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Abstract

Anchoring is a robust behavioral phenomenon modeled predominantly as a bias in individual judgment. We propose a game-theoretic model that considers players' beliefs about others' behavior as a mediator for the effect of the anchor on a player's choice. The results establish that anchoring in strategic interactions reported in the literature can be rationalized by anchored beliefs about the opponents' intentions. Notwithstanding, we also demonstrate that a player might adjust away from rather than toward the anchor in games where choices are strategic substitutes.

Keywords: Anchoring Bias; Auctions; Games; Incomplete Information; Strategy
JEL Codes: D01, D91, C72

1 Introduction

Anchoring describes a cognitive bias toward an irrelevant value. Starting with the seminal work of Tversky and Kahneman (1974), it has been extensively studied in the context of judgment with a focus on individual decision-making.[1] There is also plenty of empirical evidence showing that irrelevant numbers influence strategic choices, for example, in asset markets (Baghestanian and Walker, 2015), negotiations (Galinsky and Mussweiler, 2001), and auctions (Holst et al., 2015; Medcalfe, 2016; Wolk and Spann, 2008; Peeters et al., 2016; Trautmann and Traxler, 2010; Ku et al., 2006). Using a controlled laboratory experiment aimed at identifying anchoring effects and their robustness in games of incomplete information, Ivanova-Stenzel and Seres (2021) show that bids in auctions are adjusted to irrelevant numbers in different contexts and different auction mechanisms.

Despite the vast evidence on anchoring influencing decisions, a theoretical framework allowing for an analysis of its impact in strategic settings is missing. This paper offers a plausible game-theoretic foundation of anchoring that can be used to explain the existing empirical evidence. We suggest a model that considers an anchor as a source of bias in the evaluation of the opponents'

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[1] Examples include trivial judgments (Wilson et al., 1996), price estimates for lotteries (Chapman and Johnson, 1994), and purchasing decisions (Wansink et al., 1998).

strategies. The idea is inspired by the explanation of anchoring as an attitude change which is based on the premise that anchoring may less likely be the result from direct numeric priming but rather indirectly influence the information processing (Wegener et al., 2001, 2010; Blankenship et al., 2008).

In a game of incomplete information, we define belief anchoring as the effect of an irrelevant number on the beliefs of a player about the strategies of the opponents. We characterize the sufficient conditions under which a shift in beliefs toward the anchor leads to a shift in the equilibrium strategies in the same direction. This holds if the best-response curve of a player is upward-sloping. As we demonstrate with an example, anchored beliefs can affect the equilibrium strategies of players in different directions if the best-response curve is downward-sloping. Our argument suggests that exposing individuals to irrelevant values in strategic choices might result in an adjustment away and not toward these values, for example, in games where choices are strategic substitutes.

2 Anchoring Bias in Equilibrium

Consider the normal form representation of a static game of incomplete information with n players (Mas-Colell et al., 1995) that has the following features. Other than the type, all players observe a payoff-irrelevant value $c \in C \subseteq \mathbb{R}^+$. The players' type is a random variable x_i drawn from a compact interval $[x_L, x_H]$ according to a quasiconcave, twice continuously differentiable, cumulative distribution function $F(x_i)$.

A (pure) strategy is an upper semi-continuous function that specifies player i's strategy choice s_i for each realization of a player's type, chosen from a compact set S_i.[2] The player's utility function $u(x_i, s_i, s_{-i})$ over certain outcomes, is assumed to be continuous, quasiconcave, and twice differentiable with respect to s_i and s_{-i}, where s_{-i} denotes the strategy profile of the $n-1$ opponents of player i.

In the game, the player privately learns x_i, and their payoff is not directly influenced by the anchor, i.e., the value c. However, player i can hold either the wrong priors or the wrong expectations about the opponents' strategies. Thus, player i maximizes their objective function with adjusted beliefs about the cumulative distribution function of the opponents' strategies $\Theta_i(x_i, s_{-i}, c)$. Note that the beliefs' adjustment is player-specific, i.e., being exposed to the same anchor c does not imply that $\Theta_i = \Theta_j$, where i, j are different players.

A player solves a maximization problem by choosing a (pure) strategy s_i upon learning their type x_i.

$$\max_{s_i} \mathbb{E}[u(x_i, s_i, s_{-i})|c] = \max_{s_i} \int_{s_{-i} \in S_{-i}} u(x_i, s_i, s_{-i}) d\Theta_i(x_i, s_{-i}, c). \tag{1}$$

[2]Mas-Colell et al. (1995), p. 255.

