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Abstract

This paper analyzes the interactions between redistribution and unemployment insurance policies and their implications for the optimal design of tax-benefit systems. In a setting where individuals with different earnings abilities are exposed to unemployment risk on the labor market, I characterize the optimal income tax schedule and the optimal unemployment benefit schedule in terms of empirically estimable sufficient statistics. I provide a Pareto-efficiency condition for tax-benefit systems that implies a tight link between optimal redistribution and optimal unemployment insurance: the steeper the profile of income taxes is, the flatter the profile of unemployment benefits should be, and vice versa. Optimal replacement rates are therefore monotonically decreasing with earnings, from 1 at the bottom of the earnings distribution to 0 at the top, and redistribution through unemployment benefits is efficient. Empirical applications show that these interactions between redistribution and unemployment insurance have important quantitative implications.

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Introduction

The provision of redistribution and social insurance has become a defining feature of modern welfare states, and the design of such programs a focal point in public and political discourse. Policymakers and academics alike tend to consider the design of these programs as separate policy questions, echoing the separation between the entities in charge of operating and managing them. This paper questions this separation. Analyzing the interactions between redistribution and social insurance policies in the context of unemployment insurance, it shows that they have important implications for the optimal design of tax-benefit systems.

There are three major sources of interactions between redistribution and social insurance in this context. First, social insurance policies like unemployment insurance affect income tax revenue through their effect on, e.g., unemployment. In other words, the moral hazard costs of social insurance impact the financing of the entire tax-benefit system. Second, the moral hazard costs of social insurance are in turn affected by income taxes, because income taxes directly affect, e.g., the incentives to search for jobs when unemployed. Third, social insurance has a well-targeted redistributive value when benefits depend on (past) earnings.

To analyze these interactions and their policy implications, this paper develops a conceptual framework bridging canonical models of optimal income taxation (Mirrlees, 1971; Saez, 2001, 2002) and optimal unemployment insurance (Baily, 1978; Chetty, 2006a, 2008). I consider a population of individuals with heterogeneous earnings abilities making endogenous labor supply decisions, which generates a motive to provide redistribution at the cost of disincentivizing work. These individuals are exposed to unemployment risk on the labor market and must search for jobs when unemployed, which gives a motive to provide unemployment insurance at the cost of disincentivizing search.

A key conceptual difficulty is that the canonical optimal income tax model is static, while there is a fundamental dynamic element to unemployment insurance: the unemployment benefits received when unemployed depend on past earnings when employed. This has led previous optimal income tax papers featuring involuntary unemployment to assume that all unemployed individuals receive the same amount of benefits, independent of their earnings on the job.[1] In contrast, this paper introduces unemployment benefits that depend on earnings on the job by analyzing the steady-state representation of a dynamic model, where unemployment insurance is well defined.

This steady-state representation yields a simple and tractable framework, where individuals who decide to participate in the labor market spend a fraction of their time employed

[1]This includes Boone and Bovenberg (2004), Hungerbühler, Lehmann, Parmentier, and Van der Linden (2006), Lehmann, Parmentier, and Van der Linden (2011), Sleet and Yazici (2017), Kroft, Kucko, Lehmann, and Schmieder (2020), Hummel (2021).

and the remaining fraction of their time unemployed, while individuals who decide not to participate remain inactive and receive social assistance. In this setting, I characterize the optimal nonlinear tax-transfer schedule when employed and the optimal nonlinear benefit schedule when unemployed in terms of empirically estimable sufficient statistics.

Optimal income taxation trades off equity and efficiency. The optimal tax-transfer schedule thus depends on redistributive concerns and labor supply elasticities as in canonical results. It also depends on unemployment rates, search elasticities, and unemployment benefits through two additional channels. First, higher unemployment rates reduce the size of the income tax base. Second, higher income taxes reduce the job search incentives of the unemployed, which prolong unemployment and generate fiscal externalities.

These two channels are particularly important for the optimal design of taxes and transfers at the bottom of the earnings distribution, where the unemployment rate is high. An Earned Income Tax Credit (a larger transfer to the working poor than to the inactive; hereafter EITC) is more desirable than a Negative Income Tax (a larger transfer to the inactive than to the working poor; hereafter NIT) when search elasticities are large, because an EITC boosts job search among the unemployed.[2]

I show the existence of a Pareto-efficiency condition for tax-benefit systems, which follows from the efficient allocation of consumption between employment and unemployment. This Pareto-efficiency condition transparently extends the Baily-Chetty formula (Baily, 1978; Chetty, 2006a, 2008) and shows that optimal unemployment insurance trades off the insurance and redistributive value of unemployment benefits against their moral hazard costs. These moral hazard costs depend on search elasticities and on net contributions to the entire tax-benefit system (taxes when employed net of benefits when unemployed).

This Pareto-efficiency condition provides a tight link between redistribution and unemployment insurance policies: the steeper the tax-transfer profile is, the flatter the optimal profile of benefits should be, and vice-versa. Optimal net replacement rates (net benefits when unemployed as a fraction of net income when employed) are therefore monotonically decreasing with earnings, starting from 1 at the bottom of the earnings distribution. Moreover, a more redistributive tax-benefit system features higher replacement rates at the bottom and lower replacement rates at the top. Last but not least, efficiency can be reached through Pareto-improving reforms: if the replacement rate is, say, too high at a given earnings level, there exists a joint reduction in taxes when employed and in benefits when unemployed that raises resources for the government, while keeping individuals' utility constant.

[2]An advantage of this framework to study the optimality of an EITC versus a NIT is that the optimal tax-transfer schedule is here continuous at the origin, whereas it features an initial discontinuity in static optimal tax frameworks with participation decisions (see Jacquet, Lehmann, & Van der Linden, 2013). As a result, negative participation taxes go here hand-in-hand with negative marginal tax rates.

The above characterization of optimal tax-benefit schedules is derived in a baseline model that makes a number of simplifying assumptions. First, I abstract from savings in the baseline and assume that individuals are hand-to-mouth. Second, I abstract from eligibility requirements to unemployment insurance and assume that unemployment benefits are not limited in time. Third, I abstract from earnings decisions and assume that labor supply decisions are concentrated along the participation margin. Next, I generalize the previous results in extensions that relax these assumptions.

In a first extension, I introduce liquid savings and illiquid assets and provide a general characterization of optimal policies nesting different savings and assets models. Since liquid savings provide private insurance against unemployment, they reduce the insurance value of unemployment benefits. Assuming that low earners have very little savings (if any) and that top earners have enough savings to fully self-insure against unemployment, savings reinforce the decreasing pattern of optimal replacement rates, from 1 at the bottom of the earnings distribution to 0 at the top, while pushing for marginally lower taxes.

In a second extension, I introduce earnings decisions and derive a sufficient statistics formula for optimal marginal tax rates that extends canonical results in optimal income taxation (Diamond, 1998; Saez, 2001). It shows that negative marginal tax rates (an EITC) may be optimal if there is a large positive correlation between search elasticities and employment taxes (taxes when employed plus benefits when unemployed) across earnings. The Pareto-efficiency condition is almost unaffected, implying that earnings decisions mostly impact optimal replacement rates through their effects on the optimal income tax schedule.

Last, I complement these sufficient statistics results with mechanism design results. First, eligibility requirements to unemployment insurance are key to maintain incentive-compatibility in the presence of earnings decisions. Indeed, conditioning the receipt of unemployment benefits to spending a minimal fraction of time employed allows to eliminate upward (non-local) deviations that consists in low ability individuals spending one day employed at a high-paying job and the rest of their time unemployed with high unemployment benefits. The previous sufficient statistics results thus implicitly assume the existence of such eligibility requirements.[3] Second, redistribution through unemployment benefits is efficient because it relaxes (local) downward-binding incentive-compatibility constraints. The reason is that individuals with higher earnings ability find it optimal to search more and spend more time employed.

An empirical application to the U.S. reveals that actual net replacement rates decrease

[3] These eligibility requirements rule out off-equilibrium deviations and are thus not binding in equilibrium, by assumption. One can extend these results to settings where some of the unemployed do reach benefit exhaustion (although this typically leads to bunching at the threshold).

with earnings, from 1 at the bottom of the earnings distribution to 0 at the top. At the bottom, this is because individuals with extremely low income receive similar transfers from social assistance programs, regardless of their labor market status.[4] At the top, this comes from the fact that unemployment benefits are linearly increasing with earnings up to a cap, above which replacement rates steadily decrease with earnings. While actual replacement rates decrease with earnings, results show that the structure of linearly increasing benefits up to a cap is not Pareto-efficient, providing a scope for Pareto-improving reforms.

To simulate counterfactual policies, I adapt common parametric specifications used in prior work and calibrate the model to match key sufficient statistics as well as observed distributions of unemployment, participation, and earnings, under the existing U.S. tax-benefit system. Analyzing Pareto-efficient tax-benefit systems with varying degrees of redistribution, simulation results suggest that the tight link between redistribution and unemployment insurance has a stabilizing effect on unemployment: unemployment rates are barely affected by changes in redistribution along the Pareto-frontier.

Related literature. This paper contributes to the analysis of optimal redistribution and optimal social insurance policies. A first key contribution is to provide a simple and tractable framework bridging canonical models of optimal income taxation (Mirrlees, 1971; Saez, 2001, 2002) and optimal unemployment insurance (Baily, 1978; Chetty, 2006a, 2008). Nesting canonical results in both literatures allows to transparently analyze how these two problems interact and how these interactions affect optimal policies.

A second key contribution is to show that these interactions have crucial policy implications, starting with their impact on optimal unemployment insurance.[5] While the Baily-Chetty formula is often used to characterize *the* optimal replacement rate, I find that optimal replacement rates monotonically decrease with earnings, in a way that is shaped by redistribution. This speaks to the issue of "*optimal differentation*" of unemployment insurance identified by Spinnewijn (2020) as a key avenue for future research. It also helps connecting unemployment insurance theory to actual policy since replacement rates decrease with earnings in practice. This paper further relates to prior work studying the impact of redistributive concerns on the optimal replacement rate in calibrated macro or life-cycle models (Uren, 2018; Haan & Prowse, 2019; Setty & Yedid-Levi, 2021). Last, the rationale for eligibility requirements to unemployment insurance provided in the mechanism design analysis

[4]Since the unemployed who receive low unemployment benefits (if any) also receive means-tested transfers, these transfers provide unemployment insurance. This is an important fourth source of interactions between redistribution and unemployment policies in practice.

[5]Recent work on optimal unemployment insurance includes Michelacci and Ruffo (2015), Kroft and Notowidigdo (2016), Lawson (2017), Kolsrud, Landais, Nilsson, and Spinnewijn (2018), Landais, Michaillat, and Saez (2018b), Landais and Spinnewijn (2021), Barnichon and Zylberberg (2022).

echoes the one provided by Hopenhayn and Nicolini (2009), which is that the government cannot distinguish opportunistic job quits from involuntary separations.

This paper is closely related to prior work in optimal income taxation featuring involuntary unemployment. This strand of the literature focuses on the optimal tax implications of labor market frictions, leading to equilibrium unemployment, assuming that all the unemployed receive a constant lump-sum transfer (Boone & Bovenberg, 2004; Hungerbühler et al., 2006; Lehmann et al., 2011; Sleet & Yazici, 2017; Kroft et al., 2020; Hummel, 2021).[6] This paper adopts a complementary focus by introducing unemployment benefits that depend on earnings when employed, assuming away general equilibrium effects. It contributes to this literature by analyzing the interactions with unemployment insurance and showing that it is efficient to redistribute through unemployment benefits. It also extends the conditions, analyzed in settings without involuntary unemployment (e.g. Saez, 2002; Lockwood, 2020; Hansen, 2021) and with involuntary unemployment but constant unemployment benefits (e.g. Kroft et al., 2020; Hummel, 2021), under which an EITC or a NIT is optimal.

Last, this paper relates to a broader literature studying redistribution and social insurance.[7] The new dynamic public finance literature analyzes the redistribution and insurance value of taxes in dynamic economies with stochastic shocks, but does not generally consider social insurance programs.[8] Exceptions include Golosov and Tsyvinski (2006) who study optimal disability insurance, but abstract from interactions with redistribution, as well as Michau (2014) and Ndiaye (2020) who study the optimal design of pensions, but focus on the optimal age profile of taxes and benefits.[9] A key distinction is that both disability and retirement are (modelled as) absorbing states which is a crucial element of these dynamic models, whereas unemployment insurance precisely seeks to avoid that unemployment becomes an absorbing state. This paper contributes to this literature by providing a Pareto-efficiency condition for tax-benefit systems that transparently links optimal redistribution and optimal unemployment insurance in a sufficient statistics approach. This could be fruitfully applied to other social insurance policies (e.g. pensions) and extended to dynamic environments.

[6] An alternative explored in Boone and Bovenberg (2006), Boadway and Cuff (2018) is to assume that earnings abilities are observable and introduce ability-specific unemployment benefits. In a second-best setting, da Costa, Maestri, and Santos (2022) show that ability-specific unemployment benefits can be incentive-compatible but are never optimal in the environment that they consider.

[7] A classic result is that social insurance can be an efficient redistributive tool when risk is negatively correlated with earnings and individuals are pooled within the same insurance contract (Rochet, 1991; Cremer & Pestieau, 1996; Boadway, Leite-Monteiro, Marchand, & Pestieau, 2006; Netzer & Scheuer, 2007). Analyzing earnings-specific unemployment benefits, this paper leaves little scope for this pooling logic.

[8] See Golosov, Kocherlakota, and Tsyvinski (2003), Golosov, Tsyvinski, Werning, Diamond, and Judd (2006), Kocherlakota (2010), Weinzierl (2011), Farhi and Werning (2013), Golosov, Troshkin, and Tsyvinski (2016), Findeisen and Sachs (2017), Chang and Park (2021) and the recent review by Stantcheva (2020).

[9] See also Diamond and Mirrlees (1978), Cremer, Lozachmeur, and Pestieau (2004), Golosov, Shourideh, Troshkin, and Tsyvinski (2013), Choné and Laroque (2018), Moser and Olea de Souza e Silva (2019).

Outline. Section 1 introduces the setting. Section 2 characterizes optimal policies under baseline assumptions. Section 3 generalizes these results in extensions. Section 4 presents empirical applications.

1 Setting

This section describes the setting of the model, making a number of simplifying assumptions that are later relaxed in the extensions. First, it presents the steady-state representation used to bridge optimal income tax and optimal unemployment insurance models. Second, it introduces individuals' and the government's problems.

Steady-state representation. Consider a population of individuals with heterogeneous earnings abilities, translating in heterogeneous earnings when employed, z. At any point in time, t, an individual who is active on the labor market can be either employed or unemployed. The government levies a tax-transfer schedule on the employed, $T_e(z)$, and provides unemployment benefits to the unemployed, $B_u(z)$, which depend on earnings in the previous employment spell.[10]

An individual who is employed with earnings z values with utility function $u_e(.)$ its consumption $c_e(z)$, incurs a cost of working $k(z)$, and faces a probability $q(z)$ of becoming unemployed next period. When unemployed, this individual values with utility function $u_u(.)$ its consumption $c_u(z)$, and incurs a cost of searching for jobs $\tilde{\psi}(p,z)$, which increases with the probability p of becoming employed next period. Job search decisions then lead to a probability $p(z)$ of becoming employed next period.

By assumption, this dynamic model is stationary (time-invariant), implying that it converges to a steady state.[11] In the steady state, normalizing the discount factor to one, the utility of an individual with earnings when employed, z, is

$$\underbrace{\frac{p(z)}{q(z)+p(z)}}_{:=e(z)}\Big[\underbrace{u_e\left(c_e(z)\right)-k(z)}_{\text{employed}}\Big]+\underbrace{\frac{q(z)}{q(z)+p(z)}}_{:=1-e(z)}\Big[\underbrace{u_u\left(c_u(z)\right)-\tilde{\psi}(p(z),z)}_{\text{unemployed}}\Big] \tag{1}$$

where the steady-state probability of employment, $e(z)$, which can be interpreted as the fraction of time spent employed on the labor market, is influenced by search efforts. This steady state representation is used throughout the analysis.

[10] To avoid problems of initialization which have generally little impact on the steady state, one can assume that individuals who are active on the labor market start initially employed at $t = 1$.

[11] The assumption of stationarity is made to simplify the exposition. Extensions to non-stationary dynamic models are straightforward, provided that they converge to a steady state.

Baseline assumptions. A number of simplifying assumptions are made in the baseline. First, I assume that there is no savings and that individuals are hand-to-mouth consumers, implying that their consumption when employed is $c_e(z) = z - T_e(z)$. Second, I abstract from problems of eligibility to unemployment insurance and assume that unemployment benefits are not limited in time, implying that consumption when unemployed is $c_u(z) = B_u(z)$. Third, I assume that individuals only make two decisions: they decide on their search efforts when unemployed and on whether to participate in the labor market (extensive margin of labor supply), but earnings when employed which reflect underlying earnings abilities are exogenously given. These assumptions are later relaxed in extensions (see Section 3).

Individuals. Upon participation to the labor market, individuals' search efforts determine the fraction of time spent employed, $e(z)$, and thus indirect utility, $V(z)$, through

$$V(z) := \max_{e} \; e\Big[\underbrace{u_e\left(z - T_e(z)\right) - k(z)}_{\text{employed}}\Big] + (1-e)\Big[\underbrace{u_u\left(B_u(z)\right) - \psi\left(e, z\right)}_{\text{unemployed}}\Big], \tag{2}$$

where the utility functions $u_e(.)$ and $u_u(.)$ are increasing, twice differentiable and concave, and the search cost function $\psi(e, z)$ is increasing, twice differentiable and convex with e.[12] Individuals thus trade off the gains from being more often employed against the costs of searching more when unemployed.

Participation in the labor market requires paying a fixed cost, χ, which is not observed by the government. An individual who remains inactive derives utility, $u_0(.)$, from social assistance which takes the form of a lump-sum transfer, R_0. As a result, an individual with earnings when employed, z, participates in the labor market if and only if

$$\underbrace{V(z) - \chi}_{\text{participating}} \;\; \geq \;\; \underbrace{u_0\left(R_0\right)}_{\text{not participating}} \;, \tag{3}$$

meaning that individuals participate whenever their fixed cost of participation, χ, is lower than the cutoff, $\tilde{\chi}(z) := V(z) - u_0(R_0)$.

The population size is normalized to unity, and the distribution of earnings abilities and participation costs in the population is described by the cumulative and probability distribution functions $F_{\chi,z}(\chi, z)$ and $f_{\chi,z}(\chi, z)$, which are known to the government. The *potential* density of individuals with earnings z when employed is then $f_z(z) := \int_{\chi} f_{\chi,z}(\chi, z) d\chi$. Taking participation decisions into account, the *actual* density of individuals who participate in the labor market with earnings z when employed is $h_z(z) := \int_{\chi \leq \tilde{\chi}(z)} f_{\chi,z}(\chi, z) d\chi$.

[12] Since earnings z are exogenous in the baseline, dependence of $k(z)$ and $\psi(e, z)$ on earnings z are theoretically irrelevant, they are only used in the empirical application to match key empirical moments.

Government. The policymaker uses the tax-benefit system $\{T_e(z), B_u(z), R_0\}$ for redistribution and unemployment insurance. It seeks to maximize a welfarist objective given by the sum of utilities up to a transformation, $G(.)$,

$$\int_z \Big[\underbrace{\int_{\chi \leq \tilde{\chi}(z)} G\Big(V(z) - \chi\Big)}_{\text{participating}} + \underbrace{\int_{\chi \geq \tilde{\chi}(z)} G\Big(u(R_0)\Big)}_{\text{not participating}} \Big] dF_{\chi,z}(\chi, z). \tag{4}$$

Redistribution involves the comparison of utilities across individuals, it is thus driven both by the concavity of the transformation $G(.)$ and by the concavity of individuals' utility from consumption. In contrast, insurance is only driven by the latter as it solely relates to the maximization of expected utility $V(z)$ at each earnings z.

The government's resource constraint is

$$\int_z \Big[\underbrace{\int_{\chi \leq \tilde{\chi}(z)} \Big(e(z)\, T_e(z) - (1 - e(z))\, B_u(z)\Big)}_{\text{participating}} - \underbrace{\int_{\chi \geq \tilde{\chi}(z)} R_0}_{\text{not participating}} \Big] dF_{\chi,z}(\chi, z) \geq Exp, \tag{5}$$

stating that the taxes levied on the employed must finance benefits to the unemployed, transfers to the inactive, and an exogenous expenditure requirement, Exp.[13]

2 Optimal policies

This section presents sufficient statistics characterizations of optimal tax and optimal benefit schedules in the baseline model. They are the key results of the paper, highlighting the tight link between redistribution and unemployment insurance. Policy implications are discussed in an application where optimal policies take a particularly simple form.

