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Abstract

We introduce a model of the banking sector that formally incorporates a buffer function of capital. Heterogeneous banks choose their portfolio risk, bank size, and capital holdings. Banks voluntarily hold equity when the buffer effect against the risk of default outweighs the cost advantages of debt financing. In this setting, banks with lower monitoring costs are larger, choose riskier portfolios, and have less equity. Moreover, binding capital requirements or levies on bank borrowing are shown to make higher-risk portfolios more attractive. Accounting for banks' interior capital choices can thus explain why higher capital ratios incentivize banks to undertake riskier projects.

Keywords: voluntary equity, capital requirements, bank heterogeneity

JEL Classification: G28, G38, H32

1 Introduction

Around the world, a core response to the financial crisis of 2007-2009 has been to increase banks' capital requirements. The international Basel III capital standards have raised the ratio of common equity to risk-weighted assets, and have added further capital buffers for systemically important banks. Larger equity buffers reduce the risk of individual bank failures, and with it the negative contagion effects on other banks in the financial system. This is even more important as banks have incentives for excessive risk-taking because of the limited liability they face in the presence of government-run deposit insurance schemes and, in some cases, implicit bailout guarantees. By increasing banks' 'skin in the game', higher equity holdings aim to reduce moral hazard incentives, and lead banks to pursue more prudent portfolio strategies.

Empirically, the evidence linking higher capital-to-asset ratios with improved financial stability is mixed, however.[1] According to studies which focus explicitly on the portfolio risk of banks, policy measures leading to higher capital ratios will also induce more risk-taking on the banks' asset side (loans and securities). For example, the capital injection to troubled banks offered through the TARP programme in the United States led banks to adopt riskier portfolios while simultaneously increasing their capital ratio (Black and Hazelwood, 2013; Duchin and Sosyura, 2014). Similar results are obtained by Devereux et al. (2019) who study the effects of levies on bank borrowing imposed by 14 European countries. While these levies were found to increase banks' reliance on equity funding, they simultaneously induce banks to increase the riskiness of their assets, thus keeping total risk-taking by banks virtually unchanged. Overall, therefore, there is considerable evidence that increases in banks' capital ratios are associated with riskier lending and investment strategies.

In this paper we offer a new argument for why mandated higher capital requirements can incentivize banks to undertake more risky projects, thus counteracting the desired effect of higher bank capital on financial stability. Our argument is based on the observation that many banks voluntarily hold equity capital, which acts as a buffer against

[1]While some studies find a positive effect of higher capital requirements on bank stability (Laeven and Levine, 2009) and on banks' performance in the financial crisis (Beltratti and Stulz, 2012), others have found a non-monotonous response to successively stricter capital requirements (Calem and Rob, 1999; Dias, 2021). In a recent study using long-run data from banks in 17 countries, Jordà et al. (2021) find no evidence that higher capital ratios reduce the risk of banking crises.

the possibility of default and its associated costs. There is substantial evidence that banks value equity, and often hold capital well above the required regulatory minimum. For example, Flannery and Rangan (2008) document a massive build up of equity in US banks during the 1990s, leading to capital holdings which on average exceeded the required floor by as much as 75%. Related evidence is found for Spain (Ayuso et al., 2004), and Norway (Lindquist, 2004). Finally, Gropp and Heider (2010) report large differences in the capital ratios of large U.S. and European banks, and high levels of discretionary capital in parts of the banking sector.

To study voluntary equity holdings of banks, we explicitly model the buffer function of capital against the possibility of default. This buffer function is central to the motivation of capital regulation, but it is rarely incorporated in the formal modelling of banks' incentives. Equity reduces the likelihood of bank default, and banks voluntarily hold equity when they fear the default costs associated with this event. In such a setting, when banks simultaneously choose project risk and equity capital, additional regulatory capital requirements will prompt them to re-evaluate their risk-taking decisions. Specifically, we show that the banks' responses to binding capital constraints are crucially linked to their choices regarding voluntary capital. Banks which are averse to equity holdings will choose less risky projects in response to higher capital requirements. However, banks which voluntarily hold equity capital will unambiguously move towards riskier asset portfolios. In the latter type of situation, mandated capital requirements may make banks less safe. Overall, the effects of higher capital requirements on banks' risk choices can be evaluated on the basis of empirically observable equity choices of banks.

We obtain these findings in a setting where heterogeneous banks differ exogenously in their monitoring costs, and hence in their ability to undertake risky projects. Banks make three choices. They choose the size of their operations, with more able banks being larger in equilibrium. Banks also select between a high-risk, high-return portfolio and a low-risk, low-return portfolio. Finally, banks endogenously choose the ratio of equity financing versus debt (deposit) financing. The rationale behind voluntary capital holdings is the creation of capital buffers, which reduce the probability of default and the costs of possible bankruptcy for the institution and its managers. This loss may be interpreted as the reputational and financial cost to the bank's CEO. On the other hand, debt financing has its own advantages as the owed amount is not fully being repaid in the event of bank default.[2] As better and larger banks generally have a lower risk of failure, they choose lower capital buffers for any given choice of project risk. At the same time, larger banks are more likely to choose the risky project, which is in turn associated with a larger capital buffer in the bank's optimum. These predictions are well in line with empirical evidence on the relationships between bank size and risk-taking (Bhagat et al., 2015), and between bank size and equity holdings (Rime, 2001; Ayuso et al., 2004; Lindquist, 2004).

Our central insight is to show that when banks voluntarily hold equity, tighter capital regulation will lead more banks to choose risky projects in equilibrium. Intuitively, a bank values equity only if the buffer effect is sufficiently strong to outweigh both the higher cost of equity vis-à-vis debt and the moral hazard effect that results from government guarantees. In such a scenario, any additional capital requirement hurts the bank less when it pursues a high-risk project, as high-risk projects are associated with a higher desired capital buffer in the first place. In contrast, if the moral hazard effect dominates and banks do not voluntarily hold equity capital, then the primary effect of binding capital requirements is to increase banks' liability in case of failure, which in turn reduces risk-taking. Therefore, the conventional view that higher capital ratios make banks safer, can be supported only when banks have no private interest in holding equity, which is often contradicted by empirical evidence.

The paper also studies the interaction of voluntary equity holdings with two further policy instruments. First, and perhaps counterintuitively, raising the cost of bank default (for example, through the withdrawal of government guarantees), may prompt a portfolio switch towards the more risky project. However, in such a case, the associated increase in voluntary equity holdings nevertheless makes bank failure less likely. Secondly, we study the effects of increases in the price of bank debt, caused by a tax on deposits. In this last analysis we also extend our base model of a binary choice of project risk to a continuous risk-taking decision of banks. With an interior equity choice of banks, the levy on deposits increases the voluntary capital holdings, but it simultaneously makes banks' portfolios more risky - results that correspond to the empirical evidence in Devereux et al. (2019). Hence, our main motive that higher capital ratios increase project risk reappears in this setting.

Our paper contributes to the long-standing literature on the relation between capi-

[2] This is known as the moral hazard effect of debt financing. In addition, and in presence of deposit insurance, the price of equity may exceed the deposit interest paid by banks.

tal regulation and banks' risk-taking choices.[3] This literature often studies portfolio risk in models where moral hazard arises from some form of government guarantees. Higher capital requirements usually strengthen banking sector stability by reducing banks' moral hazard (e.g. Acharya, 2003; Dell'Ariccia and Marquez, 2006).[4] In dynamic versions of this model, a counteracting effect arises because regulation lowers future profits. The associated fall in the banks' charter value implicitly reduces the costs of defaults, and makes risk taking more attractive (Keeley, 1990; Hellman et al., 2000; Repullo, 2004).[5] In our model, the risk-increasing effect of capital mandates arises instead from the banks' interior choices of voluntary equity, and the argument does not involve changes in bank default costs as a response to higher capital requirements.

Only a few papers in the theoretical literature incorporate voluntary equity choices of banks. Important contributions are Flannery (1994), Diamond and Rajan (2000) and Allen et al. (2011).[6] Diamond and Rajan (2000) derive equilibrium bank capital in a two-sided moral hazard model where banks trade off liquidity and credit creation against the risk of a bank run. In Allen et al. (2011) banks choose positive levels of capital as a commitment device to better monitor their borrowers, which in turn allows them to charge a higher loan rate. These models, however, either do not analyze capital regulation at all, or capital regulation has no effect on banks' choices (as in Allen et al., 2011). In our analysis, voluntary equity holdings have the simple objective to buffer against the costs of default, and capital requirements are binding for at least a subset of banks. In this setting we show that there is a basic complementarity between voluntary equity holdings and more risky project choices.

[3]The early literature on bank capital and risk-taking is based on portfolio choice models; see e.g. Koen and Santomero (1980), Rochet (1992), and VanHoose (2007) for a survey. Modern macroeconomic approaches quantify the effects of capital requirements in dynamic general equilibrium models that focus on the trade-off between higher banking sector stability and reduced bank lending (Van den Heuvel, 2008; Begenau, 2020).

[4]Morrison and White (2005) qualify this result in a setting where both moral hazard and adverse selection are present. They show that higher capital requirements reduce moral hazard only when the regulator has a strong screening reputation, but not in the case of 'weak' regulators.

[5]Relatedly, Hakenes and Schnabel (2011a) have shown that capital requirements can also increase risk-taking in the *firms* monitored by banks.

[6]Endogenous capital buffers have also been studied in dynamic models of capital regulation when the issuance of new capital is costly. Equity holdings above the required minimum then serve as precautionary buffers against unexpected increases in capital requirements (Repullo and Suarez, 2013), sudden inflows of deposits (Bolton et al., 2020), or to absorb losses (De Nicoló et al., 2021).

Finally, we contribute to the small theoretical literature incorporating bank heterogeneity (e.g. Kopecky and VanHoose, 2006; Haufler and Maier, 2019). In these models a bank's riskiness is typically determined by the level of monitoring intensity, and higher-ability banks are those with lower monitoring costs. Hence, better banks are mechanically characterized by a lower risk profile, contrary to empirical evidence (Bhagat et al., 2015). In our model, in contrast, banks face a project choice that is independent of their monitoring ability. In equilibrium, higher ability banks then indeed choose riskier projects, as compared to their lower-ability counterparts.

