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DECOMPOSING THE DISPOSITION EFFECT

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Abstract: We theoretically show that there is a fundamental disconnect between the disposition effect, i.e., investors' tendency to sell winning assets too early and losing assets too late, and its common empirical measure, namely a positive difference between the proportion of gains and losses realized. While its common measure cannot identify the disposition effect, it identifies the presence of some systematic bias. We further investigate the measure's comparative statics regarding markets, investors' information level, and their attention. Besides generating novel testable predictions, this analysis reveals that, in contrast to the measure's sign, variations in its magnitude are informative for its cause.

Keywords: Disposition Effect, Rational Benchmark, Investor Behavior, Behavioral Biases, Market Segments, Financial Attention, Information Level.

JEL Classification Numbers: D90, D91, D83, D84, G11, G40, G41.

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1 Introduction

One of the most robust patterns of investor behavior is the greater tendency to sell assets that have gained rather than lost value since purchase.[1] This pattern is reflected in the empirical observation that the proportion or probability of gains realized (PGR) often exceeds the proportion or probability of losses realized (PLR) and has triggered considerable interest in the literature. The standard interpretation of this pattern is that investors have a "general disposition to sell winners too early and hold losers too long." (Shefrin and Statman, 1985, p.777) In this paper, we show that there is a fundamental disconnect between the empirical observation ($PGR > PLR$) and its standard interpretation. In order to do so, we need to cleanly distinguish these commonly identified concepts. Therefore, we refer to the empirical pattern $PGR > PLR$ as positive *disposition measure*, while referring to its common interpretation of a "disposition to sell winners too early and hold losers too long" as the *disposition effect*.

Intuitively, observing a positive disposition measure seems to indicate irrational behavior: "over the horizon that these investors trade, stock returns exhibit 'momentum': stocks that have recently done well continue to outperform, on average, while those that have done poorly continue to lag. As such, investors should concentrate their selling among stocks with poor past performance – but they do the opposite." (Barberis, 2013, p.183) However, the standard interpretation of the observed pattern as a disposition effect apparently confuses the sign of the stock movement with a signal of stock quality. While it is intuitive that gains are a signal of high stock quality and losses are a signal of poor stock quality, this interpretation remains blind for the overall environment. For example, a stock that went up by 1% in a strong bull market is a "winner" but still underperformed, whereas a stock that went down by 1% in a strong bear market is a "loser" despite having outperformed the market.

This observation highlights where the disconnect between the disposition effect and its common measure stems from. The disposition effect, i.e., the tendency to sell winners *too early* and losers *too late*, pertains to some rational benchmark for when a specific asset *should be* sold. While such a benchmark is clearly correlated with the sign of the stock movement, it is *not identical* to the sign of the stock movement. In other words, the standard interpretation of the empirical disposition measure (i.e., $PGR > PLR$) as disposition effect neither distinguishes gains from good news nor losses from bad news and therefore implicitly identifies gains with a benchmark action

[1]This effect has, first and foremost, been observed for individual investors (e.g., Odean, 1998; Feng and Seasholes, 2005), but also for institutional investors (e.g., Grinblatt and Keloharju, 2001; Shapira and Venezia, 2001; Frazzini, 2006) and experimental subjects (e.g., Weber and Camerer, 1998). It applies beyond stock markets, e.g., to real estate markets (Genesove and Mayer, 2001), traded option markets (Poteshman and Serbin, 2003), executive stock options (Heath et al., 1999), futures markets (Heisler, 1994; Coval and Shumway, 2005; Locke and Mann, 2005), online betting markets (Hartzmark and Solomon, 2012), and to corporate settings (Crane and Hartzell, 2010).

of holding the asset and losses with a benchmark action of selling the asset. In this paper, we introduce a novel setup that takes the overall market environment into account and thereby enables a separation of gains from good news (i.e., news that increase the likelihood that the owned asset is expected to outperform the market) and losses from bad news. Our setup allows to explicitly establish a rational benchmark based on first-order stochastic dominance as well as individual-specific risk preferences. This benchmark prescribes for each possible state of the world – both for winners and losers – whether an investor should hold, switch, or liquidate an asset. Therefore, this benchmark enables us to cleanly distinguish the disposition effect from its common measure and to theoretically investigate the link of the two.

The basis of our theoretical investigation is a *decomposition* of PGR and PLR into frequencies of various benchmark violations. The frequency of a specific benchmark violation is the product of two components: First, the probability of the benchmark event, i.e., the probability of a specific action to be appropriate. And second, the conditional probability of choosing a different action, i.e., of violating the benchmark. While the former is shaped by the stochastic environment, the latter is determined by individual behavior. Since the stochastic environment naturally induces different probability distributions of benchmark events in gains and losses, drawing inferences on benchmark violations from observing $PGR > PLR$ is *not* straightforward.

By cleanly separating benchmark violation frequencies from individual violation propensities, the decomposition allows us to show that the disposition effect is neither necessary nor sufficient for $PGR > PLR$. Moreover, we find that investors with a propensity to hold *winners* too long and sell *losers* too early – a "violation pattern" that represents the *opposite* of the disposition effect – can still exhibit $PGR > PLR$. Also, investors whose violation pattern is neither action- nor domain-specific – the latter being a typical feature of distorted beliefs where gains and losses have no impact beyond providing different news – may still cause a positive disposition measure. This disconnect between the disposition measure and the disposition effect is especially surprising because we further confirm a common intuition about the disposition measure in the rational benchmark: the proportion of assets that *should be* realized is larger in losses than in gains for any risk attitude of the investor and for any parameterization of the stochastic environment. Intuitively, a rational investor wants her asset to outperform both the market and the safe outside option, and this is more likely to be the case for winners than for losers.

While these *model-independent* results suggest that many more violation patterns than the disposition effect are able to generate $PGR > PLR$, we further show that unsystematic benchmark violations, such as randomization, are not able to generate it. In other words, only *systematic* biases give rise to the empirical observation $PGR > PLR$. Thus, the commonly used disposition measure is indeed well suited to identify the presence of *some* behavioral bias. It is, however, neither suited to pin down the exact bias at work or which kind of violation pattern is present, nor is it suited to measure the severity of "mistakes" in general. In fact, behavioral biases beyond the ones that have been proposed so far as possible explanations may cause a positive disposition measure. In another paper (Maier and Fischer, 2021) where we investigate the link between benchmark violations and models of behavioral biases – rather than the link between benchmark violations and the disposition measure as in this paper – we find that many yet unconsidered behavioral biases indeed fulfill the necessary condition for generating $PGR > PLR$ that we identify here. These include both models with non-standard utility functions and models of non-standard belief formation.[2] Whether the benchmark violations induced by a behavioral bias are also sufficient for $PGR > PLR$ depends on the parameterization of both the stochastic environment and the model capturing the bias. This is the case for any behavioral bias, including those generating a disposition effect.

While our results on the *sign* of the disposition measure reveal that $PGR > PLR$ is not conclusive for its cause, we also show that this insight further extends to the *magnitude* of the disposition measure (captured by either $PGR - PLR$ or $\frac{PGR}{PLR} - 1$): although the disposition measure is increasing in the benchmark violations described by the disposition effect and decreasing in the opposite benchmark violations, it is also increasing in overall "mistakes," making the cause of a higher disposition measure rather ambiguous. In contrast, *variations in the magnitude* of the disposition measure turn out to be indicative of the predominant violation pattern. This metric has remained mostly unexplored, though. We derive several novel and insightful comparative statics on the disposition measure.

First, we show that, depending on the presence of a disposition effect, the magnitude of the disposition measure varies with the characteristics of the market (e.g., emerging vs. mature markets) or market segment (e.g., "startup" vs. "blue chip" segments) of an investor's (sub-)portfolio. This result suggests novel testable predictions. For instance, an investor who suffers from the disposition effect is expected to exhibit a larger (smaller) disposition measure in the more conservative (aggressive) fraction

[2] To be precise, in Maier and Fischer (2021) we discriminate among various *model-specific* behavioral biases by the kind of violation patterns they induce. While this analysis is not restricted to benchmark violations that are relevant for the disposition measure, it still identifies which behavioral biases may explain a positive disposition measure, i.e., fulfill the necessary condition for generating $PGR > PLR$. This list includes prospect theory with a status-quo and lagged-rational-expectations reference point, realization utility, regret theory, base-rate neglect, extrapolative expectations, confirmation bias, motivated beliefs (e.g., anticipatory utility, cognitive dissonance, self-attribution bias), beliefs in mean reversion, and even simple price heuristics. Note that all these models generate different violation patterns. Some models induce more of the benchmark violations that increase the disposition measure (e.g., realization utility) while others induce less of them (e.g., status-quo prospect theory), so that this analysis further helps explaining why some of these models *more readily* explain $PGR > PLR$ than others (see Barberis and Xiong, 2009; Hens and Vlcek, 2011; Meng and Weng, 2017).

of his portfolio. Since this prediction is reversed for investors suffering from an opposite disposition effect, variations in the magnitude of the disposition measure between sub-portfolios allow to infer what the prevailing violation pattern in fact is.

Second, depending on the domain-specific prevalence of benchmark violations, the disposition measure varies in magnitude with the information level of initial investment decisions. Suppose, for instance, that benchmark violations are sufficiently more prevalent in losses than gains. Then, well-informed professional traders would have lower disposition measures than less informed households, even if professionals were prone to the exact same benchmark violations as non-professionals. This alternative explanation for the common empirical finding that professionals tend to have lower disposition measures is complementary to the standard explanation that this is due to professionals being "more rational." Again, since the prediction is reversed for benchmark violations being sufficiently more prevalent in gains than losses, varying magnitudes of the disposition measure are informative for the predominant violation pattern.

Third, we show that, depending on the overall prevalence of benchmark violations, the disposition measure varies in magnitude with the investor's attention or curiosity, i.e., the duration between trading decisions.[3] If overall benchmark violations are sufficiently rare, as for professional investors, we find that more attention increases the disposition measure. On the other hand, if benchmark violations are sufficiently prevalent, as for "household" investors, more attention decreases the disposition measure. This result suggests a novel testable prediction where the effect of attention on the disposition measure depends on the group of investors. Indeed, in line with our prediction for non-professional investors, Dierick et al. (2019) find that more attentive retail investors have significantly lower disposition measures. While demonstrating that the disposition measure is not a proper measure of overall "mistakes," this result further shows that variations in its magnitude can identify their incidence since "low error" and "high error" types imply opposite signs of the disposition measure's comparative static with respect to attention.

Finally, our analysis reveals an interesting limit property: If the investor received an infinite amount of information prior to trading, the disposition measure would only depend on the benchmark violations described by the disposition effect. Thus, the common interpretation of $PGR > PLR$ in terms of a disposition effect – namely the latter being a necessary condition for the former – turns out to be accurate in the full-information limit, when the investor knows with *certainty* that a winner will outperform the market and a loser will underperform.

[3] Such financial attention is typically approximated by the frequency of trading account "log-ins" (Karlsson et al., 2009; Gherzi et al., 2014; Sicherman et al., 2016; Olafsson and Pagel, 2018; Dierick et al., 2019)

Related Literature. Our paper contributes to a vast literature in behavioral economics and finance. By providing theoretical results on the link between the disposition measure and individual behavior, our paper is most closely related to theoretical explanations of a positive disposition measure (e.g., Barberis and Xiong, 2009, 2012; Ingersoll and Jin, 2012; Henderson, 2012; Hens and Vlcek, 2011; Meng and Weng, 2017). There are two main differences to our approach.

On the one hand, while existing approaches derive model-specific results, the theoretical results we derive are model-independent and thereby offer a different and complementary perspective: Instead of investigating whether and how a given choice model can generate $PGR > PLR$, we analyze which benchmark violations are necessary and sufficient for $PGR > PLR$. Our findings show that more violation patterns than the disposition effect are able to generate $PGR > PLR$. Since potential explanations for $PGR > PLR$ have been selected on the basis of generating a disposition effect, this implies that behavioral biases in addition to the ones proposed so far are potential drivers of $PGR > PLR$.[4] Thus, by enlarging the set of potential explanations, our paper also contributes to an ongoing empirical debate on the causes of $PGR > PLR$. While knowing these causes may be helpful to "de-bias" investors or to better understand other aspects of their behavior, field as well as laboratory evidence has been inconclusive in this regard. Importantly, our decomposition and the results of our comparative statics analysis offer new ways to better distinguish among (this larger set of) potential explanations, so that the new insights of this paper help to empirically identify the cause of a positive disposition measure.[5]

On the other hand, while being more general in other dimensions, previous theoretical approaches use setups with only one risky asset of known distribution (or several identically distributed risky assets) and therefore neither take the overall market environment into account nor do they leave scope for "momentum," i.e., price drift predictability. Having a different objective, we establish a novel setup that takes the market environment into account and allows for "momentum," and thereby in-

[4]Explanations based on both distorted utility functions and distorted belief formation have been proposed. While the former mainly include variants of Kahneman and Tversky's (1979) prospect theory (for overviews see, e.g., Camerer, 2000; DellaVigna, 2009; Barberis, 2013), the latter mainly focus on an irrational belief in mean reversion. These explanations have been selected on the basis of generating a disposition effect. Also, the connection to other phenomena, such as underreaction to news, post-earnings announcement drift, or "momentum" (Frazzini, 2006; Grinblatt and Han, 2005), is based on the interpretation of $PGR > PLR$ in terms of a disposition effect. Overall, the interpretation of $PGR > PLR$ as a disposition effect has long guided the literature.

[5]While going beyond the disposition measure, in Maier and Fischer (2021) we take a first step in this direction and directly measure individuals' benchmark violation propensities using a novel experimental design. We find that our subjects are on average prone to holding losers *and* winners too long – a violation pattern different from the disposition effect but one that nevertheless fulfills the necessary condition for $PGR > PLR$ that we identify here. Since different behavioral biases are further shown to induce different violation patterns, empirically identifying the predominant violation pattern is indeed informative for the underlying psychological mechanism. For instance, our experimental subjects' buying and selling behavior is only consistent with belief distortions motivated by ownership.

corporates the main reason for why $PGR > PLR$ is considered indicative of irrational behavior (see Barberis, 2013).[6] It is this setup that enables us to separate gains (losses) from good (bad) news, so that violation patterns other than the disposition effect can be identified.

By providing a rational benchmark, our paper further relates to empirical studies which compare proportions of realized assets to a rational benchmark value (e.g., Magnani, 2015; Fischbacher et al., 2017; Frydman et al., 2014, 2017). However, while previous approaches compare behavior only to a *risk-neutral* benchmark, our rational benchmark allows for individual-specific risk preferences and therefore not only prescribes *how* agents should invest, but also *whether* they should undertake a risky investment. Note that prominent explanations, such as prospect theory, explain $PGR > PLR$ solely through a change in risk attitudes between the buying and selling decision. Thus, taking risk preferences into account is essential when studying the causes of a positive disposition measure in terms of benchmark violations. In addition, our paper is markedly distinct in using the rational benchmark to *decompose* the disposition measure, and it is this decomposition that is central to the new insights we generate.

Outline. The remainder of this paper is structured as follows. Section 2 introduces our theoretical framework. In this section, we present the general setup (Section 2.1), define the well-known disposition measure (Section 2.2), present the rational benchmark (Section 2.3), show how the benchmark disposition measure can be computed and used as comparison (Section 2.4), and, last but not least, decompose the disposition measure (Section 2.5). Section 3 illustrates our theoretical concept with a numerical example. In Section 4, we derive model-independent results on the benchmark disposition measure (Section 4.1), on the sign of the disposition measure (Section 4.2), and on the magnitude of the disposition measure (Section 4.3). Formal proofs of these results are all gathered in Appendix A, while Appendix B is devoted to the explicit computation of the benchmark disposition measure. We conclude in Section 5.

