Games

Symmetric games				Asymmetric games			
actions	2 each	3 each	4 each	actions	2 each	2 by n	3 each
2 players	√	√	X	2 players	√	√	X
3 players	√	X		3 players	X	n.a.	
n players	√			n players	X	n.a.	

Notes: √ always noise independent. X counterexample to noise independence exists. For empty cells, the existence of counterexamples follows from examples in smaller games.

It is therefore quite helpful that BDH show that some larger games can be broken down into small games, for which noise independence can be established by counting the number of players and actions. The idea behind their theorem rests on the observation that any equilibrium of a global game is a step function with equilibrium strategy profiles increasing in the relevant state variable. Hence, the GGS is also a step function increasing in the state variable. When the variance of idiosyncratic noise terms approaches zero, players may only need to consider two action profiles, namely those that are played for somewhat smaller and somewhat larger states of the world. This rough intuition cannot always be successful in describing a GGS, because symmetric 2-action games are noise independent and there are examples of larger games that are not. However, a simple criterion for games that can be broken down may reduce the workload for applied researchers to a minimum.

To get this, consider a supermodular complete information game Γ with joint action set A. For action profiles $a \leq a'$, we define the set of action profiles between these two:

$$[a,a'] = \{\tilde{a} \in A \mid a \leq \tilde{a} \leq a'\}.$$

Now, we look at the restricted game $\Gamma|[a,a']$, which is given by restricting the joint action set of Γ to the action profiles a and a' (inclusive). BDH prove the following Lemma:

Lemma (BDH, 2013): Consider a supermodular game Γ and a noise structure f. An action profile a^n is the unique GGS of Γ, if there is a sequence $0 = a^0 \leq a^1 \leq \ldots \leq a^n \leq \ldots \leq a^m = m$ s.t.

(i) a^j is the unique GGS in $\Gamma|[a^{j-1}, a^j]$ for all $j \leq n$, and

(ii) a^{j-1} is the unique GGS in $\Gamma|[a^{j-1}, a^j]$ for all $j > n$.

Corollary: If all the restricted games are noise independent, then Γ is also noise independent and a^n is the unique noise independent GGS of Γ.

This result provides a simple solution technique: If you have a game with multiple equilibria, first check whether it is supermodular, so that the result applies. Then decompose the game by defining restricted sets of action profiles that (if patched together) stretch from the lowest action profile (denoted by 0) to the largest action profile m. Note that it is not necessary that all action profiles of the original game are contained in one of the restricted sets. We only need the highest action profile of one set being the lowest action profile of the next set. So, you need to define a sequence of profiles $0 = a^0 \leq a^1 \leq \ldots \leq a^n \leq \ldots \leq a^m = m$.

Then, derive the GGS for each of the restricted games. This may sound cumbersome, but actually, the trick is in defining an appropriate sequence of profiles such that the GGS of each restricted game is easy to derive. Now, if all solutions point to the same strategy profile, this profile is a GGS of the large game. We can mark these selections by arrows as in the following example, where a^3 is the GGS of the large game[9]:

$$0 = a^0 \rightarrow a^1 \rightarrow a^2 \rightarrow a^3 \leftarrow a^4 \leftarrow a^5 = m$$

If, in addition, all small games are noise independent, the large game is also noise independent. Since noise independence is guaranteed for symmetric 2-action games and their GGS is almost trivial to derive, it is advisable to define the sequence of strategy profiles in such a way, that all restricted games fall into this class.

This is certainly not possible for all games, and even if a large game can be broken down into restricted 2-action games, the arrows may not always point into direction of the same

[9] The procedure iteratively eliminates strategy profiles as potential solutions for the large game, starting with the highest and lowest profiles. Thereby, it generalizes the iterative elimination of dominated strategies.

strategy. In the following example, the arrows point to strategy profiles a^1 and a^4 and we cannot say which or whether any of these two profiles is the GGS of the large game[10]:

$$0 = a^0 \rightarrow a^1 \leftarrow a^2 \rightarrow a^3 \rightarrow a^4 \leftarrow a^5 = m$$

In such cases, we cannot use the decomposition result and need other techniques for calculating the large game's GGS and for checking whether it is noise independent. However, BDH give some examples for symmetric games with more than 2 actions that can easily be solved by decomposition. The application in the next section demonstrates that the decomposition may also be quite helpful for solving games with asymmetric players.

4. An entry game with asymmetric payoff functions

The application is motivated by the problem of marketing a new network good and was inspired by Ruffle, Weiss, and Etziony (2015). The payoff to any agent buying a network good is increasing in the number of other agents who adopt the same good. Social media websites, crowdsourcing applications and cellphones are a few examples. Payoffs need not be the same for all agents: for some agents, strong network effects already render the good profitable when just a few others use the same good. For others, profitability requires that many others adopt the good.

The problem of asymmetric payoff functions also arises in other applications of supermodular games, like financial crises, where the network effects differ between different banks depending on how connected they are with the inter-bank market. Depositors of banks that are highly connected and dependent on the stability of other financial institutions have incentives to withdraw their deposits, while depositors of other, less vulnerable banks may find it more profitable to retain their deposits in the bank, even if both depositor groups have the same expectation about the number of withdrawn deposits. Here, the expected payoff to any agent who keeps his deposits in the bank is increasing in the number of other agents who choose the same action.

Table 2 gives an example of such a game. Each entry refers to the player's net payoff when he purchases the good as a function of the total number of purchasers. Positive numbers mean that the payoff from the network good is higher than its price. For players A, B, and C, for example, purchase of the good is profitable if at least 3 players adopt it, whereas players J, K, and L profit from buying the good only if all 12 players adopt it. In the banking interpretation "adopting" is equivalent to keeping deposits and positive numbers mean that the expected returns from keeping deposits are higher than the return from an immediate withdrawal.

[10] Presumably, the GGS (eventually depending on the noise distribution) is some strategy profile within $[a^1, a^4]$, because smaller and larger strategy profiles can be eliminated.

Table 2. Payoff table of an asymmetric entry game

$v_i(n)$ Player i	Number of adopters n 1	2	3	4	5	6	7	8	9	10	11	12
A	-4	-1	2	5	8	11	14	17	20	23	26	29
B	-4	-1	2	5	8	11	14	17	20	23	26	29
C	-4	-1	2	5	8	11	14	17	20	23	26	29
D	-13	-10	-7	-4	-1	2	5	8	11	14	17	20
E	-13	-10	-7	-4	-1	2	5	8	11	14	17	20
F	-13	-10	-7	-4	-1	2	5	8	11	14	17	20
G	-25	-22	-19	-13	-10	-7	-4	-1	2	5	8	11
H	-25	-22	-19	-13	-10	-7	-4	-1	2	5	8	11
I	-25	-22	-19	-13	-10	-7	-4	-1	2	5	8	11
J	-34	-31	-28	-25	-22	-19	-13	-10	-7	-4	-1	2
K	-34	-31	-28	-25	-22	-19	-13	-10	-7	-4	-1	2
L	-34	-31	-28	-25	-22	-19	-13	-10	-7	-4	-1	2

For an abstract description of this type of entry games and for deriving the GGS, we introduce some notation: $v_i(n)$ = agent's payoff from entry (adopting) if n players enter in total. We assume that $v_i(n)$ is increasing in n for any player i. Thus, our game is supermodular. There are M types of players. Agents with the same payoff function belong to the same type, while players with different payoff functions belong to different types. We consider games in which we can arrange all of the types according the following order: i belongs to a higher type than j iff $v_i(n) \geq v_j(n)$ for all n with at least one strict inequality.

In a one-shot game of this type, a pure strategy for a player is a decision to either enter the game (adopt the network good) or not. We define a player's strategy by $a_i = 1$ if player i enters, and $a_i = 0$ if player i does not enter. Strategy combinations are partially ordered by the relation: $a \geq a'$ iff $a_i \geq a'_i$ for all i. Define a^0 as the strategy combination where everybody stays out; a^1 as the strategy combination where only players of the highest type (Type 1, equal to Players A, B, C in Table 2) enter, others stay out; and a^k as the strategy combination where all players of the high types 1 to k enter and players of lower types stay out. Since there are M different types, a^M is the strategy combination in which all players enter. Note that $a^{k-1} < a^k$ for all $k = 1, \ldots, M$.

Following Basteck, Daniëls, and Heinemann (2013), we can decompose the game into restricted games $[a^{k-1}, a^k]$ for $k = 1, \ldots, M$. The restricted game $[a^{k-1}, a^k]$ consists of the strategy combinations a^{k-1}, a^k, and all strategy combinations $\tilde{a}$ that are strictly in between these two, i.e. $a^{k-1} < \tilde{a} < a^k$. The strategy combinations in the restricted game share the feature that all players of the high types 1 to k–1 enter and all players of the low types k+1 to M stay out. Thus, only players of Type k have a choice in this restricted game. For these active players, the restricted game is a symmetric binary-action game with payoffs given by the k'th block diagonal of the payoff matrix. For the game displayed in Table 2, block diagonals are the same for all types. Table 3 shows this block diagonal, which is the relevant part of the payoff function of the restricted game:

Table 3. Payoff table for the restricted game of Type 1 (Players A,B,C)

$v_i(n)$ Player i	number of adopters n 1	2	3
A	-4	-1	2
B	-4	-1	2
C	-4	-1	2

In symmetric binary-action games, the GGS is given by the best response to a uniform distribution on the number of entrants among the other players. For the restricted game, displayed in Table 3, the expected payoff given a uniform distribution on the number of other players adopting is –1 < 0. Hence, the global game selects the lowest strategy combination in the restricted game, such that $a_i=0$ for all players i of Type k. Because the block diagonals are the same for all 4 groups of our game, the GGS for each of the restricted games $[a^{k-1}, a^k]$ for $k = 1, \ldots, M$ is the respective lowest strategy combination,

$$0 = a^0 \leftarrow a^1 \leftarrow a^2 \leftarrow a^3 \leftarrow a^4 .$$

It follows that the GGS of the entire game is a^0, the strategy combination where no player enters. Since the restricted games are noise independent, so is the entire game (Basteck, Daniëls, and Heinemann, 2013).