This paper proceeds as follows. Section 2 describes the setup of our model. Section 3 carries out the analysis of banks' choices. Section 4 analyzes the effects of capital regulation on the banking equilibrium. Section 5 studies bailouts and bank levies as further policy instruments that affect banks' choices. Section 6 concludes.

2 The model setup

We envision a banking sector which consists of heterogenous banks, distinguished by an exogenous quality, or ability, level θ.[7] Banks invest into 'projects', where the number of projects constitutes a bank's size. For concreteness, a bank's type determines the likelihood of successfully operating its projects. Each bank has access to two types of projects: a high-risk, high-return project H and a low-risk, low-return project L. Banks of higher ability level feature higher success rates whatever project type they choose. Banks and their managers are risk-neutral, being concerned only with the expected profit of their projects. The banking industry operates in a small open economy that takes world prices as given.

An important feature of our model is that banks have an incentive to hold equity capital in order to reduce the likelihood of bankruptcy and its associated cost. To incorporate this motive, we assume that if a project 'fails', it still yields a stochastic return, but this return may be so small that the bank becomes insolvent and has to declare default. Whether default actually occurs in case of a project failure, depends not only on the realization of returns, but also on the bank's capital structure. Hence, a larger equity ratio serves as a safety buffer to avoid bankruptcy. If the returns on a failed project

[7]Bank quality derives from (the inverse of) some underlying cost component, such as the cost of monitoring borrowers. Lower-cost banks thus have a higher ability θ to carry out risky projects.

do not allow a bank to repay its outstanding debt, it incurs a *per-project* default cost D. Total default costs are therefore proportional to bank size, but default costs do not vary in the profitability of each project.

We can interpret the default costs D from the perspective of the bank's management, as the expected reputational and financial losses which are inflicted on the bank's managers in the event of bankruptcy. Recent evidence suggests that roughly half of all CEO turnovers are induced by bad firm performance (Jenter and Lewellen, 2021). More specifically, Eckbo et al. (2016) estimate that two thirds of corporate CEOs whose firms go bankrupt leave the executive labor market, and they face total losses with a median present value of USD 7 million by the time of retirement. These figures suggest that CEOs face substantial expected income losses after a default, which may provide them with strong incentives to hedge against bankruptcy risk.[8]

Given this basic framework, a bank of type θ is assumed to maximize its aggregate profits Π_i, choosing a project $i \in \{H, L\}$, its equity ratio k, and the number of projects Q:

$$\Pi(\theta, i, k, Q) = \pi^P(\theta, i, k)\, Q - C(Q). \tag{1}$$

In (1), the variable $\pi^P(\cdot)$ denotes the bank's per-project operating profits, which is explained below. All projects are identical and their returns are perfectly correlated. Therefore, total profits are obtained by scaling the per-project profits with the number of projects Q, which we interpret as the bank's size.[9] The costs $C(Q)$ are the bank's total operating and administrative costs. These costs are increasing and convex in the number of projects Q, i.e., $C'(Q) > 0$, $C''(Q) \geq 0$, and we assume the Inada conditions to hold.

The bank's (expected) per-project operating profit $\pi^P(\cdot)$ first depends on the bank's innate type or 'ability' θ. We consider a banking industry in which bank types are independently distributed according to some continuous density function $f(\theta)$, on the interval $\theta \in [\underline{\theta}, \bar{\theta}]$ with $\bar{\theta} > \underline{\theta}$. Per-project profits also depend on two types of decisions

[8] Alternatively, and viewing banks as being run by their equity holders rather than management, the per-project loss D is a simple measure for the bank's 'charter value' that is lost in case of default. As we show below bank size, and hence the total loss in charter value, is positively correlated with the bank's ability level θ. In Appendix B, we endogenize the per-project default cost D and discuss the conditions under which the results from our benchmark analysis carry over to this extended setting.

[9] In principle, banks could invest in a portfolio of projects with different risk-return profiles. However, this will in equilibrium not be the case in our simple model.

taken by each bank.

First, a bank can choose its capital structure. Specifically, and normalizing the investment costs per project to unity, $k \in [0,1]$ determines the bank's equity ratio. For bank size Q, kQ is thus the total amount of its equity. The remaining share of the bank's financing, $1-k$, is covered by bank debt, which takes the form of savings deposits. In line with actual practice in virtually all OECD countries, we assume that savings deposits are insured by the government.[10] For analytical simplicity, we further assume that the coverage of deposit insurance is complete. This implies that depositors face no risk and the deposit rate d equals the exogenously given world interest rate. The underlying deposit guarantee gives rise to a *moral hazard* effect, incentivizing banks to rely on deposit finance to maximize the value of the government's insurance (see Demirgüç-Kunt and Detriagiache, 2002, for empirical evidence). The bank's cost of equity is instead $\rho > d$, as bank owners face a loss of their equity if the banks' projects fail. For simplicity, we treat ρ as exogenous.[11] Therefore, an increase in the equity ratio k will always increase the bank's cost of financing a given project. However, it will also increase the capital buffer and therefore reduce the expected cost of default, as we show below.

Secondly, each bank can also choose the riskiness of its projects. For simplicity, our baseline model assumes that each bank faces a discrete choice between a high-risk, high return project $i = H$, or a low-return, low-risk project L.[12] For each project $i \in \{H, L\}$ there are two states of nature $j \in \{S, F\}$ and hence two possible types of monetary returns: a successful outcome R_i^S, and a 'failed' outcome $R^F (< R_i^S)$. All project returns are exogenous, being determined in the large world market. Project H generates a larger expected per-project return in case of success; hence, $R_H^S > R_L^S$. In

[10]See Barth et al. (2006) for an overview of deposit insurance schemes around the world. The main reason for deposit insurance is that it prevents bank runs and thereby stabilizes the banking system (Diamond and Dybvig, 1983).

[11]In a more general setting, the (unit) cost of equity can be expected to fall when capital requirements are increased, and hence the bank's risk would be divided among more equity owners. In such a case, the effects of higher capital requirements on a bank's output and monitoring would then be partially offset by a simultaneous fall in ρ. However, empirical studies show that a *full* offset does not occur, so that stricter capital requirements continue to increase the banks' costs of capital even when ρ is endogenous (Baker and Wurgler, 2015). Therefore, fixing the cost of equity at an exogenous rate above the cost of savings deposits is a standard simplification in the literature (e.g. Dell' Ariccia and Marquez, 2006; Allen et al., 2011).

[12]In Section 5.2 we extend this setting to a continuous choice of project risk.

contrast, failed projects of both the H and L types generate identical stochastic returns. The common 'failed' return R^F is drawn from a continuous distribution $g(R^F)$ defined on the $]0, R^F_{max}]$ interval, where the return generated by a 'failed' project falls below its investment cost with positive or even full probability. This definition of project returns allows for a simple representation of the probability of default.

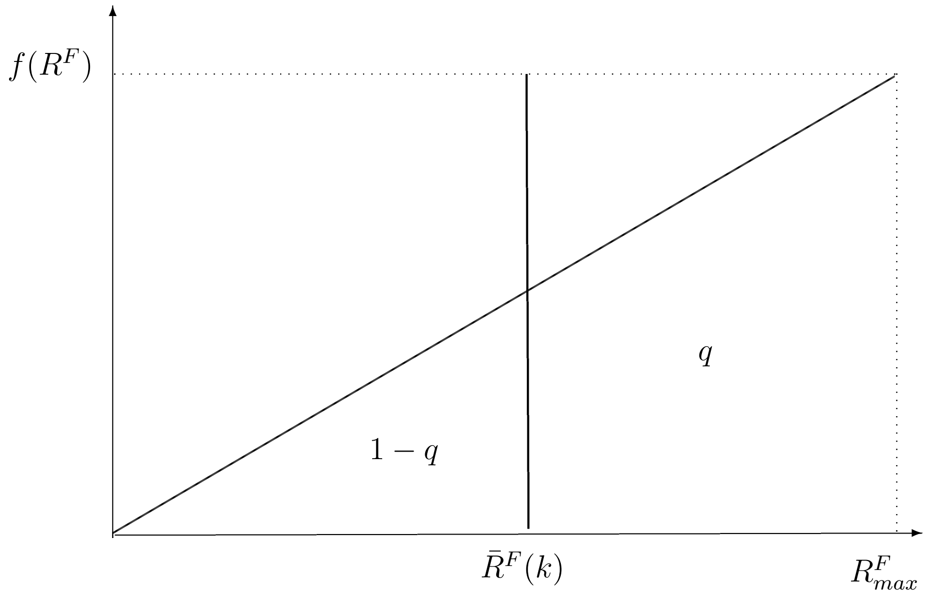
For project i, these outcomes are realized with probabilities $p_i(\theta)$ for successful returns, and with probabilities $1 - p_i(\theta)$ in case of failure. A trade-off for each bank's project choice results from the fact that project L has the larger success probability for any given type θ; hence $p_L(\theta) > p_H(\theta)$. Since a larger θ represents a more productive bank, probabilities $p_i(\theta)$ are increasing in θ for each project $i = H, L$. Finally, we let $p'_H(\theta) \geq p'_L(\theta)$ so that a change in ability θ changes the success probability of the H project by at least as much as it changes the probability of the L project. A special case consistent with this assumption is that bank quality matters only for the high-risk project, i.e. $p'_H(\theta) > 0$ and $p'_L(\theta) = 0$. Hence, in case of project success, an increase in θ yields a stronger boost in gross returns, when the bank chooses an H project.

For each project $i \in \{H, L\}$, the net return in state $j \in \{S, F\}$ is given by the gross return R^j_i minus the repayments to depositors and equity holders. This net return thus equals $R^j_i - (1-k)d - k\rho$, provided that the bank is able to pay its debtholders. In the successful state, the return R^S_i is always sufficient to repay the bank's debt, for both projects H and L. If the bank's project fails, though, the bank may be unable to repay its total debt to depositors, given by $(1-k)dQ$. In this case, insolvency occurs and the bank defaults. The government then steps in to pay out the bank's debtors, but the bank suffers a fixed per-project loss of $D \geq 0$, which we interpret as the reputational damage to its top management. Note that D is a *per-project* loss. A bank's total default costs are given by DQ, and hence are rising in proportion to the bank's size.