2 Theoretical Framework

2.1 Setup

We model a stylized asset market with two risky assets A and B and a safe outside option O for a finite horizon in discrete time, $t \in \{0, 1, ..., T\}$, from the viewpoint of an *individual* investor.[7] That is, asset prices are determined *exogenously* by indepen-

[6]Another reason why PLR *should be* larger than PGR is that differential tax rates on short-term versus long-term capital gains and losses provide an incentive to sell losers quickly, but hold on to winners (Constantinides, 1983, 1984).

[7]The opportunity to invest in a safe outside option is necessary to allow for individual-specific risk preferences in the rational benchmark. Without this outside option, the rational benchmark could only

dent stochastic processes F_h and F_l with known distributions, but it is unknown to the investor which of the two risky assets adheres to which of the processes.[8] Specifically, F_h and F_l are binomial processes with corresponding appreciation probabilities $p_h, p_l \in (0,1)$, where $p_h > p_l$. While the prices of the risky assets evolve along two different binomial trees, the return rate of the safe asset is $r \in \mathbb{R}$ per period. For simplicity, we normalize r to zero so that choosing O corresponds to *not investing*. While our setup could accommodate any return rate of the safe asset, the two natural definitions of what a gain and loss are – either in relation to the returns of the safe asset or relative to the asset's purchase price – coincide for the special case $r = 0$, which we use as a simplification throughout.

The investor is assumed to have an uninformative prior $q_0 = 1/2$ about the likelihood that asset A adheres to process F_h. As the two price processes differ in their appreciation probabilities, observing price paths of both assets is informative about which asset follows which process, i.e., price changes are noisy signals of asset quality. Hence, there is scope for learning.[9] Specifically, when counting the number of price appreciations of assets A and B *until* period t as a_t and b_t, the Bayesian posterior in period t, q_t, is fully determined by the difference $\Delta_t := a_t - b_t$, i.e.,

$$q_t = \frac{(p_h(1-p_l))^{\Delta_t}}{(p_h(1-p_l))^{\Delta_t} + (p_l(1-p_h))^{\Delta_t}}.$$

Since the prior probability that asset A adheres to F_h is $q_0 = 1/2$, $\Delta_t = 0$ induces an uninformative Bayesian posterior in period t, i.e., $q_t = 1/2$. Observing more (less) price appreciations for asset A than B, i.e., $\Delta_t > (<)\ 0$, corresponds to a Bayesian posterior of $q_t > (<)\ 1/2$. The larger the absolute value of $\Delta_t \in \mathbb{Z}$ is, the more extreme is the Bayesian posterior.

While asset prices are observed in every period, trading is restricted to periods τ and τ', with $0 < \tau < \tau' < T$. Hence, our setup can be interpreted as capturing an investment episode, which represents one element of a longer aggregation of many such elements. To simplify our further investigations, we impose equal time horizons for both investment choices, and denote this common time horizon as $n := T - \tau' = \tau' - \tau$.[10] Together with the novel feature of having *two* risky assets, the fact that in-

comprehend first-order stochastic dominance. On the other hand, two risky assets are necessary not only for the rational benchmark to comprehend first-order stochastic dominance, but also to separate gains from good news and losses from bad news (see below).

[8] Asset A adheres to process F_h if and only if asset B adheres to process F_l.

[9] Thus, we capture "momentum," i.e., price drift predictability, in the simplest possible way, namely by inducing an auto-correlation from the perspective of an individual investor without imposing a complex path dependence of appreciation probabilities. Note that we do not require "momentum" to be particularly strong. In fact, our results also hold when the "signal-to-noise ratio" is rather low (i.e., p_l is close to p_h), so that the scope for learning is limited.

[10] Note that equality of investment horizons is an implicit assumption in settings where investors trade in every period.

vestors *cannot* trade in every period (in discrete time) allows for the existence of states where an asset is in losses or gains for both good and bad news, so that we can separate asset returns from quality signals.[11] Note that for $r = 0$, an asset is in gains (losses) when its price at $t = \tau'$ is above (below) its price at $t = \tau$, independently of whether gains and losses are defined with respect to the safe outside option or with respect to the asset's purchase price. Good (bad) news refers to an increase (decrease) of the likelihood of the investor's asset to follow process F_h based on observed asset price changes between $t = \tau$ and $t = \tau'$. Let

$$g := \min\{k \in \mathbb{N} | u^k\, d^{n-k} > 1\}$$

denote the minimum number of price appreciations within n periods for an asset to be in gains (with u and d defined below).[12] Then, the condition $1 < g < n$ guarantees the existence of states where an asset is in losses (gains) despite good (bad) news. Importantly, the separation of asset returns from quality signals is necessary for *allowing* investors to make *any* kind of benchmark violation, not only the ones described by the disposition effect. In particular, the possibility of realizing too early in *losses* or keeping for too long in *gains* is precluded *by assumption* without such a separation, despite being a pervasive feature of real-world trading.[13] Note that $n > \tau$ ($n \geq 2\tau$) is a necessary (sufficient) condition for allowing investors to make *any* kind of benchmark violation.

With appreciation and depreciation increments of each process being *proportional* to its value, the common time horizon n further assures that the same posterior beliefs in periods τ and τ' translate into identical gambles (proportional to wealth). As this feature allows us to identify a rational benchmark incorporating individual-specific risk attitudes, we assume that each process appreciates by a factor of $u > 1$ and depreciates by a factor of $d < 1$.[14] For simplicity, we further assume that an asset purchased

[11] In reality, investors cannot trade in subsequent periods either as some (continuous) time will always have passed until the next trading decision.

[12] With an arbitrary return rate of the safe asset, and gains and losses being defined with respect to the safe outside option, we would have $g := \min\{k \in \mathbb{N} | u^k\, d^{n-k} > (1+r)^n\}$. Thus, with $r = 0$, the two natural candidates for the reference point (see Barberis and Xiong, 2009) coincide.

[13] To see that, suppose the investor has invested in a risky asset. Until her next investment decision, the asset's price may go up or down. If there is no other risky asset, or if the next investment decision is already in the subsequent period, a price increase (decrease) can never be bad (good) news, in the former case because any gain (loss) is good (bad) news and in the latter case because any gain (loss) is either good (bad) or no news (when the other asset's price went in the same direction). Thus, it can never be rational to sell (hold) an asset in gains (losses), given that the initial purchase decision was rational. As a result, *by construction* an investor can only hold losers too long and sell winners too early, but she can never hold winners too long or sell losers too early. Having two risky assets as well as more than one period in between investment decisions relaxes this severe restriction, because the other asset's price increasing (decreasing) more often than the own asset's price induces bad (good) news.

[14] These multipliers are necessarily identical for F_h and F_l, because otherwise observing price paths for only one period would suffice to deterministically infer which asset follows which process. There are three additional reasons for using relative rather than absolute price changes: First, for absolute price increments, the asset that follows the low process can rationally be preferred in case it is sufficiently

at time τ is always either in gains or in losses at time τ', but never breaks even, i.e., that $u^\alpha d^\beta \neq 1$ for all $\alpha + \beta = n$.

Finally, trading is restricted to binary choices, i.e., the investor must put her *entire* current wealth in either asset A, B, or O. This simplification is crucial for establishing the rational benchmark respecting both first-order stochastic dominance and individual-specific risk attitudes, as it reduces the degrees of freedom in the investment decision.[15] Furthermore, such all-or-nothing investment decisions prevent fully rational investors from exhibiting $PGR > PLR$ by mere portfolio re-balancing, which constitutes an important prerequisite when interpreting such an observation as a mistake.[16] Note that since the investor is invested in at most one risky asset, the second risky asset can be interpreted as representing the overall market environment. In the final period T, all investments are automatically liquidated at current prices.[17]

In order to reduce notational load, we drop the time index of the above introduced notations and use the following shorthands instead:

$$\begin{array}{llll} a_\tau =: a & b_\tau =: b & \Delta_\tau =: \Delta & q_\tau =: q \\ a_{\tau'} =: a' & b_{\tau'} =: b' & \Delta_{\tau'} =: \Delta' & q_{\tau'} =: q'. \end{array}$$

2.2 Disposition Measure

Going back to Odean (1998), the literature commonly uses a positive disposition measure $PGR > PLR$ to identify the disposition effect, where PGR and PLR denote either the *proportion* or the *probability* of gains and losses realized, respectively. In the former case, PGR and PLR are defined from an "ex-post" perspective, i.e., as the number of assets that were realized in gains and losses divided by the number of all assets in gains and losses, respectively. In the latter case, PGR and PLR are defined from an "ex-ante" perspective, i.e., as the probability to realize an asset conditional on being in

cheap. In contrast, using relative price changes makes the asset following process F_h always more attractive for a rational investor, regardless of the absolute price level. Second, relative price changes guarantee that asset prices always remain positive for arbitrary realizations of the price processes and any number of periods. And third, the *number* of ups and downs still uniquely determines an asset's final price, regardless of the *order* of ups and downs. This is due to the commutativity of multiplication.

[15]While all-or-nothing selling decisions represent a common simplifying assumption in theoretical approaches (e.g., Ingersoll and Jin, 2012), Henderson (2012) shows that for prospect theory investors it is indeed an optimal strategy. Also, empirical evidence supports this assumption as individual investors indeed sell entire asset positions most of the time (Feng and Seasholes, 2005; Shapira and Venezia, 2001; Kaustia, 2010; Calvet et al., 2009).

[16]Note that the empirical literature has ruled out portfolio re-balancing (e.g., of CRRA investors) as an explanation for $PGR > PLR$, as the disposition measure is similar when restricting the data to sales of entire asset holdings of a stock (Odean, 1998). Moreover, re-balancing is a rather sophisticated strategy, but it is the less sophisticated (Dhar and Zhu, 2006) and low-IQ (Grinblatt et al., 2012) investors who have larger measures. Also, disposition measures tend to be larger for individual rather than institutional or professional investors (Brown et al., 2006; Barber et al., 2007; Chen et al., 2007; Choe and Eom, 2009; Calvet et al., 2009).

[17]We follow the literature and prohibit leverage, which is uncommon for individual investors.

gains and losses, respectively. While in our setup the ex-ante approach is equivalent to the ex-post approach at the aggregate level for an infinite number of homogenous investors, it avoids problems that arise with the ex-post approach at the individual level for heterogeneous investors (see, e.g., Feng and Seasholes, 2005). Our investigation therefore uses the more robust ex-ante approach: we calculate an ex-ante expected disposition measure of an individual investor in a single trading period τ', based on complete contingent choice rules and the true underlying price processes of the assets.

First, we define an investor's complete contingent choice rules of which asset to choose in periods τ and τ', conditional on (the differences in) both assets' price appreciations until then, as

$$
\begin{aligned}
&f : \{-\tau, ..., \tau\} \longrightarrow \{A, B, O\},\ \Delta \longmapsto z, \\
&f' : \{-\tau, ..., \tau\} \times \{A, B, O\} \times \{-\tau', ..., \tau'\} \longrightarrow \{A, B, O\},\ (\Delta, z, \Delta') \longmapsto z',
\end{aligned}
$$

where z and z' denote the investment decision in periods τ and τ', respectively.[18] We calculate the disposition measure as the difference of the ex-ante probabilities to realize a gain or loss. That is, realizing gains or losses are probability events with respect to the probability measure that is induced by the true price processes of the risky assets.

Second, we (slightly abuse notation and) define the auxiliary events of basic actions of an investor who adheres to choice rules f and f' as

$$
\begin{aligned}
A &:= \{\text{buy } A \text{ at } t = \tau\} \ = \{(a, b, a', b') | f(\Delta) = A\}, \\
A' &:= \{\text{buy } A \text{ at } t = \tau'\} = \{(a, b, a', b') | f'(\Delta, f(\Delta), \Delta') = A\}.
\end{aligned}
$$

The events B, B', O, and O' are defined analogously. Note that the actions of choosing either asset A, B, or O are state contingent, which is captured by the individual choice rules f and f'. That is, each event is the set of precisely those *contingencies*, in which the respective action is undertaken. As the above three basic actions capture the entire universe of possible states of the world, $\Pr(A \cup B \cup O) = \Pr(A' \cup B' \cup O') = 1$ holds.

[18]E.g., given that $\mathbb{E}[F_h|n] + \mathbb{E}[F_l|n] > 0$ the choice rules of a risk-neutral Bayesian EUT agent are

$$
f(\Delta) = \begin{cases} A & \text{if } \Delta > 0 \\ (0.5 \circ A, 0.5 \circ B) & \text{if } \Delta = 0 \\ B & \text{if } \Delta < 0 \end{cases} \quad \text{and} \quad f'(\Delta, z, \Delta') = \begin{cases} A & \text{if } \Delta' > 0 \\ z & \text{if } \Delta' = 0 \\ B & \text{if } \Delta' < 0. \end{cases}
$$

Note that the choice rule f' of the risk neutral Bayesian EUT type in period τ' is independent of its argument Δ, and depends on its argument z only by convention. However, these arguments are crucial for the choice rules of non-standard models.

Third, we deduce events of *relative actions*, i.e., actions at time τ' in relation to the initial purchase decision at time τ:

$$\begin{aligned}
K_A &:= \{\text{buy } A \text{ at } t = \tau \text{ and keep it at } t = \tau'\} &&= A \cap A', \\
S_A &:= \{\text{buy } A \text{ at } t = \tau \text{ and switch to } B \text{ at } t = \tau'\} &&= A \cap B', \\
Q_A &:= \{\text{buy } A \text{ at } t = \tau \text{ and liquidate it at } t = \tau'\} &&= A \cap O'.
\end{aligned}$$

The events K_B, S_B, and Q_B are defined analogously. Since PGR and PLR are not tied to specific assets (i.e., A or B), we need to define the following *compound events*:

$$\begin{aligned}
K &:= \{\text{buy } A \text{ or } B \text{ at } t = \tau \text{ and keep it at } t = \tau'\} &&= K_A \cup K_B, \\
S &:= \{\text{buy } A \text{ or } B \text{ at } t = \tau \text{ and switch to the other at } t = \tau'\} &&= S_A \cup S_B, \\
Q &:= \{\text{buy } A \text{ or } B \text{ at } t = \tau \text{ and liquidate it at } t = \tau'\} &&= Q_A \cup Q_B.
\end{aligned}$$

Furthermore, PGR and PLR do not treat the actions switch and liquidate differentially, but view them both as *realizations*,

$$R := \{\text{buy } A \text{ or } B \text{ at } t = \tau \text{ and realize (i.e., sell) it at } t = \tau'\} = S \cup Q.$$

While differentiating S and Q is not necessary for the disposition measure, the distinction will become crucial for defining our benchmarks and for decomposing PGR and PLR.

Fourth, we introduce gain and loss states: An asset is *in gains* if its price at time τ' exceeds its price at time τ. Otherwise, it is *in losses*. As already introduced, g denotes the minimum number of price appreciations within n periods for an asset to be in gains, so we can define the following *gain* and *loss events* as

$$\begin{aligned}
G_A &:= \{\text{asset } A \text{ is in gains}\} &&= \{(a, b, a', b') | a' - a \geq g\}, \\
L_A &:= \{\text{asset } A \text{ is in losses}\} &&= \{(a, b, a', b') | a' - a < g\}, \\
G &:= \{\text{own asset is in gains}\} &&= (A \cap G_A) \cup (B \cap G_B), \\
L &:= \{\text{own asset is in losses}\} &&= (A \cap L_A) \cup (B \cap L_B).
\end{aligned}$$

As above, G_B and L_B are defined analogously to G_A and L_A in the definitions of G and L.

