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Konrad Stahl and Roland Strausz*

Abstract

In markets with quality unobservable to buyers, third-party certification is often the only instrument to increase transparency. While both sellers and buyers have a demand for certification, its role differs fundamentally: sellers use it for signaling, buyers use it for inspection. Seller induced certification leads to more transparency, because it is informative – even if unused. By contrast, buyer induced certification incentivizes certifiers to limit transparency, as this raises demand for inspection. Whenever transparency is socially beneficial, seller certification is preferable. It also yields certifiers larger profits, so that regulating the mode of certification is redundant.

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1 Introduction

A market exhibits limited transparency when sellers are privately informed about the quality of their product, but lack the ability to convey credibly that information to the buyers. As a result, a market with opaque product quality obtains, resulting in economic inefficiencies due to adverse selection or moral hazard. These inefficiencies create a demand for independent certifiers who increase market transparency by verifying quality. Examples abound. Labeling institutions and commercial testing agencies certify the quality of final and intermediate goods, credit-rating agencies certify modern financial products, real estate appraisers certify the quality of housing units.[1]

The examples all have in common that, in principle, there is demand for transparency through certification from both sides of the market. High-quality sellers have a demand for certifiers in order to obtain an appropriately high price for their product, and buyers have a demand for certification to ensure that they do not overspend on low quality.

With demand arising from either side of the market, we ask to what extent differences between the two business models, *buyer certification* vs. *seller certification*, affect market transparency and subsequent economic outcomes.[2] At first sight, one might expect that, *all other things equal*, the question of who initiates and pays for certification is immaterial. Our main insight is however that, even though the basic role of certification – revealing information publicly and thereby increasing market transparency – remains the same under either business model, its economic use differs drastically. In particular, we argue that seller certification acts as a *signaling device*, whereas buyer certification acts as an *inspection device*.

Resulting from this difference alone, we show that seller certification is more effective in raising market transparency than buyer certification, due to two effects. First, the decision to certify provides more information under seller certification than under buyer certification. Second, buyer certification provides the certifier with the perverse incentive to actively obstruct market transparency, which is not the case for seller certification. The two crucial ingredients leading to these two effects are 1) the importance of private information about the good's quality on the part of the seller, and 2) an imperfect ability of the seller to signal its quality in the absence of

[1]Sellers on *Alibaba*, the world's largest online business-to-business trading platform for small businesses, explicitly post online copies of their certification, see, for instance, `https://www.alibaba.com/product-detail/2-5-inch-USB-3-0_60428570133.html?spm=a2700.7724857.0.0.0jEPJk` (last retrieved August 20, 2016).

[2]In the financial sector, the two alternatives are discussed under the terms *investor pays* vs. *issuer pays*. See White (2010) for a comprehensive survey of certification in this sector.

certification.

In order to see the first effect, it is instructive to consider what a buyer learns when, somewhat paradoxically, certification does *not* take place. Under buyer certification, the buyer clearly learns nothing about the good's quality and, hence, her beliefs remain unchanged. Under seller certification, however, the fact that certification did not take place reveals to her that the seller wants to conceal the true – intuitively, the low – quality of the good. Thus, seller certification provides information to the buyer even when certification does *not* take place. This makes seller certification more informative than buyer certification.

This difference in the informational content of certification is linked directly to our observation that seller certification acts as a signaling device, whereas buyer certification acts as an inspection device. By its very nature, inspection can only be informative if it actually takes place, whereas in a signaling context, not only the presence of a specific signal, but also its absence has informational value.[3]

Also the second effect is directly linked to our observation that buyer certification acts as an inspection device. Intuitively, the buyer's demand for inspection is high when she is unsure about product quality. Therefore, a profit-maximizing certifier induces seller behavior that maximizes the buyer's uncertainty. As we make precise in our analysis, the certifier is induced to set a price of certification that *minimizes* market transparency. This perverse incentive does not arise under seller certification, where certification is used as a signaling device.

Furthermore we show that the certifier's equilibrium profits are larger under seller certification, so that the certifier's incentives are aligned with promoting market transparency. This result brings us to the normative statement that if transparency is socially beneficial, then, all other things equal, the seller-certification model should be adopted. The same reason, namely that a certifier also obtains larger profits when it offers its services to the seller rather than the buyer, leads us to the positive statement that, all other things equal, a certifier indeed does opt for seller certification. The result implies that the certifier's preferences are in line with enhancing market transparency, so that an active regulation in this respect is not required.

Importantly, we obtain these results for markets with commercial certification services, i.e. for certifiers, who set a price of certification above marginal costs. As we discuss in more detail below, our theoretical results are consistent with the empirical observation that in such markets certifiers tend to adopt the seller-certification

[3] In settings with private but verifiable information (e.g. Grossman, 1981) this informational difference between seller and buyer certification is even more apparent, because unraveling occurs exactly because the lack of a signal affects beliefs adversely, which only occurs under seller certification.

business model. Yet, we also show that buyer-certification may yield higher social welfare if certification is subsidized so that it is offered at prices below costs.

We derive our results formally by first studying a parsimonious but generic version of Akerlof's adverse-selection problem of one buyer and one privately informed seller who sells a good with only two potential qualities. Within this setup, we fully characterize the equilibrium outcomes for two models which differ only by the fact that in the first one only the seller, and in the second one, only the buyer can buy certification.

In the equilibrium of the seller-certification model, only the seller of the high-quality good demands certification and thereby convinces the buyer to pay a high price. Thus the seller uses certification as a signaling device to overcome the imperfectness of his pricing signal. This results in a fully transparent market outcome, so that Akerlof's lemons problem disappears and all gains of trade are realized.

In contrast, in the equilibrium of the buyer-certification model, the high-quality seller picks a high price to signal high quality, which the low-quality seller mimics with positive probability. Upon seeing this high price, the buyer is unsure which type of seller she faces. In order to prevent herself from overspending on low quality, she demands certification with positive probability. Hence, the buyer uses certification as an inspection device to verify the quality claim implicit in the seller's high price. The equilibrium exhibits the typical logic underlying inspection games: only a mixed-strategy equilibrium exists, where the buyer certifies with positive probability, and the low-quality seller mixes between charging a low and a high price.[4] As a result, the buyer remains uninformed with a positive probability, so that full market transparency does not obtain. In addition, we show that, in order to induce a high demand for certification by the buyer, the profit-maximizing certifier sets a price that minimizes market transparency.

The remainder of this paper is organized as follows. Section 2 contains a discussion of the related literature. In Section 3, we develop our baseline model. In Section 4, we derive the results for seller, and in Section 5 for buyer certification. In Section 6 we compare profits and welfare under seller and buyer certification. In Section 7 we discuss extensions of our baseline model, the extent to which the results are robust, and its limitations. In Section 8 we discuss examples involving third-party certification. We summarize and conclude with Section 9. All proofs are relegated to the Appendix.

[4]See Avenhaus et al. (2002) for a survey on inspection games.

2 Related Literature

In their survey on certification, Dranove and Jin (2010) point out that third-party certification is mostly viewed as a means for *sellers* to credibly disclose information. In the terms of this paper, the literature therefore typically focuses on certifiers, who use seller certification rather than buyer certification as their business model.

The literature that explicitly compares the two models is small and we are aware of only two (unpublished) papers dealing with this question. In an older working paper, Durbin (1999) examines an intermediary's choice between selling guidebooks to buyers, privately informing them about seller quality, and selling certificates to the sellers, publicly certifying the quality of their goods. Focusing on rating agencies, Fasten and Hofmann (2010) discuss the provision of certification to a seller versus individual buyers. In both papers the seller can, by assumption, not make any (non-verifiable) claims about the quality of his product under buyer-induced certification. They however arise naturally in the form of, for example, initial price quotations.

Implicitly restricting to seller certification, Lizzeri (1999) shows that a monopolistic certifier maximizes profits by designing certificates that, in equilibrium, do not reveal any information. In his setup, the non-transparent equilibrium outcome is, moreover, unique. Our analysis sheds new light on these results. First, we can interpret Lizzeri's non-transparent equilibrium as a precursor of our insight that, as a signalling device, seller certification provides information also when not being used. Indeed, in Lizzeri's equilibrium, only the absence of a certificate is informative, signaling the worst possible quality to consumers. Second, uninformative certificates maximize profits only in a framework in which the market outcome without certification already maximises aggregate surplus, and with it welfare. Indeed, we note that certification in Lizzeri (1999) has a distributive effect but no efficiency effect. Finally and in line with recent literature on the disclosure of public information (e.g., Koessler and Renault, 2012 and Yamashita, 2016), we show in Section 7 that fully informative certificates *always* maximize the certifier's profits under seller-certification, leading to full transparency. In general, this transparency result requires however certificate-specific prices. Hence, the uniqueness of the non-informative equilibrium in Lizzeri (1999) obtains because certificate-specific prices are excluded in his setup.

We follow the literature on honest certification, in which it is assumed that the certifier can commit to certify truthfully. This effectively requires that the certified information is verifiable. Our paper is therefore much related to the literature on the revelation of verifiable information with its powerful unraveling results that lead to full disclosure (e.g., Grossman and Hart 1980, Grossman 1981, Milgrom 1981, and Okuno-Fujiwara et al. 1990), the literature on mechanism design with verifiable information (e.g., Green and Laffont 1986; Bull and Watson 2004; Deneckere and Severinov 2008; Ben-Porath and Lipman 2012) and, more generally, generic Bayesian games with preplay communication of certifiable information (e.g., Hagenbach et al. 2014). Since in these contributions it is irrelevant whether the informed party discloses the verifiable information directly, or indirectly through a certifier, our focus on seller vs. buyer certification clarifies that the unraveling results implicitly rely on seller certification, where the privately informed rather than the uninformed party decides to disclose verifiable information – or have it disclosed by a third party. However, an important difference in our setup is the natural fact that the revelation of verifiable information is costly. Moreover, by focusing on the role of firms and prices we follow an approach in the tradition of industrial organization rather than mechanism design.

A second, somewhat more recent literature on certification investigates the incentives to manipulate the certification process (e.g., Strausz 2005 and Mathis et al. 2009).[5] While we view capture and information manipulation as a primary concern for certification, we abstract from these issues, because the link between the mode of certification and the threat of capture is a sophisticated one and depends much on the institutional details of the market under consideration. First, the certification process frequently necessitates the seller to supply information to the certifier. A natural worry is, therefore, that the seller could manipulate this information, leading to biases in the certification result. If however the seller's provision of information is crucial for the certification, then this type of manipulation is primarily due to the characteristics of the good itself and the certifier's certification technology rather than the certifier's business model.[6]

Second, also the seller's willingness to pay for manipulating a specific certificate is, in principle, independent of the business model. Hence, irrespective of whether the buyer or the seller asks for a certification, a low quality seller would like to bribe the certifier to hand out a favorable certificate. Finally, if the certificate affects the price of the transaction, then, naturally, both the seller and the buyer have an incentive to bribe the certifier, albeit in contrary directions. Given these issues, linking the seller's and buyer's ability to manipulate directly with the certifier's business model may not

[5] See also Faure-Grimaud et al. 2009, Skreta and Veldkamp 2009, Bar-Isaac and Shapiro 2011, Bolton et al. 2012, and Opp et al. 2013 for studies on capture in the market of rating agencies.

[6] In certain markets, certifiers can partially circumvent this dependence on the seller's information, while in others they cannot. For example, *Stiftung Warentest*, a state-subsidized certifier of consumer products, used to order the products for its tests directly from the producers. After observing the producer's manipulation of these test products, it now buys the test items from the shelf. This approach seems less applicable in financial markets. As discussed in Bolton et al. (2012), independently of the adopted business model, rating agencies crucially depend on the information of the issuer to certify complex financial products such as tranched securities.

fully reflect the main problems of manipulation and capture in these markets.

Next to abstracting from manipulation, we also do not investigate the incentives of economic agents to become certifiers (e.g., Biglaiser 1993), the effect of certifiers on market structure (e.g., Board 2009, Guo and Zhao 2009), or from interactions between the acquisition and the disclosure of information (e.g., Shavell 1994).

Since we stress the role of signaling, our paper is related to the vast literature on signaling and, in particular, on signaling of unobservable quality through prices (e.g., Wolinsky 1983). Equilibrium refinements on out-of-equilibrium beliefs are common in this literature. While we do not need such refinements for the analysis of seller certification, we resort, for the analysis of the buyer-certification model, to an equilibrium refinement of Bester and Ritzberger (2001), which extends the intuitive criterium of Cho and Kreps (1987) to nondeterministic beliefs. Considering a static environment, we abstract from dynamic signaling of quality (e.g., Bar-Isaac 2003).

3 The Setup

We consider certification in an Akerlof adverse-selection setup between one seller (he) and one buyer (she). The good's quality q represents the buyer's willingness to pay and can either be high, q_h, or low, q_l, where $\Delta q \equiv q_h - q_l > 0$ and $q_l > 0$. High quality has production costs $c_h > 0$, while low quality has costs $c_l = 0$. The exact quality level is known only to the seller, but it is common knowledge that high quality obtains with probability λ and low quality with probability $1-\lambda$. High quality delivers higher social surplus, $q_h - c_h > q_l$, but its production costs exceed average quality, $c_h > \bar{q} \equiv \lambda q_h + (1-\lambda)q_l$. Outside options are zero: the seller obtains zero if he does not produce the good, and the buyer obtains zero if she does not buy.