Strikingly, this selection does not depend on the values of the payoff matrix in the off-diagonal blocks! The experiment, however, shows that these off-diagonal payoffs affect behavior in an intuitive manner. The GGS fails to predict this.

The same procedure can also be applied to asymmetric supermodular games with different block diagonals. If the block-diagonal payoffs of high types are weakly higher than those of low types, the procedure derives the unique and noise independent GGS. If there is only one player per type, the procedure is identical to the iterative elimination of dominated strategies.

5. Experimental design

The experiment consists of four sessions, each with 12 participants, conducted in the experimental economics laboratory of Technische Universität Berlin. Subjects were invited via ORSEE (Greiner 2015), the experiment was programmed with z-tree (Fischbacher 2007).

Each subject played 20 different supermodular one-shot games in random order without feedback[11]. In each game, each subject had to decide between two options: “enter” or “not enter”. The payoff for not entering was 34 experimental currency units (ECU), independent of what the other participants decided. This number was the same in all games and appeared on the screen for every game. The payoff for entering depended on the subject’s, role in the particular game and on the other participants’ decisions in the same game.

In each game, there were 12 roles, called A, B, C, … , L. Roles were randomly assigned to the 12 participants such that each role was assumed by one participant. The random role assignment was done for each game independently of the roles or decisions in previous games, except that no subjects ever got the same role in two consecutive games. The payoffs for “enter” were displayed on the screen as in the in the following Table 4:

Table 4. Sample payoff table shown in the instructions for the experiment

	number of entrants											
Role	1	2	3	4	5	6	7	8	9	10	11	12
A	39	40	41	42	43	44	45	46	47	48	49	50
B	37	38	39	40	41	42	43	44	45	46	47	48
C	35	36	37	38	39	40	41	42	43	44	45	46
D	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**
E	31	32	33	34	35	36	37	38	39	40	41	42
F	29	30	31	32	33	34	35	36	37	38	39	40
G	27	28	29	30	31	32	33	34	35	36	37	38
H	25	26	27	28	29	30	31	32	33	34	35	36
I	23	24	25	26	27	28	29	30	31	32	33	34
J	21	22	23	24	25	26	27	28	29	30	31	32
K	19	20	21	22	23	24	25	26	27	28	29	30
L	17	18	19	20	21	22	23	24	25	26	27	28

Each row displays the payoffs to the subject with the corresponding role. The player’s own role in each game was highlighted as Role D in Table 4. For each player, there were 12 possible payoffs. The payoff each participant received for a game was determined by the total number of participants that decided to enter in this game. For example, if the player with role D entered in this game and there were, for example, 6 players (including himself) who entered in this game, then he received a payoff of 38 ECU.

All of these rules were given to subjects in written instructions (see Appendix C) and read aloud at the start of a session. Before subjects could make decisions in the 20 games, they had to answer comprehensive questions to make sure that they understood how to read the payoff tables. In each of the 20 rounds, each subject decided for one of the 20

[11] So, behavior in any game cannot be affected by the outcome of another game.

games. Once all 12 subjects made and confirmed their decisions, the round ended and the next game started. Subjects did not receive any feedback about others' behavior between games. Once all 20 games had been completed, each subject received a list containing the results of the games. This list showed the game number (1-20) and displayed for each game: the subject's own role (A-L), her or his decision ("enter" or "not enter"), the number of participants who chose to "enter" in this game, and her or his own payoff for this game. The screen also showed the sum of the subject's payoffs over all 20 games. At the end of the experiment, subjects were paid 1 Euro for every 40 ECU they earned in the experiment. Sessions took less than an hour and subjects earned between 13 and 19 Euros each.

The payoff functions for the 20 games used in this experiment are displayed along with subjects' choices in Appendix A. The games varied in the number of types (1, 2, 4, or 12), in whether the GGS predicts entry or no entry (same prediction for all types except for Game 20), and in the off-diagonal payoffs. 2 games have symmetric payoffs for all players (1 type) and 4 games have 12 different types and, thus, a unique equilibrium that can be calculated by iterative elimination of dominated strategies.

Hypotheses: Given the results of previous experiments on coordination games, in particular from HNO, we hypothesize that (i) changes in payoffs that leave the GGS unaffected have no significant impact on subjects' behavior and (ii) playing the GGS strategy yields a higher expected payoff than actual behavior or other pre-specified strategies. The next section shows that both hypotheses can be clearly rejected.

6. Experimental results

This section first gives a descriptive review of subjects' decisions in the 20 games. Then, we analyze how well the GGS predicts observed choices and compare this measure of predictive power with other selection theories for binary-choice coordination games. Following HNO, we then compare the expected payoff of a player who plays either of these selections given the observed distribution of choices by others. Thereby, we test which theories are best-suited to provide a recommendation for an individual player. Then, we test the comparative statics properties of the different solution concepts. Finally, we estimate a global game with positive variance of private signals and compare its predictive power to an estimated quantal response equilibrium (QRE) and to probabilistic responses to Laplacian and level-k beliefs. We also look at the predictive power and at the expected payoffs of best responses to the estimated probabilistic solution concepts. We compare those with predictive power and expected payoffs from the best selection theory. Thereby, we analyze whether the effort of gathering data and estimating a probabilistic model helps in achieving good point predictions of behavior or recommendations for an individual player.

Appendix A displays the payoff tables for the 20 games. These tables also state for each game and for each role how many subjects in this role decided to enter. As there were 4 sessions, the maximum number of entrants per role is 4. The maximum number of entrants per game is 48. Since all games were played without feedback, we can aggregate the results across all four sessions.

- Games 1 to 10 have 4 types of players and 5 Nash equilibria in pure strategies.[12] Games 1 to 4, 9, and 10 have the same payoffs in the block diagonals that determine the GGS. Here, the GGS is that no player enters. The games vary in their off-diagonal payoffs and so did the observed total number of entries that varied from 10 subjects (out of 48) in Game 1 to 30 subjects in Game 10. Games 5 to 8 have higher payoffs in the block diagonals and here, the GGS is that all players enter. The observed total number of entries varied from 22 to 29.
- Games 11 to 14 have 2 types of players and 3 Nash equilibria in pure strategies. The GGS for Game 11 is that all players enter. For the other 3 games, the GGS is that no player enters. The observed number of entrants is 31 in Game 11. It varies from 16 to 20 in Games 12 to 14.
- Games 15 and 16 are symmetric games (1 type of players). In both games everybody entering and nobody entering are the 2 Nash equilibria in pure strategies. The GGS is the same for both games: no player enters. We observe 27 entries in Game 15 and 9 in Game 16.
- Games 17 to 20 have 12 types each. These games have unique equilibria that can be derived by iterative elimination of dominated strategies. In Game 17, the equilibrium is that everybody enters. We observe that 20 (out of 48) subjects enter. Games 18 and 19 have the equilibrium that nobody enters. We observe 32 and 16 entries, respectively. Game 20 has a unique equilibrium in which Types A to H enter and Types I to L do not enter. We observe that 27 out of 32 players in the roles A to H enter and none of the 16 players in roles I to L.

6.1 Predictive Power of Selection Theories

This subsection compares selection theories that prescribe a pure strategy to any generic binary-choice game with an eventual indifference between the two actions for a zero-set of parameters, in which case we attribute probability .5 to both actions.

The predictive power of a selection theory is measured by the proportion of observed decisions that is correctly predicted by the respective theory. Since for many games and roles, different subjects choose different actions, selection theories cannot perfectly fit the data. At best, a selection predicts for each game and for each role the action that is chosen

[12] The equilibria are (i) nobody enters, (ii) only Players A-C enter, (iii) Players A-F enter, (iv) Players A-I enter, (v) all players enter.

by the majority of subjects. The average proportion of decisions that is taken by the majority of subjects across all games and roles, thus, serves as an upper benchmark for a selection's predictive power. The lower benchmark is given by a random prediction (0.5) for all games and roles. We compare the predictive power of the GGS with four other selection theories:

- "Laplacian", defined as the best response to a uniform distribution (Laplacian belief[13]) on the number of other players who chose B in the same game. In symmetric binary choice games with strategic complementarities, the Laplacian strategy coincides with the GGS.
- "Level-1", defined as the best response to every other subject choosing either action with probability .5. The concept of levels of reasoning has been shown to predict many subjects' choices in some supermodular games like the guessing game (Nagel, 1995).
- Payoff-dominant equilibrium (PDE), defined as the equilibrium strategy combination that yields the highest payoffs for all players, amongst all equilibria.
- Maximin, defined as the strategy that yields the highest payoff for an individual player if no other player enters.
- Best possible prediction, defined as the strategy that is taken by the majority of subjects in the same game and role. As explained above, no selection can predict a larger proportion of observed decisions than this benchmark.