2.1 Sufficient statistics

To compute the fiscal and welfare impacts of tax-benefit reforms, one needs three types of sufficient statistics. The first two measure job search and participation responses, while the third measures the social value of individual utility changes.

Job search semi-elasticities measure the percentage increase in the time spent unemployed, $1 - e(z)$, of individuals who are active on the labor market with earnings when

[13] Introducing a separate contribution used to finance unemployment benefits does not affect the results as long as all tax-benefit instruments are designed by the government – this is no longer true if unemployment insurance is managed by a separate entity with a different objective.

employed, z, upon a unit increase in taxes when employed or in benefits when unemployed,[14]

$$\mu_e(z) := \frac{1}{1-e(z)}\frac{\partial(1-e(z))}{\partial T_e(z)}, \quad \mu_u(z) := \frac{1}{1-e(z)}\frac{\partial(1-e(z))}{\partial B_u(z)}. \tag{6}$$

Participation semi-elasticities measure the (absolute) percentage change in the density of agents, $h_z(z)$, who are active on the labor market with earnings when employed, z, upon a unit increase in taxes when employed or in benefits when unemployed,

$$\pi_e(z) := \frac{1}{h_z(z)}\frac{\partial h_z(z)}{\partial(z-T_e(z))}, \quad \pi_u(z) := \frac{1}{h_z(z)}\frac{\partial h_z(z)}{\partial B_u(z)}. \tag{7}$$

Social marginal welfare weights measure the social value of individual utility changes upon a unit increase in consumption when employed or when unemployed,

$$g_e(z) := \frac{\overline{G'(V(z)-\chi)}}{\lambda}u'(c_e(z)), \quad g_u(z) := \frac{\overline{G'(V(z)-\chi)}}{\lambda}u'(c_u(z)), \tag{8}$$

where $\overline{G'(V(z)-\chi)} := \frac{1}{h_z(z)}\int_{\chi\leq\tilde{\chi}(z)} G'(V(z)-\chi)dF_{\chi,z}(\chi,z)$ is the average social marginal value of private utility changes for participants with earnings z and λ is the social marginal value of public funds. Any motive to redistribute translates into social marginal welfare weights that decrease with incomes, and the stronger this motive for redistribution is, the stronger the decreasing pattern of welfare weights.

2.2 Optimal tax and benefit schedules

Using a perturbation approach and characterizing optimal tax and benefit schedules as those that cannot be improved through tax-benefit reforms yields the following result:[15]

Proposition 1. *At the optimum, the tax-transfer schedule when employed, $T_e(z)$, satisfies at each earnings, z,*

$$\underbrace{\left(T_e(z)+R_0\right)}_{\textit{participation tax}}\pi_e(z)-(1-e(z))\underbrace{\left(T_e(z)+B_u(z)\right)}_{\textit{employment tax}}\left(\pi_e(z)-\mu_e(z)\right) = e(z)\underbrace{\left(1-g_e(z)\right)}_{\textit{mechanical effect}}, \tag{9}$$

[14]These semi-elasticities are not independent, but linked through the structure of the model. In the baseline model, assuming interior solutions to individuals' problem, we have $\frac{\mu_e(z)}{u'(z-T_e(z))} = \frac{\mu_u(z)}{u'(B_u(z))}$.

[15]At each earnings, z, there are three optimality conditions: (i) an optimal tax formula for $T_e(z)$, (ii) an optimal benefit formula for $B_u(z)$, (iii) a Pareto-efficiency formula for $\{T_e(z), B_u(z)\}$. Since two of these conditions imply the third, I characterize the optimum with (i) and (iii), relegating (ii) to the Appendix.

and a Pareto-efficient tax-benefit system, $\{T_e(z), B_u(z)\}$*, satisfies at each earnings,* z*,*

$$\underbrace{\frac{u_u'(c_u(z))}{u_e'(c_e(z))} - 1}_{\textit{mechanical effect}} = \frac{\mu_u(z)B_u(z)}{e(z)^2}\left[1 + \frac{1}{B_u(z)}\underbrace{\Big(e(z)T_e(z) - (1-e(z))\,B_u(z)\Big)}_{\textit{net contribution to tax-benefit system}}\right]. \tag{10}$$

Optimal tax formula (9) generalizes that of, e.g., Saez (2002). Absent unemployment, a unit increase in taxes, $T_e(z)$, generates a unit increase in tax revenue but reduces the utility of each individual at that earnings level, translating into a mechanical effect, $1 - g_e(z)$. This reform also decreases participation by a factor $\pi_e(z)$, which in turn decreases revenue by the *participation tax*, $T_e(z) + R_0$, measuring foregone taxes and additional transfers provided to the newly inactive. Balancing these two effects yields the optimal tax formula,

$$\big(T_e(z) + R_0\big)\pi_e(e) = \big(1 - g_e(z)\big), \tag{11}$$

which formalizes a trade-off between equity, encapsulated in welfare weights, and efficiency, related to the impact of participation on the size of the tax base (e.g. Saez, 2002).

Introducing unemployment and unemployment insurance affects both equity and efficiency. First, unemployment dampens the mechanical effect because individuals are only employed a fraction of the time, $e(z)$. This pushes for lower taxes. Second, unemployment reduces the revenue losses from participation responses by a factor $(1-e(z))(T_e(z)+B_u(z))$, because individuals do not pay taxes and receive benefits when unemployed. This pushes for higher taxes. Third, a tax increase has a negative effect on job search and increases unemployment by $(1-e(z))\mu_e(z)$. This in turn decreases revenue by the *employment tax*, $T_e(z)+B_u(z)$, measuring foregone taxes and additional benefits provided to the unemployed. This pushes for lower taxes. Overall, the impact on optimal taxes is thus ambiguous, and depends on the relative importance of participation and search responses, on the unemployment rate, and on the schedule of unemployment benefits.

Pareto-efficiency condition (10) generalizes the Baily-Chetty formula for optimal unemployment insurance. Consider a unit increase in benefits, $B_u(z)$, combined with an increase in taxes, $T_e(z)$, that leaves expected utility, $V(z)$, constant. By construction, this joint reform mechanically raises revenue in proportion to $\frac{u_u'(c_u(z))}{u_e'(c_e(z))} - 1$ and does not affect welfare nor participation (left-hand side).[16] It however affects search negatively and increases unemployment in proportion to $\frac{\mu_e(z)}{e(z)}$, which in turn decreases revenue by the employment tax, $T_e(z) + B_u(z)$. Rewriting this fiscal externality leads to Pareto-efficiency condition (10),

[16] By the envelope theorem, $dV(z) = e(z)u_e'(c_e(z))dT_e(z) - (1-e(z))u_u'(c_u(z))dB_u(z)$, and the mechanical effect on revenue is $e(z)dT_e(z) - (1-e(z))dB_u(z)$. Setting $dT_e(z) = \frac{(1-e(z))u_u'(c_u(z))}{e(z)u_e'(c_e(z))}dB_u(z)$ gives the result.

featuring the net contribution to the tax-benefit system (right-hand side).[17]

The standard Baily-Chetty formula corresponds to the case with a representative earnings level, where taxes (or contributions) exactly finance benefits implying that this net contribution is zero. This yields

$$\frac{u_u'(c_u)}{u_e'(c_e)} - 1 = \frac{\mu_u^{elast}}{e^2}, \tag{12}$$

where $\mu_u^{elast} := \mu_u B_u$ is the elasticity of unemployment duration with respect to the amount of unemployment benefit, holding taxes constant.[18] This formula characterizes optimal unemployment insurance through the optimal replacement rate, $\frac{c_u}{c_e}$. It trades-off the consumption smoothing value of insurance, measured by marginal utility of consumption across states, against the moral hazard cost of insurance measured by the search elasticity, μ_u^{elast}, and the (un)employment rate.

Introducing heterogeneous earnings and redistribution affects both sides of the trade-off. First, consumption smoothing benefits become earnings-specific, because individuals with higher earnings have higher consumption. This not only reflects the insurance value of unemployment benefits, but also their redistributive value, implying that the total value of providing unemployment insurance decreases with earnings. Second, efficiency costs also become earnings-specific, because redistribution implies that the net contribution to the tax-benefit system increases with earnings. The total cost of providing unemployment insurance thus increases with earnings. Overall, this implies that optimal replacement rates decrease with earnings, in a way that depends on redistribution.

2.3 Policy implications

To analyze the policy implications of these results, let assume a log utility from consumption, $u_e(c) = u_u(c) = \log(c)$. Introducing the participation elasticity, $\pi_e^{elast}(z) := (z - T_e(z))\pi_e(z)$,

[17] Indeed, the total change in unemployment is $d(1-e(z)) = (1-e(z))\mu_e(z)dT_e(z) + (1-e(z))\mu_u(z)dB_u(z)$ where $\frac{\mu_e(z)}{u'(c_e(z))} = \frac{\mu_u(z)}{u'(c_u(z))}$, while $T_e(z) + B_u(z) = \frac{B_u(z)}{e(z)}\left[1 + \frac{1}{B_u(z)}\left(e(z)T_e(z) - (1-e(z))B_u(z)\right)\right]$.

[18] This is formula (14) in Chetty and Finkelstein (2013), with the slight modification that they use a *total* elasticity, $\varepsilon = \frac{\mu_u^{elast}}{e}$, incorporating changes in both the benefit level and the contribution used to finance it.

the optimal tax-benefit system then satisfies,

$$T_e(z)=\frac{e(z)}{e(z)+\pi_e^{elast}(z)}\left[z-\frac{\overline{G'(V(z)-\chi)}}{\lambda}+(1-e(z))\frac{\pi_e^{elast}(z)-\mu_u^{elast}(z)}{e(z)+\mu_u^{elast}(z)}z\right]-\frac{\pi_e^{elast}(z)}{e(z)+\pi_e^{elast}(z)}R_0, \tag{13}$$

$$B_u(z)=\frac{e(z)}{e(z)+\mu_u^{elast}(z)}z-T_e(z). \tag{14}$$

EITC vs NIT. Equation (13) characterizes the optimal tax-transfer schedule $T_e(z)$ in a Pareto-efficient tax-benefit system. It shows that unemployment interacts with the standard redistribution-participation trade-off and adds in particular a novel efficiency term weighting participation and search elasticities. This has interesting implications for the shape of the tax-transfer schedule at the bottom of the earnings distribution where an important policy question is whether transfers should increase with earnings (EITC) or decrease with earnings (NIT) at the optimum (Saez, 2002; Jacquet et al., 2013; Kroft et al., 2020; Hansen, 2021; Hummel, 2021).[19]

At the origin of the earnings distribution, (13) implies that the tax-transfer is continuous, $T_e(0)=-R_0$, whenever individuals find it optimal to always remain unemployed, $e(0)=0$.[20] Since disposable incomes when employed and unemployed converge at the origin, individuals do find it optimal such that the optimal tax-transfer schedule is continuous. This contrasts with pure extensive margin models which feature a discontinuity at the origin (Jacquet et al., 2013). Intuitively, individuals with extremely low earnings who decide to participate in the labor market are always employed in a pure extensive margin model, whereas they are here mostly unemployed which smoothes out this discontinuity. This continuity property makes this framework particularly attractive to study the optimality of an EITC versus a NIT and implies that a negative participation tax can only arise through negative marginal tax rates.

Whether transfers at the origin increase (EITC) or decrease (NIT) with earnings in the optimum depends here on many elements. As identified by Hansen (2021) it depends first on the evolution of participation elasticities and redistribution concerns across earnings. But it also crucially depends on the evolution of the (un)employment rate and of search elasticities across earnings. A necessary condition for an EITC, $T_e(z) \leq -R_0$, to be optimal is that the first square bracket term in (13) be negative. The larger search elasticities are, the more likely this condition is met and the more likely an EITC is optimal. If the second term

[19] Note that policy implications for the optimal tax-transfer schedule when employed are here conditional on the existence of a Pareto-efficient benefit schedule when unemployed that adjusts in the background.

[20] $\lim_{z\to 0} T_e(z) = -\frac{e(0)}{e(0)+\pi_e^{elast}(0)}\frac{\overline{G'(V(0)-\chi)}}{\lambda} - \frac{\pi_e^{elast}(0)}{e(0)+\pi_e^{elast}(0)}R_0$, which is equal to R_0 if and only if $e(0) = 0$, under the assumption that $\pi_e^{elast}(0)$ is finite.

was always equal to $-R_0$, this condition would be both necessary and sufficient. However, as earnings increase, the second term diverges from $-R_0$ at a rate that depends on the evolution of unemployment rates and participation elasticities across earnings. Empirically, there exists a number of estimates of participation elasticities at low incomes, but estimates of search elasticities across earnings seem to be lacking. Actual policy implications thus remain an open empirical question.

Unemployment benefits & replacement rates. The equation characterizing $B_u(z)$ follows from Pareto-efficiency condition (10) and highlights the tight link between redistribution and unemployment insurance. To see this link most transparently, let further assume that the tax-transfer system is linear, $T_e(z) = \tau z - R_0$, to obtain

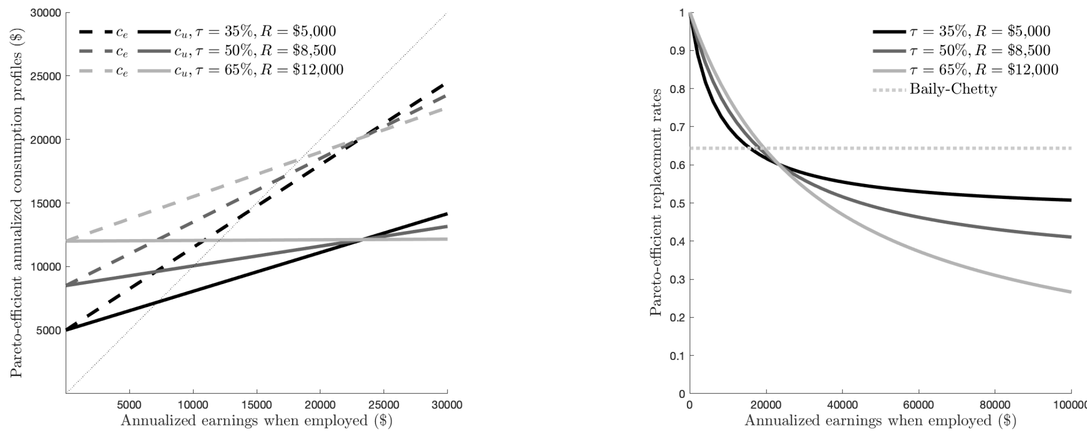
$$B_u(z) = \left(\frac{e(z)}{e(z) + \mu_u^{elast}(z)} - \tau \right) z + R_0. \tag{15}$$

First, this condition shows that unemployment benefits converge to the demogrant R_0 at the origin, where earnings z go to zero. Second, it shows that the higher the redistribution through the demogrant R_0, the higher unemployment benefits should be (and vice-versa). In other words, it is efficient to redistribute both when individuals are employed and unemployed, and thus through unemployment benefits. Third, this condition implies that the higher the tax rate τ is, the steeper the tax-transfer schedule is, and the flatter the benefits schedule should be (and vice-versa). With a high enough tax rate, it can even be optimal to replace unemployment insurance with a flat benefits system that provides a lump-sum transfer equal to the demogrant R_0. For instance, assuming a constant search elasticity $\mu_u^{elast} = 0.5$ (Schmieder & Von Wachter, 2016) and a constant 5% unemployment rate, it is optimal to do so when the tax rate is $\tau = 65\%$. These properties are illustrated on the left panel of Figure 1, which represents Pareto-efficient benefit profiles for different tax rates τ and demogrants R_0.

The tight link between redistribution and unemployment insurance implies that any Pareto-efficient tax-benefit system features net replacement rates, $\frac{B_u(z)}{z - T_e(z)}$, that decrease with earnings in a way that is shaped by redistribution.[21] At the bottom, the optimal replacement rate converges to 1, because both $T_e(z)$ and $B_u(z)$ converge to R_0. Intuitively, individuals with no earnings when employed find it optimal to stay always unemployed, they are thus observationally equivalent to individuals who stay inactive, and Pareto-efficiency implies treating both in the same way. At the top, the optimal replacement rate converges to $\frac{1}{1-\tau} \left(\frac{e}{e + \mu_u^{elast}} - \tau \right)$, which is strictly lower than 1 and decreasing with the tax rate τ. Overall,

[21] In contrast, the Baily-Chetty formula implies a constant replacement rate, $\frac{B_u(z)}{z - T_e(z)} = \frac{e^2}{e^2 + \mu_u^{elast}}$.

Figure 1: Pareto-efficient benefit profiles (left) and replacement rates (right)



Note: Both Pareto-efficient benefit profiles, $c_u(z) = B_u(z)$, represented on the left panel and Pareto-efficient net replacement rates, $\frac{B_u(z)}{z - T_e(z)}$, represented on the right panel follow from condition (15), derived assuming a log consumption utility and a linear tax-transfer schedule $T_e(z) = \tau z - R_0$. For illustration purposes, different tax rate τ and demogrant R_0 are considered, while holding $\mu_u^{elast} = 0.5$ and $e = 95\%$ constant.

this implies that in economies with more redistributive tax-transfer schedules, net replacement rates should be higher at low earnings and lower at high earnings, as illustrated on the right panel of Figure 1.

3 Extensions

This section extends previous results, relaxing baseline assumptions. First, I show that introducing savings has small downwards effects on optimal income taxes, and that it reinforces the decreasing pattern of optimal replacement rates. Second, I consider endogenous earnings decisions and derive an extended ABC formula for optimal marginal tax rates. It highlights that when search responses are large, the optimal tax-transfer schedule may feature negative marginal tax rates (e.g. an EITC). Endogenous earnings have otherwise little impact on the previous Pareto-efficiency condition, and thus on the link between optimal redistribution and optimal unemployment insurance. Third, I analyze the problem in a mechanism design approach and show that eligibility requirements to unemployment insurance are key to maintain incentive compatibility with endogenous earnings.

3.1 Savings reinforce the decreasing pattern of replacement rates

In the baseline, unemployment insurance is the only mean of insurance against unemployment, by assumption. I now extend the analysis to account for savings as a private mean of insurance against unemployment.

Setting. Consider liquid savings (e.g. money on a bank account) that can be used to smooth consumption when unemployed as well as illiquid assets (e.g. retirement plans) that cannot be used when unemployed but provide other benefits. Let $s(z)$ and $a(z)$ denote the liquid savings and illiquid assets accumulated by an individual with earnings z when employed, such that consumption when employed is $c_e(z) = z - T_e(z) - s(z) - a(z)$. Normalizing interest rates to zero, if this individual spends a fraction of time e employed, consumption when unemployed is then $c_u(z) = B_u(z) + \frac{e}{1-e} s(z)$ and the total amount of illiquid assets accumulated is $ea(z)$.[22]

Individuals' expected utility when participating in the labor market becomes

$$V(z) := \max_e \; e\Big[u_e\left(c_e(z)\right) - k\left(z\right)\Big] + (1-e)\Big[u_u\left(c_u(z)\right) - \psi\left(e, z\right)\Big] + U\left(ea(z)\right) \qquad (16)$$

where $U(ea(z))$ is the utility derived from illiquid assets (e.g. utility in retirement) and search decisions factor in their potential effects on savings and assets accumulation.

