# Cooperation, Competition, and Welfare in a Matching Market

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Discussion Paper No. 332

July 21, 2022

Collaborative Research Center Transregio 190 | www.rationality-and-competition.de
Ludwig-Maximilians-Universität München | Humboldt-Universität zu Berlin
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# Cooperation, Competition, and Welfare in a Matching Market*

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July 2022

**Abstract**

We investigate the welfare effect of increasing competition in an anonymous two-sided matching market, where matched pairs play an infinitely repeated Prisoner's Dilemma. Higher matching efficiency is usually considered detrimental as it creates stronger incentives for defection. We point out, however, that a reduction in matching frictions also increases welfare because more agents find themselves in a cooperative relationship. We characterize the conditions for which increasing competition increases overall welfare. In particular, this is always the case when the incentives for defection are high.

**JEL classifications:** C72, C73, C78, D6

**Keywords:** Cooperation, Prisoner's Dilemma, Competition, Welfare, Matching, Trust Building

*We wish to thank Roberto Burguet for comments and suggestions. Helmut Bester acknowledges support by the University of the Balearic Islands and the Deutsche Forschungsgemeinschaft through CRC TRR 190 (project number 280092119). József Sákovics thanks the Spanish Government for a Beatriz Galindo grant (BG20/00079) and grant PID2020-115018RB-C33 funded by MCIN/AEI/ 10.13039/501100011033.

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# 1 Introduction

Does increasing competition increase welfare? While this is widely believed to be true for the contractible exchange of goods and services, the answer is less clear for interactions that rely on implicit trust and reputation.[1] In this paper we address this question by investigating the implications of competition on cooperation and welfare in an anonymous matching market environment: agents in a bilateral match play the repeated Prisoner's Dilemma until they are exogenously separated. In a highly competitive market with low matching frictions the delay to find a new partner for cooperation is short. This positive effect of competition on welfare is offset by the fact that it lowers the incentives for cooperation: a defector can be punished only by having to enter the matching market again after cheating leads to a break–up of the current partnership. Thus, to prevent defection from becoming attractive with low matching frictions, it is well-known that some welfare losses in a match are unavoidable. We first analyze how competition affects the feasible gains from cooperation. Then we determine the welfare implications of competition by investigating how in equilibrium the agents' expected lifetime utilities depend on the level of matching frictions.

Voluntary cooperation is often a necessary ingredient in the generation of surplus. As it gives rise to the possibility of opportunistic behavior, trust becomes a key component of welfare creation. When moral imperatives, institutions or exogenous enforcement cannot be relied upon, trust needs to be instrumentalized via an implicit/relational contract. Such a contract includes credible threats as consequence of misbehavior. By the Folk Theorem of repeated games,[2] it has long been established that if the players value future interaction sufficiently then – as long as their actions are mutually observable – cooperation is sustainable in the infinitely repeated Prisoner's Dilemma as a subgame-perfect equilibrium.

This result, however, looks at the Prisoner's Dilemma in isolation, where the players

[1] Huck et al. (2012) find in an experiment that competition in combination with information fosters trust. But, in an anonymous setting – as in our paper – trust building is poor.

[2] A standard reference is Friedman (1971).

do not have the option to leave. Indeed, it is considerably weakened when cooperation is embedded in a (matching) market and voluntary severance is allowed, especially when the new partner does not learn about the past behavior. The possibility to anonymously switch partners stymies the efforts to sustain cooperation, as a player can escape punishment for defection by the current partner by exiting the relationship, while the new partner is unaware of it. The consequence of the option to quit, however, is not the complete absence of cooperation, rather it is that the surplus from cooperation that can be sustained in equilibrium is reduced: only a part of the full gains from cooperation may be attainable in a market with low matching frictions. But, from a welfare perspective, this does not take into account that higher market efficiency leads to more people in a cooperative relationship, thereby increasing social welfare. In this paper we evaluate the trade-off between these two, opposing, effects of market efficiency on social welfare.

More precisely, we model a two–sided[3] anonymous matching market with a continuum of agents, where agents from both sides of the market are randomly matched to play a repeated Prisoner's Dilemma. The participation of players in the Prisoner's Dilemma is voluntary: at the end of every period they can unilaterally break off and return to the matching market. The key parameter of our interest is the efficiency of the matching market, $\alpha$, measuring the probability that an agent on the short side finds a partner immediately. We focus on the steady state of this market. To avoid corner solutions (and for realism) we assume that a partnership can also break up for exogenous reasons, with probability $\beta$. Then the steady-state proportion of agents in a match is a function of these two parameters in a cooperative equilibrium. In particular – and agreeing with intuition – the measure of matched agents is increasing in $\alpha$ and decreasing in $\beta$.

As our first result, we show that there exists a threshold, that increases in the agents' common discount factor $\delta$, with the following property: the full gains from

[3]When the two sides contain the same measure of agents, our results can be interpreted as those for a "one-sided" market.

cooperation can be attained in a subgame-perfect equilibrium only if the matching market efficiency, as measured by $\alpha$, lies below this threshold. For higher values of $\alpha$, the surplus available in a match has to be reduced to prevent opportunistic behavior. The literature presents several forms of such inefficiencies to build trust.[4] For example giving gifts at the beginning of a relationship may lead to trust and cooperation. The idea is that gift giving at each new match is more costly for a defector, because he changes partners more frequently than a cooperator.[5] Similarly, reducing the scale of interaction in the beginning or delaying cooperation can make defection unattractive.

For simplicity and tractability, we use "money burning" as a generic shortcut to describe trust building efforts: for values of $\alpha$ above the critical threshold, in a new match either one or both agents have to burn some amount of money in an equilibrium with *conditional cooperation*. In such an equilibrium, the players still cooperate in every period, but only if at the beginning of the first period of their match they have burned enough money; otherwise, they break up. Moreover, the aggregate amount of the required deadweight loss is strictly increasing in $\alpha$. However, it is not symmetrically shared by the two sides of the market: since agents on the short side of the market have a higher probability of a quick rematch, they have stronger incentives to deviate and as a consequence they are required to waste more effort on trust building.

Our results on the sustainability of cooperation with and without money burning allow us to calculate welfare as the expected discounted utility of agents on both sides of the market. We find that for a wide range of parameter constellations welfare is increasing in $\alpha$. In particular, this is always true when $\alpha$ is low enough so that cooperation can be sustained without money burning. But, it is also true for higher values of $\alpha$ whenever the per period deviation payoff in the Prisoner's Dilemma game, denoted as $B$, is more than twice as high as the cooperation payoff, denoted as $C$. If $B \geq 2C$, the welfare gain from a higher fraction of agents being matched outweighs the potential

[4]See the related literature part of this section for references.

[5]This function of gift giving is rather different from the gift exchange model proposed by Akerlof (1982), where firms willingly pay a wage above the market clearing wage and workers reciprocate by supplying higher effort.

increase of wasteful investments in trust building.