Table 5 states the proportion of observed choices that coincides with the predictions by each of these selection theories. We distinguish asymmetric games with multiple equilbria, (Games 1-14), symmetric games (15-16), and games with a unique equilibrium (17-20). Note that GGS and Laplacian are identical concepts in symmetric games, and GGS and PDE are both equilibrium refinements. Thus, they select the same strategy combination if the game has a unique equilibrium, while Laplacian, Level-1, and Maximin may select non-equilibrium strategies. Figure 2 normalizes the predictive power with random prediction as zero and maximum possible probability as 100.

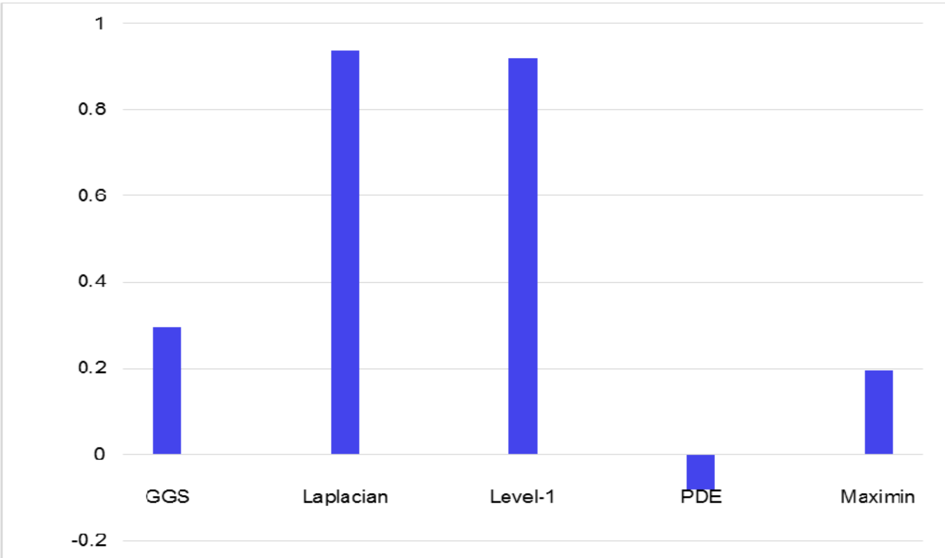
Table 5. Proportion of observed choices in line with the respective selection theory

	GGS	Laplacian	Level-1	PDE	Maximin	Best possible
Asymmetric games with multiple equilibria	.595	.815	.815	.455	.544	.827
Symmetric games	.625	.625	.625	.375	.625	.687
Games with unique equilibrium	.589	.859	.833	.589	.599	.891
All games	.597	.805	.800	.474	.564	.826

[13] The term „Laplacian" for this strategy was coined by Morris and Shin (2003), p.57.

While only 60% of subjects' decisions in the experiment are in line with the GGS, 80% of observed decisions are in line with a best response to random behavior (Level-1) and with a best response to Laplacian beliefs. These results are remarkable as they are rather close to the maximum possible predictive power that a selection can have (benchmark).

Figure 2: Predictive power of selection theories relative to benchmarks (random = 0, best possible = 1)



Another measure for the quality of selection theories is the percentage of games and roles for which a theory predicts the choice taken by the majority of subjects. The Laplacian strategy yields the best possible prediction in 92% of all games and roles, Level-1 in 90%, GGS and PDE in 60%, and Maximin in 63%. A random selection would give the best prediction in 51% of all games and roles[14].

Furthermore, in 234 out of 240 games and roles (97.5%) the Laplacian strategy is a weakly better predictor of observed behavior than the GGS. For Level-1, this number is 235 (97.9%). It should be noted that Laplacian and Level-1 strategies coincide in 230 (96%) of our games and roles[15]. Thus, it is not surprising that their predictive power is almost the same.

In order to check how robust these results are, we compare the predictive power of the same solution concepts for the symmetric coordination games used in the experiment by HNO using the same criteria. In their experiment, there were 299 subjects in total, each playing 30 different binary-choice games. Here, the GGS correctly predicted 75% of the 299x30=8970 choices and gave the best prediction in 77 games (86%), Level-1 predicted 71% of all choices and gave the best prediction in 70 games (78%)[16]. For PDE the respective

[14] This number exceeds 50%, because in 7 out of 240 games and roles, exactly half of all subjects of the respective type chose the risky option, so that either prediction is the best possible.

[15] Out of the 10 cases, in which Laplacian and Level-1 differed, Laplacian gave the better prediction in 6 cases, Level-1 in the other 4.

[16] In 8 out of 9 games, in which GGS and Level-1 prescribe different actions, the GGS predicted the majority of choices.

numbers are 60.5% (62%), and for Maximin 47.8% (48%). The benchmark was 78.1% (100%). Thus, in symmetric binary choice games, the GGS has the highest predictive power. Since the GGS is identical to the Laplacian strategy in symmetric games, we may conclude that the best response to a Laplacian belief about the proportion of players who take the higher action gives the best point predictions for symmetric *and* asymmetric binary-choice games with strategic complementarities.

Result 1: *The best response to a uniform distribution on the proportion of other players who take the higher action ("Laplacian" strategy) predicts more choices correctly than the GGS, Level-1, PDE, or Maximin.*

6.2 Expected Payoffs resulting from Selection Theories

Suppose you want to give an advice to a depositor of a bank or to a firm manager who considers adopting a new software. A player who happens to participate in a one-shot coordination game, would like to know which strategy is associated with the highest expected payoffs. As he does not know yet the behavior of other players who decide simultaneously, such a strategy must be defined ex-ante and the selection theories that we discussed in the last subsection are obvious candidates. Table 6 provides the expected payoffs that a player obtains when following either of these strategies in a randomly selected game and role with randomly selected players from our subject pool. We compare these values to the expected payoff from *random* behavior (entry with probability .5), subjects' *actual choices*, and *best response* to the observed distribution of choices. The third line normalizes these payoffs to those from the best response (100%) and random behavior (0%).

Table 6. Expected payoff associated with the respective strategies

	Random	GGS	Laplacian	Level-1	PDE	Maximin	Actual choices	Best response
Exp'd payoff	31.39	33.95	35.97	36.08	28.97	34.20	35.51	36.27
Exp'd payoff (normalized)	0%	52%	94%	96%	-50%	58%	84%	100%

Level-1 and Laplacian yield higher expected payoffs than subjects' average choices and come rather close to the maximum possible expected payoff, given by a best response to observed behavior. The GGS, instead, provides a poor recommendation by which subjects would obtain lower expected payoffs than they actually achieved (on average).

In HNO, the GGS yielded the highest expected payoff. Using their data, expected payoffs from the GGS (normalized) were € 11.75 (97%), Level-1 € 11.43 (84%), PDE € 8.29 (-40%), Maximin € 8.25 (-42%), and random € 9.31 (0%). Actual choices yielded an average of

€ 10.36 (42%) and a best response to the observed distribution would have given € 11.82 (100%). As Laplacian equals the GGS in symmetric games, we may conclude that Laplacian and Level-1 give the best recommendations for an individual player in symmetric and asymmetric binary-choice games with strategic complementarities.

Result 2: *The expected payoff for a player who follows either the Laplacian or the Level-1 strategy is higher than the expected payoff from the GGS, PDE, Maximin, or actual behavior.*

6.3 Comparative Statics

The different payoff functions in the 20 games of the experiment were designed in such a way that we can test several comparative statics properties on which the aforementioned selection theories give different predictions. In all games with different types, we observe that high-type players (A, B, C…) tend to opt for B more frequently than low types (… J, K, L). Thus, our first test is whether subjects of higher types enter more frequently than subjects of lower types within the same game. In the asymmetric games with multiple equilibria (Games 1-14), GGS, PDE and Maximin predict the same decisions by all types, while Laplacian and Level-1 predict entry by high types and no entry by low types in each of these games. If we look at the correlation between types and entries, we find that the proportion of subjects who enter is highly correlated with their type. We run linear regressions with fixed effects for each game:

$$\textit{Number of entrants} = \alpha + \beta \times \textit{Type-number}.$$

For games with 4 types the estimated $\alpha = 11.05$ (1.03), and $\beta = -3.42$ (.24). For games with 2 types we estimate $\alpha = 38$ (.74), and $\beta = -15$ (.4). Numbers in parentheses are standard errors. In both regressions "*Type*" is highly significant (*p*= .1%).

Another (non-parametric) test just counts how often higher types enter more often than the next lower type in the same game. In Games 1-14, higher types enter more often than the next lower type in 29 cases, less often in 2 cases, and equally often in 3 cases, thereof 2 cases with zero entries for both types. Thereby, we can reject that the proportion of entries is the same across types in the same game (Wilcoxon signed rank test, p<1%).