Viscusi (1978) shows that Akerlof's framework creates a demand for an external certifier, who raises market transparency. We assume that such a certifier (it) is available and can, at some fixed cost $c_c \geq 0$, reveal truthfully and publicly the seller's quality at a price p_c for its services. We assume that the cost of certification is low enough so that the high-quality good is socially preferable even *net* of certification costs: $q_h - c_h - c_c > q_l$.

Our main research question is to understand the extent to which the mode of certification affects market transparency, all other things equal. We do so by first studying the equilibrium outcomes of the two games, Γ^s and Γ^b, as illustrated in Table 1. In line with our ceteris paribus perspective, the two games differ only in stage 4, where under seller certification the seller decides whether to certify, whereas

The seller-certification game (Γ^s):	The buyer-certification game (Γ^b):
1. Certifier sets certification price p_c.	1. Certifier sets certification price p_c.
2. Seller learns quality $q \in \{q_l, q_h\}$.	2. Seller learns quality $q \in \{q_l, q_h\}$.
3. Seller sets a price p.	3. Seller sets a price p.
4. *Seller* decides whether to certify.	4. *Buyer* decides whether to certify.
5. Buyer decides whether to buy.	5. Buyer decides whether to buy.

Table 1: Timing of the seller- and buyer-certification game.

under buyer certification the buyer decides.[7] Moreover, the underlying certification game itself is kept as generic as possible so that it can capture the essence of many different certification procedures in practice. Because the certifier is to physically inspect the good, we assume that the production costs are incurred at stage 2 upon the seller's decision whether or not to produce, i.e. before the certification costs arise at stage 4.

As argued, we are especially interested in the effectiveness of certification in both attaining market transparency and realizing potential gains of trade. For this reason, we say that a certification model is *information-effective* if it leads to an equilibrium outcome where the buyer perfectly learns the seller's quality before buying the good. When certification is information-effective, it achieves full market transparency. In addition, we say that a certification model is *trade-effective* if it leads to an equilibrium outcome in which all potential gains of trade are realized, which in our setting means that the good is always produced and sold.

In our certification game, the certifier's price p_c set at $t = 1$ triggers a proper subgame, which is a Bayesian game in extensive form. Clearly, the equilibrium outcome of this subgame plays a crucial role in the determination of the certifier's optimal price p_c. For this reason, our approach is as follows. We first study, for a given p_c, the outcome of the seller-certification subgame $\Gamma^s(p_c)$, where at $t = 4$ the seller decides about certification. After characterizing this outcome, we solve for the monopolistic certifier's optimal price under seller certification. We then contrast this analysis by studying the buyer-certification subgame $\Gamma^b(p_c)$, where at $t = 4$ the buyer rather than the seller decides about certification.

[7] In the seller-certification game, the extensive-form representation of separate stages 3 and 4 has no strategic relevance; we could simply reverse the order of the two decisions. The explicit separation is only chosen to enable a direct comparison to the buyer-certification game.

4 Seller Certification

We start with characterizing the equilibrium outcome of the seller certification subgame $\Gamma^s(p_c)$. In this subgame, the seller picks a price p and decides to offer the good certified or uncertified. Observing the seller's decision and, possibly, the outcome of certification, the buyer decides whether to buy.

Allowing for mixed strategies, we denote the seller's strategy as a probability distribution over prices p and whether to certify the good. In particular, let $\sigma_i^c(p)$ denote the probability that a seller with quality q_i offers the good certified at a price p, and $\sigma_i^u(p)$ the probability that he offers the good uncertified at that price.[8] The seller's strategy σ_i is then a combination $(\sigma_i^c, \sigma_i^u), i \in \{l, h\}$ such that

$$\sum\nolimits_j \sigma_i^c(p_j) + \sum\nolimits_j \sigma_i^u(p_j) = 1.$$

After observing the seller's price and his decision to certify, the buyer forms a belief about the probability that the good has high quality. If the seller has his good certified, the buyer learns its true quality, and thus her beliefs after certification reflect the true quality q_i. Consequently, she buys a certified good whenever $p \leq q_i$. If the good is uncertified, the buyer's belief that it is of high quality is, in general, uncertain. It depends on the price p, since the buyer may interpret the price p as a signal of quality. In equilibrium, the belief must follow Bayes's rule whenever possible. Consequently, we say that *the buyer's belief* $\mu(p)$ *is consistent with the seller's strategy* (σ_l, σ_h) if for any $\sigma_i^u(p) > 0$ it satisfies

$$\mu(p) = \frac{\lambda \sigma_h^u(p)}{\lambda \sigma_h^u(p) + (1-\lambda)\sigma_l^u(p)}. \tag{1}$$

Facing an uncertified good at a price p, the buyer's belief equals $\mu(p)$, and it is optimal for her to buy when the expected quality $\mu(p)q_h + (1 - \mu(p))q_l$ exceeds the seller's price p. When that price exceeds expected quality, it is optimal not to buy, and when expected quality coincides with the price, any random buying behavior is optimal. Let $\sigma(s_b|p, \mu) \in [0, 1]$ denote the probability that the buyer buys the good uncertified, i.e., takes the action s_b, given the seller has quoted the price p and the buyer's belief is μ. We say that buying behavior σ *is optimal* if for any (p, μ), the decision to buy an uncertified good with probability $\sigma(s_b|p, \mu)$ is optimal.

Let π_i^u denote the expected payoff of a seller with quality q_i, who offers the good uncertified. Given the buyer's belief $\mu(p)$ and her buying behavior $\sigma(s_b|p, \mu)$, a high-quality seller and a low-quality seller expect the following respective payoffs from

[8] To avoid measure-theoretical issues, we let the seller randomize over only countably many prices.

offering the good uncertified at a price p:

$$\pi_h^u(p) = \sigma(s_b|p, \mu(p))p - c_h \quad \text{and} \quad \pi_l^u(p) = \sigma(s_b|p, \mu(p))p. \tag{2}$$

Hence, a strategy $\sigma_i = (\sigma_i^c, \sigma_i^u)$ yields the seller of quality q_i the expected payoff

$$\pi_i(\sigma_i) = \sum\nolimits_j \sigma_i^u(p_j)\pi_i^u(p_j) + \sum\nolimits_j \sigma_i^c(p_j)[p_j \mathbf{1}_i(p_j) - p_c - c_i],$$

where $\mathbf{1}_i(p)$ is an indicator function which equals 1 if $p \leq q_i$ and 0 otherwise. We say that *the seller strategy* σ_i^* *is optimal* if it maximizes $\pi_i(\sigma_i)$.

A perfect Bayesian equilibrium (PBE) of the subgame $\Gamma^s(p_c)$ is a combination $\{\sigma_l^*, \sigma_h^*, \mu^*, \sigma^*\}$ for which the seller's strategies σ_l^* and σ_h^* are optimal, the belief μ^* is consistent, and the buyer's strategy σ^* is optimal. With this definition the following lemma characterizes the equilibrium outcomes corresponding to the subgame $\Gamma^s(p_c)$.

Lemma 1 *Consider the subgame* $\Gamma^s(p_c)$ *with seller certification.*

i. For $p_c \leq q_h - c_h$*, a PBE exists for which the certifier obtains the payoff* $\lambda(p_c - c_c)$*, the good is always sold, the seller with quality* q_h *always certifies, whereas the seller with quality* q_l *does not. For* $p_c < q_h - c_h$*, this equilibrium outcome is unique.*

ii. For $p_c > q_h - c_h$*, the high and the low-quality seller do not certify in any PBE and the outcome coincides with the market outcome without a certifier.*

The lemma shows that for a low enough price of certification, the high-quality seller certifies to reveal his high quality. Hence, certification is used as a signaling device and the buyer interprets an uncertified good as revealing bad quality. For all certification prices different from $q_h - c_h$, the equilibrium outcome is unique. Note that this is in line with results about certification in competitive adverse-selection markets (e.g., Viscusi 1978).

The lemma has the following direct implication.

Corollary 1 *For* $p_c < q_h - c_h$*, seller certification is information- and trade-effective.*

When choosing its price of certification, the certifier will take into account the extent to which it affects demand as stated in the lemma. Let Π^s denote the certifier's payoff under seller certification. The following proposition characterizes the outcome under seller certification when we include the price-setting decision of the certifier.

Proposition 1 *The game with seller certification has a unique equilibrium outcome* $\bar{p}_c^s = q_h - c_h$ *with equilibrium expected payoffs* $\Pi^s = \lambda(q_h - c_h - c_c)$ *to the certifier, and* $\pi_h^* = 0$ *and* $\pi_l^* = q_l$ *to the seller. Moreover, the high-quality seller certifies with certainty, the low-quality seller does not certify, and the good is always traded.*

Unsurprisingly, the monopolistic certifier extracts all economic rents from certification. Consequently, the high-quality seller is just as well off as without certification and obtains zero profits. Yet, in equilibrium all gains of trade are realized and the seller's quality is fully revealed. This yields the following corollary.

Corollary 2 *Monopolistic seller certification is information- and trade-effective.*

5 Buyer Certification

We first consider the buyer certification subgame $\Gamma^b(p_c)$ for a given price of certification p_c. In this subgame, the seller first picks a price p and the buyer then decides whether to certify the good and to buy it. Let $\sigma_i(p_j)$ denote the probability that the seller with quality q_i sets a price p_j. Thus, for both $i \in \{l, h\}$,

$$\sum_j \sigma_i(p_j) = 1.$$

As under seller certification, observing the price p, the buyer forms belief $\mu(p)$ about the probability that the good has high quality. Again, the buyer's belief follows Bayes's rule whenever possible, and we say that *it is consistent with the seller's strategy* (σ_h, σ_l) if for any $\sigma_i(p) > 0$ it satisfies

$$\mu(p) = \frac{\lambda\sigma_h(p)}{\lambda\sigma_h(p) + (1-\lambda)\sigma_l(p)}. \tag{3}$$

Given the price p and belief μ, the buyer has three relevant actions:

1. Action s_b: The buyer does not certify but buys the good. This yields payoff $U(s_b|p,\mu) = \mu q_h + (1-\mu)q_l - p$.

2. Action s_n: The buyer does not certify, nor buy the good. This yields payoff $U(s_n|p,\mu) = 0$.

3. Action s_h: The buyer certifies the good and buys only if certification reveals the high quality q_h. This yields payoff $U(s_h|p,\mu) = \mu(q_h - p) - p_c$.

The other three actions open to the buyer – to certify and always buy, to certify but never buy, and to certify and buy only if quality is low – are clearly suboptimal. We therefore disregard them.

The action s_n is optimal whenever $U(s_n|p,\mu) \geq U(s_b|p,\mu)$ and $U(s_n|p,\mu) \geq U(s_h|p,\mu)$. Hence, the set of (p,μ) combinations for which s_n is optimal is

$$S(s_n|p_c) \equiv \{(p,\mu)|p \geq \mu q_h + (1-\mu)q_l \ \wedge \ p_c \geq \mu(q_h - p)\}.$$

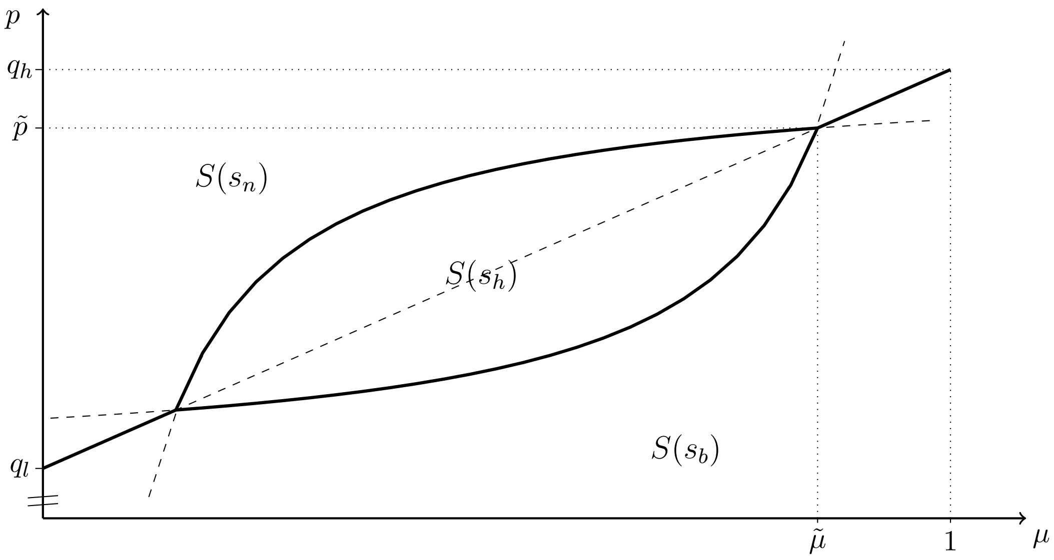

Figure 1: Buyer's buying behavior for given $p_c < \Delta q/4$.