Finally, we are able to state

Definition 1 (Disposition Measure) *The investor's (ex-ante expected) disposition measure DM_i with $i \in \{1,2\}$ is defined as*

$$DM_1 := PGR - PLR \quad \textit{and} \quad DM_2 := \tfrac{PGR}{PLR} - 1, \quad \textit{where}$$
$$PGR := \Pr(\textit{realize own asset}|\textit{own asset in gains}) = \Pr(R|G) = \tfrac{\Pr(G \cap R)}{\Pr(G)},$$
$$PLR := \Pr(\textit{realize own asset}|\textit{own asset in losses}) = \Pr(R|L) = \tfrac{\Pr(L \cap R)}{\Pr(L)}.$$

As both measures are used in the literature, throughout we investigate the disposition measure defined either as a difference between PGR and PLR (DM_1) or as a ratio of the two (DM_2). Note that in both cases $DM_i > 0 \iff PGR > PLR$ holds. All our results hold independently of which definition is used.[19]

2.3 Rational Benchmark

A key contribution of our paper is its ability to derive a complete contingent benchmark of what action should be undertaken in which state of the world. Our benchmark is based on first-order stochastic dominance as well as individual-specific risk preferences.

First, we define benchmark events of *first-order stochastic dominance (FOSD) violations*, i.e., we collect states in which some specified action constitutes such a violation. We impose the simplifying convention that switching assets is a first-order violation in case of uninformative posteriors.[20] Then,

$$\begin{aligned}
V_A &:= \{A \text{ is } 1^{\text{st}}\text{-order dominated at } t = \tau'\} &&= \{(a,b,a',b')|a' < b'\},\\
V_K &:= \{\text{own asset is } 1^{\text{st}}\text{-order dominated at } t = \tau'\} &&= (A \cap V_A) \cup (B \cap V_B),\\
V_S &:= \{\text{other asset is } 1^{\text{st}}\text{-order dominated at } t = \tau'\} &&= (A \cap V_A^C) \cup (B \cap V_B^C),
\end{aligned}$$

where V_B is defined analogously to V_A in the above definitions of V_K and V_S, and V_A^C and V_B^C denote the complements of V_A and V_B, respectively. Note that these events do not collect states where the investor chooses an action which *is* a first-order violation, but states where some specified action *would be* a violation. For instance, V_K collects the states in which keeping one's own asset at time τ' would constitute a first-order violation.

[19] Further, note that although PGR and PLR are defined as probabilities, we sometimes refer to them as proportions for reasons of readability.

[20] This assumption ensures that there is a first-order benchmark in all states of the world. It becomes necessary only due to the discrete time structure of our setup – in a time-continuous world, uninformative states are zero-probability events. In our time-discrete world, the convention that a rational investor should keep her asset for uninformative Bayesian posteriors would be implied, for instance, by an arbitrary small (physical or mental) transaction cost. The larger n is, the less likely are states in which the Bayesian posterior is uninformative, so that the convention is not restrictive in practice.

Second, besides violations of FOSD, investors may exhibit inconsistent risk preferences between the two trading periods τ and τ'. This gives rise to an individual-specific second-order benchmark of which risks should be accepted at time τ', given own choices at time τ. Note that changing risk preferences are one possible non-rational explanation of the disposition effect, e.g., in prospect theory. Therefore, we need to abstract away from such changes in the rational second-order benchmark. However, this second-order benchmark is only meaningful for investors whose initial purchase decisions are monotonic in the informativeness of states. Thus, while our first-order benchmark is always applicable as both assets can always be ordered in terms of FOSD, applying our second-order benchmark requires an assumption on the investor's choice rule in period τ: choice rule f is monotonically increasing, i.e.,

$$\exists\, \overline{q} \in [1/2, 1) \quad \text{with either} \qquad \overline{q} > 1/2 \;\text{ s.t. }\; f(\Delta) = \begin{cases} A & \text{if } q \geq \overline{q} \\ O & \text{if } 1 - \overline{q} < q < \overline{q} \\ B & \text{if } q \leq 1 - \overline{q}, \end{cases}$$

$$\text{or} \qquad \overline{q} = 1/2 \;\text{ s.t. }\; f(\Delta) = \begin{cases} A & \text{if } q > 1/2 \\ (0.5 \circ A, 0.5 \circ B) & \text{if } q = 1/2 \\ B & \text{if } q < 1/2. \end{cases}$$

This assumption imposes *minimal* requirements on rationality that are fulfilled by all models in which agents, prior to asset ownership, update in the same direction as Bayesians, do so similarly for both assets, and have monotone preferences.[21] As we show in Maier and Fischer (2021), these minimal requirements are fulfilled by almost all behavioral biases with a potential to generate $DM_i > 0$.

We can now define benchmark events of *second-order risk preference (SORP) violations*, which collect states where some specified action constitutes a risk preference inconsistency:

$$\begin{aligned} V_Q &:= \{\text{liquidate own asset is a 2}^{\text{nd}}\text{-order violation at } t = \tau'\} \\ &= \{(a, b, a', b') | f(\text{sgn}(\Delta') \min\{|\Delta'|, \tau\}) \neq O\}, \\ V_I &:= \{\text{invest in risky asset is a 2}^{\text{nd}}\text{-order violation at } t = \tau'\} \\ &= \{(a, b, a', b') | f(\text{sgn}(\Delta') \min\{|\Delta'|, \tau\}) = O\}. \end{aligned}$$

[21] The assumption that the choice rule is symmetric (i.e., the same $\overline{q}$ applies to both risky assets) is only used for simplicity. All results we derive would equally hold if the thresholds for assets A and B were different. However, since this would imply an irrationality which is independent of asset ownership – e.g., because initial priors would not be uninformative or one of the assets is somehow "favored" over the other – we impose symmetry for the rational second-order benchmark.

Again, these events do not refer to states where the investor chooses an action which *is* a second-order violation, but to states where some specified action *would be* a violation. Thus, V_Q collects states where investment at time τ' offers an identical (proportional) gamble as some initial investment opportunity at time τ, which would have been *accepted* according to choice rule f, so that *liquidating* constitutes a risk preference inconsistency. Analogously, V_I collects states where investment at time τ' offers an identical (proportional) gamble as some initial investment opportunity at time τ, which would have been *rejected* according to choice rule f. This benchmark is normative from the point of view of a Bayesian EUT investor with CRRA preferences.[22]

Combining the first- and second-order benchmarks provides us with the *full benchmark*. Intuitively, in each state of the world there is a first-order benchmark of stochastic dominance as well as a second-order benchmark of risk preference. As above, we introduce *benchmark events* of combinations of FOSD and SORP violations:[23]

$$\begin{aligned}
V_{KQ} &:= \{\text{keep is a } 1^{\text{st}}\text{-order and liquidate a } 2^{\text{nd}}\text{-order violation}\} &&= V_K \cap V_Q,\\
V_{KS} &:= \{\text{keep is a } 1^{\text{st}}\text{-order and switch a } 2^{\text{nd}}\text{-order violation}\} &&= V_K \cap V_I,\\
V_{SK} &:= \{\text{switch is a } 1^{\text{st}}\text{-order and keep a } 2^{\text{nd}}\text{-order violation}\} &&= V_S \cap V_I,\\
V_{SQ} &:= \{\text{switch is a } 1^{\text{st}}\text{-order and liquidate a } 2^{\text{nd}}\text{-order violation}\} &&= V_S \cap V_Q.
\end{aligned}$$

Again, these events do not refer to states where the investor chooses an action which *is* a violation, but to states where two specified actions *would be* a first- or second-order violation. These events constitute a full benchmark in the sense that they pin down a unique action that should be undertaken for each state of the world. For instance, in the event V_{KQ}, switching to the other risky asset is the only action that neither constitutes a first-order nor a second-order violation and therefore represents the unique appropriate (i.e., rational) action. As usual, we specify these benchmark events per domain. For example, $GV_{KQ} := G \cap V_{KQ}$ is the event where keeping is a first-order and liquidating a second-order violation *in gains*. Throughout the paper, we use the concatenation of events as a shorthand for their intersection.

[22] Note that preferences satisfying expected utility theory (EUT) and constant relative risk aversion (CRRA) constitutes *the* standard assumption in the literature for rational investors.

[23] Note that the second-order investment violations are tied to investing into the first-order dominant asset. While investing into the first-order dominated asset could be seen as a second-order violation as well, we chose to define the various violations without any overlap. That is, for each of the four benchmark events, exactly one action is a first-order violation, exactly one (other) action is a second-order violation, and the remaining third possible action is the unique appropriate action. Therefore, first- and second-order violations are in fact hierarchical, i.e., we regard a decision as a second-order violation only if it is no first-order violation. For liquidation, this is trivial, as liquidating can never be a first-order violation. For investing, however, we have to impose the above restriction.

Furthermore, we write PGX and PLX as shorthands for the conditional probabilities $\Pr(X|G)$ and $\Pr(X|L)$ of an event X, conditional on domain G and L, respectively.[24] We call these conditional probabilities of the above introduced benchmark events *violation possibilities*. For example, $PGV_{KQ} := \Pr(V_{KQ}|G)$ is the probability mass of all states in which keeping and liquidating are possible violations, conditional on being in gains.

2.4 Benchmark Disposition Measure

These violation possibilities (or benchmark event probabilities) allow us to derive proportions of gains and losses that *should be realized*, which we denote as $\overline{PGR}$ and $\overline{PLR}$, respectively.

Definition 2 (Benchmark Disposition Measure) *The investor's (ex-ante expected) benchmark disposition measure $\overline{DM}_i$ with $i \in \{1,2\}$ is defined as*

$$\overline{DM}_1 := \overline{PGR} - \overline{PLR} \quad \textit{and} \quad \overline{DM}_2 := \frac{\overline{PGR}}{\overline{PLR}} - 1, \quad \textit{where}$$

$$\begin{aligned}
\overline{PGR} := \Pr(\textit{should realize own asset}|\textit{own asset in gains}) &= \sum_{V \in \{V_{KQ}, V_{KS}, V_{SK}\}} \Pr(V|G) \\
&= PGV_{KQ} + PGV_{KS} + PGV_{SK}, \\
\overline{PLR} := \Pr(\textit{should realize own asset}|\textit{own asset in losses}) &= \sum_{V \in \{V_{KQ}, V_{KS}, V_{SK}\}} \Pr(V|L) \\
&= PLV_{KQ} + PLV_{KS} + PLV_{SK}.
\end{aligned}$$

Intuitively, an asset should be realized whenever either switching or liquidating is the unique appropriate action. This is the case when either keeping and liquidating (V_{KQ}), keeping and switching (V_{KS}), or switching and keeping (V_{SK}) constitute the respective first- and second-order violation possibilities. Analogously, we define the proportions of gains and losses that *should be kept* as $\overline{PGK} = PGV_{SQ}$ and $\overline{PLK} = PLV_{SQ}$, respectively. Our benchmark is complete in the sense that it specifies a unique appropriate action for each state of the world, both in gains and losses. Hence, all violation possibilities of a domain add up to one, and the proportions of gains or losses that should be realized and that should be kept add up to one as well:

$$\begin{aligned}
PGV_{KQ} + PGV_{KS} + PGV_{SK} + PGV_{SQ} &= 1 \quad \text{so that} \quad \overline{PGR} + \overline{PGK} = 1, \\
PLV_{KQ} + PLV_{KS} + PLV_{SK} + PLV_{SQ} &= 1 \quad \text{so that} \quad \overline{PLR} + \overline{PLK} = 1.
\end{aligned}$$

Importantly, our framework allows to fully formalize $\overline{PGR}$ and $\overline{PLR}$ (and hence also $\overline{PGK}$ and $\overline{PLK}$), so that we can explicitly compute an investor's benchmark disposition

[24] Note that this notation is in line with our above definitions of PGR and PLR as probabilities of realizing one's asset, conditional on being in gains and losses, respectively.

measure $\overline{DM}_i$. Appendix B shows how this computation can be performed for any given parameterization of the stochastic environment and for any given risk attitude of the investor.

While observing $DM_i > 0$ tells us that relatively more gains than losses are realized, it cannot identify whether this is due to the fact that more gains or less losses are realized than should be realized. For such an identification, we need to define a *disposition measure in gains* (DMG_i) and *losses* (DML_i) as the deviations in actually realized gains and losses compared to the domain-specific benchmark: $DMG_1 := PGR - \overline{PGR}$, $DML_1 := PLR - \overline{PLR}$ and $DMG_2 := \frac{PGR}{\overline{PGR}} - 1$, $DML_2 := \frac{PLR}{\overline{PLR}} - 1$. The common understanding is that $DMG_i > 0$ and $DML_i < 0$, i.e., more winners *and* fewer losers are actually realized than should be realized. Note, however, that even if $DMG_i > 0$ and $DML_i < 0$, it may still be the case that $DM_i < 0$. The reason is that $\overline{DM}_i$ is always negative (as we show below in Proposition 1) and, indeed, can be so negative that both domain-specific disposition measures are simply not large enough to induce an aggregate disposition measure $DM_i > 0$. Measuring

$$DM_i - \overline{DM}_i \quad \text{or} \quad \frac{DM_i}{\overline{DM}_i} - 1$$

instead allows to resolve this problem, because $DMG_i > 0$ and $DML_i < 0$ implies $DM_i > \overline{DM}_i$.[25] Thus, comparing DM_i to its negative rational benchmark value $\overline{DM}_i$ clearly yields a more sensitive measure than comparing it to zero (as usually done), as it can identify aggregate disposition-prone behavior even for cases where $DM_i < 0$.

This insight seems particularly relevant for experiments: Since the value of $\overline{DM}_i$ depends on the parametrization of the stochastic environment *chosen* by the experimenter, a constant positive deviation of DM_i from its rational benchmark value $\overline{DM}_i$ may lead to $DM_i > 0$ in some environments, but not in others. As a result, differences in calibrations may help explain why some experimental studies find $DM_i > 0$ while others do not. Thus, in experimental studies, comparing DM_i to its rational benchmark value seems more appropriate than comparing it to zero in order to identify aggregate disposition-prone behavior. Indeed, some experimental studies provide such a benchmark comparison under the assumption of risk neutrality (e.g., Fischbacher et al., 2017; Frydman et al., 2014, 2017).

In contrast, when the value of $\overline{DM}_i$ is unknown, as is typically the case in field studies, it is natural to compare DM_i to zero. Actually, the comparison to zero even has an important advantage: as we show below (in Proposition 5), unsystematic benchmark violations, such as randomization, are not able to generate $DM_i > 0$, whereas DM_i is still larger than its rational benchmark value for any (parametrization of the)

[25] This is easily verified by noting that $DM_i > \overline{DM}_i \iff DMG_i > DML_i$. Thus, $DMG_i > 0$ *and* $DML_i < 0$ implies $DM_i > \overline{DM}_i$, while $DM_i > \overline{DM}_i$ implies $DMG_i > 0$ *or* $DML_i < 0$. Also, note that $DM_i > 0$ is sufficient but not necessary for $DM_i > \overline{DM}_i$, because $\overline{DM}_i < 0$ (see Proposition 1 below).

stochastic environment. Thus, the only way to identify *systematically* biased behavior via the disposition measure's sign is by comparing DM_i to zero.

While the comparisons of DM_i to both zero and $\overline{DM}_i$ have their eligibility, the central disconnect to the disposition effect that we uncover in this paper applies to both measures: all our results on benchmark violation propensities (Propositions 2 to 4 below) hold similarly no matter whether DM_i is compared to zero or its negative rational benchmark value. Thus, the disconnect we uncover is *not* a result of comparing DM_i to zero, but is rather due to misinterpreting an observed aggregate frequency as individual propensity.[26] Our decomposition presented next facilitates this insight.

2.5 Decomposition

We now introduce violations of the above introduced benchmarks. A benchmark violation occurs when an action is chosen that constitutes a violation. Hence, the intersections of the four full benchmark events with the events of one of their respective violating actions give rise to eight *violation events*. Analogously, we define the four events of non-violation as intersections of the benchmark events with their respective unique appropriate action. Table 1 summarizes these action-benchmark combinations.