Result 3: *Players of higher types enter more often than players of lower types.*

For comparisons between games, let us say that a game is "higher" than another game if all payoffs in the first game are weakly larger than the payoffs in the second game with at least one strict inequality. The second comparative statics property concerns the off-diagonal payoffs. As explained above, only entries in the block-diagonal matrices affect the GGS. Games 1 to 4, 9 and 10 have the same block diagonals, according to which the GGS predicts no entries, but vary in their off-diagonal payoffs. Game 2 has higher payoffs than Game 1 in cells above the diagonal blocks, Game 3 has higher payoffs than Game 1 below the diagonal blocks, Game 4 has higher payoffs than Games 2 and 3 on either side. Game 9 has higher payoffs than Game 3 below the diagonal, and Game 10 has higher payoffs than

Games 4 and 9 on either side. Games 5 to 8 can be ordered in the same way as Games 1 to 4. Amongst the games with 2 types, Game 13 has higher payoffs above the diagonal than Game 12. Finally, Game 18 has higher payoffs below the diagonal than Game 19. Table 7 summarizes the independent comparisons between games that are driven by variations in off-diagonal payoffs. The partial order of games is displayed in the first and fourth column of Table 7. The columns to the right of these pairs of games indicate the number of entrants in the lower and in the higher game(s) of the respective pairs. Dependent comparisons have been left out. We find that in 9 of these independent pairs higher off-diagonal payoffs lead to more entries with one opposing case. The positive impact of off-diagonal payoffs on the number of entries is significant at 5% according to the Wilcoxon signed-rank test.

Result 4: *Higher off-diagonal payoffs raise the number of entrants.*

Table 7. Effect of higher off-diagonal payoffs on the number of entries

Pair of games	entrants in lower game	entrants in higher game(s)	Pair of games	entrants in lower game	entrants in higher game(s)
1 < 2	10	17	5 < 6	26	22
1 < 3	10	15	5 < 7	26	29
2, 3 < 4	17, 15	21	6, 7 < 8	22, 29	29
3 < 9	15	21	12 < 13	19	20
4, 9 < 10	21, 21	30	19 < 18	16	32

The third comparative statics prediction that we want to test is whether an overall increase in payoffs (including the block diagonals) raises the number of entrants. Here, we compare games for which the payoffs in one game are equal to the payoffs in another game plus some constant, $v_i'(n) = v_i(n) + \delta$, and for which the GGS predicts entry in the higher game and no entry in the lower gamer. Table 8 summarizes these comparisons. Here, the higher games always have more entrants, which is significant at 5% according to the Wilcoxon signed-rank test.

Result 5: *A constant increase in all payoffs, such that the GGS predicts entry in the higher game and no entry in the lower game leads to more entries in the higher than in the lower game.*

Table 8. Effect of a constant increase in payoffs on the number of entries

Pair of games (δ)	entrants in lower game	entrants in higher game(s)
1 < 5 (3)	10	26
2 < 6 (3)	17	22
3 < 7 (3)	15	29
4 < 8 (3)	21	29

Pair of games (δ)	entrants in lower game	entrants in higher game(s)
12 < 11 (5)	19	31
14 < 13 (3)	16	20
16 < 15 (8)	9	27

The GGS only predicts the comparative statics stated in Results 5, while Results 3 and 4 go against its predictions. PDE and Maximin strategy predict none of the three results. The Laplacian and the Level-1 strategy, instead, predict all three results.

6.4 Modelling heterogeneous behavior

Selection theories may predict a large number of choices correctly, but they can neither account for nor predict the heterogeneity of individual choices within the same game and role. To account for heterogeneity, probabilistic models assign some probability for either choice to each game and role. Ideally, such a probabilistic model would predict the observed distribution of choices.

There are many possible ways of formulating probabilistic models. HNO fitted logistic functions and two global games to their data. As explained in Section 2, the estimated global games provided a nice fit of these data, even though there were some systematic deviations, and they also gave reasonable predictions out of sample. Alternatives to these approaches are quantal response equilibria (QRE) or any choice function that adds some noise to the prescriptions of a selection theory.

Here, we estimate a global game with positive variance of private signals on monetary payoffs and compare its predictive power to an estimated QRE. We also define noisy responses to Laplacian beliefs and Level-1, by assuming that subjects choose the action with the higher expected payoff (given their beliefs) with some probability defined by a logit response function. Each of the four models has one free parameter that we estimate using the data from the experiment. We compare their log-likelihoods to evaluate the predictive power of these estimated models.

In order to judge whether modelling the distribution of choices helps a participant of a one-shot game to obtain higher payoffs, we calculate the expected payoffs resulting from best responses to the four estimated models and compare those with the expected payoffs of selection theories displayed in Table 6.

First, we define a global-game that embeds the coordination games used in the experiment. Following BDM, we define the payoff for entry as $v_i(n)+\theta$, where θ is a random variable with infinite variance. For any given θ, players receive i.i.d. private signals with normal distribution $x_i \sim N(\theta,\sigma^2)$. The standard deviation of private signals σ allows heterogeneous behavior and can be estimated to fit the observations of an experiment.

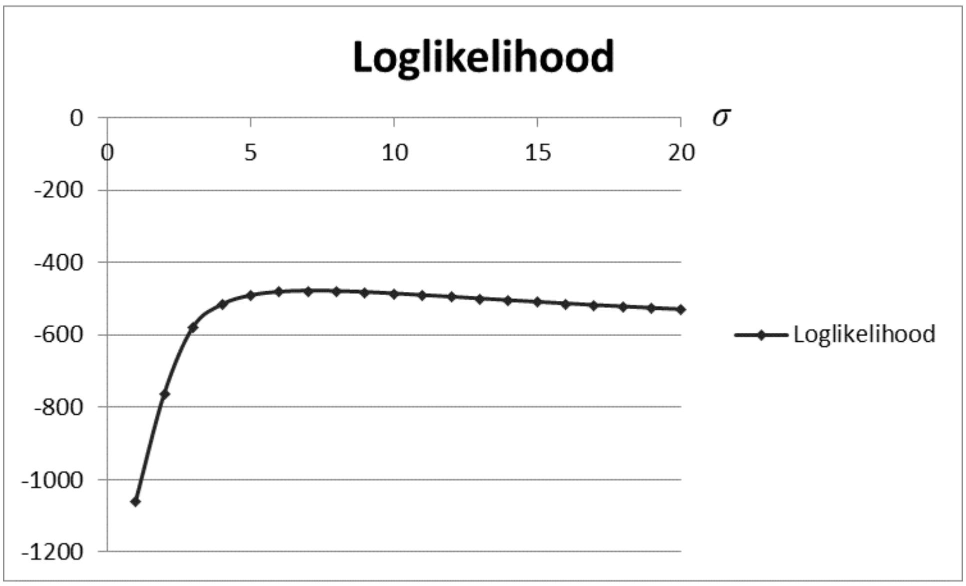
A global-game equilibrium is a vector of thresholds $(\bar{x}^i)_{i=1,\dots 12}$ for each of the 12 players. Of course, all players who belong to the same type (i.e. have the same payoff function), also have the same equilibrium threshold so that a global game has M different thresholds for the M types. The equilibrium is characterized by the following two conditions

(1) Given that the true realization is $\theta = 0$, the probability that player j enters is $prob(x_j \geq \bar{x}^j) = 1 - \Phi(\bar{x}^j / \sigma)$, with Φ denoting the cumulative standard normal distribution.

(2) Player i is indifferent at signal $\bar{x}^i$, if and only if $E(v_i(n)+\theta \mid x_i = \bar{x}^i) = 0$.

Appendix B provides the details of how the likelihood function is constructed. Maximizing the loglikelihood, we estimate a standard deviation of signals σ of about 7. The loglikelihood at this value is –478.9 (average loglikelihood = –.499). Figure 3 displays the loglikelihood depending on σ. One gets the impression that it is rather flat at the top and does not fall much for an increasing σ, so that the estimate of σ is rather imprecise[17]. Since there are 960 decisions in total, the loglikelihood of random predictions is 960 x $ln(.5)$ = –665.

Figure 3: Loglikelihood



[17] The likelihood is not significantly lower for σ=6 or σ=8.

For the QRE, we use a standard definition with a logistic choice function according to which a subject in Role i enters with probability

$$p_i = \frac{\exp(\lambda \cdot E(v_i(n) \mid p_{-i}))}{\exp(\lambda \cdot E(v_i(n) \mid p_{-i}) + \exp(\lambda \cdot 34)}, \quad (1)$$

where p_{-i} is the vector of entry probabilities for all other subjects. Subjects of the same type enter with the same probability. Hence, the QRE of a game is given by M probabilities that solve the M equations (1) simultaneously. The computational complexity of solving these equations is lower than for solving the equations defining a global-game equilibrium, because the equations need to be evaluated only at the true payoffs, while a global game needs to evaluate similar equations for all possible realizations of θ. The maximum-likelihood estimation of the rationality parameter in the QRE yields λ = .07. The loglikelihood at this value –521.0 (average loglikelihood = –.543), which is clearly lower than the result for the estimated global game. Reason is that the estimated QRE predicts rather small differences between entry probabilities of high and low types of the same game. The estimated global game also predicts smaller differences between entry probabilities of different types than observed in the experiment, yet it captures these differences better than the QRE.

Noisy responses to Laplacian and Level-1 beliefs are modelled by a logit response function to the respective beliefs. Their construction does not require solving complicated equation systems. Here, a subject in Role i enters with probability

$$p_i = \frac{\exp(\lambda \cdot E(v_i(n))}{\exp(\lambda \cdot E(v_i(n)) + \exp(\lambda \cdot 34)},$$

where the expected payoffs $E(v_i(n)$ are simply derived from a uniform distribution on n for Laplacian beliefs (that can be calculated by head) and from a binomial distribution for Level-1 beliefs. The maximum-likelihood estimates are λ = .184 for Laplacian beliefs and λ = .170 for Level-1 beliefs. The maximum loglikelihood is –456.8 (average loglikelihood = –.476) for Laplacian and –463.8 (–.483) for Level-1 beliefs. Both numbers are clearly higher than the maximum likelihood of the estimated global game[18]. They correspond to an average likelihood per observation of 62.1% and 61.7% respectively, compared to 60.7% in the estimated global game and 58.1% in the estimated QRE. It is stunning that the model with lowest computational complexity ("noisy Laplacian") yields the best fit of data.