Likewise, the action s_b is optimal whenever $U(s_b|p,\mu) \geq U(s_n|p,\mu)$ and $U(s_b|p,\mu) \geq U(s_h|p,\mu)$. Hence, the set of (p,μ) combinations for which s_b is optimal is

$$S(s_b|p_c) \equiv \{(p,\mu)|p \leq \mu q_h + (1-\mu)q_l \ \wedge \ p_c \geq (1-\mu)(p-q_l)\}\,.$$

Finally, the action s_h is optimal whenever $U(s_h|p,\mu) \geq U(s_n|p,\mu)$ and $U(s_h|p,\mu) \geq U(s_b|p,\mu)$. Hence, the set of (p,μ) combinations for which s_h is optimal is

$$S(s_h|p_c) \equiv \{(p,\mu)|p_c \leq \mu(q_h-p) \wedge p_c \leq (1-\mu)(p-q_l)\}\,.$$

Figure 1 illustrates the buyer's optimal actions. For low product prices p, the buyer buys the good uncertified, $(p,\mu) \in S(s_b)$, whereas for high prices p the buyer refrains from buying, $(p,\mu) \in S(s_n)$. It turns out that as long as $p_c < \Delta q/4$, there is an intermediate range of prices p and beliefs μ such that the buyer demands certification, i.e., $(p,\mu) \in S(s_h)$. In this case, the buyer only buys the product when certification reveals it to be of high quality. Note that apart from points on the thick, dividing lines, the buyer's optimal action is uniquely determined so that mixing over different actions is suboptimal.

For future reference we define

$$\tilde{p} \equiv \left(q_h + q_l + \sqrt{\Delta q(\Delta q - 4p_c)}\right)/2 \text{ and } \tilde{\mu} \equiv \left(1 + \sqrt{1 - 4p_c/\Delta q}\right)/2. \qquad (4)$$

If the seller quotes the price $\tilde{p}$ and the buyer has beliefs $\tilde{\mu}$, then the buyer is indifferent between all her three actions.

Let $\sigma(s|p,\mu)$ denote the probability that the buyer takes action $s \in \{s_b, s_n, s_h\}$ given price p and belief μ. We can then denote the buyer's (mixed) strategy by probabilities $\sigma(s|p,\mu)$ such that

$$\sigma(s_b|p,\mu) + \sigma(s_n|p,\mu) + \sigma(s_h|p,\mu) = 1.$$

We say that *the strategy* σ^* *is optimal* if it randomizes among those actions that are optimal: $\sigma^*(s|p,\mu) > 0$ implies that $(p,\mu) \in S(s|p_c)$.

Given buyer's belief μ and her strategy σ, a seller with quality q_h and a seller with quality q_l expect the following respective payoffs from offering the good at a price p:

$$\pi_h(p,\mu|\sigma) = [\sigma(s_b|p,\mu) + \sigma(s_h|p,\mu)]p - c_h \text{ and } \pi_l(p,\mu|\sigma) = \sigma(s_b|p,\mu)p.$$

Given that a price p leads to the belief $\mu(p)$, a seller with quality q_h and a seller with quality q_l expect the following respective payoffs from offering the good at a price p:

$$\pi_h^b(p) = \pi_h(p,\mu(p)|\sigma) \quad \text{and} \quad \pi_l^b(p) = \pi_l(p,\mu(p)|\sigma). \tag{5}$$

We say that *the seller's pricing strategy* σ_i *is optimal* (*with respect to the buyer's behavior* (σ^*, μ^*)) if any price $\hat{p}$ such that $\sigma_i(\hat{p}) > 0$ maximizes $\pi_i^b(p)$:

$$\sigma_i(p) > 0 \;\Rightarrow\; \pi_i(p,\mu^*(p)|\sigma^*) \geq \pi_i(p',\mu^*(p')|\sigma^*), \;\forall p'. \tag{6}$$

A perfect Baysian equilibrium (PBE) of the subgame $\Gamma^b(p_c)$ is a combination $\{\sigma_l^*, \sigma_h^*, \mu^*, \sigma^*\}$ for which the sellers' strategies σ_l^* and σ_h^* are optimal, the belief μ^* is consistent and the buyer's strategy σ^* is optimal.

It follows that in a PBE $(\sigma_h^*, \sigma_l^*, \mu^*, \sigma^*)$ the high-quality seller's and the low-quality seller's payoffs, respectively, are

$$\pi_h^* = \sum\nolimits_j \sigma_h^*(p_j)\pi_h(p_j,\mu^*(p_j)|\sigma^*) \quad \text{and } \pi_l^* = \sum\nolimits_j \sigma_l^*(p_j)\pi_l(p_j,\mu^*(p_j)|\sigma^*).$$

Corollary 1 showed that seller certification at a price $p_c < c_h - q_h$ is both information- and trade-effective. In contrast, if buyer certification is to be trade effective, the buyer cannot opt for certification at any price chosen with positive probability by the low quality seller. But then certification is altogether useless because it is always chosen at prices that must have been set by the high quality seller. Hence, the buyer will never choose to have the good certified if the price p_c of certification is strictly positive. Therefore, trade-effectiveness is incompatible with costly certification. The following lemma gives precision to these arguments.

Lemma 2 *If buyer certification is offered at a price* $p_c > 0$*, then it is not trade-effective.*

Because a monopolistic certifier obtains positive profits only with a strictly positive price exceeding its costs $c_c \geq 0$, the lemma implies that buyer certification is an imperfect tool for achieving market efficiency. When certification involves no costs ($c_c = 0$), this result allows us to conclude directly that welfare under seller certification is higher than under buyer certification (using the usual definition of welfare as the sum of all the agents' surplus). Moreover, because under seller certification, the certifier is able to extract all the rents from certification, its profits must then also be larger. We therefore obtain the following corollary.

Corollary 3 *Suppose the certifier incurs no cost of certification* ($c_c = 0$). *Then seller certification is welfare superior to buyer certification and yields the certifier larger profits, so that its preferences concerning the certification model are in line with welfare.*

Lemma 2 is insufficient to make similar claims when certification is costly ($c_c > 0$). Although the indirect gains are higher under seller certification, we cannot exclude a priori that, due to a higher certification intensity, these higher gains are offset by larger certification costs. In order to address this question, we first need to fully characterize the equilibrium outcome in the subgame $\Gamma^b(p_c)$. This characterization will also enable us to show a further perverse effect of buyer certification: it induces certifiers to artificially limit market transparency. This implies that the market outcome under buyer certification also fails to be informative-effective.

The next lemma derives intuitive properties of the equilibrium outcome that hold in any perfect Bayesian equilibrium of the subgame $\Gamma^b(p_c)$ with a positive certification price p_c. First, the seller's expected profits increase when the buyer is more optimistic about the good's quality. Second, the seller, no matter his type, is shown to never set a price below q_l, and the low-quality seller never a price above q_h. Finally, the low-quality seller is shown to never lose from the presence of asymmetric information, since he can always guarantee himself the payoff q_l that he obtains with observable quality. By contrast, the high-quality seller loses from the presence of asymmetric information; his payoff is strictly smaller than $q_h - c_h$.

Lemma 3 *In any PBE* $(\sigma_l^*, \sigma_h^*, \mu^*, \sigma^*)$ *of the subgame* $\Gamma^b(p_c)$ *with* $p_c > 0$ *we have i) equilibrium payoffs* $\pi_h(p, \mu|\sigma^*)$ *and* $\pi_l(p, \mu|\sigma^*)$ *are nondecreasing in* μ*; ii)* $\sigma_l^*(p) = 0$ *for all* $p \notin [q_l, q_h]$ *and* $\sigma_h^*(p) = 0$ *for all* $p < q_l$*; iii)* $\pi_l^* \geq q_l$ *and* $\pi_h^* < q_h - c_h$*.*

The concept of perfect Bayesian equilibrium does not place any restrictions on the buyer's out-of-equilibrium beliefs. Hence, as is typical for signaling games, without any restrictions on these beliefs we cannot pin down behavior in the subgame $\Gamma^b(p_c)$

to a specific equilibrium outcome. Especially by the use of extreme out-of-equilibrium beliefs, one can sustain many pricing strategies in a PBE.

In order to reduce the arbitrariness of equilibrium play, it is necessary to strengthen the solution concept of PBE by introducing more plausible restrictions on out-of-equilibrium beliefs. A common belief restriction is the intuitive criterion of Cho and Kreps (1987), which in its standard formulation only has bite in an equilibrium where the signaling player fully reveals himself so that $\mu \in \{0, 1\}$ results. Since the sellers' use of mixed strategies typically leads to intermediate beliefs $\mu \notin \{0, 1\}$, we use Bester and Ritzberger (2001)'s extension of the intuitive criterium to such intermediate beliefs:

Belief Restriction (BR): A perfect Bayesian equilibrium $(\sigma_h^*, \sigma_l^*, \mu^*, \sigma^*)$ satisfies *BR* if, for any $\mu \in [0, 1]$ and any out-of-equilibrium price p, we have

$$\pi_l(p, \mu) < \pi_l^* \wedge \pi_h(p, \mu) > \pi_h^* \;\Rightarrow\; \mu^*(p) \geq \mu.$$

The belief restriction states intuitively that if a pessimistic belief μ gives only the high-quality seller an incentive to deviate, then the restriction requires that the buyer's actual belief should not be even more pessimistic than μ. It extends the intuitive criterion of Cho and Kreps, which obtains for the special case $\mu = 1$. Indeed, the restriction extends the logic of the Cho-Kreps criterion to situations where the deviation to a price p is profitable only for the high-quality seller when the buyer believes that the deviation originates from the high-quality seller with probability μ.

The next lemma characterizes equilibrium outcomes that satisfy the belief restriction (BR). In particular, the refinement implies that the high-quality seller can sell his product at a price of at least $\tilde{p}$.

Lemma 4 *Any perfect Bayesian equilibrium $(\sigma_l^*, \sigma_h^*, \mu^*, \sigma^*)$ of the subgame $\Gamma^b(p_c)$ that satisfies BR exhibits i) $\sigma_h^*(p) = 0$ for all $p < \tilde{p}$ and ii) $\pi_h^* \geq \tilde{p} - c_h$.*

By combining the previous two lemmas, we are now able to characterize the equilibrium outcome.

Proposition 2 *Consider a PBE $(\sigma_l^*, \sigma_h^*, \mu^*, \sigma^*)$ of $\Gamma^b(p_c)$ that satisfies BR. Then:*

i. For $\tilde{\mu} > \lambda$ and $\tilde{p} > c_h$ it is unique. The high-quality seller sets the price $\tilde{p}$ with certainty, $\sigma_h^*(\tilde{p}) = 1$, while the low-quality seller randomizes between price $\tilde{p}$ and q_l and the buyer randomizes between s_b and s_h upon observing the price $\tilde{p}$. The respective probabilities with which the low-quality seller picks $\tilde{p}$ and the buyer certifies are

$$\sigma_l^*(\tilde{p}) = \frac{\lambda(1-\tilde{\mu})}{\tilde{\mu}(1-\lambda)} \text{ and } \sigma^*(s_h|\tilde{p}, \tilde{\mu}) = \frac{\tilde{p} - q_l}{\tilde{p}}.$$

ii. For $\tilde{\mu} < \lambda$ or $\tilde{p} < c_h$, certification does not take place in equilibrium.

iii. For $\tilde{\mu} \geq \lambda$ and $\tilde{p} \geq c_h$, an equilibrium outcome as described under i. exists.

The proposition formalizes our insight that buyer certification serves as an inspection device to discipline the low-quality seller. Indeed, the high-quality seller signals his quality by announcing $\tilde{p}$, while the buyer and the low-quality seller play the mixed strategies typical of an inspection game: By choosing the low price q_l, the low-quality seller provides an honest signal, whereas he cheats by picking the high price $\tilde{p}$. Whenever the buyer observes $\tilde{p}$, she cannot identify the good's quality. Therefore she certifies with positive probability.

In line with the logic underlying inspection games, a pure equilibrium does not exist. On the one hand, if the buyer would always certify when seeing the high price, the low-quality seller would not cheat by asking such a price; but without any cheating certification is suboptimal. On the other hand, if the buyer would never certify, then the low-quality seller would have a strict incentive to cheat and to quote the high price; but with such cheating the buyer would want to certify. Hence, only a mixed equilibrium exists, where the buyer's certification probability keeps the low-quality seller indifferent between cheating and honestly pricing his good, while at the same time the cheating probability of the low-quality seller keeps the buyer indifferent between buying the good uncertified and asking for certification. In order to satisfy both indifference conditions, the high price must equal $\tilde{p}$ and the buyer's belief must equal $\tilde{\mu}$.

In Proposition 2 we characterize the equilibrium outcome under buyer certification for a given price of certification p_c. The proposition allows us to derive the demand for buyer certification by taking into account that $\tilde{\mu}$ and $\tilde{p}$ depend on p_c according to (4). We therefore write these dependencies explicitly as $\tilde{p}(p_c)$ and $\tilde{\mu}(p_c)$. Because the equilibrium probability of buyer certification is the compounded probability that the seller picks the price $\tilde{p}$ and the buyer certifies, we can write demand as

$$x^b(p_c) = [\lambda + (1-\lambda)\sigma_l^*(\tilde{p}(p_c))]\sigma^*(s_h|\tilde{p}(p_c), \tilde{\mu}(p_c))$$

whenever $\tilde{\mu}(p_c) \geq \lambda$ and $\tilde{p}(p_c) \geq c_h$, and as zero otherwise. Inserting $\sigma_l^*(\tilde{p})$ and $\sigma^*(s_h|\tilde{p}, \tilde{\mu})$ from Proposition 2, the certifier's profit under buyer certification is

$$\Pi^b(p_c) = x^b(p_c)(p_c - c_c) = \frac{\lambda(\tilde{p}(p_c) - q_l)}{\tilde{\mu}(p_c)\tilde{p}(p_c)}(p_c - c_c), \tag{7}$$

whenever $\tilde{\mu}(p_c) \geq \lambda$ and $\tilde{p}(p_c) \geq c_h$, and zero otherwise. In the next proposition we derive the monopoly price of buyer certification.