This setting nests many different savings and assets models, among which two benchmark that are important in this context. A first benchmark is privately-optimal savings and assets, i.e., savings and assets levels that exactly maximize expected utility. First-order conditions for savings and assets choices imply that in this case the marginal utility of consumption is equalized across states of the world:

$$u_e'(c_e(z)) = u_u'(c_u(z)), \qquad u_e'(c_e(z)) = U'(e(z)a(z)). \qquad (17)$$

A second benchmark is one where savings and assets are exogenous to the tax-benefit system, implying that they do not respond to tax-benefit reforms:

$$\frac{\partial s(z)}{\partial T_e(z)} = \frac{\partial s(z)}{\partial B_u(z)} = 0, \qquad \frac{\partial a(z)}{\partial T_e(z)} = \frac{\partial a(z)}{\partial B_u(z)} = 0. \qquad (18)$$

Optimal policies. Savings and assets responses to tax-benefit reforms become new relevant sufficient statistics that enter the characterization of optimal tax-benefit schedules:

Proposition 2. *At the optimum, the tax-transfer schedule when employed, $T_e(z)$, satisfies*

[22] Implicitly, this formulation of the problem defines liquid savings as funds used for consumption when unemployed. This can be seen as a convention where unused residual savings are turned into illiquid assets.

at each earnings, z,

$$\big(T_e(z)+R_0\big)\pi_e(z) - (1-e(z))\big(T_e(z)+B_u(z)\big)\big(\pi_e(z)-\mu_e(z)\big) = e(z)\big(1-g_e(z)\big) \quad (19)$$
$$+\, e(z)g_e(z)\left[\left(\frac{u_u'(c_u(z))}{u_e'(c_e(z))}-1\right)\frac{\partial s(z)}{\partial T_e(z)} + \left(\frac{U'(ea(z))}{u_e'(c_e(z))}-1\right)\frac{\partial a(z)}{\partial T_e(z)}\right],$$

and a Pareto-efficient tax-benefit system, $\{T_e(z), B_u(z)\}$*, satisfies at each earnings, z,*

$$K_r(z)\frac{u_u'(c_u(z))}{u_e'(c_e(z))}-1=\left[1+K_\mu(z)K_r(z)\frac{1-e(z)}{e(z)}\right]\frac{\mu_u(z)B_u(z)}{e(z)}\left[1+\frac{e(z)T_e(z)-(1-e(z))B_u(z)}{B_u(z)}\right], \quad (20)$$

with $K_r(z)$ *and* $K_\mu(z)$ *defined such that a unit increase in benefits and a* $K_r(z)\frac{1-e(z)}{e(z)}\frac{u_u'(c_u(z))}{u_e'(c_e(z))}$ *increase in taxes leave utility constant, and such that* $K_\mu(z)\frac{\mu_u(z)}{u_u'(c_u(z))}=\frac{\mu_e(z)}{u_e'(c_e(z))}$.

Optimal tax formula (19) shows that the introduction of savings and assets adds a corrective term measuring the welfare impact of savings and assets responses to tax reforms (second line). Since tax increases tend to decrease the amount of savings and assets accumulated, thereby reducing expected utility, this corrective term generally pushes for lower taxes. However, this term vanishes when savings and assets are at their privately optimal level as it implies that changes in savings and assets do not affect expected utility, or when savings and assets are exogenous to the tax-benefit system as it implies that they do not respond to tax increases. In these two benchmarks, the optimal tax formula thus remains unchanged (although the value of sufficient statistics might change). As a result, at earnings z, savings and assets meaningfully impact the optimal amount of tax and transfer only when they are sufficiently responsive to taxes and, yet, low enough to trigger important welfare changes.[23] This suggests that the presence of savings and assets may push for marginally lower optimal levels of tax and transfer (all else equal).

Savings and assets enter Pareto-efficiency condition (20) in three distinct ways. First, because changes in benefits and changes in taxes might trigger savings and assets responses with first-oder welfare effects, there is a wedge $K_r(z)$, defined such that a unit increase in benefits combined with a $K_r(z)\frac{1-e(z)}{e(z)}\frac{u_u'(c_u(z))}{u_e'(c_e(z))}$ increase in taxes leaves indirect utility $V(z)$ constant. In the two benchmarks where assets and savings are either utility-maximizing or exogenous, this wedge disappears, $K_r(z) = 1$. Otherwise, it tends to be lower than 1 and pushes for lower unemployment benefits, as it captures the negative welfare impact induced by the crowing-out of private insurance by public insurance.

Second, by the same token, there is a wedge $K_\mu(z)$ in the structural relationship linking

[23]This is most likely to be the case for individuals with median earnings who do save but are nonetheless constrained in their ability to do so. Very low earners are likely too financially constrained to exhibit strong savings responses, while the savings responses of high earners are unlikely to imply large welfare changes.

semi-elasticities $\mu_e(z)$ and $\mu_u(z)$, used to compare the unemployment responses induced by changes in benefits and changes in taxes. This wedge is defined such that $K_\mu(z)\frac{\mu_u(z)}{u'_u(c_u(z))} = \frac{\mu_e(z)}{u'_e(c_e(z))}$ and it only disappears when savings and assets are exogenous and, in addition, savings are equal to zero (baseline assumptions). Otherwise, this wedge tends to be lower than 1 and pushes for higher unemployment benefits, as savings tend to reduce total unemployment responses to joint reforms by providing a private mean of insurance.

Third, and most importantly, savings and assets enter on the left-hand side of (20) through $c_u(z)$ and $c_e(z)$. More savings to smooth consumption when unemployed imply a smaller gap in marginal utilities between employment and unemployment, which pushes for lower unemployment benefits. When the amount of savings is privately optimal, the gap in marginal utilities disappears: individuals perfectly self-insure themselves and there is no unemployment benefits at the optimum. This is often the case studied in the macro literature concluding that the optimal replacement rate is approximately 0%. In contrast, the public literature tends to assume that savings are exogenous (sometimes with the justification that they are accumulated prior to the unemployment spell) and insufficient, thereby concluding that the optimal replacement rate is much higher.

This framework allows to reconcile these two views by assuming that individuals with low earnings when employed have insufficient savings (if any), confirming the optimality of high replacement rates at the bottom, and that individuals with high earnings when employed are able to perfectly self-insure themselves, providing a justification to provide no unemployment benefits at the top. Hence, savings tend to reinforce the decreasing pattern of optimal replacement rates, with optimal replacement rates converging to 0 at the top.

3.2 Endogenous earnings and negative marginal tax rates

In the baseline, labor supply decisions were restricted to participation (extensive margin) and I now consider an extension to earnings decisions (intensive margin).

Setting. Following Mirrlees (1971), individuals in the population are endowed with heterogeneous earnings ability, ω. Assume that the cost, $k(z;\omega)$, associated with working at an earnings level z is increasing and convex with earnings, and decreasing with ability. Further assume that the search cost, $\psi(e,z;\omega)$, required to spend a fraction of time e employed at earnings z is increasing and convex with the time spent employed, increasing with earnings, and decreasing with ability.

Going back to the case without savings, consumption when employed is $c_e(z) = z - T_e(z)$ and consumption when unemployed is $c_u(z) = B_u(z)$. Individuals who participate in the labor market choose their earnings and their search efforts through

$$V(\omega) := \max_z \left\{ \max_e \; e\Big[u_e\left(c_e(z)\right) - k(z;\omega)\Big] + (1-e)\,\Big[u_e\left(c_u(z)\right) - \psi(e,z;\omega)\Big]\right\}, \tag{21}$$

and the solution to this maximization problem defines earnings as function of ability, $z(\omega)$, and time spent employed as a function of earnings and ability, $e(z;\omega)$.

Monotonicity. Assume that at all ability levels the solution to this problem is strictly interior, meaning that first-order conditions for e and z hold and that second-order conditions are satisfied with strict inequality, $(SOC)_e < 0$ and $(SOC)_z < 0$. Then, it can be shown that

$$\frac{dz(\omega)}{d\omega} = \frac{1}{(SOC)_z}\left[e\frac{\partial^2 k(z;\omega)}{\partial z\partial\omega} + (1-e)\frac{\partial^2\psi(e,z;\omega)}{\partial z\partial\omega} + (SOC)_e\frac{\partial e}{\partial z}\frac{\partial e}{\partial\omega}\right]\Big|_{e=e(z;\omega),\, z=z(\omega)} \tag{22}$$

meaning that earnings increase monotonically with ability when the term in the square bracket is negative. In the absence of unemployment, $e=1$, this comes down to the Spence-Mirrlees single-crossing condition *for work*, $\frac{\partial^2 k(z;\omega)}{\partial z\partial\omega} < 0$, stating that individuals with higher ability need a lower compensation to marginally increase their earnings when employed.

In the presence of unemployment, this condition alone does not guarantee earnings monotonicity. First, one needs to assume a Spence-Mirrlees single-crossing condition *for search*, $\frac{\partial^2\psi(e,z;\omega)}{\partial z\partial\omega} < 0$, stating that individuals with higher ability need a lower compensation to search for marginally higher paying jobs when unemployed. Second, in the "normal" case where search decisions are such that the time spent employed decreases with earnings, $\frac{\partial e(z;\omega)}{\partial z} \leq 0$, and increases with ability, $\frac{\partial e(z;\omega)}{\partial\omega} \geq 0$, the laste term in (22) is positive. One thus needs to assume that the sum of these three terms remains negative.

Assumption 1. *At all ability levels, (i) first- and second-order conditions for z and e hold strictly; (ii) preferences satisfy a Spence-Mirrlees single-crossing conditions for work, $\frac{\partial^2 k(z;\omega)}{\partial z\partial\omega} < 0$, and for search, $\frac{\partial^2\psi(e,z;\omega)}{\partial z\partial\omega} < 0$; (iii) search decisions are such that at $z = z(\omega)$,*

$$(SOC)_e\frac{\partial e(z;\omega)}{\partial z}\frac{\partial e(z;\omega)}{\partial\omega} < -e(z;\omega)\frac{\partial^2 k(z;\omega)}{\partial z\partial\omega} - (1-e(z;\omega))\frac{\partial^2\psi(e(z;\omega),z;\omega)}{\partial z\partial\omega}. \tag{23}$$

Under these assumptions, there exists a strictly monotonic mapping between earnings z and ability ω. We exploit this monotonicity property to abuse notations and eliminate the dependence on ability ω in the rest of the sufficient statistics analysis.

Sufficient statistics. Additional sufficient statistics are necessary to measure earnings responses to tax-benefit reforms as well as search responses to changes in earnings.

Compensated earnings semi-elasticities measure earnings responses to changes in marginal net-of-tax rates when employed and marginal benefit rates when unemployed,

$$\zeta_e(z) := \frac{1}{z}\frac{\partial z}{\partial(1-T_e'(z))}, \qquad \zeta_u(z) := \frac{1}{z}\frac{\partial z}{\partial B_u'(z)}. \tag{24}$$

Income effects semi-parameters measure (absolute) earnings responses to changes in the amount of tax-transfer when employed and of benefit when unemployed,

$$\eta_e(z) := \frac{\partial z}{\partial T_e(z)}, \qquad \eta_u(z) := -\frac{\partial z}{\partial B_u(z)}. \tag{25}$$

The cross-effect parameter measures responses in the fraction of time spent unemployed, $1-e(z)$, to changes in earnings,

$$\xi_z^{1-e}(z) := \frac{\partial(1-e(z))}{\partial z}. \tag{26}$$

Optimal policies. Equipped with these sufficient statistics, I provide the following characterization of optimal tax-benefit schedules in the presence of endogenous earnings decisions:

Proposition 3. *At the optimum, the tax-transfer schedule when employed, $T_e(z)$, satisfies at each earnings, z,*

$$\begin{aligned}&\Big[\big(e(z)T_e'(z) - (1-e(z))B_u'(z)\big) - \big(T_e(z)+B_u(z)\big)\xi_z^{1-e}(z)\Big]\zeta_e(z)\,z\,h_z(z) \\ &= \int_{x\geq z}\Big\{e(x)(1-g_e(x)) - \big(T_e(x)+R_0\big)\pi_e(x) + (1-e(x))\,\big(T_e(x)+B_u(x)\big)\,(\pi_e(x)-\mu_e(x)) \\ &\qquad + \Big[\big(e(x)T_e'(x)-(1-e(x))B_u'(x)\big) - \big(T_e(x)+B_u(x)\big)\xi_z^{1-e}(x)\Big]\eta_e(x)\Big\}h_z(x)dx\end{aligned} \tag{27}$$

and a Pareto-efficient tax-benefit system, $\{T_e(z), B_u(z)\}$, satisfies at each earnings, z,

$$\begin{aligned}\frac{u_u'(c_u(z))}{u_e'(c_e(z))}-1 &= \frac{\mu_u(z)B_u(z)}{e(z)^2}\left[1+\frac{1}{B_u(z)}\Big(e(z)T_e(z)-(1-e(z))B_u(z)\Big)\right] \\ &\quad + \frac{\eta_u(z)}{1-e(z)}\Big[\big(e(z)T_e'(z)-(1-e(z))B_u'(z)\big) - \big(T_e(z)+B_u(z)\big)\xi_z^{1-e}(z)\Big] \\ &\qquad \times\left[1-\frac{1-T_e'(z)}{B_u'(z)}\frac{u'(B_u(z))}{u'(z-T_e(z))}\frac{u''(z-T_e(z))}{u''(B_u(z))}\right].\end{aligned} \tag{28}$$

Optimal tax formula (27) is an ABC-type formula for optimal marginal tax rates (Diamond, 1998; Saez, 2001). Absent unemployment, $e(z)=1$, this formula boils down to the formula derived in Jacquet et al. (2013),

$$T_e'(z)\zeta_e(z)zh_z(z) = \int_{x\geq z} \Big\{(1-g_e(x)) - \big(T_e(x)+R_0\big)\pi_e(x) + T_e'(z)\eta_e(x)\Big\} h_z(x)dx. \qquad (29)$$

The left-hand side translates the fact that an increase in the marginal tax rate around earnings level z triggers substitution effects around this earnings level. This induces negative earnings responses, $\zeta_e(z)zh_z(z)$, that decreases government revenue in proportion to the marginal tax rate $T_e'(z)$. The right-hand side measures the impacts of the corresponding increase in tax liability at all earnings above z. The first term measures mechanical effects, the second term measures fiscal externalities from participation responses, and the third term measures fiscal externalities from earnings responses due to income effects. In this setting, negative marginal tax rates can be optimal at the bottom when social marginal welfare weights, g_e, are sufficiently above 1 and participation semi-elasticities are sufficiently small (resp. large) at earnings levels with a positive (resp. negative) participation tax, $T_e(x)+R_0$.

Introducing unemployment and unemployment insurance implies that negative earnings responses around earnings level z, $\zeta_e(z)zh_z(z)$, decrease government revenue through foregone tax revenues only when individuals are employed. Moreover, negative earnings responses paradoxically increase government revenue through the provision of lower benefits when unemployed (when $B_u'(z) \geq 0$) and through an increase in the time spent employed (when $\xi_z^{1-e}(z) \geq 0$). Looking at the impact of changes in tax liability at earnings above z on the second line of (27), one can recognize the terms that appear in the baseline optimal tax formula: unemployment dampens mechanical effects, calls for a correction of the fiscal externalities induced by participation responses, and introduces novel fiscal externalities induced by search responses. Last, fiscal externalities from earnings responses due to income effects on the third line of (27) are also dampened.

Overall, these corrections have ambiguous effects on marginal tax rates and heavily influence the desirablity of negative marginal tax rates. At earnings with a negative employment tax, $T_e(z)+B_u(z)$, participation responses larger than search responses, $\pi_e(z) \geq \mu_e(z)$, contribute to the optimality of negative marginal tax rates. In contrast, at earnings where the employment tax is positive, it is larger search responses that contribute to the optimality of negative marginal tax rates.

Pareto-efficiency condition (28) retains the same structure as in the baseline. Earnings responses only adding a corrective term stemming from income effects, proportional to the income effect parameter $\eta_u(z)$. Here, joint changes in tax-benefit levels are engineered through joint two-bracket reforms and while earnings responses related to substitution effects exactly cancel out each other, those related to income effects do not.[24] These earnings responses generate fiscal externalities proportional to the expected marginal tax-benefit rate, $e(z)T_e'(z) - (1-e(z))B_u'(z)$, and to the employment tax scaled by adjustments in search decisions, $\big(T_e(z)+B_u(z)\big)\xi_z^{1-e}(z)$. The last term contrasts the positive income effects from the increase in taxes with negative income effects from the increase in benefits.

Empirically, these corrections seem quantitatively small at low earnings. At high earnings the presence of savings implies very low benefits of providing unemployment insurance, regardless of the strength of these fiscal externalities. As a result, accounting for endogenous earnings decisions does not seem crucial for optimal unemployment insurance. In particular, the tight link betwen redistribution and unemployment insurance highlighted in previous sections and its impact on optimal benefit profiles and the decreasing pattern of replacement rates are unaffected.

3.3 Mechanism design: Eligibility and implementation

Previous sufficient statistics results characterize optimal policies under the assumption that solutions to individuals' maximization problems are interior (Assumption 1). Turning to a mechanism design approach, I characterize optimal allocations subject to feasibility and incentive-compatibility constraints. This allows to shed light on the conditions under which the previous results are valid and to study the issue of implementation, that is the set of policy instruments necessary to decentralize optimal allocations.

I show that eligibility thresholds to unemployment insurance are necessary to eliminate upward deviations and maintain incentive-compatibility while providing insurance. This implies that previous sufficient statistics results derived in the presence of earnings responses implicitly assume the existence of large enough eligibility thresholds. Moreover, I show that it is efficient to redistribute through progressive unemployment benefits because this relaxes threats of downward deviations.

Planner's problem. The economy is populated with individuals characterized by their earnings ability and fixed participation costs (ω, χ). A direct mechanism asks individuals to report their type and then assigns them, based on their report, a bundle that consists in a participation status, together with a consumption level c_0 for non-participants, and for

[24] F. Bierbrauer, Boyer, and Hansen (2020) use *two-bracket reforms* of a unique income tax schedule, trading-off efficiency effects across earnings levels, to characterize Pareto-efficient income taxation. Ferey, Lockwood, and Taubinsky (2021) use *joint one-bracket reforms* of income and savings taxes, trading-off efficiency effects across tax bases, to derive Pareto-efficiency conditions for nonlinear tax systems. I use *joint two-bracket reforms* of taxes when employed and benefits when unemployed, trading-off efficiency effects across states, to derive Pareto-efficiency conditions for tax-benefit systems.

participants an earnings level when employed z, a time spent employed e, a consumption when employed c_e, a consumption when unemployed c_u. This mechanism induces truthful reporting only if it satisfies incentive-compatibility constraints, that is, if individuals find the bundle designed for their type preferable to any other bundle.

Incentive-compatibility has a few basic implications that simplify the exposition of the planner's problem. First, bundles (z, e, c_e, c_u) must be independent of participation costs χ, otherwise participating individuals would report the most advantageous χ at their ability level.[25] Second, consumption c_0 must be independent of types, otherwise non-participating individuals would report the type with the highest c_0. Third, participation must follow a cutoff rule: if an individual finds it optimal to participate given participation costs χ, then all individuals with the same ability level ω and lower participation costs must find it optimal too. Denoting this participation cut-off, $\tilde{\chi}(\omega)$, these properties imply that we can write without loss of generality an allocation as a set $\{(z(\omega), e(\omega), c_e(\omega), c_u(\omega), \tilde{\chi}(\omega))_\omega, c_0\}$.

The planner's problem is to maximize its objective,

$$\int_\omega \left[\int_{\chi \leq \tilde{\chi}(\omega)} G\Big(V_m(\omega;\omega) - \chi\Big) + \int_{\chi \geq \tilde{\chi}(\omega)} G\Big(u_0(c_0)\Big) \right] dF_{\omega,\chi}(\omega, \chi), \tag{30}$$

subject to the resource constraint,

$$\int_\omega \left[\int_{\chi \leq \tilde{\chi}(\omega)} \Big(e(\omega)(z(\omega) - c_e(\omega)) - (1 - e(\omega))c_u(\omega) \Big) - \int_{\chi \geq \tilde{\chi}(\omega)} c_0 \right] dF_{\omega,\chi}(\omega, \chi) \geq Exp, \tag{31}$$

and subject to incentive-compatiblity constraints upon participation in the labor market,

$$\forall \omega, \forall \omega', \quad V_m(\omega;\omega) \geq V_m(\omega';\omega), \tag{32}$$

where $V_m(\omega';\omega)$ is the utility of an individual with ability ω reporting ability ω',

$$V_m(\omega';\omega) := e(\omega') \left[u_e(c_e(\omega')) - k(z(\omega');\omega) \right] + (1 - e(\omega')) \left[u_u(c_u(\omega')) - \psi(e(\omega'), z; \omega) \right], \tag{33}$$

and where (individual-rationality implies that) the participation cutoff is defined by,

$$\tilde{\chi}(\omega) := V_m(\omega;\omega) - u_0(c_0). \tag{34}$$

Unrestricted mechanism. When the mechanism is unrestricted, the planner chooses the allocation $\{(z(\omega), e(\omega), c_e(\omega), c_u(\omega), \tilde{\chi}(\omega))_\omega, c_0\}$, meaning that it can choose time spent

[25] This relies on the assumption that fixed participation costs are additively separable and do not affect utility when employed or unemployed.

employed, $e(\omega)$, independently of consumption levels when employed and unemployed. The planner can thus eliminate moral hazard and provide full insurance against unemployment:[26]

Lemma 1. *In the second-best (unrestricted mechanism), an optimal allocation features first-best insurance against unemployment, $u_u'(c_u(\omega)) = u_e'(c_e(\omega))$, at each earnings ability ω.*

Intuitively, the steady-state representation of the problem induces a deterministic link between search efforts and time spent employed e, which enables the planner to eliminate moral hazard by imposing a particular e. This result thus follows from the combination of a steady-state representation and an unrestricted mechanism.