Result 6: *The estimated noisy response to Laplacian beliefs about the number of entrants fits the observed distribution of choices better than estimated global game, QRE or noisy response to Level-1 beliefs.*

All four models of heterogeneous behavior give the observed comparative statics predictions stated in results 3 to 5. A global game with positive variance of private signals predicts higher entry probabilities by higher types, because the off-diagonal payoffs lead to

[18] Loglikelihoods of the four models are summarized in Table 9

different threshold values for different types. This is an important difference to the GGS that fails to make this distinction.

Does it help a player who participates in a randomly selected one-shot coordination game to know any of the estimated models? The expected payoffs of best responses to either of the estimated models of heterogeneous behavior are given in Table 9. The third line relates these payoffs to those from the best response (100%) and random behavior (0%). All four models yield similar expected payoffs. Comparing them with expected payoffs stated in Table 6 shows that all of these best responses provide higher expected payoffs than following the GGS and they all provide improvements over actual choices. However, the Level-1 strategy yields a (slightly) higher expected payoff than best responses to any of the estimated models of heterogeneous behavior and the Laplacian strategy that requires the lowest computational effort is only slightly outperformed by the best response to the estimated noisy Level-1 model. Thus, we may conclude that an estimated model of heterogeneous behavior would not have helped a player in the experiment to make better decisions, even though these models have been fitted to match observations.

Result 7: *Best responses to different estimated models of heterogeneous behavior yield similar expected payoffs, comparable to but not higher than expected payoffs for the Laplacian and the Level-1 strategy.*

Table 9. Expected payoffs of best responses to estimated models

Estimated Model	Global game	QRE	Noisy Laplacian	Noisy Level-1
Expected payoff from best response	35.87	35.83	35.95	36.03
Expected payoff (normalized)	92%	91%	93%	95%
Predictive power of best response	.786	.786	.801	.799

In the last line, Table 9 also states the predictive power in terms of the proportion of observed decisions that are correctly predicted by the respective best response. Note that this prediction would only be useful out of sample, because it requires the data from the current experiment. As the models are fitted to observations, the predictive power out of sample is likely to be lower. A comparison with the predictive power of abstract selection theories in the last row of Table 5 shows that the best responses to random behavior (Laplacian strategy and Level-1) yield predictions that are equally good as the in-sample predictions from best responses to fitted models of heterogeneous behavior. This underlines the robustness of Result 1.

7. Conclusions

Table 10. Comparison of solution concepts

	GGS	Laplacian	Level-1	Noisy Laplacian	Noisy Level-1	Global game	QRE
% correct predictions	59.7%	80.5%	80.0%				
Loglikelihood	- infinity	- infinity	- infinity	- 457	- 464	- 479	- 521
Expected payoff in % of maximum	52%	94%	96%	93%	95%	92%	91%
Qualitative comparative statics	no	yes	yes	yes	yes	yes	yes
Computational complexity	Very low	Lowest	low	low	low	Very high	high

Table 10 summarizes the main results of this experiment. Given these results and the previous tests of the theory of global games in symmetric games, we can draw the following conclusions for global games as a descriptive theory for supermodular binary-action coordination games with complete information:

1. The predictive power of global games depends on the nature of the game. For symmetric games, the GGS correctly predicts most actions, yields the highest expected payoff, and also predicts the qualitative comparative statics that are observed in experiments. For asymmetric games this need not be true. The present paper provides a counter-example. Here, the best response to Laplacian beliefs about the number of agents who take either action provides the best prediction of actions, a high expected payoff for an individual player, and a correct prediction of the qualitative comparative statics. In symmetric games, the GGS is identical to the best response to Laplacian beliefs. Thus, the latter may dominate the GGS in its predictive power. Finding out whether this generally holds for binary-action supermodular games with complete information, requires further experiments on other asymmetric games.
2. In symmetric one-shot coordination games, the equilibrium of a global game with positive variance yields a good fit of the heterogeneity of observed choices and can, thus, be used as a descriptive theory. In this experiment on an asymmetric game, the predictive power of a fitted global game is outperformed by a logit response function to Laplacian beliefs.

3. In repeated symmetric coordination games, groups converge to an equilibrium that is somewhere between the GGS and the payoff-dominant equilibrium. As repeated coordination games most often convergence to the best response of choices in the first period, this result cannot be expected to hold for repeated asymmetric coordination games. As observed choices in the asymmetric one-shot games of the experiment were often below the GGS, we hypothesize that repetitions of these games may lead to equilibria that are lower than the GGS. Whether the Laplacian strategy, instead, can serve as a lower bound, is a matter of future research.

For theorists working with supermodular coordination games, these results are good news. In order to assess the comparative statics properties, they do not need to construct a global game but can get reliable predictions from the best response to Laplacian beliefs that is much easier to calculate and analyze also in algebraic form. The best response to Laplacian beliefs also provides the best point predictions and this seems to be true also for asymmetric games (at least the ones tested in the present experiment). This allows theorists to easily extend their models to asymmetric set-ups, e.g. bank-run models with banks or depositors of different size.

Depending on the goals, a theorist may require a probabilistic model that yields a prediction not only about the final aggregate outcome, i.e. whether a banking crisis occurs or not, but also yields a description of the distribution of actions, i.e. which share of depositors (depending on size) may be expected to withdraw deposits. This distribution may be important for evaluating how much liquidity needs to be provided in order to avoid a banking crisis or what the best limits are for a deposit insurance. For these questions, noisy Laplacian beliefs give the best answer.

Consider, for example, a regulator who wants to assess the contribution of an individual bank's failure to a systemic crisis or a central bank that needs to decide whether it should let a distressed bank fail, support it as lender of last resort, or inject liquidity into the market. Here, the main question is whether the failure of the distressed institution with or without liquidity provision to the market leads to a systemic crisis. Thus, the aggregate outcome is of primary importance. The precise distribution of withdrawn deposits is of secondary importance. However, miscoordination among depositors is also associated with losses to those who are on the wrong side (withdraw, although the bank survives or vice versa), even if these losses are small in comparison to the welfare losses caused by a contagious financial crisis.

Thus, the choice of an appropriate solution concept may depend on the goal. If we want to predict behavior of a single depositor or a single agent whom we want to get as a customer for a network good, a simple selection theory, like the best response to Laplacian beliefs, may be sufficient and, given the computational burden associated with other concepts, the best concept to apply. The same may be true if we want to give advice to an individual agent in a one-shot coordination game.

If we are interested in the distribution of choices, a probabilistic model accounting for the heterogeneity of choices is indispensable. Note however, that all models that can be used to describe heterogeneous behavior need a parameter steering the diversity of choices. In a global game, this is the precision of private signals. The true variance of private signals is likely too small, because we see that behavior in games with complete information (zero variance of private signals) is best fitted by a global game with a positive variance. Unfortunately, estimated variances of private signals have a dimension that limits their external validity. For example, I can hardly take the estimated standard deviation of 7 Euros that maximizes the loglikelihood of observations in the presented experiment and assume that the same standard deviation gives me a good prediction for a bank run game, where payoffs are in the magnitude of thousands of Euros. Noisy Laplacian beliefs, instead, need a parameter for rationality. This has no dimension and may have a higher external validity. Hence, one can use the rationality parameter estimated with data from past financial crises on very distinct markets or even from experiments like the one presented here, and use them to calibrate a new model.

The obvious downside of Laplacian beliefs as a selection theory is that they are only defined in binary choice games (e.g., extend or withdraw credit). When it comes to more than two decision alternatives, level-1 can still be applied, but we do not know yet how well its predictions perform in comparison to other concepts when there are more than two decision alternatives. As shown for the present experiment on binary-choice games, level-1 may provide predictions that are almost as good as Laplacian beliefs, and the computational complexity is not much higher. Noisy level-1 also yields a good fit of observed heterogeneity and requires the same dimensionless rationality parameter.

The models tested in this experiment can be ordered by their computational complexity that also matters for the decision which concept to use for analyzing an applied coordination problem. The best response to Laplacian beliefs can be calculated by head: it just requires to compare the average payoffs of the two actions, A and B, over the potential number of other agents choosing the same action. The Level-1 strategy requires calculating a binomial distribution. Their probabilistic versions plug the expected payoffs in a logit function. QRE involves solving a system of equations of logit functions, and the global game equilibrium needs to evaluate such a system of equations not only for the payoffs of the actual game, but simultaneously for all other potential payoffs in the conditional support of the state space. Surprisingly, the present results support the least complex solution concepts.

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Appendix A. Games used in the experiment and observed choices

The following tables aggregate the payoff for players of the same type in one line. So, for example, Game 1 has 4 types: Players with roles A, B and C are of the same type and, thus, have the same payoffs conditional on the total number of entrants. Note that there were 4 sessions and, thus, 4 observations for each role. The last two columns display the Laplacian and the Level-1 strategy: "0" means no entry, "1" means entry.