Thus, in order to find their best response to adjusted beliefs, player i solves

$$\arg\max_{s_i} \int_{s_{-i}\in S_{-i}} u(x_i, s_i, s_{-i}) d\Theta_i(x_i, s_{-i}, c) = s_i(x_i, \Theta_i(x_i, s_{-i}, c)), \forall i, \tag{2}$$

where $s_i(\cdot)$ is the best-response function to the adjusted beliefs of i. For technical reasons, we assume that there is a solution $(s^*(\cdot), s^*_{-i}(\cdot))$ to this system of n equations and that this solution is unique.[3]

In the spirit of Tversky and Kahneman (1974), we define anchoring as the influence of an irrelevant number on the players' beliefs about the other players' strategies.

Definition 1: Player i's *beliefs* about the strategies of their opponents is *anchored* if for any $c, c' \in C$ with $c' \succ c$ holds that $\Theta_i(x_i, s_{-i}, c')$ first-order stochastically dominates $\Theta_i(x_i, s_{-i}, c)$, i.e., $\Theta_i(x_i, s_{-i}, c') \leq \Theta_i(x_i, s_{-i}, c)$, and with strict inequality at some x_i.

Note that the no-anchor case, i.e., $c = \emptyset$, is also covered by Definition 1.

For example, consider the first-price auction with two bidders where an anchor $c \in C$ is introduced. The maximization problem of a bidder i with valuation x_i, facing an opponent j who uses an identical bidding strategy, is

$$\max_{b_i}(x_i - b_i)Prob(b_i \geq b_{-i}|x_i, c),$$

Hence, i's payoff $(x_i - b_i)$ conditional on winning is not influenced by the anchor. However, if the player's beliefs about the distribution of opposing bids is anchored, player i optimizes an objective function with a biased probability of winning such that if $c' \succ c$ then

$$Prob(b_i \geq b_{-i}|x_i, c') < Prob(b_i \geq b_{-i}|x_i, c).$$

A higher anchor increases beliefs about opposing bids, and hence, reduces the chance of winning.

Anchored beliefs can imply equilibrium strategies $(s^*_i(x_i, c), s^*_{-i}(x_i, c))$ that are shifted by the anchor c. However, this does not imply a direction of the effect of the anchor on the equilibrium strategies. For example, in the auction game, the best-response curve is upward-sloping, hence, anchored beliefs do increase equilibrium bids and prices. The same logic applies to the Bertrand oligopoly game when, for example, players have private information about their marginal costs and the prices are strategic complements, i.e., a player's best response is increasing in the price of the opponents'. As this game is isomorphic to a first-price auction, a higher anchor shifts the player's beliefs about the distribution of the opponents' prices upwards. However, anchored beliefs do not necessarily lead to an upward shift of equilibrium strategies.

The following is an example of anchored beliefs where the effect on the best-response functions is negative. Consider a Cournot duopoly game of incomplete information in which players $1, 2$ again

[3] The reason behind this choice is so that there is a clear definition of the effect of an anchor on the equilibrium.

have privately known constant marginal cost types k_1, k_2, set quantities q_1, q_1, and face a downward-sloping demand curve $p(q_1, q_2)$, such that their payoff equals $\Pi_i(q_i, q_j) = (p(q_1, q_2) - k_i)q_i$. As the best-response function $q_i(k_i, \Theta_i(k_i, q_j, c))$ is downward-sloping in q_j, the effect of the anchored beliefs would be the opposite. Intuitively, if an anchor increases the belief about the opponent's quantity, the best response will be to choose a smaller quantity. Note that this link does not depend on the information set concerning the players' cost types. For example, the same reasoning also applies to the special case with complete information, i.e., when the marginal costs are common knowledge.

Lemma 1 formalizes these observations.