Threshold mechanism. In practice, governments do not mandate individuals to spend a particular amount of time employed, but rather mandate them to spend a sufficient amount of time employed in order to receive unemployment benefits when unemployed. Given the incentives implied by this *threshold mechanism*, individuals privately decide on their search efforts leading to time spent employed, e, which restores moral hazard.

Consider the threshold $\underline{e}(\omega')>0$, defined such that an individual with ability ω, employed a fraction of time e, and reporting ability ω' obtains utility,

$$V_t(e,\omega';\omega) := \begin{cases} e\left[u_e(c_e(\omega')) - k(z(\omega');\omega)\right] + (1-e)\left[u_u(c_u(\omega')) - \psi(e,z(\omega');\omega)\right] & \text{if } e \geq \underline{e}(\omega'), \\ \frac{e}{\underline{e}(\omega')}\left\{\underline{e}(\omega')\left[u_e(c_e(\omega')) - k(z(\omega');\omega)\right] + (1-\underline{e}(\omega'))u_u(c_u(\omega'))\right\} \\ \quad + (1-\frac{e}{\underline{e}(\omega')})u_u(c_0) - (1-e)\psi(e,z(\omega');\omega) & \text{if } e \leq \underline{e}(\omega'). \end{cases} \tag{35}$$

If individuals spend a time $e \geq \underline{e}(\omega')$ employed, they are always eligible to receive $c_u(\omega')$ when unemployed. Otherwise, they receive c_0 in the subperiod of unemployment in which they are not eligible to $c_u(\omega')$. Utility $V_t(e,\omega';\omega)$ is thus continuous (but kinked) at $e = \underline{e}(\omega')$ and smoothly goes to $u_u(c_0)$ as the time spent employed converges to zero. Denoting $e_t(\omega';\omega)$ the time spent employed that maximizes utility $V_t(e,\omega';\omega)$, an individual with ability ω reporting ability ω' obtains utility $V_t(\omega';\omega) := V_t(e_t(\omega';\omega),\omega';\omega)$ in a threshold mechanism.

Now, if this threshold is infinitesimal, $\underline{e}(\omega') \approx 0$, unemployed individuals are always eligible to receive $c_u(\omega')$, provided that they are employed at some point in time. Assuming job search costs $\psi(e,z(\omega');\omega)$ go to zero as e goes to zero, anyone can then obtain $c_u(\omega')$ by reporting ability ω', spending an infinitesimal time (e.g. a day) employed at earnings $z(\omega')$,

[26] Interestingly, da Costa et al. (2022) come to the opposite conclusion in the framework that they study. In a second-best setting, they show that while type-dependent benefits can be incentive-compatible, benefits are independent of types in the optimum, implying very limited insurance against unemployment.

and staying unemployed the rest of the time. The provision of unemployment insurance is thus bounded by the utility level of the individual who is the worst-off on the labor market:

Lemma 2. *In a threshold mechanism, if* $\underline{e}(\omega') \approx 0$*, then* $u_u(c_u(\omega')) \leq \min_{\omega} V_t(\omega;\omega)$.

In the absence of meaningful eligibility requirements, unemployment insurance provision is thus limited by upward deviations, where low ability types ω find it optimal to report high ability $\omega' > \omega$ and exactly spend a fraction of time $\underline{e}(\omega')$ employed. Sufficiently high eligibility thresholds are thus necessary for the provision of type-specific unemployment insurance. At the same time, if eligibility thresholds are too high, then individuals are not always eligible to receive $c_u(\omega')$ when unemployed and thresholds affect both decisions and welfare.[27] The analysis abstracts from this possibility relying on the following assumption:

Assumption 2. *Eligibility thresholds are exogenously given and are both (i) sufficiently large such that only local incentive-compatibility constraints are binding, (ii) sufficiently low such that they do not affect individuals' job search decisions at the allocation designed for them.*

Part (i) implies that incentive-compatibility can be replaced by a first-order condition for ω' (and a monotonicity condition). Part (ii) guarantees that optimal search efforts are defined by first-order conditions for e, leading to $e_t(\omega;\omega) > \underline{e}(\omega)$, and that eligibility thresholds do not directly affect decisions nor welfare. Overall, Assumption 2 ensures smoothness and allows to solve the mechanism design problem using a first-order approach.[28]

Lemma 3. *In the third-best (threshold mechanism), an optimal allocation satisfy at each earnings ability* ω *the following first-order conditions for* $c_e(\omega)$ *and* $c_u(\omega)$ *respectively,*

$$\lambda\left[e_t(\omega;\omega) - \frac{\partial e_t(\omega;\omega)}{\partial c_e(\omega)}(z(\omega) - c_e(\omega) + c_u(\omega))\right] \int_{\chi \leq \tilde{\chi}(\omega)} dF_{\omega,\chi}(\omega,\chi) \tag{36}$$

$$= \eta(\omega) e_t(\omega;\omega) u_e'(c_e(\omega)) - \mu(\omega) \frac{\partial e_t(\omega;\omega)}{\partial c_e(\omega)} \frac{\partial FOC_{e_t}}{\partial \omega}\Big|_{\omega'=\omega},$$

$$\lambda\left[(1 - e_t(\omega;\omega)) - \frac{\partial e_t(\omega;\omega)}{\partial c_u(\omega)}(z(\omega) - c_e(\omega) + c_u(\omega))\right] \int_{\chi \leq \tilde{\chi}(\omega)} dF_{\omega,\chi}(\omega,\chi) \tag{37}$$

$$= \eta(\omega)(1 - e_t(\omega;\omega)) u_u'(c_u(\omega)) - \mu(\omega) \frac{\partial e_t(\omega;\omega)}{\partial c_u(\omega)} \frac{\partial FOC_{e_t}}{\partial \omega}\Big|_{\omega'=\omega},$$

[27] This may also lead to bunching at the threshold. Recent optimal tax papers have been able to provide general results in environments with bunching and these techniques could be applied in this context (see e.g. Jacquet & Lehmann, 2017; F. J. Bierbrauer, Boyer, & Peichl, 2021).

[28] After having characterized the solution, one can check numerically that there exists a set of eligibility thresholds satisfying Assumption 2, it is thus a testable assumption.

with λ the multiplier on the resource constraint (31)*, $\mu(\omega)$ the multiplier on the first-order condition replacing incentive-compatibility constraints* (32)*, $\eta(\omega)$ the multiplier on the definition of indirect utility $V_t(\omega';\omega)$, and FOC_{e_t} the first-order condition defining $e_t(\omega';\omega)$.*

The first line of (36) and (37) measures the fiscal costs of raising $c_e(\omega)$ or $c_u(\omega)$, which depend on the shadow value of public funds λ, the time spent employed or unemployed, the induced changes in search efforts, the employment tax, and the mass of participating individuals at ability ω. On the second line, the first term measures the welfare benefits of doing so, which depend on the shadow value of indirect utility $\eta(\omega)$, the time spent employed or unemployed, and the marginal utility of consumption.

The last term of (36) and (37) measures the incentive value of raising $c_e(\omega)$ or $c_u(\omega)$. It depends on the shadow value of locally relaxing incentive-compatibility $\mu(\omega)$, interacted with the changes in search efforts at a given ability ω, and the changes in search efforts across ability ω. Since individuals with higher ability find it optimal to spend more time employed at a given bundle, we have $\frac{\partial FOC_{e_t}}{\partial \omega}\Big|_{\omega'=\omega} \geq 0$. As $\frac{\partial e_t(\omega;\omega)}{\partial c_e(\omega)} \geq 0$ and $\frac{\partial e_t(\omega;\omega)}{\partial c_u(\omega)} \leq 0$, this implies that the more binding is incentive-compatibility, the lower should $c_e(\omega)$ be and the higher should $c_u(\omega)$ be. This shows that redistributing through unemployment benefits is efficient because it relaxes incentive-compatibility.

Policy implementation. Besides its intuitive structure giving the nature of the mechanism design problem, the appeal of a threshold mechanism is that it is equivalent to the tax-benefit system studied in the sufficient statistics analysis. When earnings z are monotonically increasing with ability (Assumption 1), we can define $T_e(z(\omega)) := z(\omega) - c_e(\omega)$, $B_u(z(\omega)) := c_u(\omega)$, $\underline{e}(z(\omega)) := \underline{e}(\omega)$, and $R_0 := c_0$. When eligibility thresholds do not affect choices other than by ensuring that the solutions to individuals' problems are interior (Assumption 2), the time spent employed, earnings, and participation decisions made by individuals under this tax-benefit system exactly correspond to those made in a threshold mechanism. The implied allocation is thus exactly the same:

Lemma 4. *When Assumptions 1 and 2 hold, the tax-benefit system $\{(T_e(z), B_u(z), \underline{e}(z))_z, R_0\}$ decentralizes the optimal allocation of the corresponding threshold mechanism.*

This shows that the previous sufficient statistics analysis with endogenous earnings decisions relies on the existence of eligibility requirements enabling the provision of earnings-specific unemployment benefits. Without such eligibility requirements, the solution to individuals maximization problems would not be interior, violating Assumption 1. Beyond the importance of eligibility requirements to unemployment insurance, this result underlines the complementarity of sufficient statistics and mechanism design approaches.

4 Empirical application to the U.S.

The empirical application to the U.S. economy is divided in two parts. First, I test whether the current U.S. tax-benefit system is Pareto-efficient based on existing empirical estimates of key sufficient statistics. While replacement rates steadily decrease with earnings, results suggest that there is scope for Pareto-improving reforms. Second, adapting parametric specifications commonly used in the literature, I calibrate the model to match key sufficient statistics estimates as well as earnings, unemployment and participation distributions, and carry out numerical simulations of counterfactual policies. Preliminary results suggest non-trivial welfare gains from reforming unemployment benefits and show that the tight link between redistribution and unemployment insurance implied by Pareto-efficiency seems to stabilize unemployment rates upon increases in the degree of redistribution.

4.1 Pareto-efficiency and the structure of unemployment benefits

Assuming a log utility for consumption, $u_e(c) = u_u(c) = \log c$, one can rewrite the Pareto-efficiency condition derived in the baseline, (10), as[29]

$$\frac{B_u(z)}{z - T_e(z)} = \left[1 + \frac{\mu_u^{elast}(z)}{e(z)^2}\left[1 + \frac{1}{B_u(z)}\Big(e(z)T_e(z) - (1 - e(z))\, B_u(z)\Big)\right]\right]^{-1}. \tag{38}$$

This provides a simple formula to test whether, under U.S. tax-benefit system, the actual net replacement rate (left-hand side) is equal to the net replacement rate implied by Pareto-efficiency (right-hand side). Next, I detail the different inputs that enter the formula.

U.S. tax-benefit system. To simulate actual tax-benefit schedules in the U.S. I use the OECD Tax-Benefit model, TaxBEN. It allows to simulate income taxes, social security contributions and the major cash benefits programs, but it excludes wealth and capital income taxes, taxes on consumption and in-kind transfers. A particularity of the U.S. is that taxes and benefits, in particular unemployment benefits, vary widely across states. The solution adopted in TaxBEN, and thus in this paper, is to simulate the tax-benefit system applicable in the state of Michigan.[30]

In the legislation, amounts of taxes and transfers are sometimes computed at the indi-

[29] A log utility for consumption implies a coefficient of relative risk aversion of 1. This value is consistent with income effects estimated in the labor supply literature (Chetty, 2006b), and in the range of values often used for optimal unemployment insurance (e.g. Chetty, 2006a, 2008; Landais, Michaillat, & Saez, 2018a).

[30] TaxBEN includes personal income taxes, social contributions, earned income tax credits (EITC), family benefits and tax credits (TANF, CTC, CCDF, CDCC), social assistance programs (SNAP) and unemployment benefits, at both federal and state levels.

vidual level (e.g. social security contributions) and sometimes at the household level (e.g. social assistance programs), in which case household composition (e.g. number of children) matters. Since the theoretical model does not incorporate interrelated labor supply decisions within households, I carry out simulations for singles, focusing on childless singles.

Another simplifying assumption relates to the reference period used to compute taxes and benefits. While income tax payments are based on annual income, transfer programs usually operate on an infra-annual basis. Following TaxBEN methodology, taxes and benefits are computed for a particular month and then multiplied by 12 to be reported on an annual basis. Moreover, I assume that individuals receive unemployment benefits during their entire unemployment spell, thereby abstracting from issues related to eligibility, take-up and long-term unemployment.

Last, TaxBEN simulations include employee social contributions, exclude employer social contributions and do not differentiate social contributions by types. This may be problematic since employee social contributions typically fund old age, survivors, and disability insurance whereas employer social contributions fund pensions and unemployment insurance. Results are however quantitatively robust to including social contributions funding unemployment insurance because they are small – a federal tax of 0.6% on the first $\$7,000$ of earnings and a state tax of 3.2% on the first $\$9,000$ of earnings.

The tax-transfer schedule, $T_e(z)$, corresponds to the total amount of taxes net of transfers when employed at earnings z. Simulations show that it is extremely close ($R^2 = 99\%$) to an affine schedule with intercept $R_0^{US} = -\$4,283$ and linear tax rate $\tau^{US} = 33.20\%$.

The benefit schedule, $B_u(z)$, corresponds to the amount of benefits received when unemployed, after taxes and transfers, for individuals who were previously employed at earnings z. I simulate it using the following two-step procedure. First, I simulate unemployment benefits following the legislation. Second, I apply to this amount of unemployment benefits a restricted tax-transfer schedule that only includes the personal income tax and social assistance, since unemployment benefits are legally part of the personal income tax base and individuals with low unemployment benefits may also receive social assistance (SNAP). Results from this two-step procedure show that the benefits schedule is approximately affine at low earnings up to a cap above which benefits are constant, as shown in Figure 2.[31]

Unemployment rates. The distribution of unemployment rates across earnings relies on annual measures of unemployment rates by educational attainments from 1992 to 2019, compiled by the Bureau of Labor and Statistics (BLS) using the Current Population Survey

[31] To check the validity of this two-step procedure used to simulate the benefit schedule $B_u(z)$, I compare the implied distribution of net replacement rates across earnings with the values of net replacement rates reported by the OECD at particular earnings levels, which are all consistent.

Figure 2: Unemployment benefits and net benefits when unemployed in the U.S.

Note: Unemployment benefits and net benefits received when unemployed (after income tax payment and the receipt of social assistance) are computed using the OECD TaxBen simulation tool for 2019.

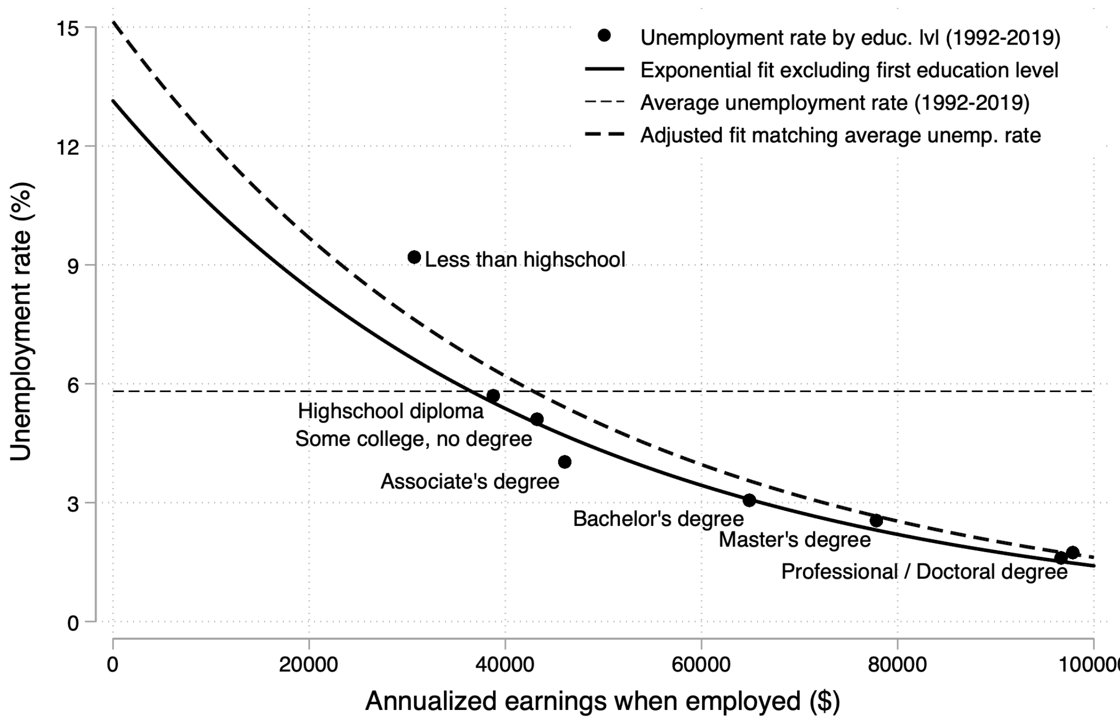
(CPS). Using the same data source, educational attainments are then mapped to earnings when employed, using median *usual weekly earnings* by educational attainment in 2019 (see the calibration of the earnings distribution below).

The data shows a significant gradient in unemployment rates across earnings. To obtain a smooth distribution of unemployment rates across earnings, I fit an exponential function to the data: $u(z) = a_u \exp(-b_u z)$. As can be seen on Figure 3, this functional form provides a relatively good fit. It also ensures that the unemployment rate is always positive and asymptotically goes to zero at top incomes.

Combining the fitted distribution of unemployment rates with a calibrated distribution of earnings yields a micro estimate of the average unemployment rate equal to 5.04%, which is close to the 5.81% average unemployment rate estimated from macro data over the same period.[32] I then adjust the intercept a_u to match the latter and obtain a distribution that is consistent with macro data, while preserving the gradient in unemployment rates measured at the micro level.

[32] These numbers diverge when performing the estimation using all 8 education levels. To remain conservative in the gradient of unemployment rates, I exclude the lowest education level and perform the estimation using 7 education levels only. Estimated parameters are then equal to $a_u = 13.14$ and $b_u = 2.235 \times 10^{-5}$.

Figure 3: Unemployment rates across earnings in the U.S.



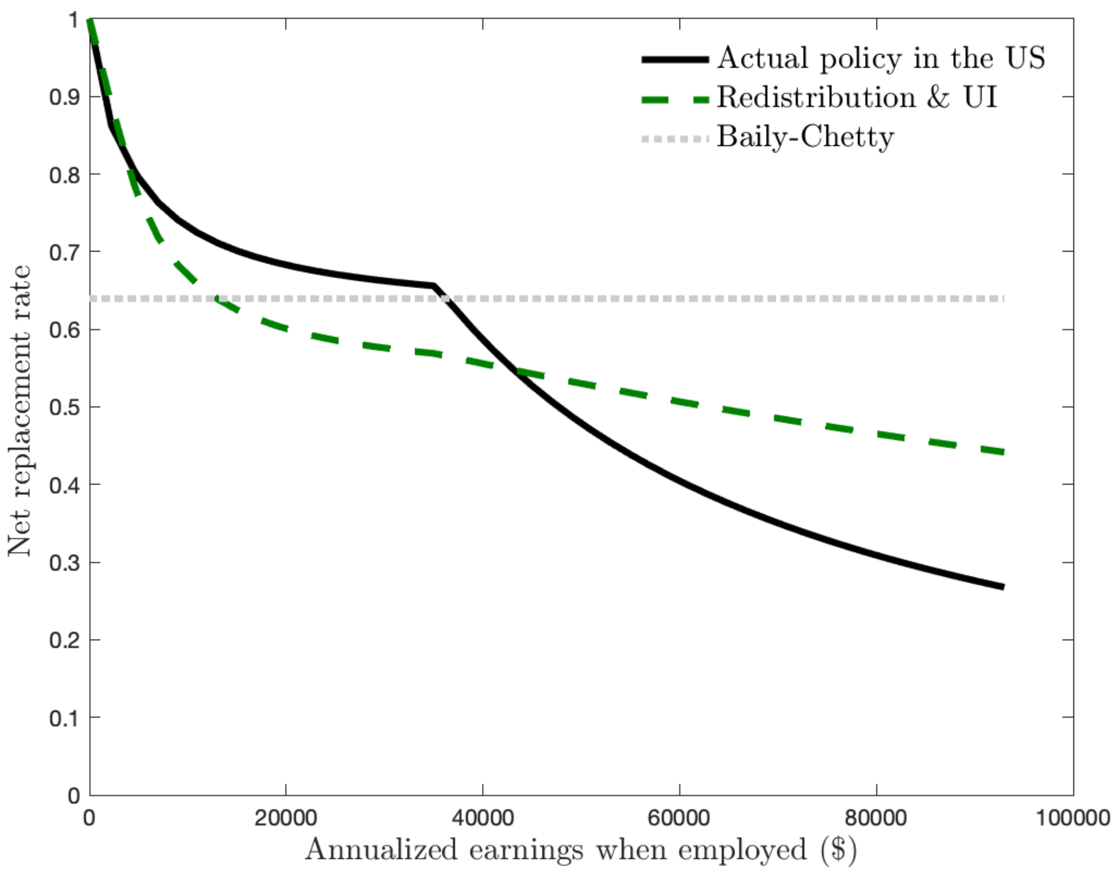
Note: Using compiled BLS statistics relying on CPS data, unemployment rates across earnings levels are obtained by combining unemployment rates across education levels for the period 1992-2019 with median usual weekly earnings by education levels in 2019. Exponential fits rely on nonlinear least-square estimation.