When a project fails ($j = F$), whether the bank actually defaults will depend on the realization of the return R^F in the bad state, as well as on the bank's equity holdings of k per project. Specifically, a bank becomes insolvent when the realized return R^F is insufficient to repay the debtholders, i.e., $R^F < \bar{R}^F \equiv (1-k)d$. The threshold return $\bar{R}^F$ that is required to avoid default is thus decreasing in the bank's capital ratio k. This reflects the buffer function of equity capital in our model: a higher k narrows the set of stochastic returns R^F, for which the bank becomes insolvent.

Let $q(k) \in [0, 1]$, with $q'(k) > 0$, denote the probability that the bank can pay out its debtholders and therefore 'survives', even under the bad project outcome $j = F$,

Figure 1: Distribution of returns R^F and the bank's default probability



whereas $1-q$ is the probability of default. Figure 1 illustrates the bank's probability of default $1-q$, conditional on the outcome $j=F$, for a specific distribution function $g(R^F) = aR^F$, with $R^F \in]0, R^F_{max}]$. In Figure 1, a higher equity level k will shift the threshold value $\bar{R}^F$ to the left and thus increase the area q, which depicts the bank's probability to avoid default even if the project fails.

We are now ready to define expected per-project profits $\pi_i^P(\theta)$. Denoting by $\hat{R}^F(k)$ the expected return of the bank for $j=F$, conditional on avoiding default, these are:

$$\begin{aligned} \pi_i^P(\cdot) &= p_i(\theta)[R_i^S - (1-k)d - \rho k] \\ &+ [1-p_i(\theta)]\{q(k)[\hat{R}^F(k) - (1-k)d - \rho k] - [1-q(k)](\rho k + D)\}. \end{aligned}$$

The first line in this expression gives the expected return when the project is successful ($j=S$), whereas the second line is the expected return when the project fails ($j=F$). In the second line, the first term in the curly bracket is the expected return when the bank is able to avoid default, whereas the second term gives the loss in case of default. Notice that in case of insolvency, the bank uses any positive gross return R^F to repay its debt obligations. Hence its own return, net of the repayment of debt, is zero regardless of the realization of gross returns R^F.[13] From the bank's point of view, repayments to depositors therefore matter only as long as the bank remains solvent.

[13]In contrast, depositors benefit from a larger R^F in case of the bank's insolvency, because the bank better meets its obligations to them.

In contrast, the bank's equity holders are residual claimants and the cost of equity ρk can be seen as the opportunity cost of equity capital. These costs arise regardless of the project outcomes, and regardless of whether default occurs or not. They matter, however, when equity capital is increased. Rewriting π_i^P therefore yields

$$\pi_i^P(\cdot) = p_i(\theta)[R_i^S - (1-k)d] + [1 - p_i(\theta)]Y^F - \rho k, \tag{2}$$

where Y^F is the expected return under project failure:

$$Y^F(k) \equiv q(k)[\hat{R}^F(k) - (1-k)d] - [1 - q(k)]D, \tag{3}$$

which is non-negative when default costs D are negligible, but negative when default costs are high. Substituting the per-project profits π^P from (2) into the aggregate profit equation (1) completes the bank's objective function.[14]

Our model is used to study minimum capital standards $\underline{k}$ as imposed by the government. This regulation is motivated by the government's deposit insurance scheme, which gives rise to the *moral hazard effect* effect discussed above. Whether the regulated capital standard is binding or not for any particular bank will depend on whether the bank's own equity choice k^* is above or below the minimum capital ratio $\underline{k}$. Our model can thus be summarized in the following stages:[15]

Stage 0: Banking authorities impose a regulatory framework, in the form of capital requirements $\underline{k}$.

Stage 1: Each bank θ chooses its equity ratio $k \in [0, 1]$. With k^* being the bank's profit maximizing choice, the implemented equity is then $k = max\{\underline{k}, k^*\}$.

Stage 2: Each bank chooses its project type $i \in \{H, L\}$ and the number of projects Q, as a function of its type θ and the equilibrium capital ratio k^*.

[14] Interpreting D as managerial default costs, the bank is run by its managers who are being compensated on the basis of firm profits. The objective (2) then assumes managerial compensation to be linear in profits with a share parameter of unity. Notice that for managerial profit shares $\alpha < 1$, the bank's objective would be $\alpha\pi_i^P(\cdot)$ with true default costs $\hat{D}$ being scaled up to $D = \hat{D}/\alpha$. As the scale parameter α does not affect any of our subsequent arguments, we let $\alpha = 1$ for simplicity. Of course, no such agency considerations arise when the bank is run by its equity holders, and D is interpreted as the bank's charter value.

[15] With no change in results, we could alternatively assume that banks take the entirety of their decisions at the same time. Presenting decisions in sequential form has the advantage that the analysis of stage 2 can be carried out irrespective of whether the capital ratio k is determined by the regulator (in stage 0), or by the bank itself (in stage 1).

Stage 3: For each project, gross returns $R_i^j \in \{R_i^S, R^F\}$ with $i = H, L$ and $j = S, F$ materialize, where the index j indicates project success ($j = S$) or project failure ($j = F$). When $j = S$, the bank repays its debt holders at the rate $d > 1$. When $j = F$, the bank stays solvent with the conditional probability $q(k)$, and still repays its debtholders. With the conditional probability $(1-q)$ the bank defaults and incurs default costs of $D > 0$ per project.

In the following we solve the model by backward induction.

3 Banks' choices

3.1 Bank size and project choice

We start in stage 2 by considering a bank of type θ whose equity ratio $k \leq 1$ has already been selected in stage 1. This bank chooses its project $i = H, L$ and its size Q to maximize profits $\Pi(i, Q; \theta, k)$. Specifically, the bank will choose project i rather than project $j \neq i$ if $\pi_i^P \geq \pi_j^P$, and its optimal size Q^* maximizes

$$\Pi(\theta, k, Q) = \max\{\pi_H^P, \pi_L^P\}\, Q - C(Q), \tag{4}$$

Differentiating (4) with respect to Q, the bank's optimal size is implicitly determined by $C'(Q^*) = \pi^P$, where $\pi^P = \max\{\pi_H^P, \pi_L^P\}$, as described in (2). Since $C''(Q) > 0$, optimal bank size $Q^*(\pi^P)$ increases in per-project profits π^P, implying that more profitable banks are also larger.[16]

For our analysis of project choice, we define $\Delta\pi^P \equiv \pi_H^P(\theta) - \pi_L^P(\theta)$ as the difference in the expected returns to a high-risk project H, as compared to a low-risk project L. Substituting from (2) gives

$$\Delta\pi^P = p_H R_H^S - p_L R_L^S - (p_H - p_L)[(1-k)d + Y^F(k)], \tag{5}$$

where $Y^F(k)$ is given in (3). To see how project choice varies with the bank type, we differentiate (5) with respect to θ and expand to get

$$\frac{d\Delta\pi^P}{d\theta} = (R_H^S - R_L^S)\,\frac{dp_H}{d\theta} + \left[\frac{dp_H}{d\theta} - \frac{dp_L}{d\theta}\right]\,[R_L^S - (1-k)d - Y^F] > 0. \tag{6}$$

[16] This corresponds to the empirical evidence. See Buch et al. (2011) for the case of German banks.

This expression is unambiguously positive because $R_H^S > R_L^S$ and $dp_H/d\theta \geq dp_L/d\theta > 0$. The last bracket on the RHS gives the difference between the successful and the failed return for the low-risk project, which must always be positive (even if $Y^F > 0$). Hence, high-productivity banks tend to choose the risky project in our setting. This is in line with the empirical observation that large banks often take on more complex risk exposures (Bhagat et al., 2015).

To allow for a non-trivial sorting of banks into different project types, we introduce:

Assumption (S): (Sorting)

$$\Delta\pi^P(\underline{\theta}, k) < 0, \qquad \Delta\pi^P(\bar{\theta}, k) > 0 \quad \forall\, k \in [0, 1].$$

For any given capital ratio k which is equal across all banks, Assumption (S) ensures the lowest-ability bank to have higher expected profits under the low-risk strategy, while the highest-ability bank will prefer the high-risk strategy. From the continuity of $\Delta\pi^P$ in θ it then follows that there must be some interior threshold type $\hat{\theta} \in (\underline{\theta}, \bar{\theta})$ which is just indifferent between the two projects. Specifically, all banks with ability types below this threshold will choose project L, whereas banks with a θ above the threshold will choose project H. We summarize our results in:

Proposition 1. (*i*) *For a given and common equity ratio k, all banks of type $\theta \leq \hat{\theta}$ choose the low-risk project L, whereas banks with ability $\theta > \hat{\theta}$ choose the high-risk project H.* (*ii*) *Optimal bank size increases in ability θ, and H banks are larger than L banks.*

Higher-ability banks are thus larger, choose high-risk projects, and obtain higher expected profits – both in per-project and in absolute terms – than lower-ability banks. On the other hand, though, the associated default probabilities do not necessarily increase in bank size, as they are shaped by two countervailing forces. While the more risky portfolio choices of higher-type banks raises the likelihood of insolvency, their superior skills make a failure outcome less likely.

3.2 Choice of equity capital

A core feature of our model is that we allow each bank of type θ to choose its optimal equity ratio $k^*(\theta)$. This decision is made in stage 1. Specifically, we first derive the necessary and sufficient conditions governing banks' capital choice and then analyze how the optimal value of k^* depends on the bank type θ and on banks' project choices $i = H, L$.

For each project type, the optimal equity choice k_i^* maximizes π_i^P in (2). Appendix 1 computes the effect of a marginal effect of a change in k on per-project profits as

$$\frac{d\pi_i^P(\cdot)}{dk} = \{\underbrace{-\rho + d}_{(1)(-)} \underbrace{-(1-p_i)(1-q)d}_{(2)(-)} + \underbrace{(1-p_i)\frac{dq}{dk}D}_{(3)(+)}. \tag{7}$$

Condition (7) introduces three effects which are of central importance for our ensuing analysis. The first term in (7) represents a *financing cost effect*. When equity is more costly than debt, an increase in the equity ratio increases the bank's nominal cost of financing a project. The second term is also negative, as a higher k increases the total costs to banks in case the project fails (with probability $1 - p_i$). This term represents a *moral hazard effect*, as the bank's gain from a partial non-repayment of debt in case of default, shrinks when its equity ratio increases. The third term in (7) is positive, however, and it represents the *buffer effect* of increased equity financing. A larger equity ratio reduces the likelihood of default and therefore helps to avoid the default cost D. This buffer effect becomes more important when the bank's private default costs D increase, or when a higher k substantially increases the chances of the bank's survival under project failure (i.e. when dq/dk is large).