Table 1: Action-Benchmark Combinations

1st-order violations	KV_{KQ}	KV_{KS}	SV_{SK}	SV_{SQ}
2nd-order violations	QV_{KQ}	SV_{KS}	KV_{SK}	QV_{SQ}
No violations	SV_{KQ}	QV_{KS}	QV_{SK}	KV_{SQ}

Note that each column of Table 1, i.e., each of the four benchmark events, is intersected with one of the three possible actions K, S, and Q, but depending on the benchmark (column), different actions induce a first-order, second-order, or no violation. In particular, all entries of the table are probability events themselves, defined according to the above introduced concatenation notation. As always, these events are defined for gains and losses, and their domain-conditional probabilities are denoted in the familiar PGX-PLX-notation.

[26]Note that this applies similarly to the domain-specific disposition measures DMG_i and DML_i. For instance, suppose that both PGR and $\overline{PGR}$ were 50%. Then, the disposition measure in gains would be zero, but every individual investment choice may still constitute a benchmark violation. Even when the exact right number of winning assets is sold, it may comprise exactly those assets that should have been kept, while the kept assets are the ones that should have been sold.

This gives rise to a decomposition of PGR and PLR into individual action-benchmark combinations:

$$
\begin{aligned}
PGR &= PGSV_{KQ} + PGSV_{KS} + PGSV_{SK} + PGSV_{SQ} \\
&\quad + PGQV_{KQ} + PGQV_{KS} + PGQV_{SK} + PGQV_{SQ}, \\
PLR &= PLSV_{KQ} + PLSV_{KS} + PLSV_{SK} + PLSV_{SQ} \\
&\quad + PLQV_{KQ} + PLQV_{KS} + PLQV_{SK} + PLQV_{SQ}.
\end{aligned}
$$

Clearly, a realization occurs whenever the investor switches to the other risky asset or liquidates her own asset, which are possible actions for each of the four benchmark events. In this decomposition of PGR (PLR), the probabilities $PGSV_{KQ}$, $PGQV_{KS}$, and $PGQV_{SK}$ ($PLSV_{KQ}$, $PLQV_{KS}$, and $PLQV_{SK}$) refer to realizations that are appropriate actions, while the other five terms refer to realizations that are benchmark violations. We call these probabilities of benchmark violations, conditional on their domain, *violation frequencies*. For instance, we refer to $PGKV_{KQ} = \Pr(KV_{KQ}|G)$ as the frequency of keeping one's asset in gains when it is a first-order violation to do so, while liquidating would have been a second-order violation, and switching the appropriate action.

Note, however, that the disposition effect – or any other benchmark violation pattern of an investor – refers to the probabilities of benchmark violations, conditional on both their domain and their benchmark, which we call *violation propensities*. We denote these propensities by the Greek letter referring to the committed violation, indexed by the benchmark on which it is conditioned:

$$
\begin{aligned}
\kappa_{KQ} &:= \Pr(K|V_{KQ}), & \lambda_{KQ} &:= \Pr(Q|V_{KQ}), \\
\kappa_{KS} &:= \Pr(K|V_{KS}), & \sigma_{KS} &:= \Pr(S|V_{KS}), \\
\kappa_{SK} &:= \Pr(K|V_{SK}), & \sigma_{SK} &:= \Pr(S|V_{SK}), \\
\sigma_{SQ} &:= \Pr(S|V_{SQ}), & \lambda_{SQ} &:= \Pr(Q|V_{SQ}).
\end{aligned}
$$

For instance, κ_{KQ} is the propensity to keep one's asset when it is a first order-violation to do so, while liquidating would have been a second-order violation, and switching the appropriate action. Note that for each of the four benchmark events, the propensities of the first-order violation, second-order violation, and appropriate action add up to one, as they cover all possible actions (keep, switch, and liquidate). Table 2 gives an overview of all propensities (of violations and appropriate actions) per benchmark event.[27]

All violation propensities are canonically defined for gains and losses. Note that each *violation frequency* is the product of its respective *violation propensity* and *violation*

[27]Note that we abstained from using the same notation for the propensities of appropriate actions (e.g., σ_{KQ}), so that Greek letters are "reserved" for benchmark violations throughout.

Table 2: Propensities per Benchmark Event

	V_{KQ}	V_{KS}	V_{SK}	V_{SQ}
K	κ_{KQ}	κ_{KS}	κ_{SK}	$1-\sigma_{SQ}-\lambda_{SQ}$
S	$1-\kappa_{KQ}-\lambda_{KQ}$	σ_{KS}	σ_{SK}	σ_{SQ}
Q	λ_{KQ}	$1-\kappa_{KS}-\sigma_{KS}$	$1-\sigma_{SK}-\kappa_{SK}$	λ_{SQ}

possibility (i.e., the likelihood of the underlying benchmark event), e.g., $PGKV_{KQ} = \kappa^G_{KQ} \times PGV_{KQ}$. Hence, we can restate the decomposition of PGR and PLR to obtain

Definition 3 (Decomposition) *The disposition measure DM_i can be decomposed such that*

$$PGR = PGV_{KQ}(1-\kappa^G_{KQ}) + PGV_{KS}(1-\kappa^G_{KS}) + PGV_{SK}(1-\kappa^G_{SK}) + PGV_{SQ}(\sigma^G_{SQ}+\lambda^G_{SQ}),$$
$$PLR = PLV_{KQ}(1-\kappa^L_{KQ}) + PLV_{KS}(1-\kappa^L_{KS}) + PLV_{SK}(1-\kappa^L_{SK}) + PLV_{SQ}(\sigma^L_{SQ}+\lambda^L_{SQ}).$$

The decomposition of PGR (PLR) consists of four components. First, the frequency of gains (losses) that should not be kept and not be liquidated, and that are not kept. Second, the frequency of gains (losses) that should not be kept and not be switched, and that are not kept. Third, the frequency of gains (losses) that should not be switched and not be kept, and that are not kept. And fourth, the frequency of gains (losses) that should not be switched and not be liquidated, but that are switched or liquidated.

Note that there are three violation propensities in both gains and losses that do not appear to be relevant for the disposition measure at all, namely σ_{SK}, σ_{KS}, and λ_{KQ}. The intuition for this is straightforward: These violations constitute some asset realization in states of the world where the other possible asset realization would have been the appropriate action. For instance, σ_{SK} is the propensity to switch assets when it is a first-order violation to do so, and liquidating the asset would have been the appropriate action. However, by its very definition, it makes no difference for the disposition measure whether the asset was appropriately realized (here, liquidated), or inappropriately so (here, switched). It only makes a difference whether it was realized or kept.

Restricting attention to those violations that appear relevant in our decomposition, for ease of interpretation we subsequently distinguish "conventional" from "unconventional" violations, thereby referring to the disposition effect.

Definition 4 (Conventional & Unconventional Violations) *The domain-specific propensities κ^L_{KQ}, κ^L_{KS}, κ^L_{SK} and σ^G_{SQ}, λ^G_{SQ} are called "conventional" violation propensities. Likewise, κ^G_{KQ}, κ^G_{KS}, κ^G_{SK} and σ^L_{SQ}, λ^L_{SQ} are called "unconventional" violation propensities.*

As stated in Definition 4, conventional violations refer to the benchmark violations that constitute the disposition effect, i.e., realizing (either switching or liquidating) a gain that should be kept and keeping a loss that should be realized. On the other hand, unconventional violations refer to the exact opposite and commonly disregarded benchmark violations of realizing a loss that should be kept and of keeping a gain that should be realized.

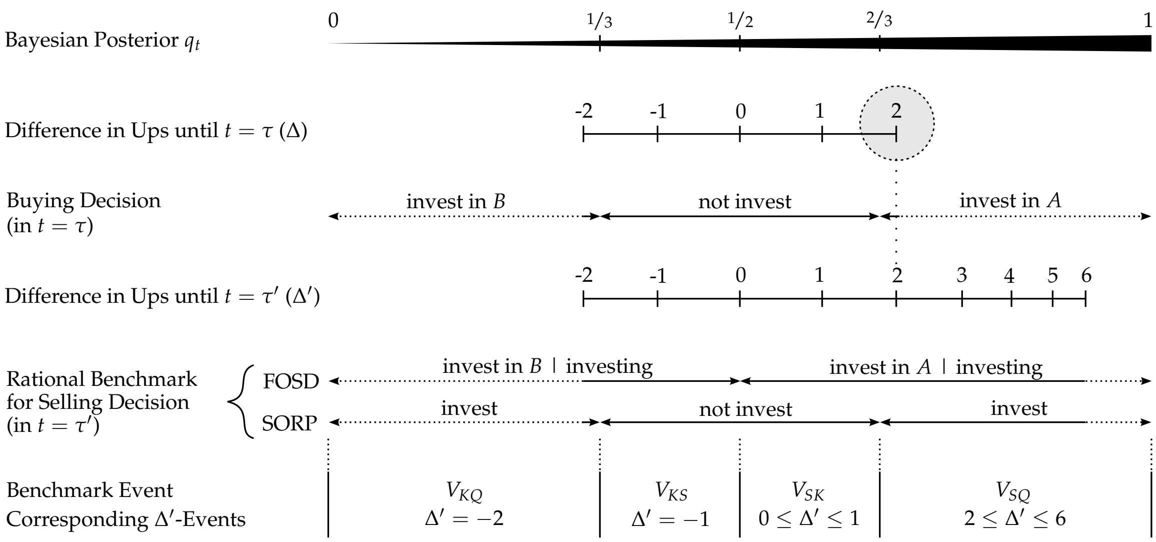
3 Illustration

In this section, we present a numeric example to illustrate our decomposition of PGR and PLR. Section 4 proceeds with our theoretical results. The example applies a specific parameterization of the stochastic environment where

$$T = 10, \quad \tau = 2, \quad \tau' = 6, \quad p_h = 0.55, \quad p_l = 0.45, \quad u = 1.3, \quad d = 0.8,$$

so that the high process F_h yields three times the expected return of the low process F_l per period.[28] We further consider a specific, risk averse investor (called Nat), who prefers to invest in a risky asset whenever its likelihood to be the "good" asset exceeds $2/3$.[29] More specifically, Nat's threshold of $\overline{q} = 2/3$ (just like any other threshold lying between 0.6 and 0.69) translates into buying asset A for $\Delta \geq 2$, asset B for $\Delta \leq -2$, and no asset for $-1 \leq \Delta \leq 1$. This buying behavior and its relation to Δ and the corresponding Bayesian posteriors is illustrated by the top three rows in Figure 1.

Figure 1: Illustrative Example



[28]Precisely, F_h yields an expected return of $0.55 \times 0.30 - 0.45 \times 0.20 = 7.5\%$ per period, whereas F_l yields only $0.45 \times 0.30 - 0.55 \times 0.20 = 2.5\%$. Note that compounding amplifies this effect over time. Over the maximum number of eight periods, the high process yields an expected return of $1.075^8 - 1 = 78.35\%$ and the low process $1.025^8 - 1 = 21.84\%$.

[29]Assuming an investor is Bayesian, any such likelihood threshold corresponds to a specific risk aversion coefficient. If the investor was not Bayesian, this coefficient would be over- or under-estimated (in terms of utility curvature), but the threshold would still measure her risk attitude appropriately (stemming from utility curvature *and* biased beliefs).

Next, we suppose that $\Delta = 2$ realizes (indicated by the grey circle around $\Delta = 2$ in Figure 1), so that Nat buys asset A in period τ according to his choice rule f. Until period τ', prices of both assets go up or down in each period, so Δ' (see the fourth row in Figure 1) can take any value between -2 (A never and B always appreciates) and 6 (A always and B never appreciates).

Rational Benchmark. The fifth row in Figure 1 shows Nat's rational benchmark for his selling decision in period τ'. The rational benchmark consists of two dimensions. First, a rational investor should never invest in (i.e., never switch to or keep holding) the asset that is *less* likely to be the "good" asset, independent of what she owned before, as doing so constitutes a *first-order stochastic dominance violation*. Thus, by first-order stochastic dominance (FOSD), *conditional* on investing in a risky asset, any rational investor should invest in asset A (B) for Bayesian posteriors q' above (below) $1/2$. In terms of our example, Nat should not keep asset A for $\Delta' < 0$ and should not switch to B for $\Delta' \geq 0$ by FOSD. Second, a rational investor with EUT preferences satisfying CRRA should not change her "uninvested" investment threshold – in our example to invest if and only if $q_t \notin (1/3, 2/3)$ – because for a fixed belief, investing induces the same gamble proportional to wealth in both trading periods τ and τ'. Thus, changing this investment strategy means the investor must have changed her risk preference from the buying to the selling decision, and doing so constitutes a *second-order risk preference violation* (according to her *individual* benchmark derived from her buying behavior).[30] Thus, while FOSD postulates *how* to invest conditional on investing, an investor's second-order risk preference (SORP) rather postulates *whether* to invest. Taking both first- and second-order concerns into account, Nat should keep asset A for $\Delta' \geq 2$, i.e., for no news or good news, but should not keep his asset for bad news: he should switch to asset B for $\Delta' = -2$ and liquidate asset A for $1 \geq \Delta' \geq -1$. Any diverging behavior constitutes *either* a first- *or* a second-order benchmark violation. Thus, both FOSD and SORP together determine the full rational benchmark of a Bayesian EUT agent with CRRA.

Benchmark Disposition Measure. Note that an investor can always keep her asset (K), liquidate her asset (Q), or switch to the other asset (S). In our example, Nat purchased asset A in period τ, so that K, Q, and S correspond to choosing A, O, and B in period τ'. A *benchmark event* collects states in which a specific action constitutes a first-order and another specific action constitutes a second-order violation. Therefore, these benchmark events pin down a full benchmark that specifies a unique appropriate action for each possible state of the world. For instance, in the event V_{KQ} switching

[30] For instance, Nat may shy away from investing when he should invest, e.g., by liquidating his asset for some likelihood larger than $2/3$. Or, he may invest when he should not according to his benchmark, e.g., by keeping his asset for some likelihood between $1/2$ and $2/3$. These are second-order benchmark violations as they require a change in risk attitude between the initial purchase and the selling decision to be rationalized (either via changed utility curvature or changed updating).

to the other risky asset is the only action that neither constitutes a first-order nor a second-order violation. In this benchmark event, keeping the asset would be a first-order violation (i.e., would violate FOSD) and liquidating would be a second-order violation (i.e., would violate SORP). Each benchmark event $V \in \{V_{KQ}, V_{KS}, V_{SK}, V_{SQ}\}$ comprises those Δ' that induce the same benchmark, and the set of all benchmark events $\{V_{KQ}, V_{KS}, V_{SK}, V_{SQ}\}$ captures all possible states of the world. The bottom row in Figure 1 displays these benchmark events and their corresponding Δ' in our example.

Note two important properties. First, while the probability mass of each Δ' is solely determined by the stochastic environment, an agent's choice rule f determines which Δ' are summarized within the same benchmark event. Second, because Δ' can result from different combinations of ups and downs of both assets, the own asset may be either in gains or losses for a given Δ'. Therefore, a given Δ' may occur with positive probability both in gains and losses.[31] As each benchmark event comprises one or several Δ', we can thus further determine the probabilities with which a benchmark event occurs in gains and losses. For instance, in our example the corresponding probability masses of $(V_{KQ}, V_{KS}, V_{SK}, V_{SQ})$ are $(0, 0, 0.101, 0.658)$ in gains and $(0.002, 0.016, 0.141, 0.082)$ in losses, so that, e.g., $\Pr(V_{SQ}|G) = \frac{0.658}{0.759}$ and $\Pr(V_{SQ}|L) = \frac{0.082}{0.241}$.[32] This implies that an investor can realize her asset (i.e., switch or liquidate) too readily or keep her asset for too long in both gains *and* losses.