Search elasticities. For the calibration of search elasticities, I rely on estimates of the elasticity of unemployment duration D – measured as the duration of unemployment benefits receipt – with respect to the amount of unemployment benefits B, that is $\varepsilon_B^D \equiv \frac{B}{D}\frac{\partial D}{\partial B}$. Following Landais et al. (2018a, footnote 5), there exists a simple relationship between this empirical elasticity concept and the one used in the theoretical analysis. Abstracting from general equilibrium effects and writing the unemployment rate as $1 - e \approx sD$, where s is the separation rate assumed exogenous and D is the average duration of unemployment, we simply have $\varepsilon_B^D = \mu_u^{elast}$.

Schmieder and Von Wachter (2016, Table 2) provide a survey of the empirical literature and report existing quasi-experimental results. All estimates lie in a range between 0 and 1, and there are two modes at 0.3 and 0.7. Using data from Missouri covering 2003 to 2013, Card, Johnston, Leung, Mas, and Pei (2015) estimate in a regression kink design an elasticity of 0.35 pre-recession and an elasticity of 0.78 in the midst and aftermath of the Great Recession. These two modes may thus reflect differences in labor market conditions, among other factors. I use 0.5 as a baseline average and assume that this search elasticity is constant across earnings given the relative scarcity of reliable estimates across different earnings levels.

Results. Figure 4 shows that actual replacement rates decrease with earnings, from 1 at the bottom of the earnings distribution to low values at the top. At the bottom, this is because individuals with extremely low income receive similar transfers from social assistance, regardless of their labor market status. At the top, this comes from the fact that unemployment benefits are linearly increasing with earnings up to a cap, above which replacement rates steadily decrease with earnings. This cap is located around an annualized level of earnings of $40,000 which causes the kink in actual replacement rates.

Figure 4: Testing the Pareto-efficiency of the U.S. tax-benefit system



Note: Actual net replacement rates (black solid line) follow from actual tax-benefit schedules. Net replacement rates implied by Pareto-efficiency (green long dash line) follow from (38). The net replacement rate implied by the Baily-Chetty formula (gray short dash line) follows from (38) with the employment rate set to its average, $e(z) = 1 - 5.81\%$, and assuming insurance is actuarially fair, $e(z)T_e(z) = (1 - e(z))B_u(z)$.

Replacement rates implied by Pareto-efficiency are also decreasing with earnings, but they are slightly lower than actual replacement rates at low earnings levels and slightly higher than actual replacement rates at high earnings levels. At low earnings levels, this result is somewhat sensitive to the curvature of the utility function and one might get higher Pareto-efficient replacement rates assuming a coefficient of relative risk aversion larger than 1, the value implied by log utility. At high earnings, this result would be greatly affected by the introduction of savings that would imply much lower Pareto-efficient replacement rates.

Regardless of the sensitivity of Pareto-efficient replacement rates to the assumptions and exact calibration, this test highlights that the structure of linearly increasing benefits up to a cap is unlikely to be Pareto-efficient.[33] This suggests a scope for (ex-ante) Pareto-improving reforms, despite the decreasing pattern of actual replacement rates.

A broader take-away is that interactions between redistribution and unemployment insurance seem important to connect optimal unemployment insurance theory with actual policy: the pattern of decreasing replacement rates aligns more closely with actual policy than the standard policy prescription obtained from the Baily-Chetty formula.

4.2 Simulating counterfactual policies

A well-known issue with the simulation of counterfactual policies is that tax-benefit reforms trigger behavioral changes proportional to labor supply and search elasticities, but potentially also trigger changes in elasticities themselves, implying the need to specify a structural model (Chetty, 2009; Kleven, 2021). Adapting common parametric specifications used in prior work, I calibrate the model to match key sufficient statistics as well as observed distributions of unemployment, participation, and earnings, under the existing US tax-benefit policy. Analyzing Pareto-efficient tax-benefit systems with varying degrees of redistribution, preliminary simulation results suggest that the tight link between redistribution and unemployment insurance has a stabilizing effect on unemployment: unemployment rates are barely affected by changes in redistribution along the Pareto-frontier.

Parametrization and calibration. I use the following parametric specification and calibration procedure. First, the utility from consumption is $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ where γ directly corresponds to the coefficient of relative risk aversion and is set to 1 in the central calibration (log utility).

Second, the disutility to work is $k(z;\omega) = \frac{k_0}{1+\varepsilon}\left(\frac{z}{\omega}\right)^{1+\varepsilon}$ where ε is set to match the earnings elasticity $\zeta_e^{elast}(z)$ and ability ω is obtained by inverting the first-order condition for earnings

[33] Noting that the cap kicks in at median earnings, this structure could be rationalized in a political economy model since it maximizes, in relative terms, benefits for the median voter. See F. J. Bierbrauer et al. (2021) for a median voter theorem in a setting with nonlinear policy instruments.

z under the existing US tax-benefit policy.[34] The distribution of ability is then calibrated to match the distribution of earnings $h_z(z)$.

Third, the disutility to search is $\psi(e,z;\omega) = \frac{\psi_0}{\kappa}\frac{z^{\psi_z}}{\omega^{\psi_\omega}}\frac{1}{(1-e)^{\kappa+1}}$ where κ is set to match the search elasticity μ_u^{elast} and parameters $(\psi_0, \psi_z, \psi_\omega)$ are calibrated to match the observed distribution of unemployment rates $1-e(z)$.

The last step is to calibrate the distribution of fixed costs χ. Following Jacquet et al. (2013), I assume an ability-specific distribution of fixed-cost $f_{\chi|\omega}(\chi|\omega)$ given by

$$\int_{x\le\chi} f_{\chi|\omega}(x|\omega)\,dx = \frac{\exp(-\phi_1(\omega)+\phi_2(\omega)\chi)}{1+\exp(-\phi_1(\omega)+\phi_2(\omega)\chi)} \tag{39}$$

where $\phi_1(\omega)$ and $\phi_2(\omega)$ are set to match the participation rate and participation elasticity at the earnings level $z(\omega)$ associated with ability level ω.

Earnings distribution. In the optimal income tax literature, the earnings distribution is usually calibrated using annual taxable income. However, annual taxable income measures both earned income when employed and unemployment benefits received when unemployed. As a result, annual taxable income mixes earnings ability and the realization of unemployment risk in ways that may lead to important composition effects.

To circumvent this issue, I construct an earnings distribution using the variable *usual weekly earnings* available in Current Population Survey (CPS) data for all individuals who participate in the labor force.[35] This variable measures earnings during weeks worked, it thus provides a proxy for earnings ability that is somewhat independent from the realization of unemployment risk. I multiply this variable by 52 to obtain an annualized measure of the earnings individuals would have earned if employed during the whole year.

Appending all monthly CPS files of 2019, I calibrate a log-normal distribution of annualized earnings when employed (Figure A1 in Appendix). The log-normal distribution broadly matches the shape of the earnings distribution and guarantees smoothness, thereby eliminating any convergence issues that may arise with less smooth distributions (e.g. kernel density). Since the top of the earnings distribution is best represented by a Pareto distribution (e.g. Atkinson, Piketty, & Saez, 2011), I append to this log-normal distribution a Pareto-tail with parameter $\alpha = 2$ for earnings above \$200,000 following Saez (2001).

[34] In the presence of unemployment, this "iso-elastic" specification does not give rise to a constant earnings elasticity $\zeta_e^{elast}(z)$. I thus calibrate ϵ such that the intensive margin elasticity at the average earnings level is equal to the targetted value.

[35] A technical note from the BLS states: *"The term "usual" is determined by each respondent's own understanding of the term. If the respondent asks for a definition of "usual", interviewers are instructed to define the term as more than half the weeks worked during the past 4 or 5 months."*

Participation rates. To calibrate the distribution of participation rates, I repeat the procedure used to calibrate the distribution of unemployment rates (with the caveat that participation rates compiled by the BLS are only available for 2016). I use participation rates by educational attainments that are then mapped to earnings when employed, using median *usual weekly earnings* by educational attainment. The data shows a significant upward gradient in participation rates across earnings (Figure A2 in Appendix) and I fit an exponential function to the data to obtain a smooth distribution, $p(z) = 100 - a_p \exp(-b_p z)$. Estimating these parameters with nonlinear least-square results in $a_u = 13.14$ and $b_u = 2.235 \times 10^{-5}$.

Combining the fitted distribution of participation rates with the calibrated distribution of earnings yields an average participation rate of 67.35%. This largely reflects the fact that the data on participation rates covers all individuals aged 25 and older – including those aged 65 and older whose participation rate is around 20%. To avoid capturing interactions between participation and retirement decisions of this older segment of the population, I adjust the intercept a_p to match the participation rate among individuals aged between 25 and 64, equal to 77.04% (OECD). This preserves the gradient in participation rates measured at the micro level while providing a distribution that is consistent with macro data.

Labor supply elasticities. Chetty, Guren, Manoli, and Weber (2013) provide a meta-analysis of participation elasticities focusing on reduced-form estimates. They conclude that 0.2 is a reasonable value for participation elasticities, although they tend to be somewhat larger for certain subgroups of the population like the young, the old, and single mothers. For instance, Eissa, Kleven, and Kreiner (2008) suggest that the participation elasticity of single mothers is likely close to 0.7, while Kroft et al. (2020) estimate a participation elasticity of 0.57 for single women. Moreover, structural estimates tend to show that extensive margin responses of both single men and single women tend to be larger at lower earnings levels (e.g. Bargain, Orsini, and Peichl, 2014). I thus assume participation elasticities decrease with earnings in line with prior work in optimal income taxation (e.g. Saez, 2002; Jacquet et al., 2013; Kroft et al., 2020). More specifically, I set the participation elasticity $\pi_e^{elast}(z)$ to 0.5 at the origin of the earnings distribution and assume it linearly decreases to 0 at an earnings level of $\$100,000$ and above.[36]

At the intensive margin, quasi-experimental studies estimating the elasticity of earnings

[36] Because many empirical studies rely on variations in taxes (or wages) when employed, I assume that they measure the participation elasticity in response to changes in taxes, $\pi_e^{elast}(z)$. An interesting implication of this model is that participation decisions also depend on the unemployment rate and the benefits received when unemployed. To the best of my knowledge, the impact of these two factors on participation elasticities has not been investigated in the empirical literature.

with respect to marginal net of tax rate often find small elasticities around 0.1 (Saez, Slemrod, & Giertz, 2012). Chetty (2012) argues that many of these small estimates are likely driven by adjustment frictions. Doing a meta-analysis of the literature and accounting for the size of tax changes used to estimate elasticities, he concludes that a central value for intensive margin elasticities is 0.33 and I use this value as a baseline for $\zeta_e^{elast}(z)$.

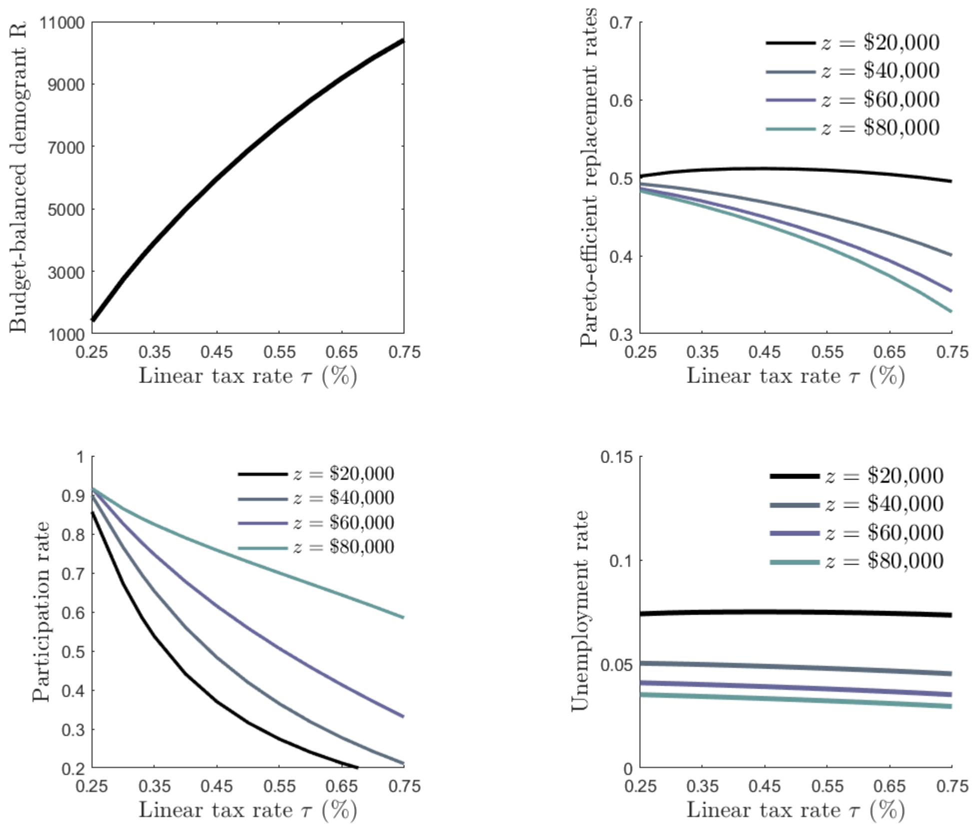
Results. This calibrated structural model is used to simulate Pareto-efficient tax-benefit systems that feature varying levels of redistribution. To have a simple one-dimensional parametrization of redistribution, consider an affine tax-transfer schedule, $T_e(z) = \tau z - R_0$. A more redistributive tax-benefit system then corresponds to one with a higher tax rate. For each tax rate τ, I then numerically solve for the fixed-point where the demogrant R_0 balances the government budget, the tax-benefit system is Pareto-efficient, and individuals' decisions are optimal given the tax-benefit system in place. This yields a characterization of the Pareto-frontier in the class of tax-benefit systems that feature affine tax-transfer schedules when employed. Preliminary simulation results, which disregard savings and earnings decisions thereby following the baseline model, are displayed in Figure 5.

The top left panel of Figure 5 shows that a higher tax rate allows the government to finance a higher demogrant R_0 , thereby increasing redistribution. The demogrant keeps on increasing at high tax rates in part because of the absence of labor supply responses above an earnings level of $\$100,000$ in this pure extensive margin model. The top right panel shows the evolution of replacement rates across tax rates for different earnings levels. Optimal replacement rates are quite similar at a low tax rate of 25% and diverge as the tax rate increases: replacement rates increase at very low earnings levels through the increase in the demogrant, but decrease at high earnings levels driven through the increase in the tax rate, reflecting the logic highlighted when discussing the benefit formula (15) and Figure 1.

The bottom left panel of Figure 5 shows that increases in the degree of redistribution induce strong participation responses that translate into stark reductions in participation rates at all earnings levels. In contrast, the bottom right panel of Figure 5 shows that unemployment rates remain strikingly stable at all earnings levels. This is all the more striking in light of the divergence in replacement rates as the tax rate increases. This suggests that the tight link between redistribution and unemployment insurance implied by Pareto-efficiency has a stabilizing effect on unemployment rates when varying the degree of redistribution, which then mostly affects participation.[37]

[37]It would be interesting to extend these results to include general equilibrium effects. Intuitively, the reduction in labor force participation should push wages up and the unemployment rate down introducing additional feedback effects from tax-benefit reforms.

Figure 5: Pareto-efficient tax-benefit systems across varying degrees of redistribution



Note: Assuming an affine tax-transfer schedule, Pareto-efficient tax-benefit systems are simulated for different linear tax rates. The Figure represents for each tax rate, the demogrant (top left), replacement rates (top right), participation rates (bottom left), unemployment rates (bottom right).

5 Conclusion

This paper analyzes the interactions between redistribution and unemployment insurance policies in a simple and tractable framework bridging canonical models of optimal income taxation and optimal unemployment insurance. Characterizing optimal policies in terms of empirically estimable sufficient statistics, I show that these interactions have important implications for the design tax-benefit systems.

First, they affect the optimal design of redistributive taxes and transfers, in particular at the bottom of the earnings distribution where an EITC may be desirable to boost the job search incentives of the unemployed. Second, they imply that redistribution through progressive unemployment benefits is efficient, even in the presence of an optimal redistributive tax-transfer schedule. Third, they imply that optimal unemployment insurance features replacement rates that decrease with earnings, from 1 at the bottom of the earnings distribution to 0 at the top, in a way that is shaped by redistribution.