Lemma 1: Suppose a player's beliefs are anchored. Then, the best response of the player is increasing in the anchor if the best-response curve is increasing in the opponents' strategies, i.e.,

$$\frac{\partial s_i(x_i, \Theta_i(x_i, s_{-i}, c))}{\partial \Theta_i(x_i, s_{-i}, c)} \cdot \frac{\partial \Theta_i(x_i, s_{-i}, c)}{\partial s_{-i}} > 0, \forall x_i$$

$$\Longrightarrow s_i(x_i, \Theta_i(x_i, s_{-i}, c')) > s_i(x_i, \Theta_i(x_i, s_{-i}, c)), \forall x_i, s'_{-i}; c', c \in C; c' > c. \tag{3}$$

Conversely, it is decreasing in the anchor if the best-response curve is decreasing in the beliefs about the opponents' strategies, i.e.,

$$\frac{\partial s_i(x_i, \Theta_i(x_i, s_{-i}, c))}{\partial \Theta_i(x_i, s_{-i}, c)} \cdot \frac{\partial \Theta_i(x_i, s_{-i}, c)}{\partial s_{-i}} < 0, \forall x_i$$

$$\Longrightarrow s_i(x_i, \Theta_i(x_i, s_{-i}, c')) < s_i(x_i, \Theta_i(x_i, s_{-i}, c)), \forall x_i, s_{-i}; c', c \in C; c' > c. \tag{4}$$

The statements in Lemma 1 holds under the definition of anchored beliefs.

The common understanding of the anchoring bias is that irrelevant information biases decisions in the direction of the anchor. For example, experimental evidence (Ivanova-Stenzel and Seres, 2021) indicates that bidding strategies in first-price sealed-bid auctions are anchored by irrelevant numbers. The announcement as well as the use of a higher irrelevant number results in higher bids. Figure 1 illustrates this by depicting the cumulative distribution function of the overbidding ratios in two treatment conditions, where the anchor is a commonly known upper bid limit $c = 95$ or $c' = 115$.[4] As one can see, the CDF in the high anchor condition ($c' = 115$) is shifted to the right.

Example 1 illustrates to what extent our model is capable of accounting for the observed behavior.

[4] The numbers used as anchors are above the upper bound of the interval from which bidder valuations are drawn, and consequently, above the upper bound of the equilibrium strategy set. The overbidding ratio is the relative deviation of the observed bids from the Bayesian Nash equilibrium bids and is equal to $\frac{b_i - b_i^*}{b_i^*}$.

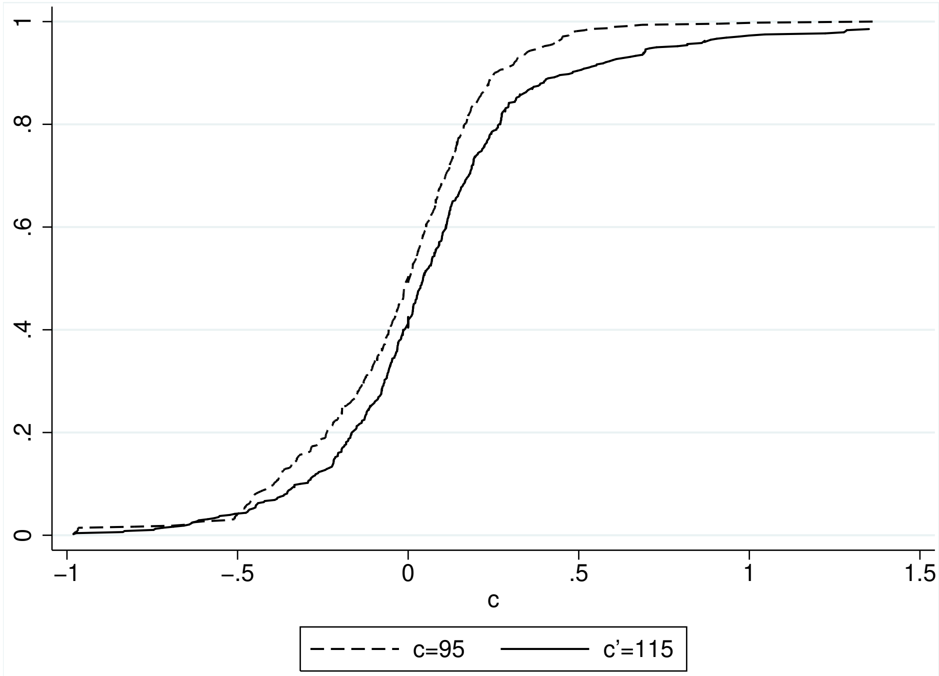

Figure 1: Cumulative distribution function of the overbidding ratio in first-price auctions with and upper bid limit $c = 95$ and $c' = 115$ (Ivanova-Stenzel and Seres, 2021)