The proportions of gains and losses that *should be* realized can then be easily constructed from the benchmark event probabilities of exactly those benchmark events for which realizing constitutes the unique appropriate action. Thus, in our example we have

$$\overline{PGR} = \sum_{V \in \{V_{KQ}, V_{KS}, V_{SK}\}} \Pr(V|G) = \tfrac{0}{0.759} + \tfrac{0}{0.759} + \tfrac{0.101}{0.759} = 0.13,$$
$$\overline{PLR} = \sum_{V \in \{V_{KQ}, V_{KS}, V_{SK}\}} \Pr(V|L) = \tfrac{0.002}{0.241} + \tfrac{0.016}{0.241} + \tfrac{0.141}{0.241} = 0.66,$$

[31] For instance, given that $\Delta = 2$, $\Delta' = 3$ can be generated through various price appreciations of asset A vs. asset B between the two investment periods τ and τ', denoted $(a' - a, b' - b)$: $(1, 0)$, $(2, 1)$, $(3, 2)$, and $(4, 3)$. Although all these states represent "good news" in the sense that the likelihood for Nat's own asset being the better asset has increased since purchase (from 69% to 77%), in the state $(1, 0)$ his asset is in losses, while his asset is in gains in the other three states. In fact, for every possible Δ', we can calculate the probabilities with which this event generates a gain and loss state: e.g., conditional on A being the better asset, the corresponding mass of $\Delta' = 3$ of the four states $(1, 0)$, $(2, 1)$, $(3, 2)$, and $(4, 3)$ is 0.018, 0.110, 0.110, and 0.018 (e.g., the last one is equal to $4 * 0.55^4 * 0.45^3 * (1 - 0.45)$). Thus, the mass of $\Delta' = 3$ is 0.018 in losses and $0.110 * 2 + 0.018 = 0.238$ in gains.

[32] While these specific probabilities are derived under the condition that asset A follows F_h, we could equally assume that B follows F_h. For all our results, it is completely irrelevant which of the two is the "good" asset.

so that $\overline{DM}_1 = \overline{PGR} - \overline{PLR} = -0.53$ and $\overline{DM}_2 = \frac{\overline{PGR}}{\overline{PLR}} - 1 = -0.80$. Note that the benchmark disposition measure is negative. Proposition 1 below establishes that this holds in general, regardless of the parameterization of the stochastic environment and the investor's risk attitude (captured by choice rule f).

Decomposition. Finally, we we apply our decomposition of PGR and PLR to the example. Both PGR and PLR consist of their respective probabilities of benchmark events, each multiplied with the probability to realize an asset (i.e., switch or liquidate) given the benchmark event, i.e.,

$$\begin{aligned} PGR = \ \Pr(R|G) &= \sum_{V\in\{V_{KQ},V_{KS},V_{SK},V_{SQ}\}} \Pr(V|G) \times \Pr(R|V,G), \\ PLR = \ \Pr(R|L) &= \sum_{V\in\{V_{KQ},V_{KS},V_{SK},V_{SQ}\}} \Pr(V|L) \times \Pr(R|V,L), \end{aligned}$$

where $\Pr(R|V,D) = \Pr(S|V,D) + \Pr(Q|V,D)$ in domain $D \in \{G,L\}$. The first factor in the products represents the probability of a benchmark event and the second factor represents the probability to realize an asset given that benchmark event. While the former is influenced by the stochastic environment, the latter is determined by individual behavior.

Importantly, since K, S, and Q cover the entire action space, we have $\Pr(K|V,D) + \Pr(S|V,D) + \Pr(Q|V,D) = 1$. As a result, the (conditional) realization probability can be expressed solely in terms of *violation propensities* for all V. More specifically, $\Pr(R|V,D)$ can be expressed as $1 - \Pr(K|V,D)$ for $V \in \{V_{KQ}, V_{KS}, V_{SK}\}$, and as $\Pr(S|V,D) + \Pr(Q|V,D)$ for $V = V_{SQ}$. This yields

$$\begin{aligned} PGR &= \sum_{V\in\{V_{KQ},V_{KS},V_{SK}\}} \Pr(V|G)\Big(1 - \Pr(K|V,G)\Big) \\ &\quad + \Pr(V_{SQ}|G)\Big(\Pr(S|V_{SQ},G) + \Pr(Q|V_{SQ},G)\Big), \\ PLR &= \sum_{V\in\{V_{KQ},V_{KS},V_{SK}\}} \Pr(V|L)\Big(1 - \Pr(K|V,L)\Big) \\ &\quad + \Pr(V_{SQ}|L)\Big(\Pr(S|V_{SQ},L) + \Pr(Q|V_{SQ},L)\Big). \end{aligned}$$

Thus, our decomposition enables us to express PGR and PLR as sum over the products of all violation possibilities and their corresponding violation propensities. As defined in the previous section, we denote these violation propensities by a Greek letter referring to the action of the committed violation – i.e., κ for keep, σ for switch, and λ for liquidate – indexed with the benchmark on which it is conditioned, so that in domain $D \in \{G,L\}$ we have $\Pr(K|V,D) = \kappa_V^D$ for $V \in \{V_{KQ}, V_{KS}, V_{SK}\}$, $\Pr(S|V_{SQ},D) = \sigma_{SQ}^D$, and $\Pr(Q|V_{SQ},D) = \lambda_{SQ}^D$. Applying this decomposition to our example, we get an expression for Nat's PGR and PLR that depends on his individual benchmark violation propensities:

$$PGR = \tfrac{0}{0.759}(1-\kappa^G_{KQ}) + \tfrac{0}{0.759}(1-\kappa^G_{KS}) + \tfrac{0.101}{0.759}(1-\kappa^G_{SK}) + \tfrac{0.658}{0.759}(\sigma^G_{SQ}+\lambda^G_{SQ}),$$
$$PLR = \tfrac{0.002}{0.241}(1-\kappa^L_{KQ}) + \tfrac{0.016}{0.241}(1-\kappa^L_{KS}) + \tfrac{0.141}{0.241}(1-\kappa^L_{SK}) + \tfrac{0.082}{0.241}(\sigma^L_{SQ}+\lambda^L_{SQ}).$$

To summarize, in this section we illustrated our decomposition by means of an example: We assumed a specific parameterization of the stochastic environment and considered a risk-averse investor with a specific choice rule at time τ, namely $f(\Delta \geq 2) = A$, $f(\Delta \leq -2) = B$, and $f(-2 < \Delta < 2) = O$. Then, conditional on asset A following F_h and the exemplary realization of $\Delta = 2$, we decomposed the disposition measure into the induced probabilities of the various benchmark events and their violation propensities. These violation propensities remained unspecified, i.e., we allowed for any choice rule f' at time τ'. In contrast to this example, our general framework (see Section 2) allows for any parameterization of the stochastic environment as well as for any monotonically increasing choice rule f at time τ, is independent of which asset in fact follows which process, and probabilistically accounts for any possible realization of Δ. The following results are based on the general framework.

4 Results

This section presents model-independent results that we derive from our decomposition (see Definition 3). All formal proofs are relegated to Appendix A. Our focus is on analyzing the link between the disposition measure and benchmark violation propensities. Before turning to this link, we first investigate the benchmark disposition measure.

4.1 On the Benchmark Disposition Measure

Our first result helps reconcile our investigations with the previous literature by confirming the intuition that an investor should rather realize losses than gains.

Proposition 1 *The proportion of losses that should be realized exceeds the proportion of gains that should be realized, i.e.,* $\overline{PLR} > \overline{PGR}$.

Intuitively, the positive correlations of gains with good news and losses with bad news imply that a winner is more likely to have received good news than a loser. Therefore, keeping a winner is in expectation more attractive than keeping a loser. This is the case from the viewpoints of both FOSD and SORP, the latter because the likelihood to invest increases in informativeness for any given risk preference – and given a monotonically increasing choice rule for the initial purchase decision (see Section 2.3), informativeness increases in good news and decreases in comparable bad news.

Note that Proposition 1 holds for any parameterization of the stochastic environment, in particular for any $p_l, p_h \in (0,1)$ with $p_h > p_l$. As shown in the following corollary, $\overline{DM}_i$ does not converge when the price processes become similar.

Corollary 1 $\overline{PLR} > \overline{PGR}$ *still holds in the limit when* $p_l \to p_h$.

Corollary 1 shows that Proposition 1 is robust towards deteriorations of the "signal-to-noise ratio." With p_l and p_h being close, the observed difference in price appreciations constitutes a very noisy signal. As a result, the SORP benchmark becomes negligible for the benchmark disposition measure as the investor should either invest or not invest, but similarly so in both investment periods and without being sensitive to the observed price paths of the assets. In contrast, conditional on investing, the FOSD benchmark is still relevant, since a rational investor wants her asset to outperform the market (i.e., the other asset) and therefore should still invest in the asset that is more likely to be the "good" asset (i.e., the asset with more price appreciations). For FOSD, it is irrelevant how much more likely this is or how much better the asset quality is in expected terms. Even if the price processes are similar, there is variance in observed price paths which are used to infer what the "good" asset is.

Proposition 1 directly implies that observing a positive disposition measure, either in the sense of $DM_i > 0$ or $DM_i > \overline{DM}_i$, requires some benchmark violation. Vice versa, an investor who does not commit any benchmark violation cannot exhibit a positive disposition measure and has $DM_i = \overline{DM}_i < 0$. However, while the existence of benchmark violations is necessary for a positive disposition measure, it is not sufficient. Even if benchmark violations occur, the disposition measure may still be negative. Next, we closely examine various possible benchmark violation patterns and how they relate to the sign of the disposition measure.

4.2 On the Sign of the Disposition Measure

In light of Proposition 1, observing $DM_i > 0$ or $DM_i > \overline{DM}_i$ seems to suggest that the predominant benchmark violations are the conventional ones (see Definition 4), and the literature reflects this intuition. However, as we show in this section, this intuition is inaccurate. Throughout, we derive our results both for the rational benchmark (i.e., $DM_i > \overline{DM}_i$) as well as for the zero benchmark (i.e., $DM_i > 0$). We start by investigating the link between a positive disposition measure and the disposition effect.

Proposition 2

(i) Some conventional violation is necessary for the disposition measure to be positive, i.e., $DM_i > 0 \vee DM_i > \overline{DM}_i \implies \kappa^L_{KQ} + \kappa^L_{KS} + \kappa^L_{SK} + \sigma^G_{SQ} + \lambda^G_{SQ} > 0.$

(ii) *Conventional violations are not sufficient for the disposition measure to be positive, i.e.,* $\kappa^L_{KQ} + \kappa^L_{KS} + \kappa^L_{SK} + \sigma^G_{SQ} + \lambda^G_{SQ} > 0 \not\Rightarrow DM_i > 0 \vee DM_i > \overline{DM}_i$.

(iii) *For* $DM_i > 0$*, case* (ii) *holds even if no unconventional violations occur, i.e.,* $\kappa^L_{KQ} + \kappa^L_{KS} + \kappa^L_{SK} + \sigma^G_{SQ} + \lambda^G_{SQ} > 0 \wedge \kappa^G_{KQ} = \kappa^G_{KS} = \kappa^G_{SK} = \sigma^L_{SQ} = \lambda^L_{SQ} = 0 \not\Rightarrow DM_i > 0$.

Case (i) of Proposition 2 implies that every preference- or belief-based model that explains a positive disposition measure needs to generate at least *one* conventional violation, but certainly not all of them. Thus, the disposition effect is not necessary for $PGR > PLR$. According to case (ii), not even all conventional violations (i.e., the disposition effect) are sufficient for a positive disposition measure. As illustrated by case (iii), for the zero benchmark comparison this remains true even in the absence of any countervailing unconventional violation. Thus, even if an investor exhibits all conventional violations, and only those, it may still be the case that $DM_i < 0$. Intuitively, as the proportion of losses that should be realized is larger than the proportion of gains that should be realized (see Proposition 1), the propensities of conventional violations may simply be too low to induce $DM_i > 0$.[33]

Two additional insights underscore how fragile the link between the sign of the disposition measure and the disposition effect actually is. First, a positive disposition measure can arise even in the case of *uniform* violation propensities. Note that violation propensities are necessarily capped at 1/2 when uniform, because all action propensities of one benchmark state add up to one, and there are two possible violations in each benchmark state (one first- and one second-order violation).

Proposition 3 *Let all violation propensities be equal to* $\mu \in [0, 1/2]$*. Then,* $DM_i > 0 \iff \mu > 1/3$ *and* $DM_i > \overline{DM}_i \iff \mu > 0$.

Proposition 3 shows that no specific violation pattern is required for a positive disposition measure. In particular, benchmark violations do not need to be domain- or action-specific in order to induce a positive disposition measure.[34] The former is particularly surprising, as it shows that a difference in violation propensities between gains and losses is not needed to generate a positive disposition measure.

In the literature, there is a long-standing debate whether $PGR > PLR$ is caused by non-standard "preferences" (specifically, utility functions) or non-standard beliefs.

[33] For the robustness of Proposition 2, it is instructive to consider the *hypothetical* scenario where $\overline{PLR} = \overline{PGR}$. Here, cases (i) and (ii) still hold, but not case (iii): if unconventional violations are *entirely absent*, exhibiting *some* conventional violation – though not necessarily all – becomes necessary *and* sufficient for $DM_i > 0$. Thus, even in this *hypothetical* scenario, the disposition effect is not necessary for a positive disposition measure.

[34] Benchmark violations are not domain-specific whenever $\kappa^G_{KQ} = \kappa^L_{KQ}$, $\kappa^G_{KS} = \kappa^L_{KS}$, $\kappa^G_{SK} = \kappa^L_{SK}$, $\sigma^G_{SQ} = \sigma^L_{SQ}$, and $\lambda^G_{SQ} = \lambda^L_{SQ}$. Benchmark violations are not action-specific whenever $\kappa^G_{KQ} = \kappa^G_{KS} = \kappa^G_{SK} = \sigma^G_{SQ} = \lambda^G_{SQ}$ and $\kappa^L_{KQ} = \kappa^L_{KS} = \kappa^L_{SK} = \sigma^L_{SQ} = \lambda^L_{SQ}$.

Proposition 3 speaks to this debate in showing that identical violation propensities in gains and losses do not prevent the disposition measure from being positive. This insight enlarges the set of potential explanations for $PGR > PLR$, in particular by including many belief-based biases. To see this, note that preference-based explanations make investors sell an asset too early (late) *despite their rational belief* that its price will likely increase (decline), but because experiencing gains and losses triggers certain attitudes. In contrast, belief-based explanations make investors sell an asset too early (late) *because of their irrational belief* that its price will probably decline (increase). A major difference between these two explanations is that an investor with biased preferences distorts rational behavior based on gains or losses, but not upon good or bad news. An investor with biased beliefs has a biased reaction to good or bad news, but not to gains or losses per se. Thus, in contrast to biased preferences, an investor with biased beliefs makes benchmark violations that are *not* domain-specific, as gains and losses have no impact on her decision beyond providing different news.[35]

Consider, for instance, the belief-based bias of (over-)extrapolative expectations, which has been used to explain important financial phenomena other than a positive disposition measure (for an overview see Barberis, 2019). Extrapolative expectations are able to generate over-optimistic beliefs towards the owned asset after good news and over-pessimistic beliefs after bad news (Hartzmark et al., 2020). As a result, an investor with such a bias keeps her asset after good news, but this is also what she should do given that she purchased it in the first place, so that no benchmark violations occur after such news. In contrast, after (not too) bad news, the rational benchmark may still prescribe to keep the asset, but the over-pessimistic investor may violate this benchmark and realize too early. Since bad news correspond to a decreased likelihood that the owned asset is a "good" asset, such news can occur in both gains and losses. Thus, the investor has a propensity to realize too early not only in losses but also in gains, and this latter benchmark violation implies that extrapolative expectations indeed satisfy the necessary condition for $PGR > PLR$ (as specified in Proposition 2), despite being unable to generate a disposition effect.[36]

Our second result on the fragility of the link between a positive disposition measure and the disposition effect is even more surprising.