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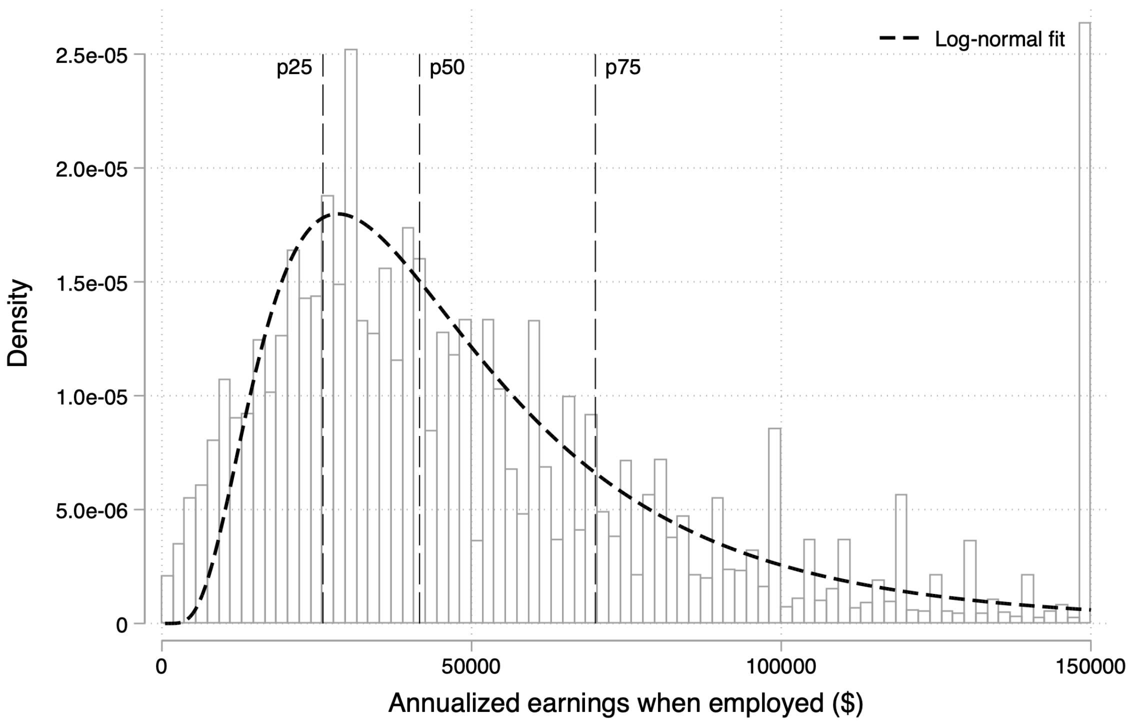
Appendix

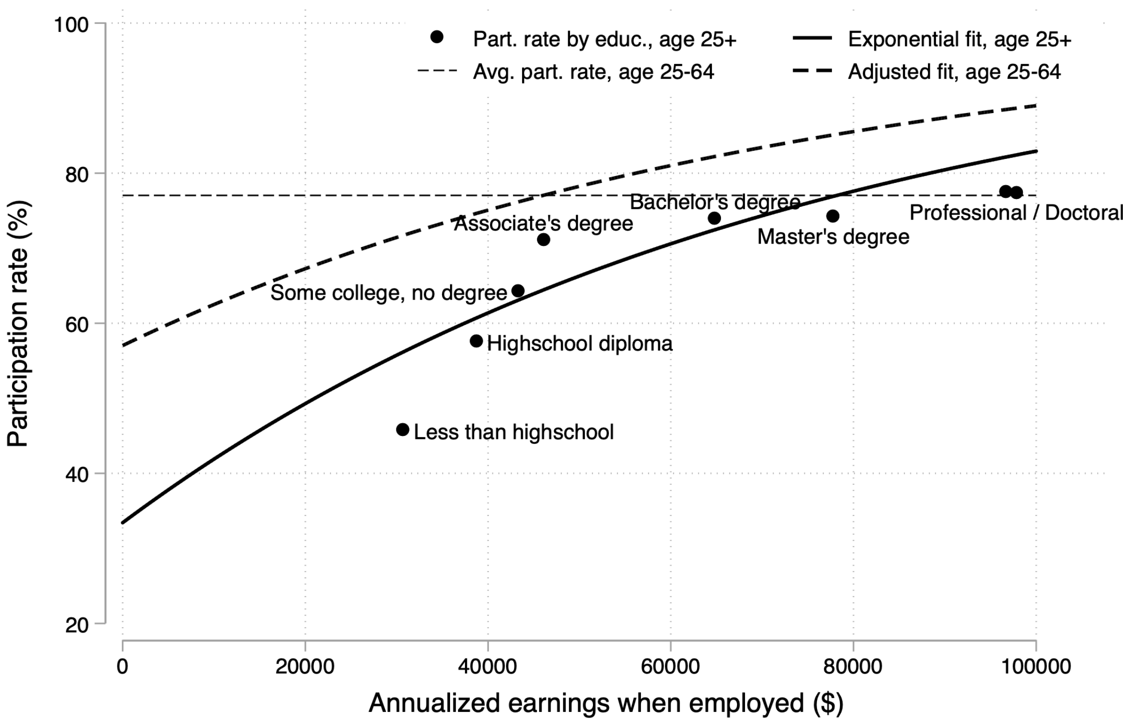
A Supplementary Tables & Figures

Figure A1: Distribution of earnings when employed in the U.S.



Note: Drawing on 2019 CPS data, the distribution of annualized earnings when employed is constructed using the variable *usual weekly earnings*, multiplied by 52. The mass at the top of the histogram reflects the topcoding of weekly earnings above $2,884.61 (annual earnings above $150,000).

Figure A2: Participation rates across earnings in the U.S.



Note: Using compiled BLS statistics relying on CPS data, participation rates across earnings levels are obtained by combining participation rates across education levels in 2016 (only year for which these statistics are readily available) with median usual weekly earnings by education levels in 2019. Exponential fits rely on nonlinear least-square estimation. Adjusted fit set to match the average participation rate of individuals aged 25-64.

B Proofs of Propositions

B.1 Optimal policies in baseline model (Proposition 1)

The Lagrangian of the government's problem, with multiplier λ, is

$$\frac{\mathcal{L}}{\lambda} = \int_z \int_{\chi \leq \tilde{\chi}(z)} \left[\frac{1}{\lambda} G\Big(V(z) - \chi\Big) + e(z)T_e(z) - (1 - e(z))\, B_u(z)\right] f_{\chi|z}(\chi) f_z(z) d\chi dz$$
$$+ \int_z \int_{\chi \geq \tilde{\chi}(z)} \left[\frac{1}{\lambda} G\Big(u_0(R_0)\Big) - R_0\right] f_{\chi|z}(\chi) f_z(z) d\chi dz. \tag{40}$$

Optimal tax. Consider a small change, $dT_e(z)$, in the tax-transfer when employed at earnings z. The change in the Langrangian implied by this reform is

$$\frac{d\mathcal{L}}{\lambda} = \int_{\chi \leq \tilde{\chi}(z)} \left[\frac{1}{\lambda} G'\Big(V(z) - \chi\Big) dV(z) + e(z) dT_e(z) + de(z)\Big(T_e(z) + B_u(z)\Big)\right] f_{\chi|z}(\chi) d\chi f_z(z)$$
$$+ \Big[e(z)T_e(z) - (1 - e(z))\, B_u(z) + R_0\Big] d\tilde{\chi}(z) f_{\chi|z}\left(\tilde{\chi}(z)\right) f_z(z), \tag{41}$$

since utility-maximizing participation decisions imply that individuals who change their participation decisions do not experience any first-order change in their utility.

Now, search decisions are also utility-maximizing, which implies by an envelope argument that the impact on expected utility when participating is,

$$dV(z) = -e(z) u_e'\left(z - T_e(z)\right) dT_e(z), \tag{42}$$

and, by definition of $\mu_e(z)$ and $\pi_e(z)$, changes in search and participation decisions are

$$de(z) = -\left(1 - e(z)\right) \mu_e(z) dT_e(z), \tag{43}$$
$$dh(z) = d\tilde{\chi}(z) f_{\chi|z}(\tilde{\chi}(z)) f_z(z) = -\pi_e(z) h_z(z) dT_e(z), \tag{44}$$

where the second equality follows from $h_z(z) = \Big(\int_{\chi \leq \tilde{\chi}(z)} f_{\chi|z}(\chi) d\chi\Big) f_z(z)$. As a result,

$$\frac{d\mathcal{L}}{\lambda} = \Bigg\{ - e(z) u_e'\left(z - T_e(z)\right) \frac{1}{\lambda} \overline{G'\Big(V(z) - \chi\Big)} + \Big[e(z) - (1 - e(z))\, \mu_e(z) \Big(T_e(z) + B_u(z)\Big)\Big]$$
$$- \Big[e(z)T_e(z) - (1 - e(z))\, B_u(z) + R_0\Big] \pi_e(z) \Bigg\} dT_e(z) h_z(z) \tag{45}$$

where

$$\overline{G'\Big(V(z) - \chi\Big)} = \frac{\int_{\chi \leq \tilde{\chi}(z)} G'\Big(V(z) - \chi\Big) f_{\chi|z}(\chi) d\chi}{h_z(z)/f_z(z)}. \tag{46}$$

Introducing social marginal welfare weights, $g_e(z)$, and characterizing optimal policy through the first-order condition $\frac{d\mathcal{L}}{dT_e(z)} = 0$, we obtain optimal tax formula (9) of Proposition 1:

$$e(z)(1-g_e(z)) - (1-e(z))(T_e(z)+B_u(z))\mu_e(z) - [e(z)T_e(z)-(1-e(z))B_u(z)+R_0]\,\pi_e(z) = 0$$
$$\iff (T_e(z)+R_0)\pi_e(z) - (1-e(z))(T_e(z)+B_u(z))(\pi_e(z)-\mu_e(z)) = e(z)(1-g_e(z)). \quad (47)$$

Optimal benefit. Consider a small change, $dB_u(z)$, in the unemployment benefits of individuals with earnings z when employed. The impact on the Langrangian is

$$\frac{d\mathcal{L}}{\lambda} = \int_{\chi\leq\tilde{\chi}(z)} \left[\frac{1}{\lambda}G'\Big(V(z)-\chi\Big)dV(z) - (1-e(z))\,dB_u(z) + de(z)\Big(T_e(z)+B_u(z)\Big)\right] f_{\chi|z}(\chi)d\chi f_z(z)$$
$$+ \Big[e(z)T_e(z) - (1-e(z))\,B_u(z) + R_0\Big]d\tilde{\chi}(z)f_{\chi|z}\left(\tilde{\chi}(z)\right) f_z(z), \quad (48)$$

since utility-maximizing participation decisions imply that individuals who change their participation decisions do not experience any first-order change in their utility.

Now, search decisions are also utility-maximizing, which implies by an envelope argument that the impact on expected utility when participating is,

$$dV(z) = (1-e(z))u_u'\left(B_u(z)\right)dB_u(z), \quad (49)$$

and, by definition of $\mu_u(z)$ and $\pi_u(z)$, changes in search and participation decisions are

$$de(z) = -\left(1-e(z)\right)\mu_u(z)dB_u(z), \quad (50)$$
$$dh(z) = d\tilde{\chi}(z)f_{\chi|z}(\tilde{\chi}(z))f_z(z) = \pi_u(z)h_z(z)dB_u(z), \quad (51)$$

where the second equality follows from $h_z(z) = \left(\int_{\chi\leq\tilde{\chi}(z)} f_{\chi|z}(\chi)d\chi\right) f_z(z)$. As a result,

$$\frac{d\mathcal{L}}{\lambda} = \Big\{(1-e(z))u_u'\left(B_u(z)\right)\frac{1}{\lambda}\overline{G'\Big(V(z)-\chi\Big)} - (1-e(z)) - (1-e(z))\,\mu_u(z)\Big(T_e(z)+B_u(z)\Big)$$
$$+ \Big[e(z)T_e(z) - (1-e(z))\,B_u(z) + R_0\Big]\pi_u(z)\Big\}h_z(z)dB_u(z). \quad (52)$$

Introducing social marginal welfare weights, $g_u(z)$, and characterizing optimal policy through the first-order condition $\frac{d\mathcal{L}}{dB_u(z)} = 0$, we obtain the following optimal benefit formula:

$$(1-e(z))(g_u(z)-1) - (1-e(z))\mu_u(z)(T_e(z)+B_u(z)) + [e(z)T_e(z)-(1-e(z))B_u(z)+R_0]\,\pi_u(z) = 0$$
$$\iff (1-e(z))(\pi_u(z)+\mu_u(z))(T_e(z)+B_u(z)) - (T_e(z)+R_0)\pi_u(z) = (1-e(z))(g_u(z)-1). \quad (53)$$

Pareto-efficiency. Consider a joint reform that consists in a small change in the benefits when unemployed, $dB_u(z)$, accompanied by a small change in the tax-transfer when employed, $dT_e(z) = \frac{1-e(z)}{e(z)} \frac{u'_u(B_u(z))}{u'_e(z-T_e(z))} dB_u(z)$, for individuals with earnings z.

This reform does not affect the utility of individuals who do not participate in the labor market. Moreover, the change in the expected utility of individuals who participate is, by an envelope argument,

$$dV(z) = -e(z)u'_e\left(z - T_e(z)\right) dT_e(z) + (1 - e(z))u'_u\left(B_u(z)\right) dB_u(z) = 0, \tag{54}$$

implying that this reform triggers no utility changes whatsoever, and therefore no changes in participation decisions. The impact on the Lagragian is thus

$$\frac{d\mathcal{L}}{\lambda} = \int_{\chi \leq \tilde{\chi}(z)} \Big[e(z)dT_e(z) - (1 - e(z))\, dB_u(z) + de(z)\left(T_e(z) + B_u(z)\right)\Big] f_{\chi|z}\left(\chi\right) f_z\left(z\right) d\chi. \tag{55}$$

Now, changes in search decisions are given by

$$\begin{aligned} de(z) &= -\left(1 - e(z)\right) \mu_e(z) dT_e(z) - \left(1 - e(z)\right) \mu_u(z) dB_u(z) \\ &= -\frac{1 - e(z)}{e(z)} \mu_u(z) dB_u(z), \end{aligned} \tag{56}$$

where the second equality follows from $\frac{\mu_e(z)}{u'_e(z-T_e(z))} = \frac{\mu_u(z)}{u'_u(B_u(z))}$ (see proof below). As a result,

$$\frac{d\mathcal{L}}{\lambda} = (1 - e(z)) \left\{ \left(\frac{u'_u\left(B_u(z)\right)}{u'_e\left(z - T_e(z)\right)} - 1 \right) - \frac{\mu_u(z)}{e(z)}\left(T_e(z) + B_u(z)\right) \right\} h_z(z) dB_u(z), \tag{57}$$

implying that there exists a joint reform of $B_u(z)$ and $T_e(z)$ that is Pareto-improving whenever the curly bracket is non-zero. A Pareto-efficient tax-benefit system thus satisfies,

$$\begin{aligned} &\frac{u'\left(B_u(z)\right)}{u'\left(z - T_e(z)\right)} - 1 = \frac{\mu_u(z)}{e(z)}\left(T_e(z) + B_u(z)\right) \\ \iff \; &\frac{u'\left(B_u(z)\right)}{u'\left(z - T_e(z)\right)} - 1 = \frac{\mu_u(z)}{e(z)} \frac{B_u(z)}{e(z)} \left[1 + \frac{1}{B_u(z)}\Big(e(z)T_e(z) - \left(1 - e(z)\right) B_u(z)\Big)\right] \end{aligned} \tag{58}$$

which is Pareto-efficiency condition (10) of Proposition 1.

Relation between search semi-elasticities. Looking at individuals' maximization problem (2), the first-order condition pinning down $e(z)$ is

$$(FOC)_e: \; \left[u_e\left(z - T_e(z)\right) - k(z)\right] - \left[u_u\left(B_u(z)\right) - \psi(e; z)\right] - (1 - e)\frac{\partial \psi\left(e; z\right)}{\partial e} = 0 \tag{59}$$

and the second-order condition is

$$(SOC)_e : \; 2\frac{\partial \psi(e;z)}{\partial e} - (1-e)\frac{\partial^2 \psi(e;z)}{\partial e^2} < 0 \tag{60}$$

Differentiating the first-order condition yields

$$- u_e'(z - T_e(z))\, dT_e(z) - u_u'(B_u(z))\, dB_u(z) + (SOC)_e\, de = 0 \tag{61}$$

Hence,

$$\mu_e(z) = -\frac{1}{1-e(z)}\frac{\partial(1-e(z))}{\partial(z-T_e(z))} = -\frac{1}{1-e(z)}\frac{u_e'(z-T_e(z))}{(SOC)_e}, \tag{62}$$

$$\mu_u(z) = \frac{1}{1-e(z)}\frac{\partial(1-e(z))}{\partial B_u(z)} = -\frac{1}{1-e(z)}\frac{u_u'(B_u(z))}{(SOC)_e}, \tag{63}$$

such that finally

$$\frac{\mu_e(z)}{u_e'(z-T_e(z))} = -\frac{1}{1-e(z)}\frac{1}{(SOC)_e} = \frac{\mu_u(z)}{u_u'(B_u(z))}. \tag{64}$$

Application with log utility. When $u_e(c) = u_u(c) = \log(c)$, introducing the elasticity concept $\mu_u^{elast}(z) = B_u(z)\mu_u(z)$, the Pareto-efficiency condition becomes

$$\frac{z-T_e(z)}{B_u(z)} - 1 = \frac{\mu_u^{elast}(z)}{B_u(z)}\frac{1}{e(z)}(T_e(z)+B_u(z)) \iff z-T_e(z)-B_u(z) = \frac{\mu_u^{elast}(z)}{e(z)}(T_e(z)+B_u(z))$$
$$\iff B_u(z) = \frac{e(z)}{e(z)+\mu_u^{elast}(z)} z - T_e(z). \tag{65}$$

Introducing elasticity concepts $\pi_e^{elast}(z) = (z - T_e(z))\,\pi_e(z)$, the optimal tax formula becomes

$$(T_e(z)+R_0)\frac{\pi_e^{elast}(z)}{z-T_e(z)} - (1-e(z))(T_e(z)+B_u(z))\left[\frac{\pi_e^{elast}(z)}{z-T_e(z)} - \frac{\mu_e^{elast}(z)}{z-T_e(z)}\right]$$
$$= e(z)\left[1 - \frac{1}{z-T_e(z)}\frac{\overline{G'(V(z)-\chi)}}{\lambda}\right] \tag{66}$$

Using $B_u(z) = \frac{e(z)}{e(z)+\mu_u^{elast}(z)} z - T_e(z)$, multiplying both sides by z–$T_e(z)$, and rearranging yields

$$(\pi_e^{elast}(z)+e(z))T_e(z) = e(z)\left[z - \frac{\overline{G'(V(z)-\chi)}}{\lambda} + (1-e(z))\frac{\pi_e^{elast}(z)-\mu_e^{elast}(z)}{e(z)+\mu_u^{elast}(z)} z\right] - \pi_e^{elast}(z)R_0, \tag{67}$$

which corresponds to the expression in the text, noting that with log utility,

$$\frac{\mu_e(z)}{u'_e(z-T_e(z))} = \frac{\mu_u(z)}{u'_u(B_u(z))} \iff \mu_e^{elast}(z) = \mu_u^{elast}(z). \tag{68}$$

B.2 Optimal policies with savings and assets (Proposition 2)

Abstracting from savings and assets taxes, the Lagrangian associated with the government's problem is the same as before, with indirect utility defined this time by (16).

Optimal taxes. Consider a small change, $dT_e(z)$, in the tax-transfer when employed at earnings z. The change in the Langrangian implied by this reform is, as before,

$$\begin{aligned}\frac{d\mathcal{L}}{\lambda} = &\int_{\chi\leq\tilde{\chi}(z)} \left[\frac{1}{\lambda}G'\Big(V(z)-\chi\Big)dV(z) + e(z)dT_e(z) + de(z)\Big(T_e(z)+B_u(z)\Big)\right] f_{\chi|z}(\chi)d\chi f_z(z) \\ &+ \Big[e(z)T_e(z) - (1-e(z))\,B_u(z) + R_0\Big] d\tilde{\chi}(z) f_{\chi|z}\left(\tilde{\chi}(z)\right) f_z(z),\end{aligned} \tag{69}$$

since utility-maximizing participation decisions imply that individuals who change their participation decisions do not experience any first-order change in their utility.

Now, search decisions are utility-maximizing but the accumulation of savings and assets may not be, implying that the impact on indirect utility when participating is,

$$\begin{aligned}dV(z) &= e(z)\left[-u'_e(.)\left(1+\frac{\partial s(z)}{\partial T_e(z)}+\frac{\partial a(z)}{\partial T_e(z)}\right) + u'_u(.)\frac{\partial s(z)}{\partial T_e(z)} + U'(.)\frac{\partial a(z)}{\partial T_e(z)}\right] dT_e(z) \qquad (70)\\ &= e(z)\left[-u'_e(c_e(z))+(u'_u(c_u(z))-u'_e(c_e(z)))\frac{\partial s(z)}{\partial T_e(z)}+(U'(e(z)a(z))-u'_e(c_e(z)))\frac{\partial a(z)}{\partial T_e(z)}\right] dT_e(z)\end{aligned}$$

and changes in search and participation decisions are, as before,

$$de(z) = -\left(1-e(z)\right)\mu_e(z)dT_e(z), \tag{71}$$

$$dh(z) = d\tilde{\chi}(z)f_{\chi|z}(\tilde{\chi}(z))f_z(z) = -\pi_e(z)h_z(z)dT_e(z), \tag{72}$$

except that semi-elasticities now include endogenous changes in savings and assets. As a result,

$$\begin{aligned}\frac{d\mathcal{L}}{\lambda} = \Bigg\{&e(z)+e(z)\frac{1}{\lambda}\overline{G'\Big(V(z)-\chi\Big)}\left[-u'_e(.)+(u'_u(.)-u'_e(.))\frac{\partial s(z)}{\partial T_e(z)}+(U'(.)-u'_e(.))\frac{\partial a(z)}{\partial T_e(z)}\right] \\ &-(1-e(z))(T_e(z)+B_u(z))\mu_e(z)-\Big[e(z)T_e(z)-(1-e(z))B_u(z)+R_0\Big]\pi_e(z)\Bigg\}h_z(z)dT_e(z).\end{aligned} \tag{73}$$

Using social marginal welfare weights, $g_e(z)$, and characterizing optimal policy through the first-order condition $\frac{d\mathcal{L}}{dT_e(z)} = 0$ we finally obtain

$$e(z)(1-g_e(z)) + e(z)g_e(z)\left[\frac{u'_u(c_u(z))-u'_e(c_e(z))}{u'_e(c_e(z))}\frac{\partial s(z)}{\partial T_e(z)} + \frac{U'(e(z)a(z))-u'_e(c_e(z))}{u'_e(c_e(z))}\frac{\partial a(z)}{\partial T_e(z)}\right]$$
$$- (1-e(z))(T_e(z)+B_u(z))\mu_e(z) - \Big[e(z)T_e(z)-(1-e(z))B_u(z)+R_0\Big]\pi_e(z) = 0, \tag{74}$$

which, after rearranging terms, yields optimal tax formula (19) of Proposition 2.