Proposition 3 *Consider the game with buyer certification.*

i. For $c_h \leq (q_h + q_l)/2$, the certifier sets a price $\bar{p}_c^b = \Delta q/4$, which induces a subgame $\Gamma^b(\bar{p}_c^b)$ with $\tilde{\mu}(\bar{p}_c^b) = 1/2$ and a certification profit of

$$\Pi^b = \frac{\lambda \Delta q}{2(q_h + q_l)}(\Delta q - 4c_c).$$

ii. For $c_h > (q_h + q_l)/2$, the certifier sets the price $\bar{p}_c^b = (q_h - c_h)(c_h - q_l)/\Delta q$, which induces a subgame $\Gamma^b(\bar{p}_c^b)$ with $\tilde{p}(\bar{p}_c^b) = c_h$ and a certification profit of

$$\Pi^b = \frac{\lambda[(q_h - c_h)(c_h - q_l) - \Delta q c_c]}{c_h}.$$

The proposition reveals the perverse effect that buyer certification induces the certifier to minimize market transparency artificially. According to Proposition 3 *i.*, the certifier picks a price $\bar{p}_c^b$ such that after observing the price $\tilde{p}$, the buyer has beliefs $\tilde{\mu}(\bar{p}_c^b) = 1/2$. This maximizes her uncertainty about product quality (in the sense of Shannon entropy) and implies that market transparency is minimized.

To see that this perverse effect results directly from the role of buyer certification as an inspection device, observe that the value of an inspection device is typically higher when the underlying uncertainty is larger. Hence, the buyer's willingness to pay for certification and her demand are highest when, conditional upon observing the price $\tilde{p}$, market transparency is minimized. The certifier's most preferred price p_c is, therefore, such that $\tilde{\mu}(p_c) = 1/2$. The certifier must however ensure that at this price the high-quality seller does not drop out of the market. In the case specified in Proposition 3 *ii.*, this limits the certifier's ability to fully minimize transparency.

6 Profit and Welfare Comparisons

In Corollary 3 we showed that, for zero certification costs, seller certification outperforms buyer certification both from a social welfare and the certifier's perspective. By contrasting the equilibrium outcomes under seller and buyer certification as derived in Propositions 1 and 3, we now show that these two results also obtain when certification costs are positive. We first show this for the certifier's profits:

Proposition 4 *For any cost of certification $c_c \in [0, q_h - c_h]$, the certifier obtains a higher profit and charges higher prices under seller certification than under buyer certification, $\Pi^s > \Pi^b$ and $\bar{p}_c^s > \bar{p}_c^b$. Hence, the certifier prefers seller certification. The certification intensity under buyer certification exceeds the certification intensity under seller certification, whenever $c_h > (q_h + q_l)/2$ or $q_h < 3q_l$.*

Next we show that Corollary 3 extends to positive certification costs also for social welfare. We thereby ideally want to establish that social welfare is higher not only for the respective monopoly prices $\bar{p}_c^s$ and $\bar{p}_c^b$ but also for lower price combinations. In this case, our welfare result would also hold when certification markets are more competitive in that they exhibit equilibrium prices below monopoly. Under perfect competition we expect certification prices to equal marginal costs c_c. For intermediate forms of competition, where certifiers have some market power, we expect prices to exceed marginal costs but to not reach monopoly levels. Focusing our analysis first on unregulated certification markets, we therefore consider any combination of certification prices, (p_c^s, p_c^b), in between marginal costs and the respective monopoly price.

For any price of certification p_c^s that lies in between marginal cost c_c and the monopoly price under seller certification $\bar{p}_c^s$, the high-quality seller certifies and the good is always traded. Hence, welfare under seller certification is

$$W^s = \lambda(q_h - c_h) + (1 - \lambda)q_l - \lambda c_c.$$

It follows that, as long as the price of certification, p_c^s, does not exceed the monopoly price $\bar{p}_c^s$, welfare under seller certification is independent of the actual price, because for such prices demand is inelastic so that the price represents a pure welfare transfer.

This is different under buyer certification, where the certification price directly affects the gains from trade. This is because buyer certification is not trade-effective; the good is not sold when the low-quality seller picks a price exceeding q_l and the buyer certifies. According to Proposition 2, this happens with probability

$$\omega(p_c^b) = \sigma_l^*(\tilde{p}(p_c^b))\sigma^*(s_h|\tilde{p}(p_c^b), \tilde{\mu}(p_c^b)),$$

which depends explicitly on the price of certification p_c^b. For any certification price that does not exceed the monopoly price under buyer certification $\bar{p}_c^b$, the high-quality good is always sold, so that social welfare under buyer certification is

$$W^b(p_c^b) = \lambda(q_h - c_h) + (1 - \lambda)(1 - \omega(p_c^b))q_l - x^b(p_c^b)c_c.$$

The difference in welfare is therefore

$$\Delta W(p_c^b) \equiv W^s - W^b(p_c^b) = (1 - \lambda)\omega(p_c^b)q_l - [\lambda - x^b(p_c^b)]c_c. \tag{8}$$

The expression illustrates the trade-off between differences in trade effectiveness – represented by the first, positive term $(1 - \lambda)\omega(p_c^b)q_l$ – and the cost of certification – represented by the second, possibly negative term $[\lambda - x^b(p_c^b)]c_c$. For zero certification costs, the second term disappears and the expression is strictly positive. This confirms Corollary 3. With certification costs, we cannot directly draw a conclusion, because when the certification intensity under buyer certification, $x^b(p_c^b)$, is substantially lower than the certification intensity λ under seller certification, the second term outweighs the first term and renders $\Delta W(p_c^b)$ negative.

The next proposition shows, however, that for any buyer-certification price p_c^b in between marginal costs c_c and the monopoly price $\bar{p}_c^b$, this is not the case.

Proposition 5 *For any cost of certification $c_c \in [0, q_h - c_h]$ and any combination of seller certification and buyer-certification prices such that each price lie in between marginal costs and the respective monopoly price, $(p_c^s, p_c^b) \in [c_c, \bar{p}_c^s] \times [c_c, \bar{p}_c^b]$, welfare under seller certification exceeds welfare under buyer certification.*

As we discuss in more detail in Section 8, in some empirically relevant settings certification services are sold to buyers—primarily final consumers, but provided by non-profit and charitable organizations and certifiers who are subsidized by the government. Such non-commercial certifiers may set a price of certification that lies strictly below the cost of certification. We therefore show next that, in general, Proposition 5 does not extend to prices of certification that lie below cost. For low enough prices, buyer certification can lead to higher welfare than seller certification. An intuition for this result follows from considering the case that certification is costly $c_c > 0$ but the the price of certification is set at zero. Since the buyer can now ask for costless certification, buyer certification is no longer an inspection game. As a consequence, the buyer can induce the low quality seller to pick the correct price $p_l = q_l$ with probability 1 by certifying with probability 1. This inspection behavior leads to an outcome that coincides with the trade efficient equilibrium outcome under seller certification, suggesting that buyer certification does at least as well.

Yet, under buyer certification one can do even better, because the buyer can discipline the low quality seller with a certifying probability less than one. Hence, buyer certification obtains the trade efficient outcome with a lower certification probability. In the case that the true cost of certification is strictly positive, this implies that buyer certification yields higher welfare as it saves on certification costs. The next proposition shows this formally.

Proposition 6 *For any cost of certification $c_c \in (0, q_h - c_h]$ and any price of certification $p_c < \bar{p}_c$, buyer certification yields a higher welfare than seller certification, where*

$$\bar{p}_c \equiv \frac{q_l(c_c(q_h - 2q_l) + (q_h - q_l)q_l}{\Delta q(c_c + q_l)^2} c_c < c_c.$$

From a regulatory perspective, the proposition also implies that, if the cost of certification is relatively high in comparison to the gains in trade efficiency from market transparency, then there is a rationale for regulating the price of certification. Our results moreover show that price regulation is especially important when the certifier uses buyer certification, because in these markets certifiers have the perverse incentive to set a price that reduces market transparency.

7 Extensions

Taking a typical industrial organization approach, we compared the two natural business models of seller vs. buyer certification and demonstrated the superiority of seller certification from both a welfare and the certifier's profit maximizing perspective. Although we consider as rather generic the extensive form games by which we capture the two business models, our specific choices nevertheless invite questions about the robustness of our results.

7.1 Mechanism Design

Using a mechanism design approach, we can however demonstrate the optimality of seller certification both more generally and for a more general class of models but provided that the certifier's costs of certification are zero. In particular, this approach shows that a full disclosure rather than a partial disclosure of information is optimal. Moreover, using a different business model, e.g. allowing both the buyer and the seller to certify, does not lead to higher welfare or certification profits than seller certification.

The general framework allows for arbitrary many quality levels.[9] More specifically, let the (closed) arbitrary set $Q \subset \mathbb{R}$ represent the support of possible quality levels with the interpretation that if the buyer obtains a good with quality level $q \in Q$ at a price p, she obtains the utility $u^b = q - p$. Let $\underline{q} \equiv \min Q$ denote the minimum possible quality level. Moreover, let c_q represent the cost of a seller with quality q so that if a seller with quality q sells her good at a price p, she obtains the payoff $p - c_q$. The seller observes his quality level q privately, while the cumulative distribution function $F(q)$ with the support Q represents the uninformed buyer's belief about quality. The certifier observes the seller's quality at zero costs ($c_c = 0$).

Consider the certifier as a fully fledged mechanism designer, who is no longer restricted to only using buyer or seller certification. In line with the theory of mechanism design, the certifier can 1) freely determine the rules of the game according to which the seller and the buyer can exchange the good and according to which

[9] A complete characterization of the mixed equilibrium in the buyer certification model with more than two quality levels is however intractable.

information is revealed; and 2) determine the Perfect Bayesian Equilibrium (PBE) which the seller and buyer play in this game. Any PBE of the certifier's game induces an economic allocation, i.e., a probability $x \in [0,1]$ that the buyer obtains the good, a transfer $p \in \mathbb{R}$ from the buyer to the seller, and transfers $f^s \in \mathbb{R}$ and $f^b \in \mathbb{R}$ to the certifier of, respectively, the seller and the buyer.

In this more general setup, buyer and seller certification are but two feasible games which the certifier can use. In particular, *seller certification* corresponds to the seller and buyer playing the following game. The seller first selects a price p to the buyer and can then decide to acquire a fully revealing certificate $C = q$ from the certifier at a certificate dependent fee $f^s(C)$. Finally, the buyer decides whether to pay the price p for obtaining the good without a transfer to the certifier ($f^b = 0$).

We claim that seller certification (with fully revealing certificates) maximizes both aggregate welfare and the certifier's profits among all other mechanisms which the certifier could select. We show this claim by first deriving upper bounds on the certifier's profits and aggregate welfare. We subsequently specify a fee structure $f^s(C)$ so that the ensuing seller-certification game yields the two upper bounds.

Because $\underline{q} \equiv \min Q$ represents a lower bound on the belief of any rational buyer, a seller of quality q can guarantee himself a profit of $\underline{q} - c_q$ by bypassing the certifier and selling the good at a price $p = \underline{q}$ directly to the buyer. Alternatively, he can obtain a profit of 0 by not producing the good in the first place. Hence, $\underline{U}^s(q) \equiv \max\{\underline{q} - c_q, 0\}$ represents a lower bound on a seller's utility with quality q in any PBE which a certifier can induce. Similarly, $\underline{U}^b(q) \equiv 0$ is a lower bound on the buyer's utility for any PBE which a certifier can achieve.

With respect to aggregate welfare, the seller and buyer can achieve at most a surplus of $\bar{S}(q) \equiv \max\{q - c(q)\}$ given a quality level q. Hence,

$$\bar{S} \equiv \int_Q \bar{S}(q) dF(q)$$

is an upper bound on the ex ante expected welfare of any outcome which a certifier can achieve and

$$\bar{\Pi}^c = \int_Q \bar{S}(q) - \underline{U}^s(q) - \underline{U}^b(q) dF(q)$$

is an upper bound on its expected profits.

Defining the following certificate-contingent fee schedule

$$\bar{f}^s(q) = \begin{cases} q - \max\{\underline{q}, c(q)\}, & \text{if } q > \underline{q} \wedge q \geq c(q) \\ \underline{q} + 1, & \text{if } q = \underline{q} \\ 0, & \text{otherwise;} \end{cases}$$

we obtain the following result.

Proposition 7 *Seller certification with a fee schedule $\bar{f}^s(q)$ induces a game in which aggregate welfare $\bar{S}$ and the certifier's profit $\bar{\Pi}^c$ is an equilibrium outcome.*

The proposition shows that with an appropriate fee schedule, seller certification allows the certifier to attain the upper bounds on welfare and profits. It follows that seller certification is optimal with respect to both welfare and certifier's profits.