Proposition 4 *Let all unconventional violation propensities be equal to γ and all conventional violation propensities be equal to $\gamma - \nu$ (with $\nu > 0$). Then,*

$$DM_i > 0 \quad \Longleftrightarrow \quad \nu < \frac{(\overline{PLR}-\overline{PGR})(3\gamma-1)}{2+\overline{PLR}-2\overline{PGR}},$$

[35] In Maier and Fischer (2021), we provide an extensive analysis as to which kind of benchmark violations are induced by various preference- and belief-based behavioral biases.

[36] Just like for any violation pattern (including the disposition effect), whether the benchmark violations induced by over-extrapolative expectations are *sufficient* for $PGR > PLR$ depends on the parameterization of both the stochastic environment and the investor's (risk) preferences and beliefs.

which in turn implies $\nu < \gamma \in (1/3, 1/2]$. *Moreover,*

$$DM_1 > \overline{DM}_1 \iff \nu < \frac{(\overline{PLR}-\overline{PGR})3\gamma}{2+\overline{PLR}-2\overline{PGR}} \quad \textit{and} \quad DM_2 > \overline{DM}_2 \iff \nu < \frac{(\overline{PLR}-\overline{PGR})2\gamma}{\overline{PLR}(2-\overline{PGR})},$$

which in turn imply $\nu < \gamma \in (0, 1/2]$.

Proposition 4 shows that a positive disposition measure can arise even if unconventional violations are *more* prevalent than conventional ones, i.e., even if investors tend to realize *losers* too early and hold *winners* too long. Such a violation pattern reflects the *opposite* of the disposition effect. Note that the margin by which the unconventional violation propensities may exceed the conventional ones is capped by a threshold which increases in the former, i.e., γ. In other words, although unconventional violation propensities may exceed conventional ones, the latter still need to be sufficiently large to induce a positive disposition measure. However, the permissible positive difference between unconventional and conventional violation propensities increases in the prevalence of unconventional violations.

Propositions 2 to 4 suggest that we cannot learn much about benchmark violations from observing $DM_i > 0$ or $DM_i > \overline{DM}_i$. Such positive disposition measures can arise from various violation patterns, not just the disposition effect. Therefore, it is too restrictive to only consider models that predominantly generate conventional violations as potential explanations of an empirically observed positive disposition measure. One may be inclined to interpret these results as a critique of the commonly used disposition measure. However, our further results demonstrate that such an interpretation would be premature.

While our previous analysis shows that $DM_i > 0$ can hardly be attributed to specific violation patterns (such as the disposition effect), our next result shows that the commonly used disposition measure is indeed well suited to identify the presence of some *systematic* bias.

Proposition 5 *Suppose that investment decisions are independent of both the benchmark event* $V \in \{V_{KQ}, V_{KS}, V_{SK}, V_{SQ}\}$ *and the domain* $D \in \{G, L\}$, *i.e.,* $\Pr(K|V, D) = \Pr(K)$, $\Pr(S|V, D) = \Pr(S)$, *and* $\Pr(Q|V, D) = \Pr(Q)$. *Then,* $DM_i = 0$ *and* $DM_i > \overline{DM}_i$.

Proposition 5 shows that behavior which is independent of the rational benchmark as well as the domain of gains and losses yields the *neutral* result $DM_i = 0$. Thus, an investor who chooses her actions unconditionally – e.g., to keep the asset in any state of the world – will have $DM_i = 0$. Another example captured by Proposition 5 is *random* behavior: an investor who randomizes over the three possible actions keep, switch, and liquidate will have $DM_i = 0$.[37] Therefore, Proposition 5 shows that $DM_i > 0$ can

[37]For instance, uniform randomization over all possible actions would mean to choose K, S, and Q with probability 1/3 each. Or, sequential uniform randomization, i.e., first choosing to invest or not with

only be caused by *systematic* biases of investors. It cannot be caused by behavior that is independent of both the domain of gains and losses and the rational benchmark, such as purely random behavior. In this respect, the zero benchmark has a clear advantage over the rational benchmark, since such unsystematic benchmark violations still induce a positive measure $DM_i > \overline{DM}_i$.[38] Hence, when comparing DM_i to its rational benchmark value, it is important to additionally measure individual violation propensities in order to exclude the possibility that observing $DM_i > \overline{DM}_i$ is due to unsystematic rather than systematic biases.

4.3 On the Magnitude of the Disposition Measure

Propositions 2 to 5 are concerned with conclusions that *can and cannot* be drawn on violation patterns from observing a positive disposition measure. In this section, we show that our decomposition further allows us to derive new insights with respect to the comparative statics of the disposition measure. Note that the *magnitude* of the disposition measure has remained mostly unexplored so far. Instead, the literature has focused on the *sign* of the disposition measure alone (see Section 4.2).

Proposition 6

(i) *The disposition measure DM_i is increasing in conventional and decreasing in unconventional violation propensities.*

(ii) *Let all violation propensities be equal to $\mu \in [0, 1/2)$. Then, the disposition measure DM_i is increasing in μ.*

(iii) *Suppose $\varepsilon \in [0, \bar{\varepsilon})$ is uniformly added to all violation propensities, where $\bar{\varepsilon} = 1/2 - \max\{\kappa^D_{KQ}, \kappa^D_{KS}, \kappa^D_{SK}, \sigma^D_{SQ}, \lambda^D_{SQ}\}$ and $D \in \{G, L\}$. Then, the disposition measure DM_1 is increasing in ε, and DM_2 is increasing in ε if $\overline{PLR} > (<)\ 2/3 \wedge PGR > (<)\ PLR$.*

Case (*i*) of Proposition 6 is very intuitive and follows directly from our decomposition. It suggests that the larger the disposition measure is, the more likely is it caused by a disposition effect rather than the opposite violation pattern. However, as shown by cases (*ii*) and (*iii*), uniformly increasing all violation propensities increases the disposition measure as well. Thus, while case (*i*) suggests that the magnitude of the disposition measure may be indicative of the benchmark violations that are predominant, cases (*ii*) and (*iii*) show that such a conclusion *cannot* be drawn. In fact, a higher disposition measure may be caused by more conventional violations, less unconventional

probability 1/2 each and, in case of investment, to choose each risky asset with probability 1/2 would mean to choose K, S, and Q with probabilities 1/4, 1/4, and 1/2, respectively. Recall that the disposition measure is defined as an ex-ante expectation, so that $DM_i = 0$ is *theoretically* possible.

[38] This is due to the fact that $\overline{DM}_i < 0$ (see Proposition 1).

violations, or by more violations overall. Thus, it is unclear whether a higher disposition measure is caused by behavior that is more in line with the disposition effect or by behavior that is more mistaken overall.[39] Since $\overline{DM}_i$ is independent of violation propensities, Proposition 6 not only applies to DM_i, but also to $DM_i - \overline{DM}_i$ or $\frac{DM_i}{\overline{DM}_i} - 1$.

In contrast to the sign or the magnitude itself, in the remainder of this section we show that *variations in the magnitude* of the disposition measure turn out to be informative for violation patterns. We investigate comparative statics of DM_i with respect to three exogenous variables of our setup, namely g, τ, and n. As we explain below, these variables may be interpreted as capturing the market segment (e.g., "old" vs. "new" economy), how informed investors are (e.g., professionals vs. households), and how attentive investors are (e.g., frequency of account "log-ins"), respectively.[40]

Markets and Market Segments. First, we are interested in the effect of changing g, which denotes the minimum number of price appreciations within n periods for an asset to be in gains. Thus, the smaller $g \in \{1, 2, \ldots, n\}$ is, the higher are the expected returns of the market (for given p_h, p_l, and either u or d). Therefore, the comparison of small and large g markets or market segments may be considered as a proxy for the comparison of emerging and mature markets, or "startup" and "blue chip" segments, respectively.

Lemma 1 *Fix some $\overline{q} \in [1/2, 1)$. Then, the proportions of gains and losses that should be realized (i.e., $\overline{PGR}$ and $\overline{PLR}$, respectively) decrease in g.*

Lemma 1 shows that a larger g decreases both $\overline{PGR}$ and $\overline{PLR}$. Intuitively, the likelihood of unpleasant surprises that would force a rational investor to sell a previously attractive stock is lower in mature than emerging markets. The reason for this in our model, and plausibly also in reality, is that gains are a stronger signal of good asset quality in mature than emerging markets (i.e., for larger g), so that a rational investor is less prone to realizing gains in mature than emerging markets. On the other hand, losses are a stronger signal of bad asset quality in emerging than mature markets, so that a rational investor is also less prone to realizing losses in mature than emerging markets.

Using our decomposition, we analyze how differences in g affect the disposition measure.

[39] Note that the upper bound of DM_i (which is 1 for DM_1 and ∞ for DM_2) is only reached when the investor makes as many conventional violations as possible and no unconventional violations at all, whereas an investor who makes both conventional and unconventional violations to the fullest extent has a positive DM_i below this upper bound, namely $DM_1 = \overline{PLR} - \overline{PGR} = -\overline{DM}$ and $DM_2 = \frac{\overline{PLR} - \overline{PGR}}{1 - \overline{PLR}}$.

[40] Note that the comparative statics with respect to g and n constitute a "partial" analysis by fixing some $\overline{q} \in [1/2, 1)$. For risk neutral investors and positive expected values of the stochastic processes this restriction is irrelevant as these investors always invest in a risky asset. For risk averse investors it implies that we neglect possible indirect effects on investors' investment thresholds (see Section 2.3).

Proposition 7 *Fix some* $\overline{q} \in [1/2, 1)$. *Let all unconventional violation propensities be equal to* $\gamma \in (0, 1/2]$ *and all conventional violation propensities be equal to* $\gamma - \nu \in (0, 1/2]$. *Suppose that* $k\nu = 3\gamma - 1$ *with* $k \in \{1, 2\}$. *Then, the disposition measure* DM_i *is increasing (decreasing) in* g *if and only if* $\nu < (>) \ 0$.

Proposition 7 shows that the effect of a larger g on the disposition measure depends on the prevalent violation pattern. Suppose, for instance, that the disposition effect is present, so that conventional violation propensities are $1/2$ and unconventional violation propensities are $1/4$. Then, as shown in Proposition 7, DM_i is increasing in g. Hence, we expect an investor who suffers from the disposition effect to exhibit a larger (smaller) disposition measure in the more conservative (aggressive) fraction of his portfolio. Conversely, an investor who suffers from an opposite disposition effect, e.g., an investor whose conventional violation propensities are $1/4$ and unconventional violation propensities are $1/2$, exhibits a smaller disposition measure in markets or market segments with larger g. Hence, we expect an investor suffering from an opposite disposition effect to exhibit a larger (smaller) disposition measure in his more aggressive (conservative) sub-portfolio.

Therefore, Proposition 7 shows two things. First, it shows that the disposition measure of an investor who suffers from sufficiently high violation propensities systematically differs between sub-portfolios that differ in expected returns. Conditional on segmenting an investor's portfolio, Proposition 7 therefore yields a novel testable prediction. Second, since the direction of this comparative static depends on the prevalent violation pattern, Proposition 7 allows to infer whether an investor suffers from the disposition effect (or its opposite) from differences in the disposition measure between sub-portfolios.

Information Level. Our next investigation concerns differences with respect to how informed initial investment decisions are, which is captured by τ in our model.

Lemma 2 *For "large* τ*," i.e.,* $\tau \to \infty$ *and* $n \in \mathbb{N}$*, we obtain* $\overline{PGR} = 0$ *and* $\overline{PLR} = 0$.

Lemma 2 shows that varying τ matters for the rational benchmark. For "large τ," the investor receives plenty of information already before the buying decision in period τ and is therefore well informed when initially investing. Such a well-informed investor is less likely to receive contradicting information to change her mind after the initial purchase, so that she should not realize her asset in period τ', independently of whether she is in gains or losses. Thus, $\overline{PGR}$ and $\overline{PLR}$ converge to zero. Intuitively, the better informed a rational investor is prior to purchasing a stock – e.g., because she has tracked the stock for a longer time or has engaged in extensive research on the stock – the longer she will keep it.

Given the effects on the rational benchmark stated in Lemma 2, we can use our decomposition to further analyze how "large τ" affects the disposition measure.

Proposition 8 *In domain $D \in \{G, L\}$, let keep violation propensities be uniform across benchmark events, i.e., $\kappa^D_{KQ} = \kappa^D_{KS} = \kappa^D_{SK} =: \kappa^D \in [0,1]$, and define ρ^D as the realization violation propensity $\sigma^D_{SQ} + \lambda^D_{SQ} =: \rho^D \in (0,1]$. Then, for "large τ," the disposition measures DM_1 and DM_2 respectively converge to $\rho^G - \rho^L$ and $(\rho^G/\rho^L) - 1$ from below (above) if benchmark violation propensities are sufficiently larger (smaller) in gains than losses, i.e., if $\kappa^G + \rho^G \geq (\leq)\ 1 \geq (\leq)\ \kappa^L + \rho^L$ with one of the inequalities being strict.*

Recall that for "large τ," Lemma 2 shows that in both gains and losses the investor should not realize but rather keep her asset. Thus, her κ^D propensity, i.e., to keep the asset when she should realize it, becomes negligible for the proportions of gains and losses realized, i.e., PGR and PLR, respectively. On the other hand, her ρ^D propensity, i.e., to realize the asset when she should keep it, becomes crucial in determining PGR and PLR, so that her disposition measure $DM_1 = PGR - PLR$ converges to the between-domain difference of this violation propensity, namely $\rho^G - \rho^L$. Likewise, her disposition measure $DM_2 = \frac{PGR}{PLR} - 1$ converges to the between-domain ratio of this violation propensity, namely $(\rho^G/\rho^L) - 1$. Thus, in the limit the magnitude of DM_i *exactly* measures the difference or ratio of this violation propensity between gains and losses. Since both κ^D and ρ^D determine DM_i outside the limit, their domain-specific sum determines whether this limit value of DM_i is approached from below or above.

Thus, similar to before, Proposition 8 shows that the effect "large τ" has on the disposition measure depends on the violation pattern. However, in contrast to Proposition 7 above, here the decisive factor is not whether the disposition effect or its opposite is present, but rather whether benchmark violation propensities are sufficiently larger in gains than losses. An investor who is better informed when purchasing a stock has a smaller (larger) disposition measure if she is sufficiently more (less) prone to benchmark violations in losses than gains. Investors who are arguably well informed are professional traders, who typically tend to have lower disposition measures (Brown et al., 2006; Barber et al., 2007; Chen et al., 2007; Choe and Eom, 2009; Calvet et al., 2009). A standard explanation for this pattern is that professionals are less prone to biases and therefore exhibit less benchmark violations, which is very plausible indeed. Proposition 8 offers a complementary explanation that seems plausible, too: even if professionals are subject to the same behavioral bias and therefore make the same benchmark violations as non-professionals, and provided that losses induce sufficiently more benchmark violations than gains, Proposition 8 predicts that professionals are still expected to have lower disposition measures than "household" investors, simply because they are better informed.

Also, note that the above prediction of Proposition 8 is reversed for gains inducing sufficiently more benchmark violations than losses. Thus, conditional on observing how well informed an investor is prior to purchasing a stock – which may, for instance, be approximated by monitoring the investor's "watch list" – Proposition 8 further yields a novel testable prediction and allows to infer whether benchmark violations are more prevalent in gains or losses from differences in the magnitude of the disposition measure.