Pareto-efficiency. The change in expected utility following a joint reform of taxes and benefits is

$$dV(z) = -\left[e(z)u'_e(.)-e(z)(u'_u(.)-u'_e(.))\frac{\partial s(z)}{\partial T_e(z)}-e(z)(U'(.)-u'_e(.))\frac{\partial a(z)}{\partial T_e(z)}\right]dT_e(z) \tag{75}$$
$$+\left[(1-e(z))u'_u(.)+e(z)(u'_u(.)-u'_e(.))\frac{\partial s(z)}{\partial B_u(z)}+e(z)(U'(.)-u'_e(.))\frac{\partial a(z)}{\partial B_u(z)}\right]dB_u(z).$$

A joint reform thus leaves utility constant when $dT_e(z) = K_r(z)\frac{1-e(z)}{e(z)}\frac{u'_u(c_u(z))}{u'_e(c_e(z))}dB_u(z)$, with

$$K_r(z) := \frac{1 + \frac{e(z)}{1-e(z)}\frac{u'_u(.)-u'_e(.)}{u'_u(.)}\frac{\partial s(z)}{\partial B_u(z)} + \frac{e(z)}{1-e(z)}\frac{U'(.)-u'_e(.)}{u'_u(.)}\frac{\partial a(z)}{\partial B_u(z)}}{1 - \frac{u'_u(.)-u'_e(.)}{u'_e(.)}\frac{\partial s(z)}{\partial T_e(z)} - \frac{U'(.)-u'_e(.)}{u'_e(.)}\frac{\partial a(z)}{\partial T_e(z)}}, \tag{76}$$

implying that $K_r(z)=1$ with either utility-maximizing or exogenous savings and assets.

This joint reform triggers no utility changes whatsoever, and therefore no changes in participation decisions. The impact of this reform on the Lagragian is thus

$$\frac{d\mathcal{L}}{\lambda} = \int_{\chi\leq\tilde{\chi}(z)}\Big[e(z)dT_e(z) - (1-e(z))dB_u(z) + de(z)(T_e(z)+B_u(z))\Big]f_{\chi|z}(\chi)f_z(z)d\chi. \tag{77}$$

with changes in search behaviors given by

$$\begin{aligned} de(z) &= -(1-e(z))\mu_e(z)dT_e(z) - (1-e(z))\mu_u(z)dB_u(z) \\ &= -(1-e(z))\left[K_r(z)\frac{1-e(z)}{e(z)}\frac{u'_u(c_u(z))}{u'_e(c_e(z))}\mu_e(z) + \mu_u(z)\right]dB_u(z) \\ &= -(1-e(z))\left[K_\mu(z)K_r(z)\frac{1-e(z)}{e(z)} + 1\right]\mu_u(z)dB_u(z) \end{aligned} \tag{78}$$

where the last equality follows from $\frac{\mu_e(z)}{u'_e(c_e(z))} = K_\mu(z)\frac{\mu_u(z)}{u'_u(c_u(z))}$ (see below). As a result,

$$\frac{d\mathcal{L}}{\lambda} = (1-e(z))\left\{K_r(z)\frac{u'_u(c_u(z))}{u'_e(c_e(z))} - 1 - \left[K_\mu(z)K_r(z)\frac{1-e(z)}{e(z)} + 1\right]\mu_u(z)(T_e(z)+B_u(z))\right\}h_z(z)dB_u(z). \tag{79}$$

implying that there exists a joint reform of $B_u(z)$ and $T_e(z)$ that is Pareto-improving whenever the curly bracket is non-zero. A Pareto-efficient tax-benefit system thus satisfies,

$$K_r(z)\frac{u'_u(c_u(z))}{u'_e(c_e(z))} - 1 = \left[1 + K_\mu(z)K_r(z)\frac{1-e(z)}{e(z)}\right]\mu_u(z)(T_e(z)+B_u(z)) \tag{80}$$

$$\Longleftrightarrow K_r(z)\frac{u'_u(c_u(z))}{u'_e(c_e(z))} - 1 = \left[1 + K_\mu(z)K_r(z)\frac{1-e(z)}{e(z)}\right]\frac{\mu_u(z)B_u(z)}{e(z)}\left[1 + \frac{e(z)T_e(z) - (1-e(z))B_u(z)}{B_u(z)}\right],$$

which is Pareto-efficiency condition (20) of Proposition 2.

Relation between search semi-elasticities. The first-order condition for e associated with individuals' maximization problem (16) is

$$\begin{aligned}(FOC)_e:\ & \left[u_e\Big(z - T_e(z) - s(z) - a(z)\Big) - k(z)\right] - \left[u_u\Big(B_u(z) + \frac{e(z)}{1-e(z)}s(z)\Big) - \psi(e(z), z)\right] \\ & + \frac{s(z)}{1-e(z)}u'_u\Big(B_u(z) + \frac{e(z)}{1-e(z)}s(z)\Big) + a(z)U'(e(z)a(z)) \qquad (81) \\ & + e(z)\left[u'_u(.) - u'_e(.)\right]\frac{\partial s(z)}{\partial e} + e(z)\left[U'(.) - u'_e(.)\right]\frac{\partial a(z)}{\partial e} - (1-e(z))\frac{\partial \psi(e(z), z)}{\partial e} = 0,\end{aligned}$$

and the second-order condition is

$$\begin{aligned}(SOC)_e:\ & 2\frac{\partial \psi(e(z), z)}{\partial e} - (1-e(z))\frac{\partial^2 \psi(e(z), z)}{\partial e \partial e} + \left[\frac{u''_u(.)\,(s(z))^2}{(1-e(z))^3} + (a(z))^2 U''(.)\right] \\ & + \left[(u'_u(.) - u'_e(.)) + \frac{u''_u(.)s(z)e(z)}{(1-e(z))^2}\right]\frac{\partial s(z)}{\partial e} + \left[(U'(.) - u'_e(.)) + e(z)a(z)U''(.)\right]\frac{\partial a(z)}{\partial e} \\ & + e(z)(u'_u(.) - u'_e(.))\frac{\partial^2 s(z)}{\partial e \partial e} + e(z)(U'(.) - u'_e(.))\frac{\partial^2 a(z)}{\partial e \partial e} < 0 \qquad (82)\end{aligned}$$

Differentiating this FOC with respect to $T_e(z)$ and rearranging we get

$$\begin{aligned}\frac{(SOC)_e}{u_e'(.)}\frac{\partial e(z)}{\partial T_e(z)} &= \Bigg\{1-e(z)\frac{u_e''(.)}{u_e'(.)}\left[\frac{\partial s(z)}{\partial e}+\frac{\partial a(z)}{\partial e}\right]-e(z)\frac{u_u'(.)-u_e'(.)}{u_e'(.)}\frac{\partial^2 s(z)}{\partial T_e \partial e}-e(z)\frac{U'(.)-u_e'(.)}{u_e'(.)}\frac{\partial^2 a(z)}{\partial T_e\partial e}\Bigg\}\\ &\quad -\Bigg\{\frac{u_u'(.)-u_e'(.)}{u_e'(.)}+\frac{u_u''(.)}{u_e'(.)}\frac{e(z)}{1-e(z)}\left[\frac{s(z)}{1-e(z)}+\frac{\partial s(z)}{\partial e}e(z)\right]+\frac{u_e''(.)}{u_e'(.)}e(z)\left[\frac{\partial s(z)}{\partial e}+\frac{\partial a(z)}{\partial e}\right]\\ &\qquad +e(z)\frac{u_u'(.)-u_e'(.)}{u_e'(.)}\frac{\partial^2 s(z)}{\partial s\partial e}+e(z)\frac{U'(.)-u_e'(.)}{u_e'(.)}\frac{\partial^2 a(z)}{\partial s\partial e}\Bigg\}\frac{\partial s(z)}{\partial T_e(z)}\\ &\quad -\Bigg\{\frac{U'(.)-u_e'(.)}{u_e'(.)}+\frac{U''(.)}{u_e'(.)}e(z)\left[a(z)+e(z)\frac{\partial a(z)}{\partial e}\right]+\frac{u_e''(.)}{u_e'(.)}e(z)\left[\frac{\partial s(z)}{\partial e}+\frac{\partial a(z)}{\partial e}\right]\\ &\qquad +e(z)\frac{u_u'(.)-u_e'(.)}{u_e'(.)}\frac{\partial^2 s(z)}{\partial a\partial e}+e(z)\frac{U'(.)-u_e'(.)}{u_e'(.)}\frac{\partial^2 a(z)}{\partial a\partial e}\Bigg\}\frac{\partial a(z)}{\partial T_e(z)}. \qquad (83)\end{aligned}$$

Similarly, differentiating the FOC with respect to $B_u(z)$ and rearranging we get

$$\begin{aligned}\frac{(SOC)_e}{u_u'(.)}\frac{\partial e(z)}{\partial B_u(z)} &= \Bigg\{1-\frac{u_u''(.)}{u_u'(.)}\left[\frac{s(z)}{1-e(z)}+e(z)\frac{\partial s(z)}{\partial e}\right]-e(z)\frac{u_u'(.)-u_e'(.)}{u_u'(.)}\frac{\partial^2 s(z)}{\partial B_u \partial e}-e(z)\frac{U'(.)-u_e'(.)}{u_u'(.)}\frac{\partial^2 a(z)}{\partial B_u\partial e}\Bigg\}\\ &\quad -\Bigg\{\frac{u_u'(.)-u_e'(.)}{u_u'(.)}+\frac{u_u''(.)}{u_u'(.)}\frac{e(z)}{1-e(z)}\left[\frac{s(z)}{1-e(z)}+\frac{\partial s(z)}{\partial e}e(z)\right]+\frac{u_e''(.)}{u_u'(.)}e(z)\left[\frac{\partial s(z)}{\partial e}+\frac{\partial a(z)}{\partial e}\right]\\ &\qquad +e(z)\frac{u_u'(.)-u_e'(.)}{u_u'(.)}\frac{\partial^2 s(z)}{\partial s\partial e}+e(z)\frac{U'(.)-u_e'(.)}{u_u'(.)}\frac{\partial^2 a(z)}{\partial s\partial e}\Bigg\}\frac{\partial s(z)}{\partial B_u(z)}\\ &\quad -\Bigg\{\frac{U'(.)-u_e'(.)}{u_u'(.)}+\frac{U''(.)}{u_u'(.)}e(z)\left[a(z)+\frac{\partial a(z)}{\partial e}e(z)\right]+\frac{u_e''(.)}{u_u'(.)}e(z)\left[\frac{\partial s(z)}{\partial e}+\frac{\partial a(z)}{\partial e}\right]\\ &\qquad +e(z)\frac{u_u'(.)-u_e'(.)}{u_u'(.)}\frac{\partial^2 s(z)}{\partial a\partial e}+e(z)\frac{U'(.)-u_e'(.)}{u_u'(.)}\frac{\partial^2 a(z)}{\partial a\partial e}\Bigg\}\frac{\partial a(z)}{\partial B_u(z)}. \qquad (84)\end{aligned}$$

Omitting arguments to economize on space, let

$$\mathcal{K}_T := 1-e\frac{u_e''}{u_e'}\left[\frac{\partial s}{\partial e}+\frac{\partial a}{\partial e}\right]-e\left[\frac{u_u'-u_e'}{u_e'}\frac{\partial^2 s}{\partial T_e\partial e}+\frac{U'-u_e'}{u_e'}\frac{\partial^2 a}{\partial T_e\partial e}\right] \qquad (85)$$

$$\mathcal{K}_B := 1-\frac{u_u''}{u_u'}\left[\frac{s}{1-e}+e\frac{\partial s}{\partial e}\right]-e\frac{u_e'}{u_u'}\left[\frac{u_u'-u_e'}{u_e'}\frac{\partial^2 s}{\partial B_u\partial e}+\frac{U'-u_e'}{u_e'}\frac{\partial^2 a}{\partial B_u\partial e}\right] \qquad (86)$$

$$\mathcal{K}_s := \frac{u_u''}{u_e'}\frac{e}{1-e}\left[\frac{s}{1-e}+e\frac{\partial s}{\partial e}\right]+\frac{u_e''}{u_e'}e\left[\frac{\partial s}{\partial e}+\frac{\partial a}{\partial e}\right]+\frac{u_u'-u_e'}{u_e'}\left[1+e\frac{\partial^2 s}{\partial s\partial e}\right]+e\frac{U'-u_e'}{u_e'}\frac{\partial^2 a}{\partial s\partial e} \qquad (87)$$

$$\mathcal{K}_a := \frac{U''}{u_e'}e\left[a+e\frac{\partial a}{\partial e}\right]+\frac{u_e''}{u_e'}e\left[\frac{\partial s}{\partial e}+\frac{\partial a}{\partial e}\right]+\frac{u_u'-u_e'}{u_e'}e\frac{\partial^2 s}{\partial a\partial e}+\frac{U'-u_e'}{u_e'}\left[1+e\frac{\partial^2 a}{\partial a\partial e}\right] \qquad (88)$$

and

$$K_\mu(z) := \frac{\mathcal{K}_T(z)+\mathcal{K}_s(z)\frac{\partial s(z)}{\partial T_e(z)}+\mathcal{K}_a(z)\frac{\partial a(z)}{\partial T_e(z)}}{\mathcal{K}_B(z)+\frac{u_e'(.)}{u_u'(.)}\left[\mathcal{K}_s(z)\frac{\partial s(z)}{\partial B_u(z)}+\mathcal{K}_a(z)\frac{\partial a(z)}{\partial B_u(z)}\right]}. \qquad (89)$$

such that we finally obtain $\frac{\mu_e(z)}{u'_e(c_e(z))} = K_\mu(z)\frac{\mu_u(z)}{u'_u(c_u(z))}$.

B.3 Optimal policies with earnings decisions (Proposition 3)

Monotonicity. Consider individuals' maximization problem:

$$V(\omega) := \max_z \left\{ \max_e \; e\Big[u_e(z - T_e(z)) - k(z;\omega)\Big] + (1-e)\Big[u_u(B_u(z)) - \psi(e,z;\omega)\Big]\right\}. \qquad (90)$$

The first-order condition for search efforts defines $e(z;\omega)$ as the solution to

$$(FOC)_e : \Big[u_e(z - T_e(z)) - k(z;\omega)\Big] - \Big[u_u(B_u(z)) - \psi\,(e,z;\omega)\Big] - (1-e)\frac{\partial\psi(e,z;\omega)}{\partial e} = 0, \quad (91)$$

and the second-order condition is

$$(SOC)_e : 2\frac{\partial\psi(e,z;\omega)}{\partial e} - (1-e)\frac{\partial^2\psi(e,z;\omega)}{\partial e^2} < 0. \qquad (92)$$

Differentiating $(FOC)_e$ with respect to z yields

$$\left[(1-T'_e(z))u'_e(.) - \frac{\partial k(.)}{\partial z}\right] - \left[B'_u(z)u'_u(.) - \frac{\partial\psi(.)}{\partial z}\right] = -(SOC)_e\frac{\partial e(z;\omega)}{\partial z} + (1-e(z;\omega))\frac{\partial^2\psi(.)}{\partial e\partial z}. \qquad (93)$$

The first-order condition for earnings z defines $z(\omega)$ as the solution

$$(FOC)_z : e(z;\omega)\left[(1-T'_e(z))u'_e(z-T_e(z)) - \frac{\partial k(z;\omega)}{\partial z}\right] + (1-e(z;\omega))\left[B'_u(z)u'_u(B_u(z)) - \frac{\partial\psi(e(z;\omega),z;\omega)}{\partial z}\right] = 0, \qquad (94)$$

and the second-order condition for earnings z is

$$(SOC)_z : \frac{\partial e(z;\omega)}{\partial z}\left\{\left[(1-T'_e(z))u'_e(.) - \frac{\partial k(.)}{\partial z}\right] - \left[B'_u(z)u'_u(.) - \frac{\partial\psi(.)}{\partial z}\right] - (1-e(z;\omega))\frac{\partial^2\psi(.)}{\partial e\partial z}\right\} + e(z;\omega)\left[-T''_e(z)u'_e(.) + (1-T'_e(z))^2\,u''_e(.) - \frac{\partial^2 k(.)}{\partial z^2}\right] + (1-e(z;\omega))\left[B''_u(z)u'_u(.) + B'_u(z)u''_u(.) - \frac{\partial^2\psi(.)}{\partial z^2}\right] < 0. \qquad (95)$$

Now, differentiating $(FOC)_z$ with respect to ω, it follows that

$$\frac{dz(\omega)}{d\omega}(SOC)_z + \frac{\partial e(z;\omega)}{\partial \omega}\left\{\left[(1-T'_e(z))u'_e(.) - \frac{\partial k(.)}{\partial z}\right] - \left[B'_u(z)u'_u(.) - \frac{\partial \psi(.)}{\partial z}\right]\right\}$$
$$- e(z;\omega)\frac{\partial^2 k(.)}{\partial z \partial \omega} - (1-e(z;\omega))\left[\frac{\partial^2 \psi(.)}{\partial e \partial z}\frac{\partial e(z;\omega)}{\partial \omega} + \frac{\partial^2 \psi(.)}{\partial z \partial \omega}\right] = 0 \tag{96}$$

Using (93), simplying $\frac{\partial^2 \psi(e,z;\omega)}{\partial e \partial z}$ terms, and rearranging yields

$$\frac{dz(\omega)}{d\omega} = \frac{1}{(SOC)_z}\left[e(z;\omega)\frac{\partial^2 k(.)}{\partial z \partial \omega} + (1-e(z;\omega))\frac{\partial^2 \psi(.)}{\partial z \partial \omega} + (SOC)_e \frac{\partial e(z;\omega)}{\partial \omega}\frac{\partial e(z;\omega)}{\partial z}\right], \tag{97}$$

which is (22) in the text.

Optimal tax. Consider a marginal increase $\Delta\tau$ in the marginal tax rate $T'_e(z)$ that applies to agents in a small bandwidth of income $[z, z+\Delta z]$ which translates in a lump-sum increase in taxes $\Delta\tau\Delta z$ on all agents employed at earnings levels $z \geq z + \Delta z$. This reform induces four different effects:

(1) A mechanical effect ΔM capturing mechanical changes in the government budget and in agents' well-being from the lump-sum tax increase $\Delta\tau\Delta z$

$$\lim_{\Delta z \to 0} \Delta M = \Delta\tau\Delta z \int_{x \geq z} e(x)(1-g_e(x))h_z(x)dx \tag{98}$$

(2) A behavioral effect ΔB_z capturing earnings responses to the reform and that can be decomposed into substitution effects in response to the $\Delta\tau$ increase in the marginal tax rate, and into income effects in response to the $\Delta\tau\Delta z$ increase in the tax level

$$\lim_{\Delta z \to 0} \Delta B_z = -Q(z)z\zeta_e(z)\Delta\tau\, h_z(z)\Delta z + \int_{x \geq z} Q(x)\eta^z_e(x)\,\Delta\tau\Delta z\, h_z(x)dx \tag{99}$$

where $Q(z) := e(z)T'_e(z) - (1-e(z))B'_u(z)$ is the marginal tax-benefit rate capturing the net fiscal externality induced by a marginal change in earnings z.

(3) A behavioral effect ΔB_e capturing job search responses to the reform and that can be decomposed into direct effects in response to the $\Delta\tau\Delta z$ increase in the tax level, and into indirect effects induced by earnings changes

$$\lim_{\Delta z \to 0} \Delta B_e = (T_e(z) + B_u(z))\xi^{1-e}_z(z)\, z\zeta_e(z)\Delta\tau\, h_z(z)\Delta z \tag{100}$$
$$- \int_{x \geq z} (T_e(x) + B_u(x))\Big[(1-e(x))\mu_e(x) + \xi^{1-e}_z(x)\eta_e(x)\Big]\,\Delta\tau\Delta z\, h_z(x)dx$$

where $T_e(z) + B_u(z)$ is the employment tax capturing the net fiscal externality induced by changes in job search.