We point out that seller certification is not necessarily the only optimal mechanism. Interestingly and at first sight somewhat paradoxically, the equilibrium outcome may also not be unique in the degree of information revelation. More precisely, in addition to a mechanism that yields a fully revealing equilibrium, there may also exist optimal mechanisms that do not lead to full information revelation. Yet such partially or non-revealing mechanisms are only optimal when the revelation of more information is not welfare-relevant.[10]

The mechanism design approach does not readily extend to the case when certification involves a cost ($c_c > 0$). The reason is that certification now involves a positive welfare cost, which should also be minimized. For our general setup with full commitment to certification probabilities, unbounded transfers, and risk neutral players this leads to an existence problem; the all powerful certifier can attain the upper bounds $\bar{S}$ and the certifier's profit $\bar{\Pi}^c$ only arbitrarily closely. The non-existence of an optimal mechanism is already known from Border and Sobel (1987) and Mookherjee and Png (1989), who emphasize that in models with verifiable but costly auditing some degree of risk aversion or bounded transfers are needed to guarantee existence. A full characterization of the optimal mechanism in such frameworks is however not available.

7.2 Moral hazard

We analyzed certification in a model in which asymmetric information generates inefficient market outcomes due to adverse selection. In order to show that our results also obtain when market inefficiencies are due to moral hazard, we extend our previous model by the possibility that a high-quality seller can also produce at low quality. Formally, we do so by introducing the following additional stage in our certification game as illustrated in Table 1:

$t = 2.5$: Seller type q_h decides whether to produce quality q_h or q_l.

The introduction of moral hazard improves the outside option of the high-quality seller, because in addition to not producing, the seller now can also decide to produce

[10] As discussed in Section 2, an extreme example is Lizzeri (1999) in which information revelation does not affect welfare and a profit-maximizing mechanism exists that induces no disclosure.

at low quality. As a result, the certifier can extract less rents and its equilibrium profits decrease. Yet, our qualitative insights about the economic effect of the mode of certification and its subsequent results remain unchanged.

To make this precise, note that with moral hazard, type q_h's relevant outside option is to produce q_l (leading to profit q_l) rather than not sell at all (leading to profit 0). As a result, the certifier's profits from seller certification reduce as follows:

Proposition 8 *Under seller certification and moral hazard the certification game has the unique equilibrium outcome* $\bar{p}_c^s = \Delta q - c_h$ *with equilibrium payoffs* $\Pi^s = \lambda(\Delta q - c_h - c_c)$, $\pi_h^* = q_l$, *and* $\pi_l^* = q_l$.

The outside option changes similarly under buyer certification, where rather than ensuring that $\tilde{p} - c_h \geq 0$, the certifier now has to ensure that $\tilde{p} - c_h \geq q_l$. The next proposition makes precise how Proposition 3 changes in the presence of this form of moral hazard.

Proposition 9 *Consider buyer certification with moral hazard.*
i. For $\lambda \leq 1/2$ *and* $c_h \leq \Delta q/2$, *the certifier sets a price* $\bar{p}_c^b = \Delta q/4$ *and obtains*

$$\Pi^b = \frac{\lambda \Delta q}{2(q_h + q_l)}(\Delta q - 4c_c).$$

ii. For $\lambda > 1/2$ *or* $c_h > \Delta q/2$, *the certifier sets a price* $\bar{p}_c^b = c_h(1 - c_h/\Delta q)$ *and obtains*

$$\Pi^b = \frac{\lambda[c_h(\Delta q - c_h) - \Delta q c_c]}{c_h + q_l}.$$

Although under seller certification the certifier's profit declines relative to the baseline version of our model, the next proposition shows that the certifier's profits remain higher under seller certification.

Proposition 10 *With moral hazard the certifier obtains a higher profit under seller than under buyer certification:* $\Pi^s > \Pi^b$.

Hence, the certifier prefers seller certification also when market inefficiencies are generated by moral hazard. Moreover, also our welfare result remains unchanged, because the equilibrium including moral hazard involves no change in the allocation but only a redistribution of rents away from the certifier towards the seller.

8 Applications

In this section we discuss how our theoretical results shed new light on empirical observations in certification markets. We focus on specific cases. Boiled down our results point to an advantage of seller certification as compared to buyer certification both from a normative and a positive perspective. It is however important to stress that our analysis focuses only on one, albeit fundamental difference between buyer and seller certification. Additional differences in the mode of certification – such as differences in costs , manipulation, and credibility of certification – strengthen, dampen, or may even overturn our results. Furthermore, we focus on the certification of objective, vertically differentiated rather than subjective, horizontally differentiated buyer specific quality.[11] Finally, our results pertain to certification in markets in which certification is a voluntary decision of buyers and sellers rather than forced on them by external regulation.

Parts for complex commodities. Because of their bilateral nature, certification markets for complex intermediate products fit our model particularly well.[12] A specific example is parts procurement in the automotive industry.[13] Because the part to be supplied is buyer-specific and therefore requires relation specific investments, the buyer-seller relationship is a bilateral monopoly. Certification is conducted after the buyer-specific development of the product and before its production. Moreover, due to the high fixed cost of testing equipment, certification in these markets constitutes a natural monopoly.[14] Key test criteria are the functionality of the part, part failure rate, and safety norms, characteristics about which the seller as the producer typically possesses private information. Because these characteristics are ex post observable, a certifier who cheats on these test is likely to found out. Hence, independently of the mode of certification, the certifier's liability and her reputational losses from found out cheating support our assumption of honest certification.

[11] Buyers may hire advisors that relate to the (optimal) match of a given product to buyer-specific preferences. Examples are portfolio specific advises in the financial market provided by specialized small rating agencies such as KMV, Egan-Jones and Lace Financial (see White (2010), p. 218), and buyer-specific advises in real estate markets.

[12] Headwaters MB (2012) values this testing, inspection, and certification (TIC) sector at 100 billion euro (125 billion dollar) for 2012.

[13] The evidence is taken from Müller et al. (2016), and from a large-scale study conducted in 2007/08 by Stahl et al. for the German association of automotive manufacturers (VDA) on upstream relationships in the automotive industry. See Felli et al. (2011).

[14] An example is EDAG, an engineering company centering on the development and prototype-construction of cars as well as on independent certification of car modules and systems. In this function it serves all major car producers worldwide. See `http://www.edag.de/en/services/engineering-services/products/pruef.html` (last retrieved October 19, 2015).

The data shows that in about 80 percent of all cases the better informed upstream supplier rather than the buyer requests the testing of car modules. Moreover, based on the certification outcome, the buyer conditions her acceptance of the part on the price quoted by the seller in the procurement auction. Our model, therefore, captures the typical procurement relationship in the automotive industry. Our equilibrium result is consistent with the observations in this industry, and these concur with our welfare evaluation.

Rating of financial products. Our results are consistent with the observation that the prevailing business model of rating agencies is "investor pays" rather than "issuer pays". Yet, in the aftermath of the 2008 financial crisis, a frequent claim is that, due to concerns of capture, credit-rating agencies (CRA) should change their business model towards "investor pays". Our contribution to this debate is to point out that, in the absence of capture, the issuer pays model leads to higher welfare, which means that a switch to the investor pays model is warranted only if the problem of capture is *significantly* more severe under the issuer pays model.[15]

Interestingly, White (2010) reports that originally, the business model of rating agencies was mainly the investor pays model and this changed to the issuer pays model only in the 1970s. He emphasizes that while several reasons have been proposed, a definite one has not been established. Fridson (p.4, 1999) points to the bankruptcy of the Penn-Central Railroad in 1970 which shocked the bond markets. He argues that this shock abruptly increased the issuers' demand for, and willingness-to-pay for certification services. White (2010) notes that Fridson's reasoning is incomplete, since the shock should have also increased the willingness to pay of investors for certification. By considering the comparative statics in the difference in profits between seller and buyer certification, $\Delta\Pi \equiv \Pi^s - \Pi^b$, our results can lend support to Fridson (1999). For instance, if the Akerlof problem is severe, i.e., if $c_h > (q_h + q_l)/2$, then the difference $\Delta\Pi$ is increasing in the cost of certification c_c. Because it is likely that the bankruptcy led to a more intensive certification effort with higher costs of certification c_c, our comparative statics then imply that seller certification became relatively more profitable than buyer certification, possibly triggering a change towards the issuer pays model.[16]

Consumer reports. Certification in markets for final products is mostly done

[15] As explained in more detail in Section 2, the link between capture and the business model much depends on institutional details. A proper analysis of these problems is therefore market specific and lies beyond the scope of this paper.

[16] An informal extension mentioned in the concluding Section 9 points at a similar effect from the rise of the copying machine.

through labeling, which corresponds to seller certification. Yet, products are also tested and reviewed by independent consumer organizations (e.g., *ConsumerReports* in the US, *Which?* in the UK, and *Stiftung Warentest* in Germany). The business model of these organizations corresponds to buyer certification; they sell their results exclusively to consumers via subsrciptions or magazines, and these magazines refrain from any advertisements by producers. This business model is usually motivated by the agency's need for independence. Yet, since these organizations are non-profit and, due to donations and governmental subsidies, may offer certification at prices below the cost of certification, our results show that the business model of buyer certification is actually preferable from a welfare point of view.[17]

9 Conclusion

In a market with opaque product quality, demand for certification to raise market transparency arises from both buyers and sellers. We provide new, elementary insights into the economic role of such third-party certification by examining the extent to which the certifier's business model of certification – seller and buyer certification – affects transparency and market outcomes. In particular, we show that sellers use certification as a device to signal their quality. In contrast, buyers use certification as an inspection device to safeguard themselves against low-quality sellers. Due to these differences, seller certification is more effective in raising market transparency than buyer certification, most importantly because signalling reveals also information when it is not used. Whenever market transparency is socially beneficial, it also generates larger gains of trade, more social welfare, and higher profits to the certifier.

For commercial certification markets, our analysis leads to a clear policy implication concerning the certifier's business model. Seller certification has natural advantages over buyer certification in promoting transparency, and therefore should be given precedence. Regulatory implications are more complex when regulators can also regulate the price at which certifiers offer certification and the certification tech-

[17] *Stiftung Warentest* receives governmental subsidies of 3.5 million Euro, while its sales revenues in 2012 totalled 39.5 million Euro (`https://en.wikipedia.org/wiki/Stiftung_Warentest`, last retrieved November 24, 2016). *ConsumerReports*'s website states that it receives "generous grants from independent and family foundations and from the government" (`http://www.consumerreports.org/cro/donate/foundations-and-grants/index.htm`, last retrieved November 24, 2016). *Which?* is a registered and regulated charity by the UK charity commission, which requires that *Which?*'s purpose is to benefit the public and cannot be profit maximization. (`http://apps.charitycommission.gov.uk/Showcharity/RegisterOfCharities/CharityWithPartB.aspx?RegisteredCharityNumber=296072&SubsidiaryNumber=0`, last retrieved November 24, 2016).

nology is costly. In this case, social welfare depends on both trade efficiency and the frequency of certification, which, depending on the price of certification, is lower under buyer certification. Because price regulation also bypasses the perverse effect of buyer certification that it induces certifiers to pick prices that reduce market transparency, optimal price regulation yields higher welfare with buyer certification if the costs of certification are high relative to the gains in trade efficiency from market transparency. All these result obtain, however, under the *ceteris paribus* assumption that the mode of certification differs only by the party that demands it.

In our formal analysis, we considered the bilateral setting of just one seller and one buyer. While this situation reflects well practices in some markets, certification is often useful in settings with one seller and many buyers. A good example is the market for financial products. Assuming that, irrespective of seller and buyer certification, buyers cannot share the certification result, then this would not change our results, because profits and surpluses under both seller and buyer certification are simply scaled up with the number of buyers. If it is harder for the certifier to prevent the sharing of the certification result under buyer certification than under seller certification, then our ceteris paribus assumption is violated and seller certification has an additional benefit to buyer certification. This would provide an additional argument in favor of seller certification.[18]

We assumed that the buyer does not purchase the good if certification reveals that the seller has quoted an inappropriately high price: in other words, we disallow renegotiation. Allowing for such renegotiation does not affect the equilibrium outcome under seller certification. Under buyer certification, the possibility of renegotiation raises the "cheating" incentive of the low quality seller, because it ensures him that he can trade even when the buyer certifies. Hence, with renegotiation the buyer has to raise his frequency of certification and this reduces the aggregate surplus from buyer certification. Therefore, the possibility of renegotiation makes buyer certification even less attractive from a welfare point of view. Since in equilibrium the certifier captures all rents, it also makes buyer certification less attractive for the certifier.

Appendix

The appendix contains all formal proofs to our lemmata and propositions.

[18]See for instance White (2010), who mentions this asymmetry between buyer and seller certification as another possible reason for the change from "investor pay" model to the "issuer pay" model of the rating agencies in the 70s due to the rise of copying machines.

Proof of Lemma 1: Consider the subgame $\Gamma^s(p_c)$ with $p_c \leq q_h - c_h$. Let the q_h-seller's strategy be the pure strategy $\sigma_h^c(q_h) = 1$, and the q_l-seller be the pure strategy $\sigma_l^u(q_l) = 1$. Moreover, let the Bayes's consistent buyer's belief satisfy $\mu(p) = 0$ for all p and let $\sigma(s_b|p, \mu)$ equal 1 if $p \leq q_l$ and zero otherwise. These strategies and beliefs describe a perfect Bayesian equilibrium of the game $\Gamma^s(p_c)$ with an outcome as described in the lemma.