Financial Attention. Our final investigation considers variations of n, i.e., the duration between trading decisions. We interpret n as a proxy for how attentive or curious an investor is. It may, for instance, be measured by the frequency of "log-ins" into one's trading account (Karlsson et al., 2009; Gherzi et al., 2014; Sicherman et al., 2016; Olafsson and Pagel, 2018; Dierick et al., 2019).

Lemma 3 *Fix some* $\overline{q} \in [1/2, 1)$. *Then, for "large n," i.e.,* $n \to \infty$, $\tau \in \mathbb{N}$, *and* $p_l < \frac{g}{n} < p_h$, *we obtain* $\overline{PGR} = 0$ *and* $\overline{PLR} = 1$, *so that* $\overline{DM}_i = -1$.

For "large n," the investor receives plenty of information between the two trading decisions. The condition $p_l < \frac{g}{n} < p_h$ assures that the high process F_h generates gains in expectation whereas the low process F_l is expected to generate losses. Thus, for "large n," the rational investor will become confident to own a "good" asset when she is in gains and to own a "bad" asset when she is in losses. If, on the other hand, the investor is very attentive or curious, so that n is small, she is less informed at her selling decision and therefore less confident to own a "good" asset in gains and to own a "bad" asset in losses. Hence, for "large n," the investor should not realize her asset in gains, but should do so in losses. As a result, $\overline{PGR}$ converges to zero and $\overline{PLR}$ converges to one, so $\overline{DM}_i$ converges to its lower bound accordingly.

Lemma 3 has implications for the interpretation of experimental results. In experiments where subjects learn more between trading decisions, it becomes less likely to observe a positive disposition measure since a lower benchmark measure $\overline{DM}_i$ requires subjects to make more (conventional) violations to generate $DM_i > 0$. On the one hand, this insight may explain why some experimental studies find $PGR > PLR$ while others do not. On the other hand, it suggests that in experimental studies it may be more meaningful to compare DM_i to $\overline{DM}_i$ rather than to zero (see Section 2.4).

Given the effects on the rational benchmark stated in Lemma 3, we can again use our decomposition to analyze how the disposition measure is affected.

Proposition 9 *Fix some* $\overline{q} \in [1/2, 1)$. *In domain* $D \in \{G, L\}$, *let keep violation propensities be uniform across benchmark events, i.e.,* $\kappa^D_{KQ} = \kappa^D_{KS} = \kappa^D_{SK} =: \kappa^D \in [0, 1)$, *and define* ρ^D *as the realization violation propensity* $\sigma^D_{SQ} + \lambda^D_{SQ} =: \rho^D \in (0, 1]$. *Then, for "large n," the disposition measures* DM_1 *and* DM_2 *respectively converge to* $\kappa^L + \rho^G - 1$ *and* $(\rho^G/1-\kappa^L) - 1$ *from below (above) if benchmark violation propensities in gains and losses are sufficiently large (small), i.e., if* $\kappa^G + \rho^G \geq (\leq)\ 1 \wedge \kappa^L + \rho^L \geq (\leq)\ 1$ *with one of the inequalities being strict.*

Recall from Lemma 3 that for "large n" the investor should not realize a gain but should realize a loss. Thus, her proportion of gains realized is solely determined by her ρ^G

propensity, i.e., to realize the asset in gains when she should keep it, so that $PGR = \rho^G$. Likewise, her proportion of losses realized is solely determined by her κ^L propensity, i.e., to keep the asset in losses when she should realize it, so that $PLR = 1 - \kappa^L$. Hence, the disposition measure $DM_1 = PGR - PLR$ converges to $\rho^G - (1 - \kappa^L)$ and $DM_2 = \frac{PGR}{PLR} - 1$ converges to $(\rho^G/1-\kappa^L) - 1$ in the limit. Note that this limit value only depends on conventional violation propensities. Thus, if *full information* was obtained prior to the selling decision, there would indeed be a strong link between a positive disposition measure and the disposition effect, i.e., the latter becomes a necessary condition for the former in the limit. Intuitively, "large n" identifies all gains with a benchmark action of keeping and all losses with a benchmark action of realizing the asset, so that the standard interpretation of a positive disposition measure as disposition effect becomes in fact accurate. However, the violation propensities that are negligible in the limit have an effect outside the limit. And as shown in Proposition 9, the non-limit value of the disposition measure is lower (higher) than its limit value if benchmark violations in gains *and* losses are sufficiently prevalent (rare).

Proposition 9 clarifies that more attention or curiosity, i.e., a shorter duration between trading decisions, may increase or decrease the disposition measure. Intuitively, waiting for a shorter period until taking a trading decision on the one hand allows for quicker adjustments so that "mistakes" have a smaller impact, but on the other hand implies that decisions are taken in response to shorter horizons so that they are less informed. Interestingly, the proposition shows that it depends on the investor's overall prevalence of benchmark violations whether the disposition measure is increased or decreased. If overall benchmark violations are sufficiently rare, as may be the case for professional investors,[41] more attention or curiosity increases the disposition measure. Here, the disadvantage of less informed trading decisions outweighs the advantage of a smaller impact of "mistakes" as they are rare anyway. On the other hand, if benchmark violations are sufficiently prevalent, as may be the case for "household" or non-professional investors, more attention or curiosity decreases the disposition measure. In this case, the advantage of a smaller impact of "mistakes" becomes dominant and outweighs the disadvantage of less informed decisions. Thus, Proposition 9 offers a novel testable prediction, where the direction of the effect that more or less attention has on the disposition measure depends on the group of investors. Indeed, in line with Proposition 9, Dierick et al. (2019) find in a dataset covering retail (i.e., non-professional) investors that more attention (measured by account "log-ins") significantly reduces the disposition measure.

[41] Note that this is the standard assumption for why professional investors exhibit lower disposition measures, and is complementary to our above explanation of better informed buying decisions of professional investors.

The fact that more attention or curiosity leads to a higher (lower) disposition measure if benchmark violations are sufficiently rare (prevalent) demonstrates that DM_i is *not* a measure of "mistakes." For given violation propensities, attention affects DM_i because it changes the distribution of benchmark events.[42] While DM_i cannot identify the overall incidence of "mistakes," changes in the magnitude of DM_i can identify it since "low error" and "high error" types imply opposite signs of DM_i's comparative static with respect to attention. Thus, observing different magnitudes of DM_i with varying attention allows to infer whether benchmark violations in gains and losses are rare or prevalent.

5 Conclusion

In this paper we theoretically investigate the link between the disposition effect, i.e., investors' tendency to sell winning assets too early and losing assets too late, and its common empirical measure, namely a positive difference between the proportion or probability of gains realized (PGR) and losses realized (PLR). While the standard interpretation of $PGR > PLR$ as a disposition effect apparently confuses the sign of the stock movement with a signal of stock quality, we propose a novel setup that takes the overall market environment into account and thereby enables a separation of gains from good news (i.e., news that increase the likelihood that the owned asset is expected to outperform the market) and losses from bad news. Our setup explicitly establishes a rational benchmark based on first-order stochastic dominance and individual-specific risk preferences, which prescribes *whether* and *how* individuals should invest in any state of the world. This benchmark allows us to decompose PGR and PLR into frequencies of various benchmark violations, where each such frequency is the product of two components: first, the probability of a benchmark event prescribing the appropriate action and, second, the conditional probability of choosing a different action, i.e., of violating the benchmark. The clean separation of the former from the latter provides the basis for our theoretical investigation.

We uncover a surprising disconnect: The disposition effect is neither necessary nor sufficient for $PGR > PLR$. Even investors with an opposite disposition effect (i.e., a tendency to hold *winners* too long and sell *losers* too early) can still exhibit $PGR > PLR$. Also, investors whose benchmark violation pattern is neither action- nor domain-specific may still cause $PGR > PLR$. While these *model-independent* re-

[42]Our decomposition shows that with benchmark violations being rare, DM_1 and DM_2 are mainly determined by $\overline{PGR} - \overline{PLR}$ and $\frac{\overline{PGR}}{\overline{PLR}} - 1$, respectively. Thus, DM_i is increasing (decreasing) with more (less) attention by Lemma 3. In contrast, with benchmark violations being prevalent, DM_1 and DM_2 are mainly determined by $1 - \overline{PGR} - (1 - \overline{PLR})$ and $\frac{1-\overline{PGR}}{1-\overline{PLR}} - 1$, respectively. Hence, in this case Lemma 3 implies that DM_i is decreasing (increasing) with more (less) attention.

sults suggest that many violation patterns other than the disposition effect are able to generate $PGR > PLR$, we further show that unsystematic benchmark violations (such as randomization) are not, so that only *systematic* biases can give rise to the empirical observation $PGR > PLR$.

Just like the *sign* of the disposition measure, a comparative statics analysis reveals that the *magnitude* of $PGR - PLR$ or $\frac{PGR}{PLR} - 1$ is also not informative for the predominant violation pattern or the overall incidence of "mistakes." Instead, *variations* in the magnitude *do* turn out to be informative for the predominant violation pattern, although this metric has remained unexplored so far. Our comparative statics analysis further generates novel testable predictions of how these magnitudes are expected to change in response to variations in the market or market segment (e.g., "old" vs. "new" economy), investors' information level (e.g., professionals vs. households), and investors' financial attention (e.g., frequency of account "log-ins"). The latter variation provides two additional insights: First, investors who are able to learn more between trading decisions are expected to have lower (benchmark) disposition measures and are therefore less likely to exhibit $PGR > PLR$, which may partly explain the large variance of experimental results regarding the disposition measure. Second, the standard interpretation of $PGR > PLR$ as a disposition effect turns out to be accurate in the *hypothetical* full-information limit, when the investor knows with certainty that a winner will outperform the market and a loser will underperform. In this limit, the sign of the stock movement becomes a perfect signal of stock quality.

Our paper should *not* be understood as a critique of the commonly used disposition measure, but rather suggests that caution is warranted regarding its interpretation. While $PGR > PLR$ identifies the existence of systematically biased behavior, neither the sign nor the magnitude of the disposition measure is suited to identify the specific way this behavior is biased. However, variations in the magnitude of the disposition measure are in fact informative in this respect. Thus, our theoretical results suggest new ways of using $PGR - PLR$ or $\frac{PGR}{PLR} - 1$ to better understand what is causing $PGR > PLR$ in terms of rational benchmark violations. This is an important task for future empirical research, as it ultimately allows to narrow down the underlying psychological mechanism (or behavioral bias) at work.

A Appendix: Proofs

Proof. [Proposition 1] This proof relies on the following lemma:

Lemma 4 *Let $(\alpha_i)_{i\in\{0,\dots,n\}}$ and $(\beta_i)_{i\in\{0,\dots,n\}}$ be positive sequences such that $\frac{\alpha_i}{\beta_i}$ is strictly decreasing in i. Then,*

$$\forall\, k \in \{0,\dots,n-1\} : \frac{\sum_{i=0}^{k}\alpha_i}{\sum_{i=0}^{k}\beta_i} > \frac{\sum_{i=k+1}^{n}\alpha_i}{\sum_{i=k+1}^{n}\beta_i}.$$

Proof. [Lemma 4] Since $\frac{\alpha_i}{\beta_i}$ is strictly decreasing in i, we know that

$$\frac{\alpha_i}{\beta_i} > \frac{\alpha_{k+1}}{\beta_{k+1}}\ \forall\, i \in \{0,\dots,k\}, \tag{1}$$

and that

$$\frac{\alpha_{k+1}}{\beta_{k+1}} \geq \frac{\alpha_i}{\beta_i}\ \forall\, i \in \{k+1,\dots,n\}. \tag{2}$$

Since (1) is equivalent to $\alpha_i\beta_{k+1} > \beta_i\alpha_{k+1}\ \forall\, i \in \{0,\dots,k\}$, it implies that

$$\sum_{i=0}^{k}\alpha_i\beta_{k+1} > \sum_{i=0}^{k}\beta_i\alpha_{k+1} \iff \frac{\sum_{i=0}^{k}\alpha_i}{\sum_{i=0}^{k}\beta_i} > \frac{\alpha_{k+1}}{\beta_{k+1}}.$$

Since (2) is equivalent to $\beta_i\alpha_{k+1} \geq \alpha_i\beta_{k+1}\ \forall\, i \in \{k+1,\dots,n\}$, it implies that

$$\sum_{i=k+1}^{n}\beta_i\alpha_{k+1} \geq \sum_{i=k+1}^{n}\alpha_i\beta_{k+1} \iff \frac{\alpha_{k+1}}{\beta_{k+1}} \geq \frac{\sum_{i=k+1}^{n}\alpha_i}{\sum_{i=k+1}^{n}\beta_i}.$$

Thus, (1) and (2) together imply that

$$\frac{\sum_{i=0}^{k}\alpha_i}{\sum_{i=0}^{k}\beta_i} > \frac{\sum_{i=k+1}^{n}\alpha_i}{\sum_{i=k+1}^{n}\beta_i}.$$

Note that for the strict inequality result in Lemma 4 to hold it is actually sufficient that sequence is decreasing with only one strict inequality. ■

This proof of Proposition 1 builds upon Appendix B, where we explicitly state the benchmark disposition measure. The proof works similarly both for $\theta > 0$ and $\theta = 0$. For brevity, in the following we focus on $\theta > 0$ only. The case of $\theta = 0$ can be shown analogously, but requires more notation due to the initial randomization for uninformative priors at $t = \tau$.

Note first that we can rewrite $\overline{DM}_i$ as

$$\overline{DM}_i = \frac{\sum_{k=\theta}^{\tau}\sum_{l=0}^{k-\theta}\Pr(a=k,b=l)(\overline{DM}_i|a=k,b=l)}{\sum_{k=\theta}^{\tau}\sum_{l=0}^{k-\theta}\Pr(a=k,b=l)+\sum_{k=\theta}^{\tau}\sum_{l=0}^{k-\theta}\Pr(b=k,a=l)} + \frac{\sum_{k=\theta}^{\tau}\sum_{l=0}^{k-\theta}\Pr(b=k,a=l)(\overline{DM}_i|b=k,a=l)}{\sum_{k=\theta}^{\tau}\sum_{l=0}^{k-\theta}\Pr(a=k,b=l)+\sum_{k=\theta}^{\tau}\sum_{l=0}^{k-\theta}\Pr(b=k,a=l)}$$

Thus, to prove that $\overline{DM}_i < 0$, it is sufficient to prove that $(\overline{DM}_i|a=k,b=l) < 0$ and $(\overline{DM}_i|b=k,a=l) < 0$ for all values of k and l satisfying $|k-l| \geq \theta$. The double sums over k and l specify an asset price combination at the initial purchase decision at $t=\tau$, and the boundaries are chosen such that only those combinations are considered where a risky asset is purchased, given the risk preference represented by θ. These sums are always non-empty:

$\theta \leq \tau$ is needed for the investor to buy an asset in some contingency, which is of course necessary for the disposition measure to exist; $k - \theta \geq 0$ is implied by $k \geq \theta$ by definition of the first sum.