Perhaps surprisingly, welfare on one side or both sides of the market may be decreasing in $\alpha$ when the per period deviation payoff in the Prisoner's Dilemma game, $B$, is relatively low. The reason is that the expenditures on trust building are a submodular function of $\alpha$ and $B$: they are increasing in $\alpha$, but this increase is the lower the higher is $B$. As a result, high values of $B$ guarantee that welfare is always increasing in $\alpha$, whereas — depending also on the break–up probability $\beta$ and the discount factor $\delta$ – it may be decreasing in $\alpha$ for low values of $B$.

Further, the welfare implications of an increase in competition may differ for the two sides of the market. The intuition is that, as we have noted above, the short side of the market may have to invest more resources in trust building than the long side. For this reason, there are parameter constellations such that the short side suffers and the long side benefits from more competition. But, it can never be the other way around: whenever an increase in $\alpha$ raises welfare on the short side, then also the long side gains.

**Related literature**

The literature on the emergence and limits of cooperation is too vast to review here. Instead we confine ourselves with briefly pointing out the papers that help most in positioning our contribution.

Several authors investigate conditions under which cooperation can be sustained in situations where agents change partners over time: after a one-shot interaction in each period they break up and are randomly matched with another opponent in the following period. When interactions with other partners are not observable, punishment cannot be meted out for unobserved defections.[6] Thus cooperation may seem impossible to achieve. But, as Kandori (1992) and Harrington (1995) discovered, even if new partners could not observe each other's past behavior in matches with other players, the ones against whom an agent previously has defected, could. Based on this insight, they have

[6] Milgrom et al. (1990) discuss institutions that facilitate community punishments when traders are not perfectly informed about each other's behavior.

constructed a contagious punishment strategy, where agents who have been betrayed, defect in the future. Within a finite population, this chain of defections infects the whole population, eventually driving payoffs to zero. Thus, unless the deviation payoff is very high, defection can be deterred for patient enough players.[7]

We consider an infinite population, where the contagion argument does not work well. Instead of one-shot interactions, we assume that the break–up probability $\beta$ in a match is sufficiently small so that repeated play of the Prisoner's Dilemma allows for some cooperation. In addition, the probability $\alpha$ of finding a new partner after break–up is less than one in each period. The associated delay cost makes it possible to punish a defector by terminating the partnership. Finally, whereas the literature on playing the Prisoner's Dilemma with changing partners looks at a one–sided market, where all agents are drawn from a common pool, we employ a two–sided market to address the differential effects of sustaining cooperation on the short and long side.

If the matching process is highly efficient, then punishing a defector by termination may not be sufficient to sustain cooperation. Instead, as noted by several authors, establishing trust may require some inefficiencies. Datta (1996) considers a repeated borrower lender relation, where the borrower can cheat by running away with the money. In his model, the amount of lending is inefficiently low at beginning, but increases over time. Carmichael and MacLeod (1997) show in an evolutionary model that costly gift giving at the beginning of a relationship leads to trust and cooperation. Although the gifts themselves are useless, they are necessary to support cooperation in the repeated Prisoner's Dilemma. The same goal can be achieved by inefficiently delaying full cooperation as shown by Kranton (1996a) and Ghosh and Ray (1996).[8] All these types of efficiency losses effectively amount to "money burning", and as a shortcut we

[7]See Ellison (1994) for an improvement, where the punishment phase is of finite (expected) length. See Heller and Mohlin (2018) for a recent contribution, where each player observes a few of the partner's past actions against previous opponents.

[8]Fujiwara-Greve and Okuno-Fujiwara (2009) introduce the case where (identical) players differ across their strategies and show that the efficiency loss necessary for sustaining cooperation can be reduced.

formalize them in this way, without detailing the specific form of wasteful activities. This approach covers different kinds of trust building activities and puts the focus on their welfare losses. It also simplifies the formal analysis and enables us to describe the differences in trust building efforts on the two sides of the market.

Kranton (1996b) analyzes a model where identical traders can choose between reciprocal exchange in personalized, long-term exchange relationships and an anonymous spot market. Market thickness is endogenously determined. Reciprocity is self–sustaining, when it is the dominant interaction, as it is hard to find a trader in the spot market. In our model, market thickness is – in part – given by the exogenous parameter $\alpha$, so we can analyze the effect of varying it. Also, we have a two-sided market, where we can study the differential effects on each side.

Some authors have used a hold–up context to model cooperation. Ramey and Watson (2001), look at relationship–specific investments followed by costly effort provision, also embedded in an anonymous random matching market. Paralleling the results for the Prisoner's Dilemma, they find that when it is easy to find a new partner, an equilibrium with cooperation requires overinvestment in order to sustain effort incentives in the ongoing relationship. Bester (2013) considers a classic hold–up situation, investment followed by bargaining, in an anonymous random matching market without relationship specificity of investments. He finds that agents on the long side of the market suffer more from hold–up and underinvest by more, as they have a worse outside option. Because search frictions are the direct cause of inefficiency, as frictions decrease efficiency of investment increases.

This paper is organized as follows: Section 2 embeds the Prisoner's Dilemma game in a two–sided matching market. In Section 3 we show that the market has a unique steady–state, which is also stable. The conditions under which cooperation can be sustained without requiring trust building activities are described in Section 4. When these conditions are not satisfied, cooperation requires wasteful investments in trust building, as analyzed in Section 5. Section 6 derives the welfare implications of competition. In Section 7 we provide concluding remarks and indicate extensions of our model.

## 2 Model

We consider a two-sided market, where in each period some agents from opposite sides of the market are randomly matched into pairs. All agents are infinitely lived, risk neutral, and identical. In a match agents do not have any information about the past actions of their partner. Consequently, they cannot condition their behavior on history previous to the match.

On the short side there is a unit mass of agents. The measure of agents on the long side is $K \geq 1$. Since all players are identical, when $K = 1$, our model also is mathematically equivalent to a "one-sided" market where pairs are formed in a homogeneous pool. Matched players play a potentially infinitely repeated symmetric Prisoner's Dilemma. Time periods are discrete, and players discount future payoffs by the discount factor $\delta \in (0, 1)$.

The details of the interaction are the following: At the end of each period, the unmatched players on the short side are matched to a partner from the long side with exogenous probability $\alpha \in (0, 1]$. This implies that the probability of being matched for agents on the long side depends on the relative measures of players on the two sides of the matching market.[9] The parameter $\alpha$ captures the competitiveness of the matching market and is inversely related to frictions in the matching process. In the absence of all frictions $\alpha = 1$: all the players on the short side find a partner. At the other extreme, the limit $\alpha \to 0$ corresponds to the case where the matching market breaks down.

When two players from opposite sides of the market are matched, they choose their actions in the Prisoner's Dilemma and collect their payoffs. At the end of the period the match may break up for two reasons. First, with probability $\beta \in (0, 1)$ the match becomes unproductive and is dissolved exogenously. Additionally, players have the option to leave their partner and reenter the matching process. In either case, *both* players join the pool of unmatched players, to be potentially rematched in the next

[9] The matching probability on the long side of the market is derived in Section 3.

| | Cooperate | Defect |
|---|---|---|
| Cooperate | $C$ , $C$ | $F$ , $B$ |
| Defect | $B$ , $F$ | $D$ , $D$ |

Figure 1: *Payoffs of Prisoner's Dilemma*

period. If the relationship continues, then in the following period they repeat the same steps.