To show uniqueness for $p_c < q_h - c_h$, note first that by certifying and charging the price $p = q_h$, the q_h-seller can guarantee himself a payoff $\pi_h^c \equiv q_h - c_h - p_c > 0$. Hence, in any equilibrium of the subgame $\Gamma^s(p_c)$ the q_h-seller must obtain a payoff of at least $\pi_h^c > 0$. Moreover, if the q_h-seller always certifies, he obtains the payoff π_h^c only if charging a price $p = q_h$. Hence, given that the q_h-seller always certifies, the equilibrium outcome is unique. We next show that there does not exist a perfect Bayesian equilibrium where the q_h-seller certifies with a probability less than 1. For suppose such an equilibrium existed, then prices $\tilde{p}$ would exist such that the high-quality seller would offer the good uncertified with positive probability, i.e., $\sigma_h^u(\tilde{p}) > 0$. For $\tilde{p}$ to be an equilibrium price, the associated profits to the q_h-seller, $\pi_h^u(\tilde{p})$, must at least match $\pi_h^c > 0$. Hence, at any such price $\tilde{p}$, the buyer must buy with positive probability: $\sigma(s_b|\tilde{p}, \mu(\tilde{p})) > 0$. This, however, requires that $\mu(\tilde{p})q_h + (1 - \mu(\tilde{p}))q_l \geq \tilde{p}$. This implies that $\mu(\tilde{p}) > \lambda$, for if not, then $\tilde{p} \leq \mu(\tilde{p})q_h + (1 - \mu(\tilde{p}))q_l < \lambda q_h + (1 - \lambda)q_l < c_h$, so that the high-quality seller would not want to offer his product at price $\tilde{p}$. Hence, by (1), it must hold that $\sigma_h^u(\tilde{p}) > \sigma_l^u(\tilde{p})$ for each price $\tilde{p}$ such that $\sigma_h^u(\tilde{p}) > 0$. Adding over all such prices, we get the contradiction

$$1 \geq \sum_{\tilde{p}:\sigma_h^u(\tilde{p})>0} \sigma_h^u(\tilde{p}) > \sum_{\tilde{p}:\sigma_h^u(\tilde{p})>0} \sigma_l^u(\tilde{p}) = 1,$$

where the last equality follows, because if the q_l-seller picks a price $\bar{p}$ with $\sigma_h^u(\bar{p}) = 0$, then by (1) $\mu(\bar{p}) = 0$, so that either $\sigma(s_b|\bar{p}, \mu(\bar{p})) = 0$ or $\bar{p} \leq q_l$. In either case, the profits to the q_l-seller are less than from a price $\tilde{p}$ such that $\sigma_h^u(\tilde{p}) > 0$, because for such a $\tilde{p}$, $\pi_l^u(\tilde{p}) = \pi_h^u(\tilde{p}) + c_h \geq \pi_h^c + c_h \geq c_h > \bar{q} > q_l$.

For a subgame with $p_c > q_h - c_h$, the q_h-seller cannot obtain a profit from certification, because after certification, he can sell the good at a price of at most q_h, which yields the negative payoff, since $q_h - p_c - c_h < 0$. Consequently, an equilibrium in which the q_h-seller certifies with positive probability does not exist, because he is better off not offering his good to the market at all. Due to the lemons problem, an equilibrium where the q_h-seller offers his good uncertified does not exist. Such an equilibrium would have a price of at most $\bar{q}$, which exceeds the seller's production costs. Q.E.D.

Proof of Proposition 1: An equilibrium in which the certifier obtains a profit strictly less than $\lambda(q_h - c_h - c_c)$ does not exist, because, by Lemma 1, the certifier can guarantee itself a payoff arbitrarily close to $\lambda(q_h - c_h - c_c)$ by setting a price p_c slightly below $q_h - c_h$. Hence, if an equilibrium exists, it must exhibit $\Pi_c^s = \lambda(q_h - c_h - c_c)$. This profit is attainable only if the certifier sets a price of certification $p_c = q_h - c_h$ and the q_h-seller always certifies. According to Lemma 1 this is indeed an equilibrium outcome of the subgame $\Gamma^s(q_h - c_h)$. Q.E.D.

Proof of Lemma 2: Suppose $p_c > 0$ and buyer certification is trade-effective. That is, the subgame $\Gamma^b(p_c)$ exhibits an equilibrium in which the good is traded with probability 1. Let $P_i \equiv \{p|\sigma_i(p) > 0\}$ denote the set of prices that the q_i-seller charges with positive probability in this equilibrium. Trade effectiveness implies that i) for any $p \in P_l$ such that $p > q_l$, we must have $\sigma(s_b|p, \mu(p)) = 1$, and ii) for any $p \in P_h$, we must have $\sigma(s_b|p, \mu(p)) + \sigma(s_h|p, \mu(p)) = 1$. But then optimality concerning type q_h's price implies that for any $p \in P_h$ we have $p \geq c_h$, since any $p < c_h$ leads to a loss to seller q_h. Now consider the intersection $P_l \cap P_h$. Suppose $P_l \cap P_h \neq \emptyset$ and let $\hat{p}$ denote the highest price in $P_l \cap P_h$. Then conidition i) implies $\sigma(s_b|\hat{p}, \mu(\hat{p})) = 1$ and condition ii) implies $\hat{p} \geq c_h$. Hence, the q_l-seller obtains an equilibrium profit of at least $\sigma(s_b|\hat{p}, \mu(\hat{p}))\hat{p} = \hat{p}$. The set P_l, therefore, cannot contain a price below the highest price $\hat{p}$. Hence, if $P_l \cap P_h \neq \emptyset$, the set P_l contains only one element. But then, $\sigma_l(\hat{p}) = 1 \geq \sigma_h(\hat{p})$ so that (3) implies that $\mu(\hat{p}) \leq \lambda$. But then $\mu(\hat{p})q_h + (1 - \mu(\hat{p}))q_l < c_h \leq \hat{p}$, which contradicts $\sigma(s_b|\hat{p}, \mu(\hat{p})) = 0$ is optimal. Hence, if a trade-effective equilibrium exists, then $P_l \cap P_h = \emptyset$. But it then follows that for any $p \in P_h$ we have $\sigma_l(p) = 0$ so that (3) implies $\mu(p) = 1$ and, due to $p_c > 0$, we must have $\sigma(s_b|p, \mu(p)) = 1$. Hence, $\sigma(s_b|p, \mu(p)) = 1$ for any $p \in P_l \cup P_h$. Moreover, since $P_l \cap P_h = \emptyset$ we have for any $p_l \in P_l$ and $p_h \in P_h$ either $p_l < p_h$ or $p_l > p_h$. If $p_l < p_h$, then p_l yields the q_l-seller less than p_h (because as established $\sigma(s_b|p, \mu(p)) = 1$ for any $p \in P_l \cup P_h$) so that we obtain the contradiction that $\sigma_l(p_l) > 0$ is not optimal. Likewise, if $p_l > p_h$ the price p_l yields the q_h-seller strictly more than p_h and, hence, we obtain the contradiction that $\sigma_h(p_h) > 0$ is not optimal. Q.E.D.

Proof of Lemma 3: i) To show that $\pi_h(p, \mu|\sigma^*)$ is nondecreasing in μ we first establish that, in any PBE, $\sigma^*(s_n|p, \mu)$ is weakly decreasing in μ. Supposed not, then we may find $\mu_1 < \mu_2$ such that $0 \leq \sigma^*(s_n|p, \mu_1) < \sigma^*(s_n|p, \mu_2) \leq 1$. $\sigma^*(s_n|p, \mu_2) > 0$ implies hat $(p, \mu_2) \in S(s_n|p_c)$ and, consequently,

$$p \geq \mu_2 q_h + (1 - \mu_2)q_l \tag{9}$$

and

$$p_c \geq \mu_2(q_h - p). \tag{10}$$

Now since $\sigma^*(s_n|p,\mu_1) < 1$ we have either $\sigma^*(s_b|p,\mu_1) > 0$ or $\sigma^*(s_h|p,\mu_1) > 0$. Suppose first $\sigma^*(s_b|p,\mu_1) > 0$, then $(p,\mu_1) \in S(s_b|p_c)$, which implies $p \leq \mu_1 q_h + (1-\mu_1)q_l$. But from $\mu_2 > \mu_1$ and $q_h > q_l$ it then follows that $\mu_2 q_h + (1-\mu_2)q_l > p$, which contradicts (9). Suppose therefore that $\sigma^*(s_h|p,\mu_1) > 0$, then $(p,\mu_1) \in S(s_h|p_c)$, which implies $\mu_1(q_h - p) \geq p_c > 0$. This requires $q_h > p$. But then, due to $\mu_2 > \mu_1$, we get $\mu_2(q_h - p) > p_c$, which contradicts (10). Hence, we establish that $\sigma^*(s_n|p,\mu)$ is weakly decreasing in μ and therefore $\sigma^*(s_b|p,\mu) + \sigma^*(s_h|p,\mu)$ must be weakly increasing in μ. This directly implies that $\pi_h(p,\mu|\sigma^*)$ is weakly increasing in μ. Next we show that in any PBE $\sigma^*(s_b|p,\mu)$ is weakly increasing in μ. Suppose not, then we may find $\mu_1 < \mu_2$ such that $1 \geq \sigma^*(s_b|p,\mu_1) > \sigma^*(s_b|p,\mu_2) \geq 0$. Since $\sigma^*(s_b|p,\mu_1) > 0$, it holds that $(p,\mu_1) \in S(s_b|p_c)$ and, consequently,

$$p \leq \mu_1 q_h + (1-\mu_1)q_l \tag{11}$$

and

$$p_c \geq (1-\mu_1)(p - q_l). \tag{12}$$

Now since $\sigma^*(s_b|p,\mu_2) < 1$ we have $\sigma^*(s_n|p,\mu_2) > 0$ or $\sigma^*(s_h|p,\mu_2) > 0$. Suppose first $\sigma^*(s_n|p,\mu_2) > 0$, then $(p,\mu_2) \in S(s_n|p_c)$, which implies $p \geq \mu_2 q_h + (1-\mu_2)q_l$. But due to $\mu_2 > \mu_1$ and $q_h > q_l$ we get $p > \mu_1 q_h + (1-\mu_1)q_l$. This contradicts (11). Suppose therefore that $\sigma^*(s_h|p,\mu_2) > 0$, then $(p,\mu_2) \in S(s_h|p_c)$, which implies $(1-\mu_2)(p-q_l) \geq p_c > 0$. This requires $p > q_l$. But then, due to $\mu_2 > \mu_1$, we get $(1-\mu_1)(p-q_l) > p_c$. This contradicts (12). Hence, $\sigma^*(s_b|p,\mu)$ must be weakly increasing in μ. This directly implies that $\pi_l(p,\mu|\sigma^*)$ is weakly increasing in μ. This concludes the proof of statement i) of the lemma.

ii) For any $\bar{p} < q_l$, $\mu \in [0,1]$ we have $(\bar{p},\mu) \notin S(s_n|p_c)$, $(\bar{p},\mu) \notin S(s_h|p_c)$, and $(\bar{p},\mu) \in S(s_b|p_c)$. Hence, $\sigma^*(s_b|\bar{p},\mu^*(\bar{p})) = 1$. Now suppose for some $\bar{p} < q_l$ we have $\sigma_i^*(\bar{p}) > 0$. This would violate (6), because instead of charging $\bar{p}$ seller q_i could have raised profits by ε by charging the higher price $\bar{p} + \varepsilon < q_l$ with $\varepsilon \in (0, q_l - \bar{p})$. At $\bar{p} + \varepsilon < q_l$ the buyer always buys, because, as established, $\sigma^*(s_b|\bar{p}+\varepsilon,\mu) = 1$ for all μ and in particular for $\mu = \mu^*(\bar{p}+\varepsilon)$.

For any $\bar{p} > q_h$, $\mu \in [0,1]$ we have $(\bar{p},\mu) \in S(s_n|p_c)$, $(\bar{p},\mu) \notin S(s_h|p_c)$, and $(\bar{p},\mu) \notin S(s_b|p_c)$. Hence, $\sigma^*(s_n|\bar{p},\mu^*(\bar{p})) = 1$. Now suppose we have $\sigma_l(\bar{p}) > 0$. This would violate (6), because instead of charging $\bar{p}$, which due to $\sigma^*(s_n|\bar{p},\mu^*(\bar{p})) = 1$ leads to zero profits, seller q_l could have obtained strictly positive profits by charging the price $q_l - \varepsilon$, where $\varepsilon \in (0, q_l)$.

iii) To show $\pi_l^* \geq q_l$, suppose to the contrary that $\delta = q_l - \pi_l^* > 0$. Now consider a price $p' = q_l - \varepsilon$ with $\varepsilon \in (0, \delta)$ then for any $\mu' \in [0, 1]$ we have $(p', \mu') \in S(s_b|p_c)$ and $(p', \mu') \notin S(s_n|p_c) \cup S(s_h|p_c)$ so that we have $\sigma^*(s_b|p', \mu^*(p')) = 1$ and, therefore, $\pi_l(p', \mu^*(p')|\sigma^*) = p' > \pi_l^*$. This contradicts (6).