To see how these effects shape a bank's optimal capital choice, notice first that $k_i^* < 1$ so that no bank will endorse full self financing. This is because the bank survives with certainty when k^* approaches unity, and hence the buffer effect of equity financing ceases to exist.[17] Accordingly, whenever the bank chooses a positive equity ratio, this ratio is defined by the first order condition for an interior optimum and (7) holds with equality at $k_i^* > 0$. We assume that $\pi_i^P(\cdot)$ is concave in k:

$$\frac{d^2\pi_i^P(\cdot)}{dk^2} = (1-p_i)\left(\frac{dq}{dk}d + \frac{d^2q}{dk^2}D\right) < 0. \tag{8}$$

This second-order condition requires d^2q/dk^2 to be negative, that is, it requires a marginal increase in k to increase the survival rate q at a decreasing rate. This condition is satisfied for given k if, at the threshold return $\bar{R}^F = (1-k)d$, the density function $f(R^F)$ has positive slope (as is true in Figure 1). When condition (8) is met, $k_i^* > 0$ is uniquely defined by the first order condition $d\pi_i^P/dk = 0$.

[17] Since $R^F \geq 0$, the condition for a bank's default $R^F < (1-k)d$ can never hold for $k^* = 1$. Hence $q(1) = 1$ and $dq/dk = 0$ at this point.

Importantly, however, concavity of the objective function is no guarantee for voluntary equity holdings, because a bank chooses $k_i^* = 0$ whenever the derivative (7) is negative at $k = 0$. Notice in particular that for a bank to benefit from a positive amount of equity, the condition

$$-[1-q(k)]d + \frac{dq(k)}{dk} D \bigg|_{k=0} > 0. \tag{9a}$$

needs to be satisfied. Hence, evaluated at $k = 0$, the *buffer effect* must outweigh the *moral hazard effect*. In words, the condition requires that for low initial equity levels, the boost in survival probability and the resulting avoidance of bankruptcy costs provided by extra equity, outweighs the negative effect of having to serve its debt holders more often. This condition is required regardless of bank type, and regardless of the bank's portfolio choice. Moreover, note from eq. (7) that (9a) is a sufficient condition for positive voluntary equity holdings only if $\rho = d$, and hence if the mechanical *financing cost effect* is absent. Reversing this argument, if firms voluntarily hold positive levels of equity and $k^* > 0$, then the sum of the two effects in (9a) must be positive at k^* and, from the concavity assumption (8), also for any $k < k^*$. Hence we can write

$$-[1-q(k)]d + \frac{dq(k)}{dk} D \bigg|_{k \le k^*} > 0 \quad \text{if} \quad k^* > 0. \tag{9b}$$

As we will see below, the comparative static results of our model critically depend on whether voluntary equity holdings are positive.

A bank's choice to hold equity also depends on its type θ and its project choice $i = H, L$. We first examine how optimal equity choices evolve in bank type θ, for a given project i. Differentiating (7) with respect to θ gives (using the envelope theorem):

$$\frac{d^2 \pi_i^P(\cdot)}{dk\, d\theta} = \frac{dp_i}{d\theta} \left\{ (1-q)d - \frac{dq}{dk}\, D \right\}. \tag{10}$$

Eq. (10) entails the two counteracting effects we encountered in our previous discussion. This time, however, the *moral hazard effect* and the *buffer effect* are evaluated for different types of banks. On the one hand, high-ability banks have a lower risk of default and therefore suffer less from the higher likelihood of debt repayment associated with an increase in their equity ratio. On the other hand, the benefits of a larger equity buffer accrue primarily to low-ability banks, as those have a higher probability of failure for any project they choose. From (9b) we can sign the net effect to be negative when the bank chooses an interior optimum $k_i^* > 0$. In this scenario the *differential buffer effect* dominates, with the consequence that high-ability banks adopt lower capital ratios than low-ability banks.

Next, we investigate how optimal equity ratios depend on the project selection for any given bank type θ. Remember that when banks voluntarily hold equity and (9b) holds, the sum of the last two terms in (7) must be positive for any $k \leq k_i^*$. Since $p_H < p_L$, the derivative $d\pi_i^P(k)/dk$ in (7) must therefore be larger for $i = H$ as compared to $i = L$, for any $k \leq \min\{k_L^*, k_H^*\}$. From the concavity of π_i^P in k it then follows that $k_H^*(\theta) \geq k_L^*(\theta)$ for any θ, with strict inequality whenever $k_H^* > 0$. Hence, a H project will be associated with a higher capital ratio k^* than an L project, for any given bank type θ. This result rests on a now familiar intuition: in an interior optimum, the buffer effect must dominate the moral hazard effect. Since H projects carry a higher probability of failure, the capital buffer effect is stronger for an H project as compared to an L project, implying higher equity ratios for H projects in each bank's optimum.

These results are summarized in:

Proposition 2. *Banks voluntarily hold equity capital only if the buffer effect dominates the moral hazard effect of equity, i.e., if condition* (9a) *holds. Moreover:* (i) *for given project choice $i = H, L$, equity capital k_i^* decreases in ability θ, and strictly so when $k_i^* > 0$;* (ii) *for each bank type θ, a high-risk project H features an equity ratio $k_H^* \geq k_L^*$, with strict inequality whenever $k_H^* > 0$.*

When banks face substantial bankruptcy costs D, they may find it in their own self-interest to hold equity capital, despite the higher financing costs of this choice. These larger equity buffers monotonically increase in the magnitude of default costs. They are more important for less able banks, which have a higher probability of failure, and for banks that choose more risky projects. Conversely, banks which hold little or no equity capital are those of high ability (and high-risk portfolios), as well as banks of intermediate quality which (just) choose low-risk portfolios.

We can now generalize Proposition 1(i), which has established the existence of a threshold ability type $\hat{\theta}$ for an exogenously given level of equity, such that all banks of lower type choose the less risky portfolio, whereas all higher types adopt a high risk strategy. When equity holdings are endogenous and depend on bank and portfolio type, the bank that is indifferent between the two projects is denoted by $\tilde{\theta}$ and is characterized by $\pi_L^P(k_L^*(\tilde{\theta}), \tilde{\theta}) = \pi_H^P(k_H^*(\tilde{\theta}), \tilde{\theta})$. Notice in particular that for homogenous equity levels k, eq. (6) has established that the per-project profit difference between high risk and low-risk projects monotonically increases in bank ability θ. Obviously, this result immediately carries over to a scenario where k is chosen endogenously but the buffer effect is so small that condition (9a) is not fulfilled and $k_H^*(\theta) = k_L^*(\theta) = 0$ for all types. We now show that the same qualitative result extends to a situation where condition (9a) *is* fulfilled, and at least a subset of banks endogenously choose positive equity levels. To see this, notice that we can apply a straightforward envelope theorem argument to a scenario in which equity levels are positive for some banks, and optimally differ across risk portfolios. We define

$$\tilde{\Delta}\pi^P(\theta) \equiv \pi_H^P(k_H^*(\theta), \theta) - \pi_L^P(k_L^*(\theta), \theta)$$

as the per-project profit difference across portfolio types for the optimal selection of equity. Using (7) and the envelope theorem to obtain $(d\pi_i^P/dk_i)(dk_i^*/d\theta) = 0$ for $i = H, L$ gives

$$\frac{d\tilde{\Delta}\pi^P(\theta)}{d\theta} = p'_H[R_H^S - (1 - k_H^*)d - Y_F] - p'_L[R_H^S - (1 - k_L^*)d - Y_F] > 0, \tag{11}$$

which is positive because $p'_H \geq p'_L$, and $k_H^* \geq k_L^*$. Using this property, an interior threshold type $\tilde{\theta}$ now exists under the sorting assumption

$$\tilde{\Delta}\pi^P(\underline{\theta}) < 0 < \tilde{\Delta}\pi^P(\bar{\theta}), \tag{S'}$$

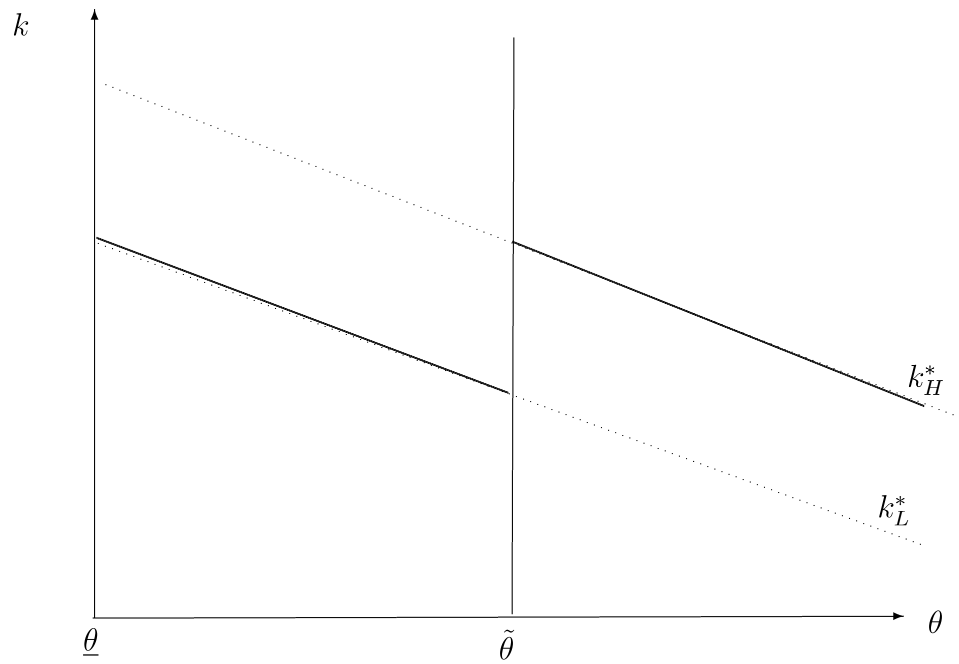
which generalizes the sorting assumption (S) to the case of endogenous equity holdings. We can then state:

Proposition 3. *Consider the banking equilibrium with endogenous capital choice, and assume that the sorting condition (S') applies. Then, there exists a threshold type $\tilde{\theta}$ such that any bank $\theta < (\geq)\tilde{\theta}$ chooses a low (high) risk portfolio.*

Notice that the indifferent bank $\tilde{\theta}$ earns the same per-project profits, and has the same size, regardless of the choice of its risk profile. Since profits are continuous in ability θ, observed bank sizes are therefore continuous everywhere, even at the threshold type $\tilde{\theta}$. In contrast to bank size, however, there is no smooth relationship between banks' ability θ and their choice of equity capital k^*. This relationship is shown in Figure 2, which illustrates the results summarized in Propositions 2 and 3.