All utilities are derived from the payoffs of the Prisoner's Dilemma, described by the one-shot payoff bi-matrix in Figure 1, where $B > C > D = 0 > F$.[10] We further assume that $B + F < 2C$, to capture that mutual cooperation is the socially optimal outcome.

As a reference point, it is useful to state the standard result that cooperation in the isolated repeated Prisoner's Dilemma – without the possibility of finding a new partner after breakdown – is feasible as a subgame-perfect Nash equilibrium if (and only if) the discount factor is high enough:[11]

$$\delta \geq \underline{\delta} \equiv \frac{B - C}{B(1 - \beta)}. \tag{1}$$

If (1) holds, cooperation can be supported by grim trigger strategies: on the equilibrium path both players cooperate in each period; but as soon as one player deviates by defecting, the other player defects in the next and all subsequent rounds. A deviation from cooperation, therefore, triggers mutual defection as the off-path equilibrium for the remainder of the relationship.

In what follows, we apply the following

[10]Setting $D = 0$ allows us to compare the critical discount factor of the isolated Prisoner's Dilemma in (1) with the discount factor that can sustain cooperation in the market environment. Also, it ensures that players find reentering the matching process more attractive than playing mutual defection for the remainder of the interaction.

[11]The equilibrium payoff from perpetual cooperation is $C/(1 - \delta(1 - \beta))$ and the payoff from unilaterally deviating is $B$, because $D = 0$. Condition (1) states that the payoff from deviating does not exceed the payoff from continuing cooperation.

**Assumption 1** *$C/B > \beta$ and $\delta > \underline{\delta}$.*

This assumption restricts the stage payoffs so that cooperation is compatible with discounting, $\underline{\delta} < 1$. It guarantees that we are always in the parameter space where, for sufficiently patient players, cooperation may occur in subgame-perfect equilibrium. Additionally, to simplify the exposition, we exclude the limiting case of $\delta = \underline{\delta}$.

# 3 Steady State Matching

We wish to analyze equilibrium behavior in a steady state of the pairwise matching market. On both sides of the market, in each period each agent is either matched and engaged in playing the Prisoner's Dilemma or unmatched and looking for a partner. In what follows, we use the index $i \in \{S, L\}$ for the short and the long side of the market, respectively. We denote by $M_M^i$ the measure of matched agents and by $M_U^i$ the measure of unmatched agents on side $i \in \{S, L\}$ of the market. In a steady state, these measures stay constant over time.

As all agents are either matched or unmatched, the following equations must hold:

$$M_M^S + M_U^S = 1, \quad M_M^L + M_U^L = K. \tag{2}$$

The transitions between states of matched and unmatched, follows a Markov process. In each period, with probability $\alpha \in (0, 1]$ each unmatched agent on the short side is matched with an unmatched agent from the long side, transitioning from unmatched to matched state. At the same time, each pair of matched agents breaks up with probability $\beta \in (0, 1)$, transitioning from matched to unmatched state. As we are interested in sustaining equilibria with cooperation, we suppose that, in equilibrium, there are no voluntary quits. Then, in the steady state, the inflow $\alpha M_U^S$ to the pool of matched agents on the short side has to be equal to the outflow $\beta M_M^S$:

$$\alpha M_U^S = \beta M_M^S. \tag{3}$$

Finally, clearly the measure of matched agents must be equal on both sides of the market:

$$M_M^S = M_M^L. \tag{4}$$

Thus, by (2)-(4), in the steady state we have

$$M_M^S = M_M^L = M_M = \frac{\alpha}{\alpha+\beta}, \quad M_U^S = \frac{\beta}{\alpha+\beta}, \quad M_U^L = K - \frac{\alpha}{\alpha+\beta}. \tag{5}$$

As $M_U^L - M_U^S = K - 1$, there are more unmatched agents on the long side than on the short side of the market – justifying our terminology – whenever $K > 1$.

Let $\alpha^L$ denote the matching probability of an agent on the long side. This probability can be derived easily from the fact that in the pairwise matching process the total number of new matches must be identical on both sides of the market. This means that $\alpha^L M_U^L = \alpha M_U^S$. By (5) we therefore obtain

$$\alpha^L = \frac{\alpha\beta}{\alpha(K-1)+\beta K}. \tag{6}$$

Thus, analogously to (3) on the short side, the inflow $\alpha^L M_U^L$ to the pool of matched agents is equal to the outflow $\beta M_M^L$ on the long side. Further, $\alpha^L < \alpha$ unless $K = 1$. As for $K > 1$

$$\frac{\partial \alpha^L}{\partial \alpha} = \frac{\beta^2 K}{(\alpha(K-1)+\beta K)^2} \in (0,1), \tag{7}$$

an increase in the efficiency of the matching technology raises the matching probabilities on both sides of the market, but at a faster rate on the short side.

By (5) there is a unique steady state. In fact, the steady state is also stable: the dynamics of the matching process implies that all variables converge to their steady state values in (5) over time. Indeed, using the period index $t$, we have for the mass of matched agents on the short side of the market that

$$M_{M,t}^S = M_{M,t-1}^S + \alpha M_{U,t-1}^S - \beta M_{M,t-1}^S, \tag{8}$$

because a fraction $\alpha$ of the unmatched agents in the previous period becomes matched, while a fraction $\beta$ of the matches in the previous period breaks down. As by (2)

$M^S_{U,t-1} = 1 - M^S_{M,t-1}$, we obtain the non-homogeneous autonomous first-order difference equation

$$M^S_{M,t} - (1 - \alpha - \beta) M^S_{M,t-1} = \alpha. \tag{9}$$

The solution of (9) is

$$M^S_{M,t} = (1 - \alpha - \beta)^t \left[ M^S_{M,0} - \frac{\alpha}{\alpha + \beta} \right] + \frac{\alpha}{\alpha + \beta}. \tag{10}$$

As $-1 < 1 - \alpha - \beta < 1$, we have $\lim_{t \to \infty} M^S_{M,t} = \alpha / (\alpha + \beta)$. Thus $M^S_{M,t}$ converges to its steady state value. Because (2) and (4) must hold in all periods $t$, this implies that $M^S_{U,t}$ and $M^L_{U,t}$ also converge to their steady state values in (5). Finally, observe that the speed of convergence is the faster, the closer $\alpha + \beta$ is to one.

# 4 Cooperation and Competition

We now investigate the conditions under which cooperation can be sustained in every match as a symmetric subgame-perfect equilibrium, when the matching market has already reached its steady state. The first difference with the isolated Prisoner's Dilemma considered at the end of Section 2 is that after a breakdown both agents can find a new partner in the matching market. In addition, each player in a match can also unilaterally end the relationship to reenter the matching process. This requires a change in the grim-trigger strategies of the players: the toughest punishment now is that, after one of the players has defected, the innocent party breaks up the partnership. Indeed, if she tried to impose anything that gave the deviator a lower continuation payoff, he would just leave. As a result – and in line with the literature that considers endogenous exit – the option to break up the relationship and return to the matching market limits the punishment that can be meted out to a defector and consequently imposes stricter constraints on cooperation.