To show $\pi_h^* < q_h - c_h$ note that for any p such that $\sigma_h^*(p) > 0$, we have $\pi_h^* = \pi_h(p, \mu^*(p)|\sigma^*) = [\sigma^*(s_b|p, \mu^*(p)) + \sigma^*(s_h|p, \mu^*(p))]p - c_h$. As argued in ii), we have $\sigma^*(s_n|p, \mu) = 1$ for all $p > q_h$ and $\mu \in [0, 1]$. Hence, $\pi_h(p, \mu|\sigma^*) = 0$ whenever $p > q_h$. But for any price $p \leq q_h$ we have $\pi_h(p, \mu|\sigma^*) \leq q_h - c_h$. Hence, it follows that $\pi_h^* \leq q_h - c_h$. Now suppose $\pi_h^* = q_h - c_h$. Then we must have $\sigma_h^*(q_h) = 1$ and $\sigma^*(s_b|q_h, \mu^*(q_h)) + \sigma^*(s_h|q_h, \mu^*(q_h)) = 1$. But, due to $\mu^*(q_h)(q_h - q_h) = 0 < p_c$, we have $(q_h, \mu^*(q_h)) \notin S(s_h|q_h)$ so that $\sigma^*(s_h|q_h, \mu^*(q_h)) = 0$. Hence, we must have $\sigma^*(s_b|q_h, \mu^*(q_h)) = 1$. This requires $(q_h, \mu^*(q_h)) \in S(s_b|p_c)$ so that we must have $\mu^*(q_h) = 1$. By (3), this requires $\sigma_l^*(q_h) = 0$. But since $\pi_l(q_h, 1|\sigma^*) = \sigma^*(s_b|q_h, \mu^*(q_h))q_h = q_h$ we must, by (6), have $\pi_l^* \geq q_h$. Together with $\sigma_l^*(q_h) = 0$, it would require $\sigma_l^*(p) > 0$ for some $p > q_h$ and leads to a contradiction with statement ii) of the lemma. Q.E.D.

Proof of Lemma 4: We first prove ii): Suppose to the contrary that $\delta \equiv \tilde{p} - c_h - \pi_h^* > 0$. Then, due to the countable number of equilibrium prices, we can find an out-of-equilibrium price $p' = \tilde{p} - \varepsilon$ for some $\varepsilon \in (0, \delta)$. Then for any belief $\mu' \in (p_c/(q_h - p'), 1 - p_c/(p' - q_l))$[19] we have $(p', \mu') \in S(s_h)$ and $(p', \mu') \notin S(s_n) \cup S(s_b)$. Consequently, $\sigma^*(s_h|p', \mu') = 1$. Hence, $\pi_h(p', \mu'|\sigma^*) = p' - c_h = \tilde{p} - c_h - \varepsilon > \tilde{p} - c_h - \delta = \pi_h^*$ and $\pi_l(p', \mu'|\sigma^*) = 0 < q_l \leq \pi_l^*$. Therefore, by BR the buyer's equilibrium belief must satisfy $\mu^*(p') \geq \mu'$. By Lemma 3 it follows $\pi_h(p', \mu^*(p')|\sigma^*) \geq \pi_h(p', \mu'|\sigma^*) = \tilde{p} - c_h - \varepsilon > \pi_h^*$. This contradicts (6). Consequently, we must have $\pi_h^* \geq \tilde{p} - c_h$. To show i) note that for all $p < \tilde{p}$ and $\mu \in [0, 1]$ we have $\pi_h(p, \mu|\sigma) \leq p - c_h < \tilde{p} - c_h \leq \pi_h^*$ so that $\sigma_h(p) > 0$ would violate (6). Q.E.D.

Proof of Proposition 2: i. First we show that for $\tilde{\mu} > \lambda$ and $\tilde{p} > c_h$ there exists no pooling, i.e., there exists no price $\bar{p}$ such that $\sigma_h^*(\bar{p}) = \sigma_l^*(\bar{p}) > 0$. For suppose there does. Then, by Lemma 4.i), we have $\bar{p} \geq \tilde{p}$ and, by Lemma **??**.i), we have $\bar{p} \leq q_h$. Yet, due to (3) we have $\mu^*(\bar{p}) = \lambda < \tilde{\mu}$ so that $q_l + \mu^*(\bar{p})\Delta q - \bar{p} < q_l + \tilde{\mu}\Delta q - \tilde{p} = 0$. Moreover, $\mu^*(\bar{p})(q_h - \bar{p}) < \tilde{\mu}(q_h - \tilde{p}) = p_c$. Therefore, $\sigma^*(s_n|\bar{p}, \mu^*(\bar{p})) = 1$ and, hence, $\pi_h(\bar{p}, \mu^*(\bar{p})) = 0$. As a result, $\sigma_h^*(\bar{p}) > 0$ contradicts (6), because, by Lemma 4.ii), $\pi_h^* \geq \tilde{p} - c_h > 0 = \pi_h(\bar{p}, \mu^*(\bar{p}))$.

[19] To see that $p_c/(q_h - p') < 1 - p_c/(p' - q_l)$ define $l(p) \equiv p_c/(q_h - p)$ and $h(p) \equiv 1 - p_c/(p - q_l)$. Then by the definition of $\tilde{p}$ we have $l(\tilde{p}) = h(\tilde{p})$. Moreover, for $q_l < p < q_h$ we have $l'(p) = p_c/(q_h - p)^2 > h'(p) = p_c/(p - q_l)^2 > 0$. Hence, $l(\tilde{p} - \varepsilon) < h(\tilde{p} - \varepsilon)$ for $\varepsilon > 0$ small. Since $p' = \tilde{p} - \varepsilon$ we have $l(p') < h(p')$.

Second, we show that for $\tilde{\mu} > \lambda$, we cannot have $\sigma_h^*(\bar{p}) > 0$ for some $\bar{p} > \tilde{p}$. Suppose to the contrary we find such a $\bar{p}$ then, by definition of $\tilde{p}$, we have $(\bar{p}, \mu) \notin S(s_h)$ for any $\mu \in [0, 1]$. Hence, $\sigma^*(s_h|\bar{p}, \mu^*(\bar{p})) = 0$ so that $\pi_l(\bar{p}, \mu^*(\bar{p})) = \pi_h(\bar{p}, \mu^*(\bar{p})) + c_h$. From Lemma 4.ii) it then follows $\pi_l(\bar{p}, \mu^*(\bar{p})) \geq \tilde{p}$ and, therefore, $\sum_{p \geq \tilde{p}} \sigma_l^*(p) = 1$. From $\bar{p} > \tilde{p}$ and $\tilde{\mu} > \lambda$ it follows $\lambda \Delta q + q_l - \bar{p} < \tilde{\mu} \Delta q + q_l - \tilde{p} = 0$ so that $\lambda \Delta q + q_l < \bar{p}$. Now suppose it also holds that $\sigma_l^*(\bar{p}) > 0$ then, by Lemma **??**.ii and (6), $0 < q_l \leq \pi_l^* = \pi_l(\bar{p}, \mu^*(\bar{p})|\sigma^*) = \sigma^*(s_b|\bar{p}, \mu^*(\bar{p}))\bar{p}$. This requires $\sigma^*(s_b|\bar{p}, \mu^*(\bar{p})) > 0$ and therefore $(\bar{p}, \mu^*(\bar{p})) \in S(s_b|p_c)$ and, hence, $\mu^*(\bar{p})\Delta q + q_l \geq \bar{p}$. Combining the latter inequality with our observation that $\lambda \Delta q + q_l < \bar{p}$ and using (3), it follows

$$\lambda \Delta q + q_l < \frac{\lambda \sigma_h^*(\bar{p})}{\lambda \sigma_h^*(\bar{p}) + (1 - \lambda)\sigma_l^*(\bar{p})} \Delta q + q_l,$$

which is equivalent to $\sigma_h^*(\bar{p}) > \sigma_l^*(\bar{p})$. Summing over all $p \geq \tilde{p}$ and using $\sum_{p \geq \tilde{p}} \sigma_l^*(p) = 1$ yields the contradiction $\sum_{p \geq \tilde{p}} \sigma_h^*(p) > 1$. Hence, we must have $\sigma_l^*(\bar{p}) = 0$ for any $\bar{p} > \tilde{p}$. But this contradicts $\sum_{p \geq \tilde{p}} \sigma_l^*(p) = 1$ and, therefore, we must have $\sigma_h^*(\bar{p}) = 0$ for all $\bar{p} > \tilde{p}$.

This second observation implies that if an equilibrium for $\tilde{\mu} > \lambda$ and $\tilde{p} > c_h$ exists then, by Lemma 4, it exhibits $\sigma_h^*(\tilde{p}) = 1$, $\pi_h^* = \tilde{p} - c_h$, and $\sigma^*(s_h|\tilde{p}, \tilde{\mu}) + \sigma^*(s_b|\tilde{p}, \tilde{\mu}) = 1$. We now show existence of such an equilibrium and demonstrate that any such equilibrium has a unique equilibrium outcome. If $\sigma_h^*(\tilde{p}) = 1$ then (3) implies that $\mu^*(\tilde{p}) = \tilde{\mu}$ whenever

$$\sigma_l^*(\tilde{p}) = \frac{\lambda(1 - \tilde{\mu})}{\tilde{\mu}(1 - \lambda)},$$

which is smaller than 1 exactly when $\lambda < \tilde{\mu}$. By definition, $(\tilde{p}, \tilde{\mu}) \in S(s_h) \cap S(s_b)$ so that any buying behavior with $\sigma^*(s_h|\tilde{p}, \tilde{\mu}) + \sigma^*(s_b|\tilde{p}, \tilde{\mu}) = 1$ is consistent in equilibrium. In particular, $\sigma^*(s_b|\tilde{p}, \tilde{\mu}) = q_l/\tilde{p} < 1$ is consistent in equilibrium. Only for this buying behavior we have $\pi_l(q_l, 0) = q_l = \pi_l(\tilde{p}, \tilde{\mu})$ so that seller q_l is indifferent between price $\tilde{p}$ and q_l. The equilibrium therefore prescribes $\sigma_l^*(q_l) = 1 - \sigma_l^*(\tilde{p})$. Finally, let $\mu^*(q_l) = 0$ and $\sigma^*(s_b|q_l, \mu^*(q_l)) = 1$ and $\mu^*(p) = 0$ for any price p larger than q_l and unequal to $\tilde{p}$. This out-of-equilibrium beliefs satisfies BR.

ii. In order to show that, in any equilibrium of $\Gamma^b(p_c)$, we have $\Pi^b(p_c) = 0$ whenever $\lambda > \tilde{\mu}$, we prove that for any $\bar{p}$ such that $\sigma^*(s_h|\bar{p}, \mu^*(\bar{p})) > 0$, it must hold $\sigma_h^*(\bar{p}) = \sigma_l^*(\bar{p}) = 0$. Suppose we have $\sigma^*(s_h|\bar{p}, \mu^*(\bar{p})) > 0$, then $(\bar{p}, \mu^*(\bar{p})) \in S(s_h)$ and, necessarily, $\bar{p} \leq \tilde{p}$. But by Lemma 4.i), $\sigma_h^*(\bar{p}) > 0$ also implies $\bar{p} \geq \tilde{p}$. Therefore, we must have $\bar{p} = \tilde{p}$. But $(\tilde{p}, \mu) \in S(s_h)$ only if $\mu = \tilde{\mu}$. Hence, we must have $\mu^*(\tilde{p}) = \tilde{\mu}$. By (3) it therefore must hold

$$\tilde{\mu} = \mu^*(\tilde{p}) = \frac{\lambda \sigma_h^*(\tilde{p})}{\lambda \sigma_h^*(\tilde{p}) + (1 - \lambda)\sigma_l^*(\tilde{p})}.$$

For $\lambda > \tilde{\mu}$ this requires $\sigma_h^*(\tilde{p}) < \sigma_l^*(\tilde{p}) \leq 1$ and therefore there is some other $p' > \tilde{p}$ such that $\sigma_h^*(p') > 0$. But if also p' is an equilibrium price, then $\pi_h(\tilde{p}, \mu^*(\tilde{p})|\sigma^*) = \pi_h(p', \mu^*(p')|\sigma^*)$. Yet, for any $p' > \tilde{p}$ it holds that $(p', \mu) \notin S(s_h|p_c)$ for any $\mu \in [0, 1]$ so that $\pi_l(p', \mu|\sigma^*) = \pi_h(p', \mu|\sigma^*) + c_h$ and, together with our assumption $\sigma^*(s_h|\bar{p}, \mu^*(\bar{p})) > 0$ yields $\pi_l(\bar{p}, \mu^*(\bar{p})|\sigma^*) < \pi_h(\bar{p}, \mu^*(\bar{p})|\sigma^*) + c_h = \pi_h(p', \mu^*(p')|\sigma^*) + c_h = \pi_l(p', \mu^*(p')|\sigma^*)$ so that, by (6), $\sigma_l^*(\bar{p}) = 0$. Since $\bar{p} = \tilde{p}$, this violates $\sigma_l^*(\tilde{p}) > \sigma_h^*(\tilde{p}) \geq 0$. As a result, $\sigma^*(s_h|\bar{p}, \mu^*(\bar{p})) > 0$ implies $\sigma_h^*(\bar{p}) = 0$.