Game 1

	Payoffs conditional on the number of entrants												Observed	Lapla	Level-
Role	1	2	3	4	5	6	7	8	9	10	11	12	no. of entrants	cian	1
A – C	30	33	36	36	36	36	36	36	36	36	36	36	9	1	1
D – F	0	0	0	30	33	36	36	36	36	36	36	36	1	0	0
G – I	0	0	0	0	0	0	30	33	36	36	36	36	0	0	0
J – L	0	0	0	0	0	0	0	0	0	30	33	36	0	0	0
												Total	10		

Game 2

	Payoffs conditional on the number of entrants												Observed	Lapla	Level-
Role	1	2	3	4	5	6	7	8	9	10	11	12	no. of entrants	cian	1
A – C	30	33	36	39	42	45	48	51	54	57	60	63	12	1	1
D – F	0	0	0	30	33	36	39	42	45	48	51	54	4	0	1
G – I	0	0	0	0	0	0	30	33	36	39	42	45	1	0	0
J – L	0	0	0	0	0	0	0	0	0	30	33	36	0	0	0
												Total	17		

Game 3

	Payoffs conditional on the number of entrants												Observed	Lapla	Level-
Role	1	2	3	4	5	6	7	8	9	10	11	12	no. of entrants	cian	1
A – C	30	33	36	36	36	36	36	36	36	36	36	36	10	1	1
D – F	21	24	27	30	33	36	36	36	36	36	36	36	5	0	1
G – I	9	12	15	21	24	27	30	33	36	36	36	36	0	0	0
J – L	0	3	6	9	12	15	21	24	27	30	33	36	0	0	0
												Total	15		

Game 4

	Payoffs conditional on the number of entrants												Observed	Lapla	Level-
Role	1	2	3	4	5	6	7	8	9	10	11	12	no. of entrants	cian	1
A – C	30	33	36	39	42	45	48	51	54	57	60	63	11	1	1
D – F	21	24	27	30	33	36	39	42	45	48	51	54	7	1	1
G – I	9	12	15	21	24	27	30	33	36	39	42	45	2	0	0
J – L	0	3	6	9	12	15	21	24	27	30	33	36	1	0	0
												Total	21		

Game 5

Role	Payoffs conditional on the number of entrants 1	2	3	4	5	6	7	8	9	10	11	12	Observed no. of entrants	Laplacian	Level-1
A – C	33	36	39	39	39	39	39	39	39	39	39	39	12	1	1
D – F	3	3	3	33	36	39	39	39	39	39	39	39	9	0	1
G – I	3	3	3	3	3	3	33	36	39	39	39	39	2	0	0
J – L	3	3	3	3	3	3	3	3	3	33	36	39	3	0	0
												Total	26		

Game 6

Role	Payoffs conditional on the number of entrants 1	2	3	4	5	6	7	8	9	10	11	12	Observed no. of entrants	Laplacian	Level-1
A – C	33	36	39	42	45	48	51	54	57	60	63	66	12	1	1
D – F	3	3	3	33	36	39	42	45	48	51	54	57	7	1	1
G – I	3	3	3	3	3	3	33	36	39	42	45	48	1	0	0
J – L	3	3	3	3	3	3	3	3	3	33	36	39	2	0	0
												Total	22		

Game 7

Role	Payoffs conditional on the number of entrants 1	2	3	4	5	6	7	8	9	10	11	12	Observed no. of entrants	Laplacian	Level-1
A – C	33	36	39	39	39	39	39	39	39	39	39	39	12	1	1
D – F	24	27	30	33	36	39	39	39	39	39	39	39	10	1	1
G – I	12	15	18	24	27	30	33	36	39	39	39	39	5	0	0
J – L	3	6	9	12	15	18	24	27	30	33	36	39	2	0	0
												Total	29		

Game 8

Role	Payoffs conditional on the number of entrants 1	2	3	4	5	6	7	8	9	10	11	12	Observed no. of entrants	Laplacian	Level-1
A – C	33	36	39	42	45	48	51	54	57	60	63	66	11	1	1
D – F	24	27	30	33	36	39	42	45	48	51	54	57	11	1	1
G – I	12	15	18	24	27	30	33	36	39	42	45	48	6	0	0
J – L	3	6	9	12	15	18	24	27	30	33	36	39	1	0	0
												Total	29		

Game 9

Role	Payoffs conditional on the number of entrants 1	2	3	4	5	6	7	8	9	10	11	12	Observed no. of entrants	Laplacian	Level-1
A – C	30	33	36	36	36	36	36	36	36	36	36	36	11	1	1
D – F	30	30	30	30	33	36	36	36	36	36	36	36	6	0	1
G – I	30	30	30	30	30	30	30	33	36	36	36	36	3	0	0
J – L	30	30	30	30	30	30	30	30	30	30	33	36	1	0	0
												Total	21		

Game 10

Role	Payoffs conditional on the number of entrants 1	2	3	4	5	6	7	8	9	10	11	12	Observed no. of entrants	Laplacian	Level-1
A – C	30	33	36	39	42	45	48	51	54	57	60	63	12	1	1
D – F	30	30	30	30	33	36	39	42	45	48	51	54	9	1	1
G – I	30	30	30	30	30	30	30	33	36	39	42	45	7	0	0
J – L	30	30	30	30	30	30	30	30	30	30	33	36	2	0	0
												Total	30		

Game 11

Role	Payoffs conditional on the number of entrants 1	2	3	4	5	6	7	8	9	10	11	12	Observed no. of entrants	Laplacian	Level-1
A – F	29	32	35	38	41	44	44	44	44	44	44	44	23	1	1
G – L	11	11	11	11	11	11	29	32	35	38	41	44	8	0	0
												Total	31		

Game 12

Role	Payoffs conditional on the number of entrants 1	2	3	4	5	6	7	8	9	10	11	12	Observed no. of entrants	Laplacian	Level-1
A – F	24	27	30	33	36	39	39	39	39	39	39	39	17	1	1
G – L	6	9	12	15	18	21	24	27	30	33	36	39	2	0	0
												Total	19		

Game 13

Role	Payoffs conditional on the number of entrants 1	2	3	4	5	6	7	8	9	10	11	12	Observed no. of entrants	Laplacian	Level-1
A – F	24	27	30	33	36	39	42	45	48	51	54	57	17	1	1
G – L	6	6	6	6	6	6	24	27	30	33	36	39	3	0	0
												Total	20		

Game 14

Role	Payoffs conditional on the number of entrants 1	2	3	4	5	6	7	8	9	10	11	12	Observed no. of entrants	Laplacian	Level-1
A – F	21	24	27	30	33	36	39	42	45	48	51	54	16	1	1
G – L	21	24	27	30	33	36	39	42	45	48	51	54	0	0	0
												Total	16		

Game 15

Role	Payoffs conditional on the number of entrants 1	2	3	4	5	6	7	8	9	10	11	12	Observed no. of entrants	Laplacian	Level-1
A – L	8	12	16	20	24	28	32	36	40	44	48	52	27	0	0

Game 16

Role	Payoffs conditional on the number of entrants 1	2	3	4	5	6	7	8	9	10	11	12	Observed no. of entrants	Laplacian	Level-1
A – L	0	4	8	12	16	20	24	28	32	36	40	44	9	0	0

Game 17

Role	Payoffs conditional on the number of entrants 1	2	3	4	5	6	7	8	9	10	11	12	Observed no. of entrants	Laplacian	Level-1
A	35	35	35	35	35	35	35	35	35	35	35	35	4	1	1
B	33	35	35	35	35	35	35	35	35	35	35	35	4	1	1
C	31	33	35	35	35	35	35	35	35	35	35	35	4	1	1
D	29	31	33	35	35	35	35	35	35	35	35	35	3	.5	1
E	27	29	31	33	35	35	35	35	35	35	35	35	2	0	1
F	25	27	29	31	33	35	35	35	35	35	35	35	0	0	1
G	23	25	27	29	31	33	35	35	35	35	35	35	1	0	0
H	21	23	25	27	29	31	33	35	35	35	35	35	0	0	0
I	19	21	23	25	27	29	31	33	35	35	35	35	0	0	0
J	17	19	21	23	25	27	29	31	33	35	35	35	1	0	0
K	15	17	19	21	23	25	27	29	31	33	35	35	1	0	0
L	13	15	17	19	21	23	25	27	29	31	33	35	0	0	0
												Total	20		

Game 18

Role	Payoffs conditional on the number of entrants 1	2	3	4	5	6	7	8	9	10	11	12	Observed no. of entrants	Laplacian	Level-1
A	33	35	37	39	41	43	45	47	49	51	53	55	4	1	1
B	33	33	35	37	39	41	43	45	47	49	51	53	4	1	1
C	33	33	33	35	37	39	41	43	45	47	49	51	4	1	1
D	33	33	33	33	35	37	39	41	43	45	47	49	4	1	1
E	33	33	33	33	33	35	37	39	41	43	45	47	3	1	1
F	33	33	33	33	33	33	35	37	39	41	43	45	4	1	1
G	33	33	33	33	33	33	33	35	37	39	41	43	3	1	0
H	33	33	33	33	33	33	33	33	35	37	39	41	3	1	0
I	33	33	33	33	33	33	33	33	33	35	37	39	2	.5	0
J	33	33	33	33	33	33	33	33	33	33	35	37	1	0	0
K	33	33	33	33	33	33	33	33	33	33	33	35	0	0	0
L	33	33	33	33	33	33	33	33	33	33	33	33	0	0	0
												Total	32		