In what follows, it will be convenient to denote by $\alpha^i$ the matching probability on side $i \in \{S, L\}$ of the market. Thus, $\alpha^S = \alpha$ and $\alpha^L$ is defined by (6).

First, let us consider the equilibrium payoff of the agents when they all cooperate in a match until their relation breaks down (and off path they play the grim trigger strategy). Denote by $V_C^i$ the expected payoff of a matched agent on side $i$ of the market at the beginning of each period. Since everyone cooperates, they collect $C$. In case the match breaks down at the end of the period, agents return to the pool of unmatched agents, where the expected payoff at the beginning of the next period is denoted as $W_C^i$. Since in equilibrium no one leaves voluntarily, we have that

$$V_C^i = C + \delta \left[ (1-\beta) V_C^i + \beta W_C^i \right]. \tag{11}$$

An unmatched agent earns no utility in the current period, capturing that search – and thus breakup – is always costly. However, in equilibrium, he enters a cooperative partnership with probability $\alpha^i$ at the end of the period, otherwise, he continues searching for a partner. Thus, he has the expected payoff

$$W_C^i = \delta \left[ \alpha^i V_C^i + (1-\alpha^i) W_C^i \right]. \tag{12}$$

Now consider a defector, who unilaterally defects after being matched. As we argued above, after the defection the relationship ends and the defector (as well as his partner) reenters the pool of unmatched agents in the next period. Let $V_D^i$ and $W_D^i$ denote a matched and an unmatched defector's expected payoff, respectively, at the beginning of a period. Then, the expected payoff from defecting in a match is equal to

$$V_D^i = B + \delta W_D^i, \tag{13}$$

as the partner will cooperate according to the hypothesized equilibrium. Note that if defecting is profitable once, it is profitable each time. Thus, a defector searching for a match obtains the payoff $\delta V_D^i$ with probability $\alpha^i$. With the remaining probability he stays unmatched in the current period and reenters the matching market again in the next period. Therefore,

$$W_D^i = \delta \left[ \alpha^i V_D^i + (1-\alpha^i) W_D^i \right] \tag{14}$$

is the analogue to (12) for a defector.

From equations (11) – (14) we obtain the solution

$$\begin{aligned} V_C^i &= \frac{(1-\delta(1-\alpha^i))\,C}{(1-\delta)\,(1-\delta(1-\alpha^i-\beta))}, \\ W_C^i &= \frac{\alpha^i \delta C}{(1-\delta)\,(1-\delta(1-\alpha^i-\beta))}, \end{aligned} \tag{15}$$

and

$$V_D^i = \frac{(1-\delta(1-\alpha^i))\, B}{(1-\delta)\,(1+\alpha^i\delta)}, \quad W_D^i = \frac{\alpha^i \delta B}{(1-\delta)\,(1+\alpha^i\delta)}, \tag{16}$$

for $i = S, L$.

Cooperation is sustainable if and only if $V_C^i - V_D^i \geq 0$ for *both* sides of the market. It is easily established that $V_C^i / V_D^i$ is strictly decreasing in the respective matching probability, $\alpha^i$. Therefore, cooperation requires that $\alpha^i$ does not exceed the critical threshold where $V_C^i = V_D^i$. Since $\alpha^S = \alpha$, on the short side of the market this requirement is satisfied if and only if[12]

$$\alpha \leq \bar{\alpha}^S(\delta) \equiv \begin{cases} \frac{C-B(1-\delta(1-\beta))}{(B-C)\delta} & \text{if } \delta < \min\left\{\frac{B-C}{C-\beta B}, 1\right\} \\ 1 & \text{otherwise} \end{cases}. \tag{17}$$

The maximal matching probability compatible with cooperation is, unsurprisingly, a function of the discount factor. More specifically, the function $\bar{\alpha}^S(.)$ has the following properties:

$$\bar{\alpha}^S(\underline{\delta}) = 0, \quad \frac{\partial \bar{\alpha}^S(\delta)}{\partial \delta} > 0, \quad \frac{\partial \bar{\alpha}^S(\delta)}{\partial (B/C)} < 0, \quad \frac{\partial \bar{\alpha}^S(\delta)}{\partial \beta} < 0, \tag{18}$$

for $\delta < \min\{(B-C)/(C-\beta B), 1)\}$, where $\underline{\delta}$ is the critical discount factor in the isolated Prisoner's Dilemma defined in (1). Thus, condition (17) cannot hold for $\alpha > 0$ if $\delta < \underline{\delta}$. But, for $\delta > \underline{\delta}$, cooperative behavior in the Prisoner's Dilemma can be sustained if $\alpha$ is not too high: as long as $\alpha$ lies in the interval $(0, \bar{\alpha}^S(\delta)]$ agents on the short side of the market may behave cooperatively if the agents on the other side also do so.[13] This implies that there is a continuous relation: the set of discount factors for which cooperation can be sustained is the same in the isolated Prisoner's Dilemma as in the one embedded in a matching market as the matching probability converges to zero. The interval of allowed matching probabilities becomes larger when $\delta$ increases: it eventually reaches the full interval $(0, 1]$ if $C/B \geq (1+\beta)/2\,(> \beta)$. Otherwise, when $C/B < (1+\beta)/2$, we have $\bar{\alpha}^S(1) < 1$, and thus, even for high values

[12]Note that $(B-C)/(C-\beta B) > (B-C)/(B-\beta B) = \underline{\delta}$.

[13]We will argue shortly that whenever the short side is willing to cooperate the long side does too.

of $\delta$, cooperation is not feasible in a market with very low matching frictions ($\alpha$ close to one). According with intuition, the lower is the basic attractiveness of defection, $B/C$, the higher is the matching efficiency compatible with cooperation. Similarly, a low breakdown probability $\beta$ makes cooperation more attractive and hence allows for higher $\alpha$.[14]

Looking at the long side, observe that the threshold for $\alpha^i = \alpha^L$ for the sustainability of cooperation is identical to the one in (17) for $\alpha^i = \alpha^S = \alpha$, because both are derived from (15) and (16). The intuition for the common threshold is that it determines the highest actual matching probability of an arbitrary player that is compatible with cooperation given that all other players play the cooperation equilibrium. Since the other players' strategy is fixed, their matching probabilities do not matter and thus the threshold is independent of the side of the market. Thus, on the long side of the market we need

$$\alpha^L \leq \bar{\alpha}^S(\delta). \tag{19}$$

Obviously, requirement (19) is satisfied if condition (17) holds for the short side because $\alpha^L \leq \alpha^S = \alpha$. We summarize our main findings in this section by the following

**Proposition 1** *If and only if $\alpha \leq \bar{\alpha}^S(\delta)$, then cooperation can be sustained in a subgame-perfect equilibrium. Moreover, the maximal market efficiency allowed, $\bar{\alpha}^S(\delta)$, is increasing in $\delta$ and decreasing in $B/C$ and $\beta$.*

If $\alpha > \bar{\alpha}^S(\delta)$, then the full gains from cooperation cannot be realized, because this would give at least the short side of the market an incentive to defect. Nonetheless, cooperation need not collapse completely. As we show in the following section, cooperation can still be reached at some cost.