As shown in Figure 2, banks of a low type θ will choose the low-risk project L. As θ continuously increases, the optimal capital ratio k^* continuously falls until the threshold ability $\tilde{\theta}$ is reached. At $\tilde{\theta}$ banks switch to the high-risk project H and the optimal capital

Figure 2: Optimal capital choices and banking equilibrium



ratio exhibits an upward jump. For $\theta > \tilde{\theta}$, project choice remains unchanged at H, and the banks' optimal capital ratio falls again.[18]

Our results in Propositions 2 and 3 are consistent with a body of empirical evidence. Flannery and Rangan (2008) analyze the reasons for the large capital build-up among U.S. banks during the 1990s and early 2000s and find that this was in large part the response to the withdrawal of implicit government guarantees. This is consistent with an increasing importance of the buffer effect for U.S. banks during this time period. Rime (2001, Table 1) shows that large Swiss banks held substantially less excess capital beyond the minimum requirement during the 1990s, as compared to smaller regional and cantonal banks. Moreover their analysis finds a positive relationship between banks' size and their risk-taking. Lindquist (2004) shows for Norwegian banks that small savings banks face a higher bankruptcy probability (lower p_i) and hold larger capital

[18]Figure 2 illustrates a situation where optimal equity choice is interior for each type of bank. Keep in mind that even if the buffer effect dominates and condition (9a) is fulfilled, some banks may decide not to voluntarily hold equity. However, when D falls so that the buffer effect loses bite, equity choices are reduced in a way that preserves the order of equity holdings across bank types. In particular, when D falls the first banks to choose zero voluntary equity are intermediate-type banks with low-risk portfolios, and high-ability banks with high-risk portfolios. Accordingly, the qualitative features of the figure extend to a situation where a subset of banks hold no equity.

buffers, as compared to large commercial banks. Large banks are also found to hold significantly less equity in a sample of Spanish banks (Ayuso et al., 2004, Table 2). Finally, Bhagat et al. (2015) study the relationship between bank size, equity holdings, and risk-taking for U.S.-based financial institutions and find that large banks hold less equity capital, and are also riskier, as compared to smaller banks.

4 Capital regulation

Our analysis of regulatory policies motivates minimum capital requirements as a response to banks' moral hazard, which is in turn caused by the existence of deposit insurance. From a social perspective, expenses for the deposit insurance fund must be added to the banks' private costs of default. In *per-project* form, the expected costs to the deposit insurance fund are $(1-p_i)(1-q)(1-k)d$, and these are higher for the high-risk project. Adding this to the banks' differential profit $\Delta\pi^P$ in (5) gives the valuation of the high-risk vs. the low-risk project from the regulator's perspective:

$$(\Delta\pi^P)^{reg} = \Delta\pi^P - (p_L - p_H)(1-q)(1-k)d. \tag{12}$$

Note that the bank manager's private default costs D (as part of $\Delta\pi^P$) are also social costs, as they represent losses in income or wealth. According to (12), the expected social return under project failure is therefore lower than the private return for bank owners, or for the bank's management. Hence $(\Delta\pi^P)^{reg} < \Delta\pi^P$.[19] As portfolio risks are monotone in bank types [eq. (6)], the threshold bank from the regulator's perspective then has a higher ability level, than the cutoff type $\hat{\theta}$ under the banks' private decisions. As a consequence, too many banks choose high-risk projects from a social welfare perspective.

We hold that bank regulators cannot fully monitor bank risk. This assumption accounts for the practical problems associated with implementing risk based capital requirements which in reality, mostly rely on the banks' internal modelling. According to the recent theoretical literature (Colliard, 2019; Hakenes and Schnabel, 2011), this approach leaves banks room for strategic misrepresentation, by allowing them to understate true risks. Recent empirical studies provide strong support for such a cautionary position (see

[19] In addition, bank defaults typically inflict further default costs on other financial institutions and the real sector. Hence, social default costs will usually exceed D, with the effect of further reducing $(\Delta\pi^P)^{reg}$ below $\Delta\pi^P$.

e.g., Begley et al., 2017; Behn et al, 2022). In our setting, banks choose among projects which from the regulator's view belong to the same risk class and are therefore, subject to the same capital requirements.[20]

Against this background, let us now suppose that the government imposes a minimum equity requirement $\underline{k}$ on each bank. The basic purpose of capital requirements is to reduce the expected costs for the deposit insurance fund by reducing banks' exposure to external debtors. This reasoning, however, commonly disregards the effect of a higher k on a bank's risk portfolio, which we will identify as an important consequence of equity regulation.

We start our discussion in Section 4.1 for the case where default costs D are so low that no bank voluntarily holds equity. Section 4.2 then analyzes the alternative case where default costs D are high enough for banks to make non-trivial choices regarding their equity holdings, and in which at least a subset of banks (or even all banks) find it advantageous to hold positive equity shares. As we will see, the effects of capital regulation on portfolio choices systematically differ across these two scenarios. In addition, as capital choices differ among heterogeneous banks, equity requirements may affect banks in different ways in our model, and our analysis will study the implications of this heterogeneity in regard to the overall effectiveness of capital regulation.

4.1 Banks hold no voluntary equity

We first study capital requirements in a situation in which no bank voluntarily holds equity capital. Hence the minimum capital requirement $\underline{k}$ will always be binding, and by definition it reduces bank profits for each type of bank. Our main interest lies in the effects of $\underline{k}$ on the portfolio choice of a bank of ability θ, which can be formally derived by differentiating the profit difference $\Delta\pi(k,\theta) = \pi_H^P(k,\theta) - \pi_L^P(k,\theta)$ with respect to k. Using (7), one obtains

$$\frac{d\Delta\pi^P(k,\theta)}{dk} = -(p_H(\theta) - p_L(\theta)) \left[-(1-q(k))d + \frac{dq(k)}{dk} D \right], \tag{13}$$

for given k. The squared bracket in (13) corresponds to the sum of the moral hazard and buffer effects as collected in condition (9a). When D is sufficiently low, condition (9a) is

[20] As Colliard (2019) notes, the reliability of internal risk measures has become a key issue in the debate on capital regulation. Leverage ratios and other non risk-based tools are expected to become more prominent in the next version of the Basel accord.

violated which renders the term in brackets and since $p_H < p_L$, the entire derivative (13) negative when voluntary capital holdings are zero, regardless of bank type θ.[21] We therefore find that higher equity capital mandated by regulation, will unambiguously make the safer project relatively more profitable for banks. Banks which reconsider their portfolio choice, will move towards the safer alternative. Intuitively, the higher equity requirement implies that banks benefit less from the non-repayment of deposit debt in case of default. Hence the *moral hazard effect* as an important motivation for adopting the high-risk portfolio becomes weaker. Moreover, for small default costs, this differential moral hazard effect dominates the differential buffer effect that works in the opposite direction.

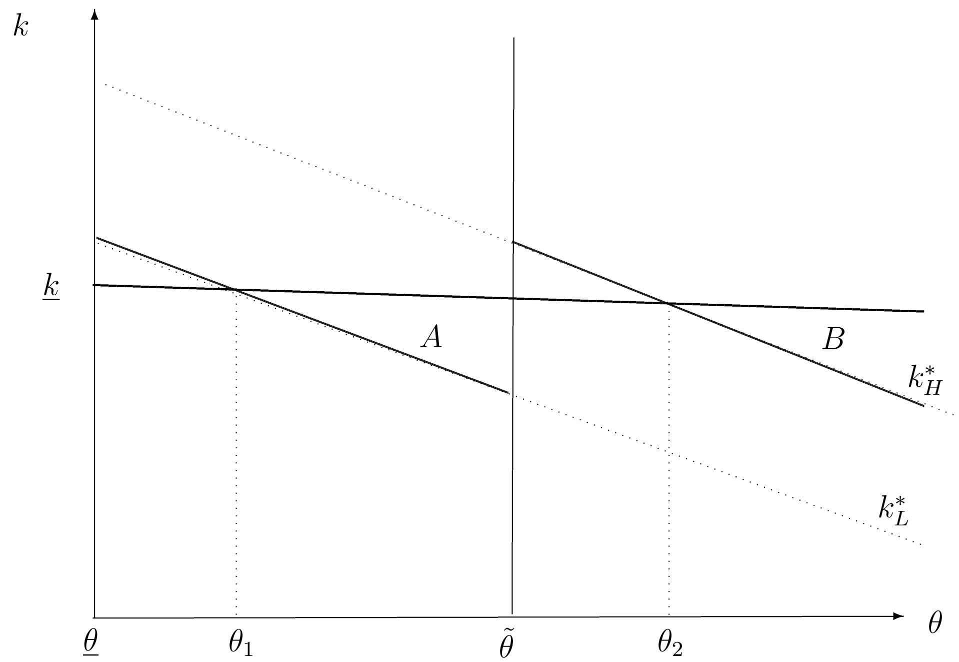
Applying our results in Proposition 1, the indifferent bank type $\hat{\theta}$ now moves to the right, that is, the bank industry as a whole reduces its risk exposure. Higher capital requirements unambiguously reduce the risks to taxpayers or depositors by increasing bank capital, and by making banks switch to less risky investments. As the wedge $\Delta\pi^P(k, \theta)$ widens for higher levels of k, this beneficial effect increases in the size of the capital floor $\underline{k}$.