Game 19

Role	Payoffs conditional on the number of entrants 1	2	3	4	5	6	7	8	9	10	11	12	Observed no. of entrants	Laplacian	Level-1
A	33	35	37	39	41	43	45	47	49	51	53	55	3	1	1
B	31	33	35	37	39	41	43	45	47	49	51	53	4	1	1
C	29	31	33	35	37	39	41	43	45	47	49	51	3	1	1
D	27	29	31	33	35	37	39	41	43	45	47	49	1	1	1
E	25	27	29	31	33	35	37	39	41	43	45	47	3	1	1
F	23	25	27	29	31	33	35	37	39	41	43	45	1	.5	.5
G	21	23	25	27	29	31	33	35	37	39	41	43	1	0	0
H	19	21	23	25	27	29	31	33	35	37	39	41	0	0	0
I	17	19	21	23	25	27	29	31	33	35	37	39	0	0	0
J	15	17	19	21	23	25	27	29	31	33	35	37	0	0	0
K	13	15	17	19	21	23	25	27	29	31	33	35	0	0	0
L	11	13	15	17	19	21	23	25	27	29	31	33	0	0	0
												Total	16		

Game 20

Role	Payoffs conditional on the number of entrants 1	2	3	4	5	6	7	8	9	10	11	12	Observed no. of entrants	Laplacian	Level-1
A	49	49	49	49	49	49	49	49	49	49	49	49	4	1	1
B	45	47	47	47	47	47	47	47	47	47	47	47	4	1	1
C	41	43	45	45	45	45	45	45	45	45	45	45	4	1	1
D	37	39	41	43	43	43	43	43	43	43	43	43	4	1	1
E	33	35	37	39	41	41	41	41	41	41	41	41	4	1	1
F	29	31	33	35	37	39	39	39	39	39	39	39	4	1	1
G	25	27	29	31	33	35	37	37	37	37	37	37	3	0	1
H	21	23	25	27	29	31	33	35	35	35	35	35	2	0	0
I	17	19	21	23	25	27	29	31	33	33	33	33	0	0	0
J	13	15	17	19	21	23	25	27	29	31	31	31	0	0	0
K	9	11	13	15	17	19	21	23	25	27	29	29	0	0	0
L	5	7	9	11	13	15	17	19	21	23	25	27	0	0	0
												Total	29		

Appendix B. Estimating the global-game equilibrium

The net payoff for entry by agent i given that a total of n agents are entering is $v_i(n)$. Extend this by a state variable, such that the payoff for entry is $v_i(n)+\theta$. Assume that the state variable has an improper uniform distribution on the reals (=> dominance regions exist). Agents receive i.i.d. private signals $x_i \sim N(\theta,\sigma^2)$. A global-game equilibrium is a vector of thresholds, such that

(1) each agent enters [does not enter] if his signal exceeds [falls short of] the threshold, and
(2) an agent receiving the threshold signal is indifferent.

Players are denoted by $i \in \{1,...,N\}$, denote the type of player i by $k(i)$. In equilibrium, players of the same type have the same threshold. Denote the equilibrium threshold of type k by x^k. We have $N=12$. The number of types varies over 1, 2, 4, and 12.

In our games, we can order types such that $k(i)<k(j)$ if and only if $v_i(n) \geq v_j(n)$ for all n, with at least one strict inequality. For example, in the games with four types, players ABC belong to Type 1, players DEF are Type 2, and so on. Due to the order of types, $x^k \leq x^{k+1}$.

(1) Given that the true realization of $\theta=0$, the probability that player j enters is

$$prob(x_j \geq x^{k(j)}) = 1-\Phi\left(x^{k(j)}/\sigma\right) \qquad (1)$$

with Φ denoting the cumulative standard normal distribution. For any given value of σ, this is the probability for a subject entering that we use in the Maximum-likelihood estimation. Before we can do so, we need to find the thresholds that are associated with a particular σ. This comes from condition (2). The second step is then to find the σ that maximizes the likelihood of observations.

(2) An arbitrary agent i is indifferent at signal $x_i = x^{k(i)}$, if and only if

$$E(v_i(n)+\theta \mid x_i)=0. \qquad (2)$$

$$E(v_i(n)\mid x_i)=\int_{-\infty}^{\infty}\left[\sum_{n=1}^{N} prob(n-1\; other\ agents\ enter \mid \theta)\cdot v_i(n)\right]\cdot f(\theta \mid x_i)\,d\theta \qquad (3)$$

and $E(\theta|x_i)=x_i$, where $f(\theta \mid x_i)=\phi\left(\frac{\theta-x_i}{\sigma}\right)$ and ϕ is the non-cumulative standard normal distribution. Note that we can reformulate

$$E(v_i(n)\mid x_i)=\sum_{n=1}^{N} v_i(n)\int_{-\infty}^{\infty} prob(n-1\; other\ agents\ enter \mid \theta)\cdot f(\theta \mid x_i)\,d\theta\ . \qquad (4)$$

Denote the conditional probability that $(n-1)$ other subjects enter <u>conditional on signal x_i</u> by

$$\hat{p}^{-i}(n-1\mid x_i)=\int_{-\infty}^{\infty} prob(n-1\; other\ agents\ than\ i\ enter \mid \theta)\cdot f(\theta \mid x_i)\,d\theta. \qquad (5)$$

The tricky part is to describe this probability.

Some helpful notation: Denote the conditional probability that another agent of type k enters given state θ by

$$p_\theta^k = prob(x_j \geq x^{k(j)} \mid \theta) = 1 - \Phi\left(\frac{x^{k(j)} - \theta}{\sigma}\right), \tag{6}$$

and the conditional probability that *m* agents of *other* types than $k(i)$ enter, for a given state θ, by $\tilde{p}^{-i}(m \mid \theta)$.

12 types with 1 agent each (Games 17-20): Note that the other games are special cases with subjects of the same type having the same threshold in equilibrium. So, I do not lay out those games explicitly.

Games 17-20 are actually dominance solvable and have a unique equilibrium. The global game may yield a better description of behavior anyway. It basically accounts for strategic uncertainty in a game where deduction should eliminate this uncertainty. We know, however, from observations that people are uncertain and do not put probability 1 on others' rationality, leave alone higher-order rationality. The logit equilibrium captures this already (see also Kübler & Weizsäcker, 2004).

So, let us continue the assumption that players are uncertain about which game they are playing (that is: θ is uncertain). As an equilibrium can be described by a vector of thresholds $x^k \leq x^{k+1}$, we can continue to use the indifference condition

$$E(v_i(n) \mid x_i) = \sum_{n=1}^{N} v_i(n) \cdot \int_{-\infty}^{\infty} prob(n-1\, other\ agents\ enter \mid \theta) f(\theta \mid x_i)\, d\theta = \sum_{n=1}^{N} v_i(n) \cdot \hat{p}^{-i}(n-1 \mid x_i) = -x_i$$

and

$$prob(n-1\, other\ \ agents\ \ enter \mid \theta)$$
$$= \tilde{p}^{-i}(n-1 \mid \theta) = prob(n-1\ \ agents\ \ from\ other\ types\ than\ i\ enter \mid \theta),$$

which is the probability that $n - 1$ of the other signals are below the individual thresholds x^k, $k \neq k(i)$. For each agent (of another type), this probability is given by p_θ^k. The probability $\tilde{p}^{-i}(n-1 \mid \theta)$ is the solution to a combinatorial problem.

Any combination of $n - 1$ of other agents must be accounted for with the probability that these $n - 1$ agents enter. The latter is the product of p_θ^k for the $n - 1$ different agents k. So, $\tilde{p}^{-i}(n-1 \mid \theta)$ is the sum_(over all possible combinations of $n - 1$ of other agents) of the products of these agents' entry probabilities,

$$\tilde{p}^{-i}(n-1 \mid \theta) = \sum_{all\ combinations\ of\ n-1\ other\ agents} \left(\prod_{k \in combination} p_\theta^k \cdot \prod_{k \notin combination} (1 - p_\theta^k) \right).$$

Let us here use the letters $i, j, k,$ ... for agents and types (because each agent is one type).

$$\widetilde{p}^{-i}(0\,|\,\theta) = \prod_{k\neq i}(1-p_\theta^k), \qquad \text{and} \qquad \widetilde{p}^{-i}(1\,|\,\theta) = \sum_{k\neq i}\left(p_\theta^k \cdot \prod_{\substack{k'\neq i\\ k'\neq k}}(1-p_\theta^{k'}) \right).$$