For what follows, it will prove useful to specify explicitly the values of the parameter $\alpha$, our measure of market efficiency, for which the long side's incentive compatibility

[14] Yet another way of interpreting this result is that when $\delta$ is high, there is a lot of slack in the incentive compatibility constraint for the isolated Prisoner's Dilemma, which can be taken up by a high probability of rematch upon breaking up.

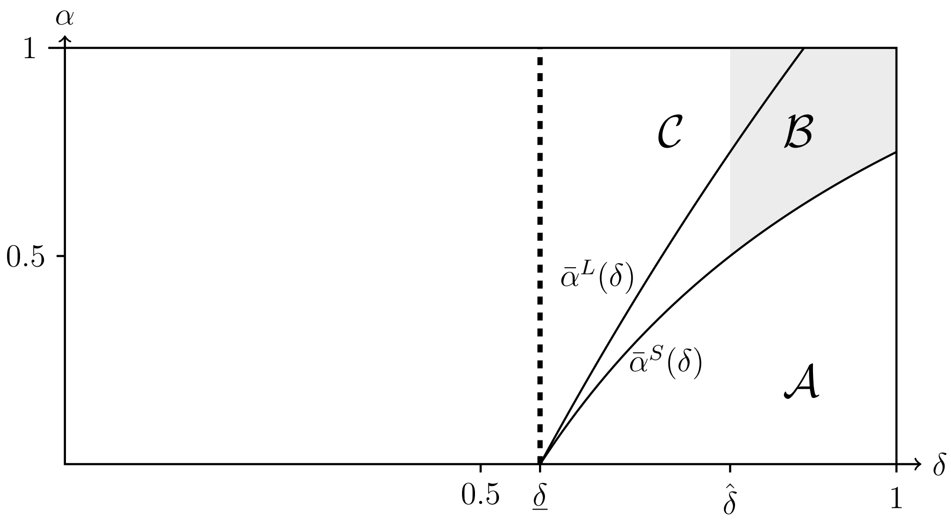


Figure 2: *Sustainability of Cooperation*

constraint (19) holds. Using the definition of $\alpha^L$ in (6), it is easy to see that $\alpha^L \leq \bar{\alpha}^S(\delta)$ is equivalent to

$$\alpha \leq \bar{\alpha}^L(\delta) \equiv \begin{cases} \frac{\bar{\alpha}^S(\delta)\beta K}{\beta - \bar{\alpha}^S(\delta)(K-1)} & \text{if } K < \frac{\bar{\alpha}^S(\delta)+\beta}{(\beta+1)\bar{\alpha}^S(\delta)} \\ 1 & \text{otherwise.} \end{cases} \tag{20}$$

Note that (20) is satisfied for all $\alpha \in (0, 1]$ if the total mass $K$ of agents on the long side is large enough. In this case, the matching probability on the long side of the market is already small enough – even for $\alpha = 1$ – to deter a deviation from cooperation: a defector on the long side is sufficiently punished by having to wait a long time (in expectation) until he can defect on a new opponent from the short side.

For the critical discount factor we have $\bar{\alpha}^L(\underline{\delta}) = \bar{\alpha}^S(\underline{\delta}) = 0$ by (18), while for $\delta > \underline{\delta}$, $\bar{\alpha}^L(\underline{\delta}) > \bar{\alpha}^S(\underline{\delta})$ unless $K = 1$. Indeed, for $K = 1$ conditions (17) and (20) are identical. In sum, condition (20) holds whenever (17) is satisfied, as we have pointed out above. Agents on the long side can then be induced to cooperate for a larger set of $(\delta, \alpha)$ pairs than agents on the short side.

Figure 2 illustrates the functions $\bar{\alpha}^S(.)$ and $\bar{\alpha}^L(.)$ for the case $K = 1.2$.[15] On the long side of the market the incentive restriction for cooperation is satisfied for all $(\delta, \alpha)$ pairs in regions $\mathcal{A}$ and $\mathcal{B}$. But, as agents on the short side can be induced to cooperate

[15]The figure is based on setting the other parameters $B = 1.4, C = 1, \beta = 0.5$. Thus $\bar{\alpha}^S(1) < 1$, whereas $\bar{\alpha}^L(\delta) = 1$ already for high enough values of $\delta$ below one.

only for $(\delta, \alpha)$ pairs in region $\mathcal{A}$, mutual cooperation is limited to region $\mathcal{A}$. We come back to Figure 2 in Section 6, where we explain the significance of the parameter $\hat{\delta}$ and the gray shaded parts of regions $\mathcal{B}$ and $\mathcal{C}$.

# 5 Conditional Cooperation and Trust Building

When sustaining full cooperation in a match is not possible, additional means of enforcement are necessary. To make switching to another partner more costly for a defector, some inefficiencies cannot be avoided. Without modeling the exact nature of these inefficiencies, we simply posit that newly matched players engage in "burning money" as a means to establish trust. Money burning could involve wasteful gift givin, delayed cooperation or reducing the scale of cooperation.[16] As a shortcut, we assume that in a new match an agent on side $i \in \{S, L\}$ is required to invest some amount $X^i \geq 0$ in wasteful activities before cooperation in the Prisoner's Dilemma begins.

More specifically, we modify the interaction in a match in the following way: in a new match, before playing the stage game, both agents simultaneously engage in money burning: players on side $i$ of the market invest $X^i$, which is observable to both partners. If $X^i$ reaches some critical value $\hat{X}^i$ for $i = S, L$, then cooperation begins and is supported by grim trigger strategies as in the previous section. If, however, one of the partners $i$ in a match invests less than $\hat{X}^i$, then both agents choose defection in the current period as a mutual best response, and at the end of the period both agents reenter the matching market. Note that the results of the previous section continue to hold if we set $\hat{X}^i = 0$ for $i = S, L$.

As we are interested in the maximum efficiency reachable in a market, in what follows we assume that agents always resort to the minimal amount of money burning $\hat{X}^i$ that sustains conditional cooperation, thereby minimizing waste.