In order to show that we must also have $\sigma_l^*(\bar{p}) = 0$, assume again that $\sigma^*(s_h|\bar{p}, \mu^*(\bar{p})) > 0$. We have shown that his implies $\sigma_h^*(\bar{p}) = 0$. Now if $\sigma_l^*(\bar{p}) > 0$ then, by (3), it follows $\mu^*(\bar{p}) = 0$. But then $q_l + \mu^*(\bar{p})\Delta q - \bar{p} - p_c = q_l - \bar{p} - p_c < q_l - \bar{p}$ so that $(\bar{p}, \mu^*(\bar{p})) \notin S(s_h)$, which contradicts $\sigma^*(s_h|\bar{p}, \mu^*(\bar{p})) > 0$.

In order to show that $\tilde{p} < c_h$ implies $\Pi^b(p_c) = 0$ suppose, on the contrary that, $\Pi^b(p_c) > 0$. This requires that there exists some $\bar{p}$ such that $\sigma^*(s_h|\bar{p}, \mu^*(\bar{p})) > 0$ and $\sigma_i^*(\bar{p}) > 0$ for some $i \in \{l, h\}$. First note that $\sigma^*(s_h|\bar{p}, \mu^*(\bar{p})) > 0$ implies $\bar{p} \leq \tilde{p}$. Now suppose $\sigma_h^*(\bar{p}) > 0$ then $\pi_h(\bar{p}, \mu^*(\bar{p})|\sigma^*) = (\sigma^*(s_h|\bar{p}, \mu^*(\bar{p})) + \sigma^*(s_b|\bar{p}, \mu^*(\bar{p})))\bar{p} - c_h < 0$ so that the high-quality seller would make a loss and, thus, violates (6). Therefore, we have $\sigma_h^*(\bar{p}) = 0$. Now if $\sigma_l^*(\bar{p}) > 0$ then (3) implies $\mu^*(\bar{p}) = 0$ so that $\sigma^*(s_h|\bar{p}, \mu^*(\bar{p})) = 0$, which contradicts $\Pi^b(p_c) > 0$. Q.E.D.

Proof of Proposition 3: In order to express the dependence of $\tilde{\mu}$ and $\tilde{p}$ on p_c explicitly, we write $\tilde{\mu}(p_c)$ and $\tilde{p}(p_c)$, respectively. We maximize expression (7) with respect to p_c over the relevant domain

$$P = \{p_c | p_c \leq \Delta q/4 \wedge \tilde{\mu}(p_c) \geq \lambda \wedge \tilde{p}(p_c) \geq c_h\}.$$

First, we show that (7) is increasing in p_c. Define

$$\alpha(p_c) \equiv \frac{\lambda(\tilde{p}(p_c) - q_l)}{\tilde{\mu}(p_c)\tilde{p}(p_c)}$$

so that $\Pi_c(p_c) = \alpha(p_c)(p_c - c_c)$. We have

$$\alpha'(p_c) = \frac{4\lambda\Delta q^2}{\sqrt{\Delta q(\Delta q - 4p_c)}\left(q_h + q_l + \sqrt{\Delta q(\Delta q - 4p_c)}\right)^2} > 0$$

so that $\alpha(p_c)$ is increasing in p_c and, hence, $\Pi_c(p_c)$ is increasing in p_c and maximized for $\max P$.

We distinguish two cases. First, for $c_h \leq (q_h + q_l)/2$ it follows $1/2 = \Delta q/(2\Delta q) \geq (c_h - q_l)/\Delta q > \lambda$, where the last inequality follows from $c_h > \bar{q}$. From $\lambda < 1/2$, it then follows $\tilde{\mu}(p_c) \geq 1/2 \geq \lambda$. Therefore,

$$P = \{p_c | p_c \leq \Delta q/4 \wedge \tilde{p}(p_c) \geq c_h\}.$$

Hence, $\max P$ is either $p_c = \Delta q/4$ or such that $\tilde{p}(p_c) = c_h$. Because $\tilde{p}(\Delta q/4) = (q_h + q_l)/2$, it follows that for $c_h \leq (q_h + q_l)/2$, the maximum obtains at $p_c = \Delta q/4$ with

$$\Pi^b = \frac{\lambda \Delta q}{2(q_h + q_l)}(\Delta q - 4c_c).$$

Second, for $c_h > (q_h + q_l)/2$ the maximum obtains for p_c such that $\tilde{p}(p_c) = c_h$ in case $\lambda \leq 1/2$. This yields $p_c = (q_h - c_h)(c_h - q_l)/\Delta q$ with

$$\Pi^b = \frac{\lambda[(q_h - c_h)(c_h - q_l) - \Delta q c_c]}{c_h};$$

while for $\lambda > 1/2$ we have

$$\tilde{\mu}(p_c) \geq \lambda \Leftrightarrow p_c \leq \lambda(1-\lambda)\Delta q.$$

Since $\lambda(1-\lambda) \leq 1/4$ the requirement $p_c < \lambda(1-\lambda)\Delta q$ automatically implies $p_c \leq \Delta q/4$. Hence for $\lambda > 1/2$ we have

$$P = \{p_c | p_c \leq \lambda(1-\lambda)\Delta q \wedge \tilde{p}(p_c) \geq c_h\}.$$

Because, $\tilde{p}(\lambda(1-\lambda)\Delta q) = \lambda q_h + (1-\lambda)q_l$, which by assumption is smaller than c_h, we have $\max P = (q_h - c_h)(c_h - q_l)/\Delta q$. Note that $c_h > \lambda q_h + (1-\lambda)q_l$ and $\lambda > 1/2$ implies that $c_h > (q_h + q_l)/2$. It follows $\tilde{\mu} = (c_h - q_l)/\Delta q$ and

$$\Pi^b = \frac{\lambda[(q_h - c_h)(c_h - q_l) - \Delta q c_c]}{c_h};$$

Q.E.D.

Proof of Proposition 4: For $c_h \leq (q_h + q_l)/2$ we have

$$\begin{aligned}\Pi^s = \lambda(q_h - c_h - c_c) &\geq \lambda(q_h - c_h - c_c)\frac{q_h - q_l}{q_h + q_l} \geq \lambda(q_h - (q_h + q_l)/2 - c_c)\frac{q_h - q_l}{q_h + q_l} \\ &= \lambda(q_h - q_l - 2c_c)\frac{q_h - q_l}{2(q_h + q_l)} \geq \lambda(q_h - q_l - 4c_c)\frac{q_h - q_l}{2(q_h + q_l)} = \Pi^b,\end{aligned}$$

where the second inequality uses $c_h \leq (q_h + q_l)/2$. Moreover, the certification intensity under buyer certification is $x^b(\bar{p}_c^b) = x^b(\Delta q/4) = \lambda \Delta q/c_h$, which exceeds the certification intensity under seller certification, λ, because due to $q_h - c_h - c_c > q_l$ it holds that $\Delta q < c_h + c_c < c_h$.

For $c_h > (q_h + q_l)/2$ it follows

$$\begin{aligned}\Pi^b &= \frac{\lambda[(q_h - c_h)(c_h - q_l) - \Delta q c_c]}{c_h} < \frac{\lambda[(q_h - c_h)(c_h - q_l) - (c_h - q_l)c_c]}{c_h} \\ &= \lambda(q_h - c_h - c_c)\frac{c_h - q_l}{c_h} \leq \lambda(q_h - c_h - c_c) = \Pi^s,\end{aligned}$$

where the first inequality uses $q_h > c_h$. Moreover, $x^b(\bar{p}_c^b) = \lambda[2\Delta q/(q_h + q_l)]$, which is smaller than λ if and only if $q_h < 3q_l$. Q.E.D.

Proof of Proposition 5: For a combination of certification prices $(p_c^s, p_c^b) \in [c_c, \bar{p}_c^s] \times [c_c, \bar{p}_c^b]$, it follows

$$\begin{aligned} \Delta W(p_c^b) &\equiv W^s - W^b(p_c^b) = (1-\lambda)\omega(p_c^b)q_l + (x^b(p_c^b) - \lambda)c_c \\ &= \frac{\lambda}{\tilde{\mu}(p_c^b)\tilde{p}(p_c^b)}[(1-\tilde{\mu}(p_c^b))(\tilde{p}(p_c^b) - q_l)(q_l + c_c) - \tilde{\mu}(p_c^b)q_l c_c] \\ &= \frac{\lambda}{\tilde{\mu}(p_c^b)\tilde{p}(p_c^b)}[(1-\tilde{\mu}(p_c^b))\tilde{\mu}(p_c^b)\Delta q(q_l + c_c) - \tilde{\mu}(p_c^b)q_l c_c] \\ &= \frac{\lambda}{\tilde{p}(p_c^b)}[(1-\tilde{\mu}(p_c^b))\Delta q(q_l + c_c) - q_l c_c] \qquad (13) \\ &\geq \frac{\lambda}{\tilde{p}(p_c^b)}[(1-\tilde{\mu}(c_c))\Delta q(q_l + c_c) - q_l c_c] \\ &= \frac{\lambda}{2\tilde{p}(p_c^b)}[(1-\sqrt{1-4c_c/\Delta q})\Delta q(q_l + c_c) - 2q_l c_c] \\ &= \frac{\lambda}{2\tilde{p}(p_c^b)}\left[\Delta q(q_l + c_c) - 2q_l c_c - \sqrt{1-4c_c/\Delta q}\Delta q(q_l + c_c)\right], \end{aligned}$$

where the inequality holds because $\tilde{\mu}$ is decreasing in p_c^b and $p_c^b \geq c_c$ if the certifier is not to make a loss. It remains to show that the term in the squared bracket is positive for any $c_c \in [0, q_h - c_c]$. That is, we need to show

$$\Delta q(q_l + c_c) - 2q_l c_c > \sqrt{1-4c_c/\Delta q}\Delta q(q_l + c_c).$$

To see this first note that the left-hand side is indeed positive, since $\Delta q \geq 4c_c$ implies $\Delta q(c_c + q_l) > \Delta q q_l/2 \geq 2c_c q_l$. Squaring both sides yields

$$\Delta q^2(q_l + c_c)^2 - 4\Delta q(q_l + c_c)q_l c_c + 4q_l^2 c_c^2 > (1 - 4c_c/\Delta q)\Delta q^2(q_l + c_c)^2,$$

which is equivalent to $c_c \Delta q(q_l + c_c)c_c + q_l^2 c_c^2 > 0$, which is evidently true. As a result, $\Delta W(p_c^b) > 0$. Q.E.D.

Proof of Proposition 6: From equation (13) in the proof of Proposition 5, welfare under buyer certification is higher if and only if

$$(1-\tilde{\mu}(p_c^b))\Delta q(q_l + c_c) < q_l c_c,$$

which, by the definition of $\tilde{\mu}$ in (4), is equivalent to

$$p_c^b < \bar{p}_c^b \equiv \frac{q_l(c_c(q_h - 2q_l) + (q_h - q_l)q_l}{\Delta q(c_c + q_l)^2} c_c.$$

Straightforward computations yield $\bar{p}_c^b < c_c$ (or, alternatively, Proposition 5 implies this indirectly). Q.E.D.

Proof of Proposition 7: We state the PBE, i.e. the seller's and buyer's strategy, and the buyer's belief of an uncertified good, that sustains the proposition's outcome. The seller of quality q sets a price $p(q) = \max\{q, c(q)\}$ and certifies if and only if $q > \underline{q}$. In equilibrium only the seller with quality $\underline{q}$ does not certify. Hence, the buyer has the degenerate belief that an uncertified good is of quality $\underline{q}$ and, therefore buys it, if and only if the price does not exceed $\underline{q}$. For any good with a certification $C = q$, the buyer buys, if and only if the price does not exceed q. Note that the PBE does not depend on any out-of-equilibrium beliefs. Q.E.D.

Proof of Proposition 8: Follows from applying the same arguments as in the proof of Proposition 1 but with the high-quality seller's outside option of $\Pi_h = q_l$ instead of $\Pi_h = 0$. The certifier therefore can at most ask for $\bar{p}_c^s = \Delta q - c_h$. Q.E.D.

Proof of Proposition 9: Mimics the arguments in the proof of Proposition 3 where the critical threshold for c_h is $\tilde{p} - q_l$ rather than $\tilde{p}$. Q.E.D.

Proof of Proposition 10: For $\lambda \leq 1/2$ and $c_h \leq \Delta q/2$, it follows

$$\begin{aligned}\Pi^b &= \frac{\lambda \Delta q}{2(q_h + q_l)}(\Delta q - 4c_c) < \lambda(\Delta q/2 - 2c_c) = \lambda(\Delta q - \Delta q/2 - 2c_c) \\ &\leq \lambda(\Delta q - c_h - 2c_c) < \lambda(\Delta q - c_h - c_c) = \Pi^s,\end{aligned}$$

where the first inequality uses $\Delta q < q_h + q_l$ and the second uses $c_h \leq \Delta q/2$.

For $\lambda > 1/2$ or $c_h > \Delta q/2$, it follows

$$\begin{aligned}\Pi^b &= \frac{\lambda[c_h(\Delta q - c_h) - \Delta q c_c]}{c_h + q_l} = \frac{\lambda[c_h(\Delta q - c_h - c_c) - (\Delta q - c_h)c_c]}{c_h + q_l} \\ &\leq \frac{\lambda c_h(\Delta q - c_h - c_c)}{c_h + q_l} \leq \lambda(\Delta q - c_h - c_c) = \Pi^s,\end{aligned}$$

where the first inequality follows because $\Delta q \geq c_h$. Q.E.D.

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