Proposition 4. *Suppose that D is sufficiently small to violate condition* (9a)*, and banks do not voluntarily hold equity capital. For each bank θ, a minimum capital requirement $\underline{k} > 0$ makes the low-risk project more attractive. Moreover, the threshold type $\hat{\theta}(\underline{k})$ increases in $\underline{k}$ so that a tighter capital requirement induces more banks to opt for low risk.*

The above scenario of zero voluntary equity holdings by banks underlies most of the existing regulatory literature, which does not incorporate a buffer effect of equity (Hellman et al., 2000; Repullo, 2004; Morrison and White, 2005; Dell'Ariccia and Marquez, 2006). In our model, this translates into a low (or even zero) level of default costs D and condition (9a) being violated. Our results show that in such a setting, imposing a mandatory capital requirement $\underline{k}$ has two compounding effects with regard to default risks. First, equity requirements have the direct effect of reducing the likelihood of default for given portfolio choice. But second, capital regulation also has the indirect effect of making it more attractive for banks to adopt the low-risk portfolio strategy, which further enhances the safety of the banking sector.

[21] This result carries over to any exogenous initial level of capital k_0. Recall that the sum of terms in the squared bracket in (13) is falling in k. Therefore, if the sum of terms in the bracket is negative for $k_0 = 0$, it will also be negative for any $k_0 > 0$.

Figure 3: Voluntary equity and capital regulation



4.2 Voluntary equity holdings

We now investigate minimum capital requirements $\underline{k}$ in the alternative environment where default costs D are large enough to meet condition (9a). In this case, some or all banks voluntarily hold positive levels of equity. The properties of this banking equilibrium were summarized in Propositions 2 and 3, and our results yield an overall picture where more able banks are larger and pursue more risky strategies. While small banks and medium-size banks with high risk portfolios are relatively well capitalized, the same cannot be said about the largest banks in the industry.

Since banks differ in their equity capital $k_i^*(\theta)$, depending on their type θ and their project choice $i \in \{H, L\}$, a uniform minimum capital requirement $\underline{k}$ will generally be binding for some, but not all banks. A possible situation is depicted in Figure 3, which introduces an exogenous requirement $\underline{k}$ to the banking equilibrium shown in Figure 2. Without any changes in project choice, the minimum requirement $\underline{k}$ will be binding for two diverse groups of banks. First, the regulation binds for banks of intermediate ability type $\theta \in \{\theta_1, \tilde{\theta}\}$, which originally hold low-risk portfolios. The increase in capital buffers for these banks is given by the area labelled A. Second, $\underline{k}$ will also be binding for the largest banks with $\theta > \theta_2$. The increase in capital holdings for these banks is labelled by the area B.

Similar to our analysis in Section 4.1 above, project choice *will*, however, change for some banks. To explore this change, notice first that for each bank of type θ, $k_H^*(\theta) \geq k_L^*(\theta)$, with strict inequality whenever $k_L^*(\theta) > 0$. Let $\pi_i^P(\theta, \underline{k})$ be bank θ's profits when pursuing project i under a minimum equity requirement of $\underline{k}$. For each bank type, we can now analyze the portfolio effects of capital requirements over several ranges of $\underline{k}$.

(1) For $\underline{k} \leq k_L^*(\theta)$, so that the capital requirement is non-binding for bank θ, capital regulation has no effect on profits and project choice.

(2) For $k_L^*(\theta) \leq \underline{k} < k_H^*(\theta)$, the capital requirement is binding for bank θ only if it chooses the low-risk project. With portfolio-dependent profits $\pi_H^P(\theta, \underline{k}) = \pi_H^P(k_H^*(\theta), \theta)$ and $\pi_L^P(\underline{k}, \theta) < \pi_L^P(k_L^*(\theta), \theta)$, introducing $\underline{k}$ increases the profit difference $\Delta\pi^P(\theta, k_L = \underline{k}, k_H = k_H^*)$ so that the high risk project becomes more attractive and $d\Delta\pi(\cdot)/d\underline{k} > 0$.

(3) For $\underline{k} > k_H^*(\theta)$, the capital mandate is binding regardless of portfolio choice, and bank θ will employ $k_H = k_L = \underline{k}$ regardless of project type. The payoff difference across projects is the same as in (6), and the derivative of this payoff difference with respect to $\underline{k}$ is described in (13). Since (9b) is positive at $\underline{k} = k_H^*$, this derivative has (for $\underline{k}$ close enough to k_H^*) a strictly positive sign. Hence, a marginal increase in $\underline{k}$ also makes the high-risk project more attractive for these bank types, even though it will not change project choice under the binary risk decision made here.

Our analysis therefore shows that when banks voluntarily hold capital, a higher capital requirement - if it has any effect - changes the incentives of *all* banks in the direction of choosing the high-risk project. This is exactly the opposite of the result in Section 4.1. Intuitively, it is now the *buffer effect* that dominates the *moral hazard effect*: as the higher equity capital k reduces the likelihood of a bank's default, the bank will be less concerned about having to pay the default cost D, and this tilts the bank's project choice in the direction of - socially undesirable - higher risk-taking.

Moreover, what applies to each individual bank, is also true for the banking industry as a whole. Of particular interest is the effect of a binding capital requirement on the bank type $\tilde{\theta}$, which was shown to be indifferent between portfolio risks in the absence of equity requirements. Introducing capital regulation, any capital requirement of size $\underline{k} > k_L^*(\tilde{\theta})$ will make some banks with low-risk projects to the left of $\tilde{\theta}$, switch towards the high-risk strategy. Moreover, the number of these banks increases as $\underline{k}$ increases. Notice that as the equity holdings of these banks will rise discretely, the minimum capital requirement $\underline{k}$ may or may not be binding for them in the new equilibrium.

But as our previous discussion has shown, introducing $\underline{k}$ makes the high-risk portfolio relatively more attractive in either case.

We summarize these findings in:

Proposition 5. *Suppose the default costs D are large enough so that banks choose interior levels of voluntary equity. Then,*
(i) for each bank θ, a capital requirement $\underline{k} > k_L^*(\theta)$ makes the high-risk portfolio relatively more attractive;
(ii) when the equity requirement $\underline{k}$ is binding for the indifferent bank $\tilde{\theta}$ (i.e., if $k_L^*(\tilde{\theta}) < \underline{k}$), low-risk banks in the vicinity of $\tilde{\theta}$ will change their strategy in favor of the high-risk portfolio. The threshold bank type $\tilde{\theta}(\underline{k})$ moves to the left, and the strength of this effect increases in $\underline{k}$.

In the light of our results, minimum capital requirements $\underline{k}$ have rather mixed effects on bank safety when the buffer effect dominates the moral hazard effect of equity holdings. First, and in contrast to the previous regime in which D was small and Condition (9a) was violated, some groups of banks may not be affected at all by the regulation, as their voluntary equity holdings exceed the mandated minimum requirement. In Figure 3, these will be the banks with ability levels $\theta \in \{\underline{\theta}, \theta_1\}$. For a second group of banks, the minimum capital ratio $\underline{k}$ will increase capital buffers, and reduce the risk of insolvency, without causing these banks to switch the riskiness of their investments. In Figure 3, these are the large banks with ability levels $\theta > \theta_2$, as well as those banks to the right of, but close to θ_1. Finally, a third group of banks will switch to the risky project H as a consequence of their higher required capital holdings. This group, shown in Figure 3 to the left of, but close to $\tilde{\theta}$, may well increase their overall solvency risk, despite the higher equity capital that they hold after the policy change.

We can also infer under which conditions the total default risk of a bank θ rises as a consequence of a capital requirement $\underline{k}$. This is the case when equity regulation induces the bank to switch towards the high-risk portfolio, and if

$$(1 - p_H(\theta))[1 - q(\max\{k_H^*(\theta), \underline{k})\}] > (1 - p_L(\theta))[1 - q(k_L^*(\theta))]. \tag{14}$$

In words, capital requirements *increase* the bankruptcy risk for a subset of banks, if the lower success probability associated with the portfolio switch towards H dominates the increased survival probability q that results from an increase in equity (to k_H^* or $\underline{k}$, whichever is higher). This condition tends to hold more often, if the distance

$p_L - p_H > 0$ is wide, or for distribution functions of R^F for which the effect of k on the default probability, $dq/d(k)$, is small in the relevant range.

Our model thus allows a more detailed assessment of capital regulation than is possible in models that do not incorporate a *buffer effect*. We show that the same buffer effect that causes banks to voluntarily hold equity also makes them more prone to adopting risky projects when governments increase capital requirements.

Our analysis can also be applied to government aid programs that increase equity capital in the supported banks. An important example of the latter is the Troubled Asset Relief Program (TARP) in the United States, which led U.S. banks to increase the riskiness of their portfolios (Black and Hazelwood, 2013; Duchin and Sosyura, 2014). Even though TARP was effectively a government bailout program, the vast majority of benefitting banks held equity ratios well above the required minimum even before receiving support (Dushin and Sosyura, 2014, p. 7). Therefore, the situation of participating banks is adequately described by a scenario of voluntary equity holdings. Our analysis shows that the increased bank capitalization is able to explain the increased risk-taking that was empirically observed among the participating banks.[22]

5 Further policy instruments

5.1 Bank bail-ins and capital requirements

It is natural to view managerial default costs D as being affected by the institutional environment, and by government policy. For example, those costs may fall when the bank is more likely to be bailed out by the government in times of financial troubles. In the following we study the implications of the reverse policy, where governments increase the expected private default costs D by reducing bailout expectations ('bail-in'). This stated policy objective underlies, for example, the common bank resolution mechanism adopted by the European banking union. Once again, we distinguish between the two scenarios where banks do or do not hold voluntary equity.

[22]This does not preclude alternative interpretations of the empirical findings, however. In particular, Duchin and Sosyura (2014) argue that the capitalization program changed banks' beliefs about *future* government bailouts and therefore increased banks' moral hazard, in addition to the direct effects of higher bank capital. In our setting, this additional effect corresponds to a reduction in the bank's expected default cost D, which is analyzed in the following.