Having to invest at the beginning of a new partnership can deter defection because a defector has to find a new partner more frequently than a cooperator. At the same

[16]See our discussion of the related literature in Section 1.

time, it clearly reduces the gains from cooperation also for cooperating agents. As the initialization of cooperation is now contingent on trust building through money burning, we refer to this type of cooperative equilibrium as *conditional cooperation.*

In what follows, we investigate the conditions under which conditional cooperation can be established, in equilibrium, for parameter values where unconditional cooperation, as studied in the previous section, is not feasible. First, consider a conditional cooperator in any period of a match *after* cooperation has already been enabled by money burning in the first period. Then, on side $i$ of the market, the expected payoff from continuing cooperation until the match is exogenously dissolved with probability $\beta$ per period is

$$V^i_{XC} = C + \delta \left[(1-\beta)V^i_{XC} + \beta W^i_{XC}\right], \tag{21}$$

where $W^i_{XC}$ denotes the payoff from joining the matching process, with the expectation of conditional cooperation upon a new match. The expected payoff from being unmatched – supposing that $V^i_{XC} > X^i$, what we will confirm later – is

$$W^i_{XC} = \delta \left[\alpha^i \left(V^i_{XC} - X^i\right) + (1-\alpha^i)W^i_{XC}\right], \tag{22}$$

similarly to equation (12), but including the waste that establishing a new cooperative relationship would entail upon finding a new partner.

Now consider a defector in a (hypothetical) conditional cooperation equilibrium. He cannot avoid burning money if he wants to induce his partner to behave cooperatively so that he can earn $B$.[17] Thus, his most dangerous defection must happen at the Prisoner's Dilemma first stage. Then his continuation payoff – post investment, as in case of cooperation – is

$$V^i_{XD} = B + \delta W^i_{XD}, \tag{23}$$

[17] In principle, the defector could not burn money and defect. In the putative equilibrium, that would lead his partner to defect, so the stage payoff would be zero, meaning that the total deviation payoff would be zero. However, as we have supposed – and later will prove – that the cooperation payoff exceeds the money burnt, the deviation payoff with money burning must also exceed it. Indeed, by definition, both payoffs coincide for the lowest amount of money burnt consistent with conditional cooperation.

because at the end of the period he returns to the pool of unmatched agents in order to continue with mischief. The expected payoff of an unmatched defector is

$$W^i_{XD} = \delta \left[\alpha^i \left(V^i_{XD} - X^i\right) + (1-\alpha^i)W^i_{XD}\right], \tag{24}$$

analogously to (22).

From equations (21)–(24) we obtain for $i \in \{S, L\}$ the solution

$$\begin{aligned} V^i_{XC} &= \frac{(1-\delta(1-\alpha^i))\,C - \alpha^i\beta\delta^2 X^i}{(1-\delta)\,(1-\delta(1-\alpha^i-\beta))}, \\ W^i_{XC} &= \frac{\alpha^i\delta\,(C-(1-\delta(1-\beta)X^i)}{(1-\delta)\,(1-\delta(1-\alpha^i-\beta))}, \end{aligned} \tag{25}$$

for a conditional cooperator, and

$$V^i_{XD} = \frac{(1-\delta(1-\alpha^i))\,B - \alpha^i\delta^2 X^i}{(1-\delta)\,(1+\alpha^i\delta)}, \quad W^i_{XD} = \frac{\alpha^i\delta(B - X^i)}{(1-\delta)\,(1+\alpha^i\delta)}, \tag{26}$$

for a defector.

We can now derive the critical level of money burning $\hat{X}^i$, $i = S, L$, that enables mutual conditional cooperation. This requires that on both sides $i$ of the market $V^i_{XC} \geq V^i_{XD}$. By (25) and (26) we obtain

$$\frac{\partial(V^i_{XC} - V^i_{XD})}{\partial X^i} = \frac{\alpha^i\delta^2(1-\beta)\,(1-\delta(1-\alpha^i))}{(1-\delta)(1+\alpha^i\delta)\,(1-\delta(1-\alpha^i-\beta))} > 0. \tag{27}$$

Further, $V^i_{XC} = V^i_{XD}$ if $X^i$ is equal to

$$\hat{X}^i \equiv \frac{B\,(1-\delta(1-\alpha^i-\beta)) - C\,(1+\alpha^i\delta)}{\alpha^i\delta^2\,(1-\beta)}. \tag{28}$$

This means that any $X^i \geq \hat{X}^i$ is a credible signal for cooperative behavior. Of course, it is not the current investment that ensures trust but the expectation that such an investment would have to be incurred (again) if the agent left his partner in search of a new one.

Evaluating $V^i_{XC}$ in (25) at $\hat{X}^i$ we obtain

$$V^i_{XC}\,|_{X^i=\hat{X}^i} \; - \; \hat{X}^i = \frac{(C-(1-\delta(1-\beta)B)\,(1-\delta(1-\alpha^i))}{\alpha^i(1-\delta)(1-\beta)\delta^2} > 0, \tag{29}$$

as $\delta > \underline{\delta}$. This confirms that the benefits from cooperation exceed the cost of establishing trust by money burning, as supposed above. Consequently, we can conclude that there exists a conditional cooperation equilibrium.

Of course, cooperation contingent on money burning is relevant only if $\hat{X}^i > 0$. By (28) we have

$$\hat{X}^i > 0 \quad \text{if and only if} \quad \alpha^i > \bar{\alpha}^S(\delta), \tag{30}$$

where $\bar{\alpha}^S(\delta)$ is defined in (17). Recall from the analysis in the previous section that side $i$ of the market can be induced to behave cooperatively without money burning only if $\alpha^i \leq \bar{\alpha}^S(\delta)$. If this restriction is not satisfied, because the matching efficiency $\alpha$ is too high, then side $i$ of the market has to burn money to become a trustworthy partner in a match.

In Figure 2 the inequality $\alpha^S = \alpha > \bar{\alpha}^S(\delta)$ holds for parameter values in regions $\mathcal{B}$ and $\mathcal{C}$. For these parameter values the short side of the market has to burn money to achieve conditional cooperation. For the long side of the market, this is the case only for parameter values in region $\mathcal{C}$, where $\alpha > \bar{\alpha}^L(\delta)$. The reason is that the incentives for defection are lower: finding a new partner after defecting takes more time for agents on the long side. Indeed, by (28)

$$\frac{\partial \hat{X}^i}{\partial \alpha^i} = \frac{C - B\left(1 - \delta(1-\beta)\right)}{(1-\beta)\left(\alpha^i\right)^2 \delta^2} > 0 \tag{31}$$

as $\delta > \underline{\delta}$. Therefore, in region $\mathcal{C}$ of Figure 2 conditional cooperation requires money burning on both sides of the market; but as $\alpha^L < \alpha^S$, the long side has to burn less than the short side, i.e., $\hat{X}^L < \hat{X}^S$.

In sum the results of this section establish the following

**Proposition 2** *If $\alpha > \bar{\alpha}^S(\delta)$, then there exists an equilibrium with conditional cooperation. The equilibrium has the following properties:*
*(a) As long as (20) holds, mutual conditional cooperation can be sustained with $\hat{X}^S(\alpha) > 0$ of money burning on the short side and without money burning on the long side.*
*(b) If (20) does not hold, then mutual conditional cooperation requires money burning on both sides: $0 < \hat{X}^L(\alpha) \leq \hat{X}^S(\alpha)$, where the second inequality is strict for $K > 1$.*

In comparison with the situation described by Proposition 1, agents benefit from a higher matching probability. But, on the downside, they incur inefficiencies by having to establish trust by money burning. We investigate the welfare implications of this trade-off in the following section.