Define the set of players excluding i, k, k', and so on by $K-\{i,k,k'\} = \{ j \,|\, j\neq i \wedge j\neq k \wedge j\neq k'\}$. Then,

$$\widetilde{p}^{-i}(2\,|\,\theta) = \sum_{k\neq i}\left(p_\theta^k \cdot \sum_{\substack{k'>k,\\ k'\neq i}}\left(p_\theta^{k'} \cdot \prod_{j\in K-\{i,k,k'\}}(1-p_\theta^j) \right)\right),$$

$$\widetilde{p}^{-i}(3\,|\,\theta) = \sum_{k\neq i}\left(p_\theta^k \cdot \sum_{\substack{k'>k,\\ k'\neq i}}\left(p_\theta^{k'} \cdot \sum_{\substack{k''>k',\\ k''\neq i}}\left(p_\theta^{k''} \cdot \prod_{j\in K-\{i,k,k',k''\}}(1-p_\theta^j) \right)\right)\right),$$

$$\widetilde{p}^{-i}(4\,|\,\theta) = \sum_{k\neq i}\left(p_\theta^k \cdot \sum_{\substack{k'>k,\\ k'\neq i}}\left(p_\theta^{k'} \cdot \sum_{\substack{k''>k',\\ k''\neq i}}\left(p_\theta^{k''} \cdot \sum_{\substack{k'''>k'',\\ k'''\neq i}}\left(p_\theta^{k'''} \cdot \prod_{j\in K-\{i,k,k',k'',k'''\}}(1-p_\theta^j) \right)\right)\right)\right),$$

$$\widetilde{p}^{-i}(5\,|\,\theta) = \sum_{k\neq i}\left(p_\theta^k \cdot \sum_{\substack{k'>k,\\ k'\neq i}}\left(p_\theta^{k'} \cdot \sum_{\substack{k''>k',\\ k''\neq i}}\left(p_\theta^{k''} \cdot \sum_{\substack{k'''>k'',\\ k'''\neq i}}\left(p_\theta^{k'''} \cdot \Big[\sum_{\substack{k''''>k''',\\ k''''\neq i}} p_\theta^{k''''} \prod_{j\in K-\{i,k,k',k'',k''',k''''\}}(1-p_\theta^j) \Big] \right)\right)\right)\right),$$

$$\widetilde{p}^{-i}(6\,|\,\theta) =$$

$$\sum_{k\neq i}\left((1-p_\theta^k) \cdot \sum_{\substack{k'>k,\\ k'\neq i}}\left((1-p_\theta^{k'}) \cdot \sum_{\substack{k''>k',\\ k''\neq i}}\left((1-p_\theta^{k''}) \cdot \sum_{\substack{k'''>k'',\\ k'''\neq i}}\left((1-p_\theta^{k'''}) \cdot \Big[\sum_{\substack{k''''>k''',\\ k''''\neq i}} (1-p_\theta^{k''''}) \prod_{j\in K-\{i,k,k',k'',k''',k''''\}} p_\theta^j \Big] \right)\right)\right)\right),$$

$$\widetilde{p}^{-i}(7\,|\,\theta) = \sum_{k\neq i}\left((1-p_\theta^k) \cdot \sum_{\substack{k'>k,\\ k'\neq i}}\left((1-p_\theta^{k'}) \cdot \sum_{\substack{k''>k',\\ k''\neq i}}\left((1-p_\theta^{k''}) \cdot \sum_{\substack{k'''>k'',\\ k'''\neq i}}\left((1-p_\theta^{k'''}) \cdot \prod_{j\in K-\{i,k,k',k'',k'''\}} p_\theta^j \right)\right)\right)\right),$$

$$\widetilde{p}^{-i}(8\,|\,\theta) = \sum_{k\neq i}\left((1-p_\theta^k) \cdot \sum_{\substack{k'>k,\\ k'\neq i}}\left((1-p_\theta^{k'}) \cdot \sum_{\substack{k''>k',\\ k''\neq i}}\left((1-p_\theta^{k''}) \cdot \prod_{j\in K-\{i,k,k',k''\}} p_\theta^j \right)\right)\right),$$

$$\widetilde{p}^{-i}(9\,|\,\theta) = \sum_{k\neq i}\left((1-p_\theta^k) \cdot \sum_{\substack{k'>k,\\ k'\neq i}}\left((1-p_\theta^{k'}) \cdot \prod_{j\in K-\{i,k,k'\}} p_\theta^j \right)\right),$$

$$\widetilde{p}^{-i}(10\,|\,\theta) = \sum_{k\neq i}\left((1-p_\theta^k) \cdot \prod_{\substack{k'\neq i\\ k'\neq k}} p_\theta^{k'} \right), \qquad \text{and} \qquad \widetilde{p}^{-i}(11\,|\,\theta) = \prod_{k\neq i} p_\theta^k .$$

Using these equations, we first calculate the vector of thresholds $(x^k)_{k=1,\dots 12}$ for each game and for a discrete grid of values for $\sigma \in \{1,2,\dots,20\}$. Then, we use Equation (1) to calculate for each σ the likelihood of entry in the respective global-game equilibrium and the likelihood for the observed number of entrants. Summing up the log-likelihoods over all games and players yields the log-likelihood function displayed in Figure 4. Because the likelihood function is rather flat, a finer grid around the maximizing integer value of σ would not substantially increase the maximum log-likelihood.

Appendix C. Instructions and Quiz

Instructions

Welcome to this experiment in the economics of decision-making. Please read these instructions carefully as they explain how you earn money from the decisions you make in today's experiment. No talking is permitted for the duration of today's session. If you have a cell phone, please turn off the ringer.

You are 12 participants in today's session and your earnings depend on your own decisions and on the decisions of the other participants.

1. Rules of the games

Today's session consists of 20 small games. In each game you have to decide between two options: "enter" or "not enter".

If you do not enter, your payoff is 34 experimental currency units (ECU), independent of what the other participants decide. If you enter, your payoff will depend on your role in this game and on the decisions of the other participants in the same game.

In each game, there are 12 roles, called A, B, C, … , L. You and each of the other 11 participants in this session will be randomly assigned to one of the 12 roles such that each role is assumed by one participant. The random role assignment will be done for each game independent of the roles or decisions in previous games, except that you will never get the same role in two consecutive games.

The payoffs for "enter" will be displayed in a table of the following format:

	number of entrants											
role	1	2	3	4	5	6	7	8	9	10	11	12
A	39	40	41	42	43	44	45	46	47	48	49	50
B	37	38	39	40	41	42	43	44	45	46	47	48
C	35	36	37	38	39	40	41	42	43	44	45	46
D	33	34	35	36	37	38	39	40	41	42	43	44
E	31	32	33	34	35	36	37	38	39	40	41	42
F	29	30	31	32	33	34	35	36	37	38	39	40
G	27	28	29	30	31	32	33	34	35	36	37	38
H	25	26	27	28	29	30	31	32	33	34	35	36
I	23	24	25	26	27	28	29	30	31	32	33	34
J	21	22	23	24	25	26	27	28	29	30	31	32
K	19	20	21	22	23	24	25	26	27	28	29	30
L	17	18	19	20	21	22	23	24	25	26	27	28

The rows display the possible payoffs for all participants. For each participant, there are 12 possible payoffs. The payoff each participant receives is determined by the total number of participants who decide to enter.

In each of the 20 games, your role will be highlighted boldface. So, for the above game, your role is "D". For example, if 6 participants (including yourself) decide to "enter" in this game, then you receive a payoff of 38 ECU. If you are the only one who decides to “enter,” you receive 33 ECU. If all 12 participants decide to “enter,” then you get 44 ECU.

If the participant with role G decides to “enter” and there are 6 participants (including participant G) who enter in this game, then participant G receives 32 ECU.

Recall that the payoff from not entering is always 34 ECU, independent of what the other participants decide.

In order to make your decision, click on one of the two circles indicating “enter” or “not enter” and confirm your decision by clicking the red OK-button. You may change your decision until you click the OK-button.

If you have not made your decision within 3 minutes, there will appear a line asking you to decide and confirm your decision.

Only when all participants made a decision and confirmed it, the session will continue with a new game.

2. End of a game

Once all 12 participants have made and confirmed their decisions, the game ends. At the completion of the game, you will not be informed of the outcome of the game. Instead, you will receive this information for all games only after the completion of all 20 games.

Once you confirm your decision by clicking the OK-button, you will see a screen asking you to wait for the next game until the other participants made their decision. If you were the last player to decide in this game, the next game will start immediately after you click the OK-button.

Once a game ends, the next game starts and all participants will be assigned new roles and see a new screen containing a payoff table with their own roles highlighted.

3. Information phase

Once all 20 games have been completed, you will receive a list containing the results of the games. This list will show the game number (1-20) and display for each game: your role (A-L), your decision (“enter” or “not enter”), the number of participants who chose to “enter” in this game, and your own payoff for this game.

The screen will also show you the sum of your payoffs over all 20 games.

Example:

Game no.	your role	your decision	number of participants who entered	your payoff (ECU)
1	D	enter	6	38
2	B	not enter	9	34
3	H	enter	3	26

…

Note that you will never learn which roles the other participants had in the various games, nor how they decided in any of the games. You will be informed only of the total number of entrants.

4. Payment

Once you complete the receipt, you will be paid. For every 40 ECU that you earned in the experiment, you will receive 1 Euro. The amount will not be rounded up or down.

5. Questions

It is important that you understand the instructions. If you have a question about any aspect of these instructions, please raise your hand and we will come to you and answer your question in private.

Never ask questions aloud!

6. Quiz

To make sure that you understood the instructions, we ask that you answer the following quiz questions in the spaces provided. The numbers in these quiz questions are merely illustrative; the actual numbers in the experiment may be quite different. In answering these questions, please feel free to consult the instructions. After all participants have completed this quiz, the first game will start.

Please look at the following payoff table.

…

a) What is your own role in this game?
b) What is you payoff if you enter and 9 participants (including yourself) enter in this game?
c) What is the payoff to player B if he enters and there are 5 players in total who enter in this game?
d) What is the payoff to the player with role A, if he does not enter?
e) What is your payoff if you do not enter?