First, consider a situation where the bank does not hold voluntary equity. Hence, default costs are sufficiently low as to violate (9a), and a marginal change in default costs leaves $k_i^* = 0$ unaffected. At the same time, the increase in D increases the attractiveness of the low-risk portfolio for any $\underline{k} \geq 0$, because

$$\frac{d\Delta\pi^P(\theta)}{dD} = (p_H - p_L)(1 - q(\underline{k})) < 0.$$

If banks do not wish to hold equity, increasing default costs have a safety enhancing effect, as the bank's management will respond to an increase in default costs by moving towards a safer portfolio. Notice further that as banks' do not respond to the measure by changing their equity holdings, higher default costs act as a complement to prudential equity requirements in this case.[23]

Conversely, consider an alternative scenario in which D is already large, and the bank voluntarily holds capital, $k_i^* > 0$. Differentiating $d\pi_i^P(\cdot)/dk = 0$ using (7) yields

$$\frac{dk_i^*}{dD} = -\frac{\frac{dq}{dk}D}{\left(\frac{dq}{dk}d + \frac{d^2q}{dk^2}D\right)} > 0. \tag{15}$$

A higher default cost D causes each bank to further raise its equity buffer as a safeguard against bankruptcy. This response directly enhances bank safety.

Furthermore, investigating the effects of changing D on the bank's portfolio choice for optimally adjusted $k_i^*(D)$, we obtain (using the definition of $\tilde{\Delta}\pi^P$),

$$\frac{d\tilde{\Delta}\pi^P(\theta)}{dD} = (1 - p_L)[1 - q(k_L^*)] - (1 - p_H)[1 - q(k_H^*)]. \tag{16}$$

Perhaps surprisingly, the effect of larger default costs on portfolio selection is now ambiguous. As $p_H < p_L$, the higher intrinsic project risk of the H portfolio continues to sway bank preferences towards the L alternative. However, since $k_H^*(D) > k_L^*(D)$ for any D, this effect is now counterbalanced by the larger equity buffer associated with the high-risk project. As (16) shows, a larger D in fact tilts bank preferences towards the more risky portfolio choice when the latter effect dominates. Hence, when banks voluntarily hold equity capital, our analysis yields the counterintuitive finding that higher default costs may induce banks to choose a riskier portfolio.

[23] Notice that when we interpret D as the bank's charter value which is negatively affected by mandatory equity holdings, the result demonstrates the charter value effect of capital regulation (see e.g, Hellman et al., 2000): a larger k lowers the charter value $D(k)$ and as such, increases the attractiveness of bank gambling.

This ambiguity result stands in contrast to conventional arguments in the literature, according to which a lower bailout probability should always be associated with a lower portfolio risk (e.g. Acharya, 2003). It is thus another example showing that policy instruments can have counterintuitive effects when banks voluntarily hold equity capital, and adjust it optimally to policy shocks. We note, however, that even when a higher D makes banks switch to the high-risk portfolio in our model, such a switch is always associated with a reduction in the overall risk of default, $(1-q)(1-p)$, taking all equilibrium effects into account.[24] We can therefore conclude that policy measures that increase managerial default costs unambiguously increase the safety of the banking industry.

5.2 Continuous portfolio choice and the effects of bank levies

A further policy instrument we consider are levies on bank's borrowing. These have been introduced in a number of countries after the financial crisis, with the simultaneous policy goals to increase government revenues from the financial sector, and to induce higher capital holdings by banks (see Devereux et al., 2019).

For the purposes of this analysis, we extend our framework to a situation where each bank has a continuous portfolio choice, rather than selecting from two discrete portfolios $i \in \{H, L\}$. Suppose portfolios are described by a parameter $r \in [\underline{r}, \bar{r}]$. The success probability of portfolio r is given as described as $p(\theta, r)$, and the successful gross return is $R^S(r)$. We assume $p_r < 0$, $p_\theta > 0$, $p_{rr} \leq 0$, $p_{r\theta} \geq 0$, $R_r^S > 0$ and $R_{rr}^S < 0$. Moreover, in alignment with the sorting assumption made in the previous sections, we impose

$$p(\underline{\theta}, \underline{r})R^S(\underline{r}) > p(\underline{\theta}, \bar{r})R^S(\bar{r}) \quad \text{and} \quad p(\bar{\theta}, \underline{r})R^S(\underline{r}) < p(\bar{\theta}, \bar{r})R^S(\bar{r}). \tag{S''}$$

Assumption (S") ensures that the weakest bank does not prefer the highest-risk portfolio, while the highest-risk portfolio gives the strongest bank a higher expected return than the lowest-risk portfolio.

Next, let us endogenize the bank's risk portfolio. Each bank θ chooses r to maximize

$$\pi^P(r, \theta, k) = p(\theta, r)A(r, k) + [1 - p(\theta, r)]B(k) \tag{17}$$

[24] To verify this, notice that (16) can be positive, and an increase in D leads banks to switch to the high-risk strategy, only if the overall default risk is lower for the high-risk project. But then the switch in project choice can never undo the direct positive effect of a higher D on bank safety via higher k^*.

where $A(r,k) = R^S(r) - (1-k)d$ and $B(k) = q[\hat{R}^F - (1-k)d] - (1-q(k))D$. The first order condition for an interior solution $r^*(\theta)$ reads

$$\frac{d\pi^P(\theta, r)}{dr} = p_r(r,\theta)[A(r,k) - B(k)] + p(r,\theta)R_r^S(r) = 0. \tag{18}$$

Under (S"), and with the second-order condition $\pi_{rr}^P(\cdot) < 0$ satisfied and proper Inada type of conditions in place, $r^*(\theta)$ is indeed interior and unique for given k.[25]

The optimal capital ratio chosen by each bank is determined by

$$\frac{d\pi_i^P(k,r,\theta)}{dk} = (-\rho + d) + (1 - p(\theta, r))[q'(k)D - (1 - q(k)d)] \leq 0. \tag{19}$$

As in our benchmark model [eq. (7)], a positive $k^*(r,\theta)$ requires the default costs D to be large enough that condition (9a) is fulfilled. If this is the case and the derivative (19) has a positive sign at $k = 0$, a bank's optimal equity choice $k^*(\cdot)$ is interior and for given r (again assuming the second order conditions are valid) uniquely defined by the first order condition $d\pi^P(k,r,\theta)/dk = 0$. In equilibrium, each bank θ's optimal choices $(r^*(\theta), k^*(\theta))$ simultaneously satisfy (18) at $k = k^*$, and (19) at $r = r^*$.

Introducing capital regulation, we now examine how r^* reacts to a change in the mandatory equity requirement $\underline{k}$. Implicitly differentiating (18) for a change in $\underline{k}$ gives

$$\frac{dr^*(\theta)}{d\underline{k}} = p_r \frac{[q'D - (1-q)d]}{\pi_{rr}^P(\cdot)} \lesseqgtr 0. \tag{20}$$

Since $p_r < 0$ and $\pi_{rr}^P < 0$, this derivative is negative when D is small, condition (9a) is violated, and banks do not voluntarily hold equity capital ($k^* = 0$). Stricter capital requirements will then unambiguously increase financial sector stability by causing each bank to choose less risky projects. To the contrary, consider the case where default costs D are substantial, condition (9a) is satisfied, and banks voluntarily hold equity. Then, the second term in (19) is positive and the derivative in (20) is also positive when evaluated at an initial equity choice $k = k^* > 0$. Hence, a binding capital requirement $\underline{k} > k^*$ will prompt the bank to adopt a more risky portfolio. In sum, the effects of tighter capital requirements are therefore very similar to those in our benchmark model where banks face a discrete choice of project risk (see Propositions 4 and 5).

We now turn to the effects of bank levies, which are typically levied on all liabilities of the bank, except for equity capital. In our setting they thus fall on deposits and we conceptualize these levies as an exogenous increase in the deposit rate d. The results of this comparative statics analysis are unambiguous and are stated in:

[25] Inada conditions which guarantee an interior optimum are $R_r^S \to \underline{r} = \infty$, and $R_r^S \to \bar{r} = 0$.

Proposition 6. *Suppose that a bank of type θ makes a continuous portfolio choice $r \in [\underline{r}, \bar{r}]$ and D is sufficiently large to yield endogenous equity $k^* > 0$. Then, an increase in the bank's deposit rate d has the following effects: (i) it increases the bank's equity choice $k^*(\theta)$, and (ii) it increases the riskiness of the bank's portfolio $r^*(\theta)$.*

Proof: See Appendix A.2.

Proposition 6 once again finds a positive correlation banks' equity holdings and their risk-taking in our model, when banks endogenously choose their equity capital k^*. A higher deposit interest rate d increases the cost of debt financing and as a consequence, increases their voluntary equity holdings k^*. In this regard, bank levies and other taxes on banks' debt serve their intended purpose. At the same time, however, the higher equity holdings change the banks' risk trade-off in the direction of more risky portfolios. Given the complementarity between risk-taking and equity in a scenario with voluntary equity, indirect effects reinforce these direct effects: the increase in k^* further boosts the bank's risk appetite, while the increased portfolio risk leads to further equity holdings in the new equilibrium [see eq. (A.3) in Appendix A.2].

Our results in Proposition 6 conform with the empirical results of Devereux et al. (2019), who analyze the effects of bank levies in 14 member countries of the European Union. They find that bank levies indeed reduced risks on the liability side of the banks' balance sheet, by inducing banks to hold more equity. However, as an unintended further effect, banks at the same time changed their asset composition in the direction of a more risky portfolio. Our model provides an intuitive explanation for this observed correlation between capital holdings and risk choice.

6 Conclusion

The paper contributes some novel insights to the debate on the relationship between capital requirements and banks' risk-taking choices. First, we have introduced a simple, heterogeneous banks framework in which the size of banks, the riskiness of their operations, and the level of voluntary equity are all chosen endogenously. Using this framework, larger and more able banks were found to select more risky portfolios and lower equity compared to their smaller counterparts. These predictions are well supported by the existing empirical evidence.