Example 1 Consider a first-price sealed-bid auction with two players i, j where types are identically and independently drawn from $[x_L, x_H]$ with uniform distribution $F(x_i)$. The players simultaneously submit their bids b_i, b_j. The winner receives $x_i - b_i$ and the loser gets 0. Suppose bidders' beliefs are anchored by value $c \in \{c', c'', \emptyset\}$:

$$\Theta(x_i, b_j, c) = \begin{cases} 0, & \text{if} \quad b_j < x_L + z(c) \\ \frac{2 \cdot x_i}{x_H - x_L} - \frac{2 \cdot b_j}{x_H - x_L}, & \text{if} \quad x_L + z(c) \leq b_j \leq \frac{x_L + x_H}{2} + z(c) \\ 1, & \text{if} \quad \frac{x_L + x_H}{2} + z(c) < b_j \end{cases}$$

where $z(\emptyset) = 0$, $z(c') = c'$ and $z(c'') = c''$. The equilibrium bid of bidder i is

$$b_i^*(x_i, c) = x_L + \frac{1}{2}(x_i - x_L) + z(c)(x_H - x_L)$$

which is increasing in the anchor.

One may wonder whether this holds in general. As it turns out, anchored beliefs are a sufficient condition to have that the players' choices in equilibrium are anchored.

Definition 2: The *equilibrium* choice of a player is *anchored* if their equilibrium strategy is increasing in the anchor c, i.e.,

$$s_i^*(x_i, \Theta_i(x_i, s_{-i}^*, c')) > s_i^*(x_i, \Theta_i(x_i, s_{-i}^*, c)) \tag{5}$$

for all x_i and $c, c' \in C$ with $c' > c$.[5]

The next proposition establishes a link between this concept and anchored beliefs.

Proposition 1: Suppose a player's beliefs are anchored and the best response is increasing in the beliefs about the opponents' strategies. Then, the equilibrium choice of the player is anchored if the player's best response function is increasing in the anchor.

Proof: Suppose the player's equilibrium choice is not anchored, i.e., there exist x_i and $c' > c$ such that

$$s_i^*(x_i, \Theta_i(x_i, s_{-i}^*, c')) \leq s_i^*(x_i, \Theta_i(x_i, s_{-i}^*, c)). \tag{6}$$

Lemma 1 implies that the best response functions are increasing in the anchor c, i.e.,

$$s_i(x_i, \Theta_i(x_i, s_{-i}, c')) > s_i(x_i, \Theta_i(x_i, s_{-i}, c)), \forall x_i, s_{-i}, c' > c. \tag{7}$$

Hence, $\Theta_i(x_i, s_{-i}, c') < \Theta_i(x_i, s_{-i}, c)$ for at least some s_{-i}. However, this contradicts the fact that the player's beliefs are anchored. □

Given Lemma 1 and Proposition 1, one might expect the opposite effect in the case of decreasing best-response functions. However, as Example 2 demonstrates, the equilibrium strategies must not necessarily be decreasing in the anchor.

Example 2 Revisit the Cournot duopoly game where the best-response functions $q_i(k_i, \Theta_i(k_i, q_j, c))$ are downward-sloping in q_j. Assume that the marginal costs of the two players are the same, $k_i = k_j = k$, the demand curve is linear $p(q_1, q_2) = a - q_1 + q_2)$, and all this is common knowledge. Thus, we can simplify the notation for the player's beliefs about the opponent's choice, i.e., $\Theta_i(k_i, q_j, c) = \Theta_i(q_j, c)$, where $c \in C$.[6] In this game, player i solves

$$\max_{q_i} \int_{q_j} \Pi_i(q_i, q_j) d\Theta_i(q_j, c) = \int_{q_j} (a - q_i + q_j - k) q_i d\Theta_i(q_j, c).$$

Let us denote the distribution of the opposing choices in the symmetric Nash equilibrium with $\Psi(q_j)$. Consider the following case: Beliefs are such that $\Theta_i(q_j, \emptyset) = \Psi(q_j)$, $\Theta_1(q_2, c) = \Psi(q_2) - \varepsilon \cdot c$, and $\Theta_2(q_1, c) = \Psi(q_1) - c$, where $c \neq \emptyset$ and ε is a small positive number. Then, $q_1^*(\Theta_1(q_2, c)) < q_1^*(\Theta_1(q_2, \emptyset))$ whereas $q_2^*(\Theta_2(q_1, c)) > q_2^*(\Theta_2(q_1, \emptyset))$, i.e., the anchor affects the equilibrium choices in different directions. Figure 2 illustrates this example graphically.