(4) A participation effect ΔP capturing negative participation responses to the reform due to the $\delta\tau\delta z$ increase in the tax level

$$\lim_{\Delta z \to 0} \Delta P = -\int_{x \geq z} \Big[(T_e(x) + R_0) - (1 - e(x))(T_e(x) + B_u(x))\Big] \pi_e(x)\, \Delta\tau\Delta z\, h_z(x)dx \quad (101)$$

where $(T_e(z)+R_0)-(1-e(z))(T_e(z)+B_u(z))$ is the participation tax net of the employment tax when unemployed, capturing the fiscal externality induced by changes in participation decisions.

Let $\Delta\mathcal{L}_e(z, \Delta z, \Delta\tau) := \Delta M + \Delta B_z + \Delta B_e + \Delta P$ be the total effect of this reform,

$$\begin{aligned}\Delta\mathcal{L}_e = \Bigg\{ &-\Big[Q(z) - (T_e(z)+B_u(z))\xi_z^{1-e}(z)\Big] z\zeta_e(z)h_z(z) \qquad (102)\\ &+ \int_{x \geq z} \Big[e(x)(1-g_e(x)) - (T_e(x)+R_0)\pi_e(x) + (1-e(x))(T_e(x)+B_u(x))(\pi_e(x)-\mu_e(x)) \\ &\qquad + (Q(x)-(T_e(x)+B_u(x))\xi_z^{1-e}(x))\eta_e(x)\Big] h_z(x)dx \Bigg\} \Delta\tau\Delta z\end{aligned}$$

with $Q(z) = e(z)T_e'(z) - (1 - e(z))B_u'(z)$. Characterizing the optimal tax schedule as one that cannot be improved upon, $\Delta\mathcal{L}_e = 0$, yields optimal tax formula (27) of Proposition 3.

Optimal benefit. Consider a marginal increase Δb in the marginal benefits rate $B_u'(z)$ that applies to small bandwidth of earnings $[z, z + \Delta z]$ which translates in a lump-sum increase in benefits $\Delta b\Delta z$ on all unemployed agents with reference earnings levels $z \geq z + \Delta z$. This reform induces four different effects:

(1) A mechanical effect ΔM capturing mechanical changes in the government budget and in agents' well-being from the lump-sum $\Delta b\Delta z$ increase in the level of benefits above z

$$\lim_{\Delta z \to 0} \Delta M = \Delta b\Delta z \int_{x \geq z} (1 - e(x))(g_u(x) - 1)h_z(x)dx \quad (103)$$

(2) A behavioral effect ΔB_z capturing earnings responses to the reform and that can be decomposed into substitution effects in response to the Δb increase in the marginal benefits rate, and into income effects in response to the $\Delta b\Delta z$ increase in the level of benefits above z

$$\lim_{\Delta z \to 0} \delta B_z = Q(z)z\zeta_u(z)\Delta b h_z(z)\Delta z - \int_{x \geq z} Q(x)\eta_u(x)\, \Delta b\Delta z\, h_z(x)dx \quad (104)$$

where $Q(z) = e(z)T_e'(z) - (1 - e(z))B_u'(z)$ is the marginal tax-benefits rate capturing the net fiscal externality induced by a marginal change in earnings z.

(3) A behavioral effect ΔB_e capturing job search responses to the reform and that can be decomposed into direct effects in response to the $\Delta b \Delta z$ increase in the benefit level, and into indirect effects induced by earnings changes

$$\lim_{\Delta z \to 0} \Delta B_e = -\left(T_e(z) + B_u(z)\right) z \xi_z^{1-e}(z) \zeta_u(z)\, \Delta b\, h_z(z) \Delta z \qquad (105)$$
$$- \int_{x \geq z} (T_e(x) + B_u(x)) \Big[(1 - e(x))\mu_u(x) - \xi_z^{1-e}(x)\eta_u(x)\Big]\, \Delta b \Delta z\, h_z(x) dx$$

where $T_e(z) + B_u(z)$ is the employment tax capturing the net fiscal externality induced by changes in job search.

(4) A participation effect ΔP capturing positive participation responses to the reform due to the $\Delta b \Delta z$ increase in the benefit level

$$\lim_{\Delta z \to 0} \Delta P = \int_{x \geq z} \Big[(T_e(x) + R_0) - (1 - e(x))(T_e(x) + B_u(x))\Big] \pi_u(x)\, \Delta b \Delta z\, h_z(x) dx \qquad (106)$$

where $(T_e(z) + R_0) - (1 - e(z))(T_e(z) + B_u(z))$ is the participation tax capturing the net fiscal externality induced by changes in participation decisions.

Let $\Delta \mathcal{L}_u(z, \Delta z, \Delta b) := \Delta M + \Delta B_z + \Delta B_e + \Delta P$ be the total effect of this reform,

$$\Delta \mathcal{L}_u = \Bigg\{ \Big(Q(z) - (T_e(z) + B_u(z)) \xi_z^{1-e}(z)\Big) z \zeta_u(z)\, h_z(z) \qquad (107)$$
$$+ \int_{x \geq z} \Big[(1-e(x))(g_u(x)-1) + (T_e(x)+R_0)\pi_u(x) - (1-e(x))(T_e(x)+B_u(x))(\pi_u(x)+\mu_u(x))$$
$$- \Big(Q(x) - (T_e(x)+B_u(x))\xi_z^{1-e}(x)\Big)\eta_u(x)\Big] h_z(x) dx \Bigg\} \Delta b \Delta z.$$

with $Q(z) = e(z)T_e'(z) - (1 - e(z))B_u'(z)$. Characterizing the optimal benefit schedule as one that cannot be improved upon, $\Delta \mathcal{L}_u = 0$, yields the following optimal benefit formula that is not reported in the main text:

$$- \Big(Q(z) - (T_e(z) + B_u(z))\xi_z^{1-e}(z)\Big)\, z \zeta_u(z)\, h_z(z) \qquad (108)$$
$$= \int_{x \geq z} \Big[(1-e(x))(g_u(x)-1) + (T_e(x)+R_0)\pi_u(x) - (1-e(x))(T_e(x)+B_u(x))(\pi_u(x)+\mu_u(x))$$
$$- \Big(Q(x) - (T_e(x)+B_u(x))\xi_z^{1-e}(x)\Big)\eta_u(x)\Big] h_z(x) dx.$$

Pareto-efficiency. In this setting, a joint increase in the level of taxes and benefits is engineered through the use of the following two-bracket reforms (F. Bierbrauer et al., 2020).

First, a two-bracket tax increase at earnings z that consists in a $\Delta\tau$ increase in the marginal tax rate in the bracket $[z, z + \Delta z]$ combined with a $\Delta\tau$ decrease in the marginal tax rate in the bracket $[z + \varepsilon, z + \varepsilon + \Delta z]$, where $\varepsilon > \Delta z > 0$.

Second, a two-bracket benefit increase at earnings z that consists in a Δb increase in the marginal benefit rate in the bracket $[z, z + \Delta z]$ combined with a Δb decrease in the marginal tax rate in the bracket $[z + \varepsilon, z + \varepsilon + \Delta z]$.

In each bracket, simultaneous changes in marginal tax and benefit rates do not trigger first-order changes in utilities, search or participation, but do trigger first-order changes in earnings through substitution effects. Letting the width of each bracket Δz go to zero, the total earnings response in the bracket $[z, z + \Delta z]$ is

$$\begin{aligned} dz &= \left[-z\zeta_e(z)\Delta\tau + z\zeta_u(z)\Delta b\right] h_z(z)\Delta z \\ &= \left[-\frac{e(z)}{(1-e(z))}\frac{u'(z-T_e(z))}{u'(B_u(z))}\Delta\tau + \Delta b\right]\zeta_u(z) z h_z(z)\Delta z \end{aligned} \tag{109}$$

where the second equality follows from $\zeta_e(z) = -\frac{e(z)}{(1-e(z))}\frac{u'(z-T_e(z))}{u'(B_u(z))}\zeta_u(z)$. A similar expression gives the total earnings response in the second bracket.

Now, these two-bracket reforms generate for all individuals with earnings in the interval $[z, z + \varepsilon]$ a $\Delta\tau\Delta z$ lump-sum increase in taxes and a $\Delta b\Delta z$ lump-sum increase in benefits. Letting the width of this interval, ε, go to zero, the former decreases their utility by $\Delta\tau\Delta z e(z)u'(z - T_e(z))$, while the latter increases their utility by $\Delta b\Delta z(1 - e(z))u'(B_u(z))$ in application of the envelope theorem.

Setting $\Delta\tau = \frac{1-e(z)}{e(z)}\frac{u'(B_u(z))}{u'(z-T_e(z))}\Delta b$ ensures that the utility of individuals in the interval $[z, z + \varepsilon]$ is unaffected by the reform. This implies that participation decisions are unaffected, but also that in each bracket the total earnings response is exactly zero. As a result, these joint two-bracket reforms only trigger fiscal effects in the interval $[z, z + \varepsilon]$ stemming from mechanical impacts, changes in job search, and changes in earnings due income effects.

The total impact of these combined two-bracket reforms is thus:

$$\begin{aligned} \Delta\mathcal{L} = &\Big\{e(z) - (1-e(z))(T_e(z)+B_u(z))\mu_e(z) + \Big[Q(z) - (T_e(z)+B_u(z))\xi_z^{1-e}(z)\Big]\eta_e(z)\Big\}\Delta\tau\Delta z h_z(z)\varepsilon \\ &+ \Big\{-(1-e(z)) - (1-e(z))(T_e(z)+B_u(z))\mu_u(z) - \Big(Q(z) - (T_e(z)+B_u(z))\xi_z^{1-e}(z)\Big)\eta_u(z)\Big\}\Delta b\Delta z h_z(z)\varepsilon \end{aligned} \tag{110}$$

Further using $\mu_e(z) = \frac{u'(z-T_e(z))}{u'(B_u(z))}\mu_u(z)$ as well as $\eta_e(z) = \frac{1-T_e'(z)}{B_u'(z)}\frac{e(z)}{1-e(z)}\frac{u''(z-T_e(z))}{u''(B_u(z))}\eta_u(z)$ and

$\Delta\tau = \frac{1-e(z)}{e(z)} \frac{u'(B_u(z))}{u'(z-T_e(z))}\Delta b$, we get

$$\Delta\mathcal{L} = \Bigg\{(1-e(z))\left[\frac{u'(B_u(z))}{u'(z-T_e(z))} - 1 - \frac{\mu_u(z)}{e(z)}(T_e(z)+B_u(z))\right] + \Big(Q(z)-(T_e(z)+B_u(z))\xi_z^{1-e}(z)\Big)\eta_u(z)\left[\frac{1-T_e'(z)}{B_u'(z)}\frac{u'(B_u(z))}{u'(z-T_e(z))}\frac{u''(z-T_e(z))}{u''(B_u(z))} - 1\right]\Bigg\}\Delta b\Delta z h_z(z)\varepsilon. \tag{111}$$

Setting $\Delta\mathcal{L} = 0$ and rewriting the employment tax in terms of the net contribution to the tax-benefit system, we obtain the Pareto-efficiency condition (28) of Proposition 3:

$$\frac{u'(B_u(z))}{u'(z-T_e(z))} - 1 = \frac{\mu_u(z)B_u(z)}{e(z)^2}\left[1 + \frac{1}{B_u(z)}\Big(e(z)T_e(z) - (1-e(z))B_u(z)\Big)\right] + \Big(Q(z) - (T_e(z)+B_u(z))\xi_z^{1-e}(z)\Big)\frac{\eta_u(z)}{1-e(z)}\left[\frac{1-T_e'(z)}{B_u'(z)}\frac{u'(B_u(z))}{u'(z-T_e(z))}\frac{u''(z-T_e(z))}{u''(B_u(z))} - 1\right]. \tag{112}$$

Going beyond the previous heuristic proof, we formally want to compute:

$$\begin{aligned}\Delta\mathcal{L} &= \Delta\mathcal{L}_e(z,\Delta z,\Delta\tau) + \Delta\mathcal{L}_e(z+\varepsilon,\Delta z,-\Delta\tau) + \Delta\mathcal{L}_u(z,\Delta z,\Delta b) + \Delta\mathcal{L}_u(z+\varepsilon,\Delta z,-\Delta b)\\ &= \left[\Delta\mathcal{L}_e(z,1,1) - \Delta\mathcal{L}_e(z+\varepsilon,1,1)\right]\Delta z\Delta\tau + \left[\Delta\mathcal{L}_u(z,1,1) - \Delta\mathcal{L}_u(z+\varepsilon,1,1)\right]\Delta z\Delta b\end{aligned} \tag{113}$$

Letting ε go to zero, and noting that by definition $\frac{\varphi(x+\varepsilon)-\varphi(x)}{\varepsilon} \to \varphi'(x)$, we get

$$\lim_{\varepsilon\to 0}\Delta\mathcal{L} = -\left[\frac{\partial}{\partial z}\left(\Delta\mathcal{L}_e(z,1,1)\right)\Delta\tau + \frac{\partial}{\partial z}\left(\Delta\mathcal{L}_u(z,1,1)\right)\Delta b\right]\varepsilon\Delta z \tag{114}$$

Now, assuming semi-elasticities $\zeta_e(z)$, $\zeta_u(z)$, and $\xi_z^{1-e}(z)$ are locally constant, we have

$$\begin{aligned}\frac{\partial}{\partial z}\left(\Delta\mathcal{L}_e\right) = &-\Big(Q'(z)-(T_e'(z)+B_u'(z))\xi_z^{1-e}\Big)zh_z(z)\zeta_e - \Big(Q(z)-(T_e(z)+B_u(z))\xi_z^{1-e}\Big)(h_z(z)+zh_z'(z))\zeta_e\\ &+\Big[e(z)(1-g_e(z)) - (T_e(z)+R_0)\pi_e(z) + (1-e(z))(T_e(z)+B_u(z))(\pi_e(z)-\mu_e(z))\\ &\quad + (Q(z)-(T_e(z)+B_u(z))\xi_z^{1-e}(z))\eta_e(z)\Big]h_z(z),\end{aligned} \tag{115}$$

as well as

$$\begin{aligned}\frac{\partial}{\partial z}\left(\Delta\mathcal{L}_u\right) = &\Big(Q'(z)-(T_e'(z)+B_u'(z))\xi_z^{1-e}\Big)zh_z(z)\zeta_u + \Big(Q(z)-(T_e(z)+B_u(z))\xi_z^{1-e}\Big)(h_z(z)+zh_z'(z))\zeta_u\\ &+\Big[(1-e(z))(g_u(z)-1) + (T_e(z)+R_0)\pi_u(z) - (1-e(z))(T_e(z)+B_u(z))(\pi_u(z)+\mu_u(z))\\ &\quad + (Q(z)-(T_e(z)+B_u(z))\xi_z^{1-e}(z))\eta_u(z)\Big]h_z(z).\end{aligned} \tag{116}$$

Making use of

$$
\begin{aligned}
\Delta\tau &= \frac{1-e(z)}{e(z)}\frac{u'(B_u(z))}{u'(z-T_e(z))}\Delta b \\
g_e(z) &= \frac{u'(z-T_e(z))}{u'(B_u(z))}g_u(z) \\
\mu_e(z) &= \frac{u'(z-T_e(z))}{u'(B_u(z))}\mu_u(z) \\
\pi_e(z) &= \frac{e(z)}{1-e(z)}\frac{u'(z-T_e(z))}{u'(B_u(z))}\pi_u(z) \\
\eta_e(z) &= \frac{1-T_e'(z)}{B_u'(z)}\frac{e(z)}{(1-e(z))}\frac{u''(z-T_e(z))}{u''(B_u(z))}\eta_u(z) \\
\zeta_e(z) &= \frac{e(z)}{1-e(z)}\frac{u'(z-T_e(z))}{u'(B_u(z))}\zeta_u(z)
\end{aligned}
$$

we then recover the following expression which completes this formal proof:

$$
\begin{aligned}
\lim_{\varepsilon\to 0}\Delta\mathcal{L} =& \Bigg\{(1-e(z))\left[\frac{u'(B_u(z))}{u'(z-T_e(z))}-1-\frac{\mu_u(z)}{e(z)}(T_e(z)+B_u(z))\right] \\
&+\Big(Q(z)-(T_e(z)+B_u(z))\xi_z^{1-e}(z)\Big)\eta_u(z)\left[\frac{1-T_e'(z)}{B_u'(z)}\frac{u'(B_u(z))}{u'(z-T_e(z))}\frac{u''(z-T_e(z))}{u''(B_u(z))}-1\right]\Bigg\}\Delta b\Delta z h_z(z)\varepsilon.
\end{aligned} \tag{117}
$$

Relations between sufficient statistics. The link between social marginal welfare weights $g_e(z)$ and $g_u(z)$ follows from the definition of these statistics. The link between search semi-elasticities $\mu_e(z)$ and $\mu_u(z)$ is exactly the same as in the baseline.

For participation semi-elasticities, we have

$$
h_z(z) = f_z(z)\int_{\chi\leq\tilde{\chi}(z)} f_{\chi|z}(\chi)d\chi \tag{118}
$$

such that

$$
dh_z(z) = f_z(z)\, d\left(\int_{\chi\leq\tilde{\chi}(z)} f_{\chi|z}(\chi)d\chi\right) = f_z(z)\, f_{\chi|z}(\tilde{\chi}(z))\, d\tilde{\chi}(z) \tag{119}
$$

with changes in participation thresholds

$$
\begin{aligned}
d\tilde{\chi}(z) &= d\Big(V(z)-u(R_0)\Big) \\
&= -e(z)u_e'(z-T_e(z))dT_e(z) + (1-e(z))\,u_u'(B_u(z))dB_u(z)
\end{aligned} \tag{120}
$$

Hence,

$$\pi_e(z) = \frac{1}{h_z(z)}\frac{\partial h_z(z)}{\partial(z-T_e(z))} = \frac{f_{\chi|z}(\tilde{\chi}(z))}{\int_{\chi\leq\tilde{\chi}(z)} f_{\chi|z}(\chi)d\chi} e(z)u_e'(z-T_e(z)) \tag{121}$$

$$\pi_u(z) = \frac{1}{h_z(z)}\frac{\partial h_z(z)}{\partial B_u(z)} = \frac{f_{\chi|z}(\tilde{\chi}(z))}{\int_{\chi\leq\tilde{\chi}(z)} f_{\chi|z}(\chi)d\chi}(1-e(z))u_u'(B_u(z)) \tag{122}$$

such that finally

$$\frac{\pi_e(z)}{e(z)u_e'(z-T_e(z))} = \frac{f_{\chi|z}(\tilde{\chi}(z))}{\int_{\chi\leq\tilde{\chi}(z)} f_{\chi|z}(\chi)d\chi} = \frac{\pi_u(z)}{(1-e(z))u_u'(B_u(z))} \tag{123}$$

For earnings semi-elasticities, differentiating the first-order condition for earnings, $(FOC)_z$, with respect to changes in tax-benefit levels and rates yields

$$\begin{aligned}(SOC)_z dz = {} & e(z;\omega)\left[u_e'(.)dT_e'(z) + (1-T_e'(z))u_e''(.)dT_e(z)\right] \\ & -(1-e(z;\omega))\left[u_u'(.)dB_u'(z) + B_u'(z)u_u''(.)dB_u(z)\right].\end{aligned} \tag{124}$$

This gives

$$\zeta_e(z) = \frac{1}{z}\frac{\partial z}{\partial(1-T_e'(z))} = -\frac{1}{z}\frac{e(z)u_e'(.)}{(SOC)_z} \tag{125}$$

$$\zeta_u(z) = \frac{1}{z}\frac{\partial z}{\partial B_u'(z)} = -\frac{1}{z}\frac{(1-e(z))u_u'(.)}{(SOC)_z} \tag{126}$$

such that

$$\frac{\zeta_e(z)}{e(z)u_e'(z-T_e(z))} = -\frac{1}{z}\frac{1}{(SOC)_z} = \frac{\zeta_u(z)}{(1-e(z))u_u'(B_u(z))}. \tag{127}$$

Similarly, we have

$$\eta_e(z) = \frac{\partial z}{\partial T_e(z)} = \frac{e(z)(1-T_e'(z))u_e''(.)}{(SOC)_z} \tag{128}$$

$$\eta_u(z) = -\frac{\partial z}{\partial B_u(z)} = \frac{(1-e(z))B_u'(z)u_u''(.)}{(SOC)_z} \tag{129}$$

such that

$$\frac{\eta_e(z)}{e(z)(1-T_e'(z))u_e''(z-T_e(z))} = \frac{1}{(SOC)_z} = \frac{\eta_u(z)}{(1-e(z))B_u'(z)u_u''(B_u(z))}. \tag{130}$$