Second, we have argued that banks voluntarily hold some level of equity as insurance against default. In the model, such an equity buffer is used when bank management faces adverse reputational or financial consequences in case of bankruptcy. As emphasized in the literature, debt financing creates incentives to invest in an overly risky portfolio because the bank can avoid debt repayment in case of default. This moral hazard effect also renders equity unattractive because the value of a portfolio 'gamble' falls when more equity is put at risk. However, the empirical evidence suggests that many banks nevertheless hold equity voluntarily. This implies that the buffer effect of bank capital dominates the moral hazard effect, at least over some initial range of equity holdings.

Third, and as an important consequence, we have shown that when banks voluntarily hold equity capital, a binding capital requirement will lead them to choose riskier projects that benefit from the larger equity buffer. An analogous argument holds when bank levies are used as an alternative policy instrument to fight banks' moral hazard. Again, these predictions are in line with the empirical evidence according to which banks responded to higher capital requirements, or the introduction of bank levies, by switching to riskier portfolios.

Our analysis thus identifies an important side effect of government policies which are introduced to improve the safety of the banking sector. The same extra capital that safeguards the bank against default, increases its temptation to take on more risks because in expectation, it makes bank default and the associated costs less likely. The surge towards higher risk is powerful when the default costs for banks and their managers are high - and as we have argued, the prevalence of voluntary equity holdings in the banking industry suggest they are.

Appendix

Appendix A: Derivations

A1: Derivation of equation (7)

Per-project profits are $\pi_i^P = p_i[R^S - (1-k)d] + (1-p_i)Y^F - k\rho$ [cf. eqs. (2)–(3)]. We define the expected profit of a failed project, net of the reputation costs of failure, as

$$\zeta \equiv q(k)[\hat{R}^F(k) - (1-k)d] = \int_{\bar{R}^F=(1-k)d}^{R^F_{max}} [R^F - (1-k)d] f(R^F) dR^F.$$

Defining $u = R^F - (1-k)d$ and $v' = f(R^F)$ and using integration by parts, one has

$$\begin{aligned} \zeta &= \int_{(1-k)d}^{R^F_{max}} [R^F - (1-k)d] f(R^F) dR^F = uv|_{(1-k)d}^{R^F_{max}} - \int_{(1-k)d}^{R^F_{max}} u'v \\ &= [(R^F_{max} - (1-k)d)] - \int_{(1-k)d}^{R^F_{max}} F(R^F) dR^F. \end{aligned}$$

Since $Y^F = \zeta + [(1-q(k)]D$, the expected return under project failure, gross of default costs, is

$$Y^F = R^F_{max} - (1-k)d - \int_{(1-k)d}^{R^F_{max}} F(R^F) dR^F - [(1-q(k)]D.$$

Using the Leibniz rule on the second term of Y^F, the derivative with respect to k is

$$\frac{dY^F}{dk} = d - (1-q)d + \frac{dq}{dk}D = dq + \frac{dq}{dk}D.$$

Hence the derivative of per-project profits with respect to k is

$$\frac{d\pi_i^P}{dk} = [p_i + (1-p_i)q]d - \rho + (1-p_i)\frac{dq}{dk}D, \tag{A.1}$$

which corresponds to eq. (7) in the main text.

A2: Proof of Proposition 6

Totally differentiating the first-order conditions (18) and (19) yields the equation system

$$\begin{bmatrix} a & b \\ b & f \end{bmatrix} \begin{bmatrix} dr \\ dk \end{bmatrix} = \begin{bmatrix} c \\ g \end{bmatrix} dd, \tag{A.2}$$

$$
\begin{aligned}
a &= 2p_r R_r^S + pR_{rr}^S < 0 \\
b &= -p_r \left[q'D - d(1-q) \right] > 0 \\
c &= p_r(1-q)(1-k) < 0 \\
f &= (1-p) \left[dq' + q''D \right] < 0 \\
g &= -[p + (1-p)q] < 0.
\end{aligned}
$$

In signing these terms, we have used $p_r < 0$, $R_r^S > 0$ and $R_{rr}^S < 0$ under continuous portfolio choices of banks, $q'D - d(1-q) > 0$ under a positive and endogenous equity choice k^*, and $dq' + q''D < 0$ from the second-order condition for k^*.

Solving the equation system (A.2) for exogenous variations in d gives

$$
\frac{dr}{dd} = \frac{cf - gb}{|A|} > 0, \qquad \frac{dk}{dd} = \frac{ag - bc}{|A|} > 0, \tag{A.3}
$$

where the Jacobian determinant $|A| = af - b^2 > 0$ must be positive. These results are stated in Proposition 6. □

Appendix B: Endogenous default costs

In our main model, default costs D per portfolio unit have been taken as exogenously given. In this extension, we endogenize default costs, by interpreting them as the lost charter value of a bank that goes into default.

In a simple dynamic set up the charter value of a bank in period t, V_t, is given by[26]

$$
V_{it} = \Pi_{it}(\theta) + s\delta V_{i,t+1},
$$

where $\Pi_t(\theta)$ are the profits of a bank of type θ in period t, δ is the discount factor for future profits and

$$
s_i(\theta, i, k) = p_i(\theta) + (1 - p_i(\theta))q(k) \tag{B.1}
$$

is the probability that a bank of type θ with project $i \in \{H, L\}$ will *not* go bankrupt, and therefore 'survive', to period $t+1$. This survival property is rising in the bank's capital buffer k.

In the steady state, we have $V_t = V_{t+1}$ and $\Pi_t = \Pi$, leading to

$$
V_i(\theta, i, k) = \frac{\Pi_i}{(1 - s_i\delta)}, \tag{B.2}
$$

[26] See Allen at al. (2011, Section 5.2) for a similar approach.

where $\Pi_i(\theta) = \pi_i^P Q - C(Q)$, as in (4). Per-project profits are $\pi_i^P = p_i Y^S + (1-p_i)\tilde{Y}^F - \rho k$, where

$$Y^S \equiv R_i^S - (1-k)d, \qquad \tilde{Y}^F \equiv q(k)[\hat{R}^F(k) - (1-k)d], \tag{B.3}$$

which corresponds to (2)–(3), except that $\tilde{Y}^F$ does not include the default cost term D.

This intertemporal variant shares many of the core features of our benchmark model. In particular, higher ability banks are larger, have a higher sum of discounted profits V_i, and are more likely to choose the high-risk project. The optimal equity choice of a bank of type θ with project choice i is determined by

$$\begin{aligned} \frac{\partial V_i}{\partial k} &= \frac{Q}{(1-s_i\delta)}\left[\frac{\partial \pi_i^P}{\partial k} + \frac{\delta \pi_i^P}{(1-s_i\delta)}\frac{\partial s_i}{\partial k}\right] \\ &= \frac{Q}{(1-s_i\delta)}\left\{-\rho + d + (1-p_i)\left[-(1-q)d + q'\frac{\delta\pi_i^P}{(1-s_i\delta)}\right]\right\} = 0, \end{aligned} \tag{B.4}$$

where the second step has used (B.1) and (B.3). The effects in (B.4) are analogous to those in (7), where the exogenous default cost D is now replaced by the expected loss in the discounted sum of future profits (the charter value).

One important difference to the benchmark model is that high-ability banks may now have higher voluntary equity holdings than less able banks. Differentiating (B.4) with respect to θ and evaluating at the optimal interior level of k^* gives:

$$\left.\frac{\partial^2 V_i}{\partial k \partial \theta}\right|_{k=k^*} = \frac{dp_i}{d\theta}\left\{\frac{-(\rho-d)}{(1-p_i)} + \frac{\delta(1-p_i)q'}{(1-s_i\delta)}\left[\frac{\delta\pi_i^P}{(1-s_i\delta)} + (Y^S - \tilde{Y}^F)\right]\right\}. \tag{B.5}$$

The first term in (B.5) corresponds to (10) in our benchmark model. It results from the fact that higher-ability banks fail less often and are less in need of a large equity buffer. The second term in the curly bracket works, however, in the opposite direction: high-ability banks are more profitable and therefore lose a higher charter value when they fail. In general, the relationship between a bank type θ and voluntary equity holdings k^* is therefore ambiguous, when default costs are endogenized. A negative relationship, as in our benchmark model, will still result under this extension, however, when the weight of future profits δ is low, or when the probability of success p_i is sufficiently large for both project types.

Finally, we determine how project choice is affected by an increase in equity under this model extension. We evaluate this change for the bank of type $\tilde{\theta}$, which is indifferent between choosing the high-risk or the low-risk project. Hence $V_H(\tilde{\theta}) = V_L(\tilde{\theta})$ and, since loan quantities are independent of project choice, $\pi_H^P/(1-s_H\delta) = \pi_L^P/(1-s_L\delta) \equiv \sum \pi^P$.

Using the definition $\Delta V(\theta) \equiv V_H(\theta) - V_L(\theta)$ we then get

$$\left.\frac{d\Delta V(\theta)}{dk}\right|_{\tilde{\theta}} = \frac{Q}{(1-s_H\delta)}\left[-\rho + d + (1-p_H)\delta\Omega\right] - \frac{Q}{(1-s_L\delta)}\left[-\rho + d + (1-p_L)\delta\Omega\right] \ ,$$

where $\Omega \equiv -(1-q)d + q'\delta \sum \pi^P$ is independent of project choice. When the critical bank voluntarily holds equity under either project choice, $\Omega > 0$ must hold from (B.4), analogous to our benchmark model.

$$\left.\frac{d\Delta V(\theta)}{dk}\right|_{\tilde{\theta}} = \frac{Q\,(p_L - p_H)}{(1-s_H\delta)}\Omega + Q[-\rho + d + (1-p_L)\Omega]\left[\frac{1}{(1-s_H\delta)} - \frac{1}{(1-s_L\delta)}\right]. \quad \text{(B.6)}$$

The first term in (B.6) must be positive under voluntary equity holdings ($\Omega > 0$), since $p_L > p_H$. The second term is zero from the first-order condition (B.4), if the bank's equity is optimally chosen for the low-risk project. In this special case the analogy to our benchmark model [eq. (13)] is exact, and a marginal increase in capital requirements that is just binding for bank $\tilde{\theta}$ will unambiguously lead it to switch to the high-risk project. More generally, the second term will be non-zero, but it will still be small. Therefore, for banks in the neighborhood of $\tilde{\theta}$, the analysis from our benchmark remains valid and banks that choose a low-risk project initially will respond to the capital increase by increasing their project risk.

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