[5]Note the difference between this concept and the concept of biased-based equilibrium. Heller and Winter (2020) consider a game by which beliefs about the opponent's strategy is the function of the opponent's strategy, hence, they are not influenced by an exogenous factor.

[6]Note that the cumulative distribution function in the case of complete information is particularly simple as q_i is deterministic.

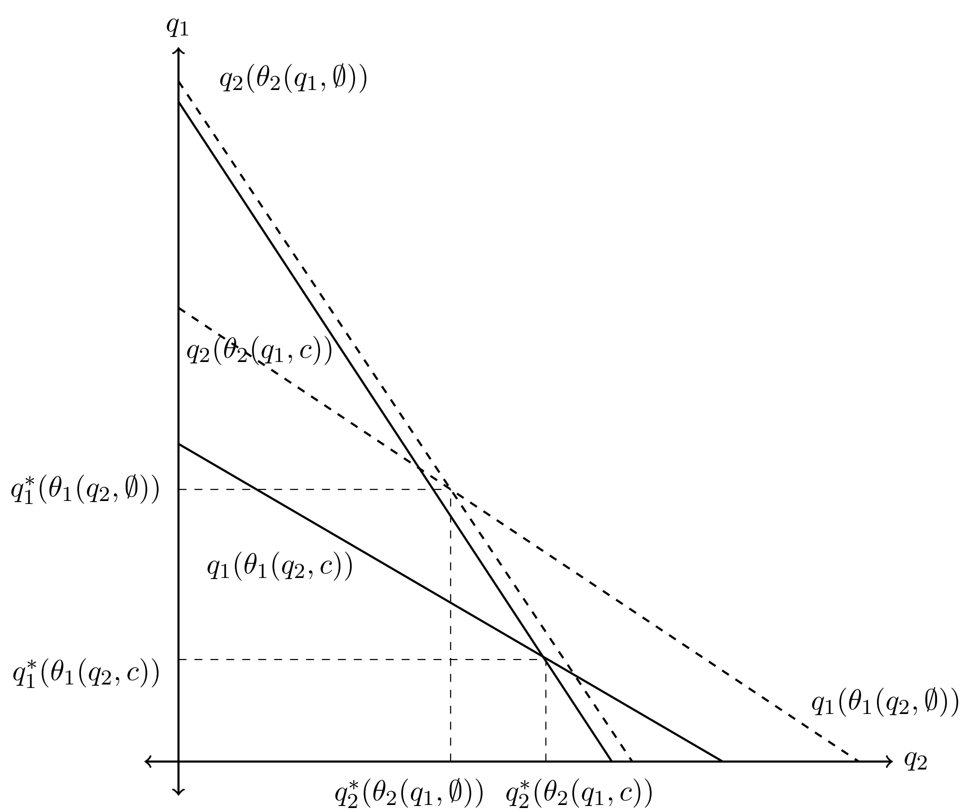

Figure 2: Cournot duopoly game with complete information. The continuous lines are best response curves with anchor c. Although beliefs are anchored, the anchor's effect has different signs on the equilibrium choices of the two players (Example 2).

3 Discussion

In this paper, we explore whether players' beliefs about the opponents' strategies can act as a mediator for the effect of the anchor on the subjects' decisions in strategic settings. The main insight of our analysis is that there is a clear link between the way in which anchors influence players' reasoning about the behavior of others and their best responses in strategic interactions.

Our results suggest that the two concepts that capture the direct and indirect effect of the anchor lead to the same adjustment of the chosen actions toward the anchor in settings where the choices are strategic complements, e.g., in the case of price competition or auctions. However, their impact may have the opposite sign if agents have a downward-sloping reaction curve, i.e., if their choices are considered strategic substitutes, such as, for example, in certain oligopoly production markets (Fudenberg and Tirole, 1984), in the case of acquisition information about industry-specific uncertain cost factors (Christen, 2005), or decision allocation games within firms (Aghion and Tirole, 1997).

This split highlights that the distinction between the two concepts is relevant and proves that the difference between them is empirically verifiable. Hence, our model is suitable for the design of an identification strategy for the driving forces of the anchoring bias in games. Our approach to defining the anchoring bias as an impairment of strategic reasoning is also of help for determining the consequences of using anchors in various settings featuring strategic interactions.

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