# 6 Welfare and Competition

In this section we analyze the welfare implications of changes in market efficiency, as measured by the parameter $\alpha$.[18] A reduction in market frictions that increases $\alpha$ has two opposing welfare effects. On the positive side, if a partnership becomes unproductive and breaks down, a higher $\alpha$ means that it is easier for both agents to find a new productive partnership, leading to a higher proportion of agents being matched – and, therefore, cooperating – in the steady state. On the negative side, a higher arrival rate of potential alternative partners may be detrimental to the sustainability of cooperation in a long-term partnership: defecting becomes more attractive when after reentering the market there is only a short delay to be matched with another victim.

We measure welfare by the expected lifetime utility of agents in the steady state. First, we look at the case where agents do not have to burn money to enter a cooperative relation, $\alpha \leq \bar{\alpha}^S(\delta)$. In this case, welfare on side $i$ of the market is

$$\mathcal{W}^i \equiv M_M V_C^i + M_U^i W_C^i, \tag{32}$$

where $M_M = M_M^S = M_M^L$ and $M_U^i$ are given by (5), and $V_C^i$ and $W_C^i$ by (15): the mass $M_M$ of matched agents has the expected utility $V_C^i$, whereas the mass $M_U^i$ of unmatched agents has the expected utility $W_C^i$. Since our welfare measure is anonymous/utilitarian, it does not matter who earns the fruits of cooperation in any given period: thus, as unmatched agents do not earn while unmatched, by (5), and (15) we can rewrite $\mathcal{W}^i$ as the discounted payoff of $M_M$ agents cooperating in every period

$$\mathcal{W}^i = M_M \frac{C}{1-\delta} = \frac{\alpha\, C}{(\alpha+\beta)(1-\delta)}. \tag{33}$$

[18]We continue to focus on the most efficient equilibrium and the market in its steady state.

Note that this value is the same for both sides of the market, because the mass of matched cooperating agents is the same for $i = S, L$. Since it is strictly increasing in $\alpha$, as long as mutual cooperation can be established without either side being required to burn money, an increase in the matching probability $\alpha$ is unambiguously beneficial for both sides $i$. This immediately implies the following result for region $\mathcal{A}$ in Figure 2:

**Proposition 3** *If $\alpha < \bar{\alpha}^S(\delta)$, then welfare $\mathcal{W}^i$ is strictly increasing in market efficiency, $\alpha$, on both sides of the market.*

The intuition is simple: as long as there is slack in the incentive compatibility constraint for cooperation, the negative effect of (marginally) increasing market efficiency does not manifest itself, while the positive effect does.

We next turn to the case where establishing (conditional) cooperation requires money burning. Recall that for $X^i = \hat{X}^i$ we have $V^i_{XD} = V^i_{XC}$. This in turn implies that (22) and (24) are identical so that $W^i_{XD} = W^i_{XC}$ as well. Moreover, both (21) and (23) do not depend on $\alpha^i$ and are identical for both sides of the market. Therefore, the equilibrium value functions are the same on both sides of the market, and are independent of the matching frictions. Thus, evaluated at $X^i = \hat{X}^i$, the solution of (21)–(24) simplifies to

$$\begin{aligned} V^i_{XC}|_{X^i=\hat{X}^i} = V^i_{XD}|_{X^i=\hat{X}^i} &= \quad \hat{V}_X \equiv \frac{C - B\beta}{(1-\beta)(1-\delta)}, \\ W^i_{XC}|_{X^i=\hat{X}^i} = W^i_{XD}|_{X^i=\hat{X}^i} &= \quad \hat{W}_X \equiv \frac{C - B(1-\delta(1-\beta))}{\delta(1-\beta)(1-\delta)}, \end{aligned} \tag{34}$$

for $i = S, L$.

Therefore, when $\hat{X}^i > 0$, welfare on side $i$ can again be written in two equivalent ways:

$$\mathcal{W}^i_X \equiv M_M\left(\hat{V}_X - \beta\hat{X}^i\right) + M^i_U\hat{W}_X = M_M\frac{C - \beta\hat{X}^i}{1-\delta}, \tag{35}$$

as in every period the mass of newly matched agents $\alpha^i M^i_U = \beta M_M$ burns $\hat{X}^i$. An increase in market efficiency, $\alpha$, has two opposing effects on welfare $\mathcal{W}^i_X$: on the one hand, there is a beneficial effect because the mass $M_M = \alpha/(\alpha+\beta)$ of matched cooperating agents is increased. On the other hand, as we have noted before, also the amount of wasteful money burning $\hat{X}^i$ for new matches is increased. Using the definition of $\hat{X}^i$ in (28) and applying $\alpha^S = \alpha$ and (6) for $\alpha^L$, we obtain that the overall effect is

$$\frac{\partial \mathcal{W}_X^i}{\partial \alpha} = \frac{\beta \left(B - C(1 + \delta(1-\beta)\right)}{\left(\alpha + \beta\right)^2 \delta^2 \left(1 - \beta\right)}, \tag{36}$$

for $i = S, L$. Note that when money burning occurs on both sides, then their welfare is affected equally by a change in market efficiency, because the right-hand side of (36) is independent of $i$.[19]

To state our welfare results for conditional cooperation with money burning, we define the discount factor

$$\hat{\delta} \equiv \frac{B - C}{C(1-\beta)}, \tag{37}$$

at which the welfare derivative in (36) – that is strictly decreasing in $\delta$ – is equal to zero. Note that, because $C < B$, $\hat{\delta}$ is greater than the critical discount factor $\underline{\delta}$ in (1), so the welfare derivative cannot be negative for all $\delta$ satisfying Assumption 1. Also, it is smaller than $(B - C)/(C - B\beta)$, the value of discounting above which $\bar{\alpha}^S(\delta)$ in (17) is equal to one in case $(B - C)/(C - B\beta) < 1$.[20] Thus, given that $\hat{\delta} < 1$, there always exist discount factors for which the welfare derivative in (36) is negative. Finally, $\hat{\delta} < 1$ if and only if $B < (2 - \beta)C$. This means that welfare $\mathcal{W}_X^i$ is *always* increasing in $\alpha$ whenever $B \geq (2 - \beta)C$.

The following two propositions summarize our findings for parameter values in regions $\mathcal{B}$ and $\mathcal{C}$ of Figure 2. In region $\mathcal{B}$ only the short side burns money in equilibrium. Thus, $\mathcal{W}^L$ in (33) is relevant for welfare on the long side and $\mathcal{W}_X^S$ in (35) on the short side:

**Proposition 4** *If* $\alpha \in \left(\bar{\alpha}^S(\delta), \bar{\alpha}^L(\delta)\right)$*, then an increase in market efficiency,* $\alpha$*, has the following welfare implications:*
*(a) Welfare on the long side of the market is increasing.*

[19]The reason is that the value functions in (34) do not depend on $\alpha$. Additionally, the difference in waste between the two sides, $\beta M_M \left(\hat{X}^S - \hat{X}^L\right)$, is also independent of $\alpha$.

[20]Note, however, that whether $\bar{\alpha}^L(\delta)$ hits one below or above $\hat{\delta}$ depends on the size of $K$.

*(b) Welfare on the short side of the market is increasing if $B \geq (2-\beta)C$. If $B < (2-\beta)C$, then it is increasing for $\delta \in (\underline{\delta}, \hat{\delta})$ and decreasing for all $\delta > \hat{\delta}$ such that $\bar{\alpha}^S(\delta) < 1$.*

The parameter values in Figure 2 satisfy $B < (2-\beta)C$ so that $\hat{\delta} < 1$.[21] The gray shaded part of region $\mathcal{B}$ indicates the values of $\delta$ and $\alpha$ for which welfare is decreasing in $\alpha$ on the short side, but not on the long side of the market.

Finally, in region $\mathcal{C}$ of Figure 2, where both sides need to burn money to sustain conditional cooperation, their welfare is given by $\mathcal{W}_X^i$, $i = S, L$, in (35). Therefore, the following result applies:

**Proposition 5** *When $\alpha > \bar{\alpha}^L(\delta)$, then welfare on both sides of the market is increasing in market efficiency, $\alpha$, whenever $B \geq (2-\beta)C$. If $B < (2-\beta)C$, then welfare on both sides is increasing for $\delta \in (\underline{\delta}, \hat{\delta})$ and decreasing for all $\delta > \hat{\delta}$ such that $\bar{\alpha}^L(\delta) < 1$.*[22]

In the gray shaded part of region $\mathcal{C}$ in Figure 2 welfare on both sides is decreasing in $\alpha$, as the second statement of the proposition applies for $\delta > \hat{\delta}$.

By Propositions (4) and (5), a sufficient condition for lower market frictions to increase welfare is that $B \geq (2-\beta)C$. In combination with Proposition (3) this guarantees that a higher level of competition for partners is welfare enhancing for all $\delta > \underline{\delta}$. The lower the matching frictions, the larger is the mass of agents who find themselves cooperating with a partner in a match. As long as $B \geq (2-\beta)C$, this positive effect outweighs the welfare losses that arise when intensified competition necessitates higher trust building efforts.

If, however, $B < (2-\beta)C$, then it may happen – for high values of $\alpha$ and $\delta$ – that an increase in competitiveness lowers welfare on one or both sides of the market. The reason is that the mass $M_M = \alpha/(\alpha+\beta)$ of matched agents is concave in $\alpha$. Thus, for high values of $\alpha$ a further increase in market efficiency has only a small impact on the surplus generated by additional partnerships. Consequently, the required increase

[21] Cf. footnote 15.

[22] Note that when $K$ is sufficiently high the decreasing region is empty, as $\alpha > \bar{\alpha}^L(\delta)$ implies $\delta < \hat{\delta}$.

in money burning can dominate the welfare implications of market efficiency. Indeed, as (31) shows, the increase in money burning due to an increase in $\alpha$ is higher for low values of $B$. This is the reason for why lower market frictions may reduce welfare if $B < (2 - \beta)C$.

For welfare to be guaranteed to be increasing in market efficiency, the values of the parameters $B$ and $-C$ have to be sufficiently high. Somewhat paradoxically, this makes cooperation harder to sustain, even in an isolated Prisoner's Dilemma. In a similar vein, in our matching environment $\partial \mathcal{W}_X^i / \partial B$ and $-\partial \mathcal{W}_X^i / \partial C$ are both easily seen to be negative. The explanation for the opposite effect on the change in welfare due to increasing competition has to do with the *cross* partial derivatives. Welfare is a supermodular function of $\alpha$ and $B$, and $\alpha$ and $-C$.[23] In other words, the variables $B$ and $-C$ are complements for market efficiency $\alpha$ regarding the welfare effects.

# 7 Conclusions

We have presented a two–sided matching model, in which pairs of matched agents play a repeated Prisoner's Dilemma until their relation is exogenously dissolved. A key parameter of the model is the level of matching frictions. We first study how this parameter in combination with the players' discount factor affects the sustainability of cooperation and the feasible gains in a match, when in a new match past behavior is not observable. Then we analyze how increasing competition, due to lower matching frictions, affects the overall welfare of players.

With low matching frictions, the option to search for a new partner limits the scope of punishing defection from cooperative behavior within a partnership. This necessitates some inefficiencies in cooperation. In our model, these welfare losses are conveniently described as money burning. Money burning as a generic shortcut for wasteful trust building activities can reestablish cooperation, because it lowers continuation payoffs more for the deviators than for the cooperators. Higher matching market efficiency

[23] Formally $\partial^2 \mathcal{W}_X^i / \partial B \partial \alpha$ and $-\partial^2 \mathcal{W}_X^i / \partial C \partial \alpha$ are both positive.

requires more resources to be wasted in order to maintain cooperation in subgame-perfect equilibrium. It turns out that the necessary trust building activities are not the same on both sides of the market: as players on the long side intrinsically have lower rematching probabilities, they need to waste less resources for trust building.

On the one hand, as matching efficiency increases, more pairs are formed in steady state, thereby increasing aggregate welfare. On the other hand, higher efficiency necessitates a higher amount of money burning, as it increases the deviation profits. It turns out that the trade-off tends to be resolved in favor of the first effect: decreasing matching frictions improve welfare over a wide range of parameter values. This result is only overturned if the deviation incentives are low. In this case, money burning occurs only if matching frictions are very low, but its amount is highly sensitive to changes in the level of frictions.

Our analysis focuses on the most efficient subgame-perfect equilibrium, in which players cooperate until they are separated and, at the beginning of their relation, invest the minimal amount of money burning that is necessary to deter defection. As noted by Datta (1996), players in a new match might be tempted to profitably renegotiate investments in trust building. The idea that prevents renegotiation is that any such proposal by one of the players in a match induces the other player to believe that his current partner would also seek to avoid money burning in future matches. This, however, would make defection profitable. Any proposal for renegotiation is therefore considered as not trustworthy. Without this belief our equilibrium for low market frictions would collapse and only mutual defection would be feasible.[24]

In our matching environment we have used the repeated Prisoner's Dilemma to analyze the interdependence between cooperation, competition and welfare. But our framework is applicable also to other bilateral games in which reputation mechanisms are necessary for optimal outcomes, as e.g. in the trust game. In this way, relational contracting and competition could be studied in a market context. In our model,

[24]Carmichael and MacLeod (1997) present an evolutionary model which selects the equilibrium in which wasteful gift giving induces trust building.

the players' behavior is based on rational self-interest. But it could be modified to incorporate behavioral preferences such as altruism, fairness, and reciprocity.[25] This could provide new insights into how different preference attitudes and competition relate in their welfare implications in a matching market.

[25] Bester (2021) analyzes fairness in bargaining and the role of market frictions in a bilateral matching market.

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