In the following, we will prove that $(\overline{DM}_i | a = k, b = l) < 0$. By "swapping" places of a and b, the proof for $(\overline{DM}_i | b = k, a = l) < 0$ works similarly and is therefore omitted. Note that

$$
\begin{aligned}
(\overline{DM}_i | a = k, b = l) < 0 \iff & (\overline{PGR} | a = k, b = l) \quad < (\overline{PLR} | a = k, b = l) \\
\iff & \frac{(\overline{NPGR} | a = k, b = l)}{(\overline{DPGR} | a = k, b = l)} < \frac{(\overline{NPLR} | a = k, b = l)}{(\overline{DPLR} | a = k, b = l)},
\end{aligned}
$$

where

$$
\begin{aligned}
(\overline{NPGR} | a = k, b = l) :=& \sum_{m=k+g}^{k+n} \sum_{j=m-\theta+1}^{l+n} \Pr\left(a' = m, b' = j | a = k, b = l\right) \\
=& \sum_{m=k+g}^{k+n} \Pr\left(a' = m | a = k, b = l\right) \sum_{j=m-\theta+1}^{l+n} \Pr\left(b' = j | a = k, b = l\right), \\
(\overline{DPGR} | a = k, b = l) :=& \sum_{m=k+g}^{k+n} \sum_{j=l}^{l+n} \Pr\left(a' = m, b' = j | a = k, b = l\right) \\
=& \sum_{m=k+g}^{k+n} \Pr\left(a' = m | a = k, b = l\right) \sum_{j=l}^{l+n} \Pr\left(b' = j | a = k, b = l\right), \\
(\overline{NPLR} | a = k, b = l) :=& \sum_{m=k}^{k+g-1} \sum_{j=m-\theta+1}^{l+n} \Pr\left(a' = m, b' = j | a = k, b = l\right) \\
=& \sum_{m=k}^{k+g-1} \Pr\left(a' = m | a = k, b = l\right) \sum_{j=m-\theta+1}^{l+n} \Pr\left(b' = j | a = k, b = l\right), \\
(\overline{DPLR} | a = k, b = l) :=& \sum_{m=k}^{k+g-1} \sum_{j=l}^{l+n} \Pr\left(a' = m, b' = j | a = k, b = l\right) \\
=& \sum_{m=k}^{k+g-1} \Pr\left(a' = m | a = k, b = l\right) \sum_{j=l}^{l+n} \Pr\left(b' = j | a = k, b = l\right).
\end{aligned}
$$

In these expressions, the first sum pins down whether the own asset is in gains or losses, i.e., it specifies the number of price increases of the own asset between periods τ and τ' as bigger or smaller than g. The second sum collects the states where the own asset should be realized in the numerators, and all possible realizations of "the other" asset in the denominators. An asset should be realized, i.e., sold, whenever it is first- or second-order dominated, i.e., whenever the overall difference in the number of ups of the own and the other asset is negative or below the investment threshold θ. Note that the first sum is always non-empty, whereas the second sum is empty for "big" m: $k + n \geq k + g$ follows from $n \geq g$; $m - \theta + 1 > l + n$ for $m = k + n$ (upper bound in gains) as $k - \theta \geq l$ by definition of the second sum, and $m - \theta + 1 \leq l + n$ for $m = k$ (lower bound in losses) as $k \leq \tau < n$. The equalities follow because the price processes are independent. Note that the probabilities in these expressions can be calculated explicitly by means of the binomial processes. Precisely,

$$\begin{aligned}
\Pr\left(a' = m, b' = j | a = k, b = l\right) &= \binom{n}{m-k}\binom{n}{j-l} p_A^{m-k}(1-p_A)^{n-m+k} p_B^{j-l}(1-p_B)^{n-j+l}, \\
\Pr\left(a' = m | a = k, b = l\right) &= \binom{n}{m-k} p_A^{m-k}(1-p_A)^{n-m+k}, \\
\Pr\left(b' = j | a = k, b = l\right) &= \binom{n}{j-l} p_B^{j-l}(1-p_B)^{n-j+l},
\end{aligned}$$

where $p_A = p_h$, $p_B = p_l$ if A follows F_h and $p_A = p_l$, $p_B = p_h$ if B follows F_h. Importantly, however, this proof does not hinge on the explicit values of these probabilities.

Next, we define $(\alpha_m | a = k, b = l)$ and $(\beta_m | a = k, b = l)$ as follows:

$$\begin{aligned}
(\alpha_m | a = k, b = l) &:= \sum_{j=m-\theta+1}^{l+n} \Pr\left(a' = m, b' = j | a = k, b = l\right) \\
&= \Pr\left(a' = m | a = k, b = l\right) \sum_{j=m-\theta+1}^{l+n} \Pr\left(b' = j | a = k, b = l\right), \qquad (3) \\
(\beta_m | a = k, b = l) &:= \sum_{j=l}^{l+n} \Pr\left(a' = m, b' = j | a = k, b = l\right) \\
&= \Pr\left(a' = m | a = k, b = l\right) \sum_{j=l}^{l+n} \Pr\left(b' = j | a = k, b = l\right). \qquad (4)
\end{aligned}$$

As above, the equalities follow because the price processes are independent. Then, $\frac{(\alpha_m | a=k, b=l)}{(\beta_m | a=k, b=l)} \leq 1$ is decreasing in m as the denominator of the reduced fraction is in fact independent of m, and the numerator shrinks in m as the sum gets smaller. In fact, $\frac{(\alpha_m | a=k, b=l)}{(\beta_m | a=k, b=l)}$ is strictly decreasing until the sum in the numerator gets empty, and then remains equal zero. Note that the sequences $(\alpha_m | a = k, b = l)$ and $(\beta_m | a = k, b = l)$ are not at all monotonic, only their ratios are. This, however, allows us to apply Lemma 4 to these ratios.

Lemma 4 implies that

$$\frac{\sum_{m=k}^{k+g-1} (\alpha_m | a = k, b = l)}{\sum_{m=k}^{k+g-1} (\beta_m | a = k, b = l)} > \frac{\sum_{m=k+g}^{k+n} (\alpha_m | a = k, b = l)}{\sum_{m=k+g}^{k+n} (\beta_m | a = k, b = l)}. \qquad (5)$$

Plugging (3) and (4) into (5) yields

$$\begin{aligned}
& \frac{\sum_{m=k}^{k+g-1} \Pr\left(a' = m | a = k, b = l\right) \sum_{j=m-\theta+1}^{l+n} \Pr\left(b' = j | a = k, b = l\right)}{\sum_{m=k}^{k+g-1} \Pr\left(a' = m | a = k, b = l\right) \sum_{j=l}^{l+n} \Pr\left(b' = j | a = k, b = l\right)} \\
> \quad & \frac{\sum_{m=k+g}^{k+n} \Pr\left(a' = m | a = k, b = l\right) \sum_{j=m-\theta+1}^{l+n} \Pr\left(b' = j | a = k, b = l\right)}{\sum_{m=k+g}^{k+n} \Pr\left(a' = m | a = k, b = l\right) \sum_{j=l}^{l+n} \Pr\left(b' = j | a = k, b = l\right)} \\
\iff \quad & \frac{(\overline{NPLR} | a = k, b = l)}{(\overline{DPLR} | a = k, b = l)} > \frac{(\overline{NPGR} | a = k, b = l)}{(\overline{DPGR} | a = k, b = l)} \\
\iff \quad & (\overline{PLR} | a = k, b = l) > (\overline{PGR} | a = k, b = l) \\
\iff \quad & (\overline{DM}_i | a = k, b = l) < 0
\end{aligned}$$

■

Proof. [Corollary 1] This proof directly follows from the proof of Proposition 1. Letting $p_l \to p_h$ implies that $\theta = 0$, given that the investor initially invests at time τ. Since the proof of Proposition 1 does not hinge on the explicit value of $\Pr(a' = m, b' = j | a = k, b = l)$, it also holds for $p_l \to p_h$. ■

Proof. [Proposition 2] We prove each case of Proposition 2 separately, and always start with $DM_i > 0$ followed by $DM_i > \overline{DM}_i$:

(i) Let $DM_i > 0$. Assume $\kappa^L_{KQ} + \kappa^L_{KS} + \sigma^G_{SQ} + \lambda^G_{SQ} + \kappa^L_{SK} = 0$ in contra-position. As all violation propensities are non-negative, this is equivalent to $\kappa^L_{KQ} = \kappa^L_{KS} = \sigma^G_{SQ} = \lambda^G_{SQ} = \kappa^L_{SK} = 0$. Then, by our decomposition, $DM_i > 0$ is equivalent to

$$\begin{aligned} & PGV_{KQ}(1-\kappa^G_{KQ}) + PGV_{KS}(1-\kappa^G_{KS}) + PGV_{SK}(1-\kappa^G_{SK}) \\ > & PLV_{KQ} + PLV_{KS} + PLV_{SQ}(\sigma^L_{SQ} + \lambda^L_{SQ}) + PLV_{SK} \end{aligned}$$

which, in turn, implies

$$\begin{aligned} \overline{PGR} = & PGV_{KQ} + PGV_{KS} + PGV_{SK} \\ \geq & PGV_{KQ}(1-\kappa^G_{KQ}) + PGV_{KS}(1-\kappa^G_{KS}) + PGV_{SK}(1-\kappa^G_{SK}) \\ > & PLV_{KQ} + PLV_{KS} + PLV_{SQ}(\sigma^L_{SQ} + \lambda^L_{SQ}) + PLV_{SK} \\ = & \overline{PLR} + PLV_{SQ}(\sigma^L_{SQ} + \lambda^L_{SQ}) \\ \geq & \overline{PLR}. \end{aligned}$$

This is a contradiction to $\overline{PGR} < \overline{PLR}$, i.e., to Proposition 1. Therefore, the assumption was wrong and the conjecture holds.

Now, let $DM_i > \overline{DM}_i$ and note that above we established that $\overline{PGR} \geq PGR$ and $\overline{PLR} \leq PLR$. The latter contradicts the former because $DM_1 > \overline{DM}_1 \iff PGR - \overline{PGR} > PLR - \overline{PLR}$ and $DM_2 > \overline{DM}_2 \iff {PGR}/{\overline{PGR}} > {PLR}/{\overline{PLR}}$. Thus, the assumption was wrong and the conjecture holds.

(ii) For $DM_i > 0$ this is a direct implication of case (iii): DM_i decreases in unconventional violation propensities by Proposition 6, so this case is less restrictive than case (iii).

For $DM_i > \overline{DM}_i$, let $\kappa^L_{KQ} + \kappa^L_{KS} + \sigma^G_{SQ} + \lambda^G_{SQ} + \kappa^L_{SK} > 0$, i.e., at least one of the conventional violation propensities has to be strictly positive. Further, let all unconventional violation propensities be equal to $\gamma \in (0, 1/2]$, i.e., $\kappa^G_{KQ} = \kappa^G_{KS} = \sigma^L_{SQ} = \lambda^L_{SQ} = \kappa^G_{SK} = \gamma$. We first prove the proposition for $DM_1 - \overline{DM}_1$. Let

$$\max\{\kappa^L_{KQ}, \kappa^L_{KS}, \sigma^G_{SQ}, \lambda^G_{SQ}, \kappa^L_{SK}\} \leq \gamma \frac{\overline{PGR} + 2(1-\overline{PLR})}{\overline{PLR} + 2(1-\overline{PGR})},$$

where $\frac{\overline{PGR}+2(1-\overline{PLR})}{\overline{PLR}+2(1-\overline{PGR})} < 1$ by Proposition 1. Then, our decomposition implies

$$\begin{array}{rl}
& DM_1 - \overline{DM}_1 \\
= & PGV_{KQ}(1-\gamma) + PGV_{KS}(1-\gamma) + PGV_{SQ}(\sigma^G_{SQ} + \lambda^G_{SQ}) + PGV_{SK}(1-\gamma) - \overline{PGR} \\
- & PLV_{KQ}(1-\kappa^L_{KQ}) - PLV_{KS}(1-\kappa^L_{KS}) - PLV_{SQ}(\gamma+\gamma) - PLV_{SK}(1-\kappa^L_{SK}) + \overline{PLR} \\
\leq & \overline{PGR}(1-\gamma-1) + PGV_{SQ}(\sigma^G_{SQ} + \lambda^G_{SQ}) \\
- & \overline{PLR}(1-\max\{\kappa^L_{KQ}, \kappa^L_{KS}, \kappa^L_{SK}\} - 1) - PLV_{SQ} * 2\gamma \\
= & \overline{PLR} * \max\{\kappa^L_{KQ}, \kappa^L_{KS}, \kappa^L_{SK}\} + (1-\overline{PGR})(\sigma^G_{SQ} + \lambda^G_{SQ}) - \overline{PGR} * \gamma - (1-\overline{PLR})2\gamma \\
\leq & \overline{PLR} * \gamma \frac{\overline{PGR}+2(1-\overline{PLR})}{\overline{PLR}+2(1-\overline{PGR})} + (1-\overline{PGR})2\gamma \frac{\overline{PGR}+2(1-\overline{PLR})}{\overline{PLR}+2(1-\overline{PGR})} - \overline{PGR} * \gamma - (1-\overline{PLR})2\gamma \\
= & \gamma \left(\frac{\overline{PGR}+2(1-\overline{PLR})}{\overline{PLR}+2(1-\overline{PGR})} (\overline{PLR} + 2(1-\overline{PGR}) - \overline{PGR} - 2(1-\overline{PLR}) \right) \\
= & \gamma \big(\overline{PGR} + 2(1-\overline{PLR}) - \overline{PGR} - 2(1-\overline{PLR}) \big) \\
= & 0.
\end{array}$$

We next prove the proposition for $DM_2 - \overline{DM}_2$. Let

$$\max\{\kappa^L_{KQ}, \kappa^L_{KS}, \sigma^G_{SQ}, \lambda^G_{SQ}, \kappa^L_{SK}\} \leq \gamma \frac{2 * \overline{PGR} - \overline{PGR} * \overline{PLR}}{2 * \overline{PLR} - \overline{PGR} * \overline{PLR}},$$

where $\frac{2*\overline{PGR}-\overline{PGR}*\overline{PLR}}{2*\overline{PLR}-\overline{PGR}*\overline{PLR}} < 1$ by Proposition 1. Since $DM_2 - \overline{DM}_2 \leq 0 \iff PGR * \overline{PLR} - PLR * \overline{PGR} \leq 0$, our decomposition implies $DM_2 - \overline{DM}_2 \leq 0$ if and only if

$$\begin{array}{rl}
& \Big(PGV_{KQ} * (1-\gamma) + PGV_{KS} * (1-\gamma) + PGV_{SQ} * (\sigma^G_{SQ} + \lambda^G_{SQ}) + PGV_{SK} * (1-\gamma) \Big) \overline{PLR} \\
\leq & \Big(PLV_{KQ} * (1-\kappa^L_{KQ}) - PLV_{KS} * (1-\kappa^L_{KS}) - PLV_{SQ} * (\gamma+\gamma) - PLV_{SK} * (1-\kappa^L_{SK}) \Big) \overline{PGR},
\end{array}$$

so that $DM_2 - \overline{DM}_2 \leq 0$ if

$$\begin{array}{rl}
& \Big(\overline{PGR}(1-\gamma) + (1-\overline{PGR})(\sigma^G_{SQ} + \lambda^G_{SQ}) \Big) \overline{PLR} \\
- & \Big(\overline{PLR}(1-\max\{\kappa^L_{KQ}, \kappa^L_{KS}, \kappa^L_{SK}\}) + (1-\overline{PLR})2\gamma \Big) \overline{PGR} \leq 0,
\end{array}$$

which is fulfilled since

$$\begin{array}{rl}
& \Big(\overline{PGR}(1-\gamma) + (1-\overline{PGR})(\sigma^G_{SQ} + \lambda^G_{SQ}) \Big) \overline{PLR} \\
- & \Big(\overline{PLR}(1-\max\{\kappa^L_{KQ}, \kappa^L_{KS}, \kappa^L_{SK}\}) + (1-\overline{PLR})2\gamma \Big) \overline{PGR} \\
\leq & \Big(\overline{PGR}(1-\gamma) + (1-\overline{PGR})2\gamma \frac{2*\overline{PGR}-\overline{PGR}*\overline{PLR}}{2*\overline{PLR}-\overline{PGR}*\overline{PLR}} \Big) \overline{PLR} \\
- & \Big(\overline{PLR}(1-\gamma \frac{2*\overline{PGR}-\overline{PGR}*\overline{PLR}}{2*\overline{PLR}-\overline{PGR}*\overline{PLR}}) + (1-\overline{PLR})2\gamma \Big) \overline{PGR} \\
= & \gamma \Big(\frac{2*\overline{PGR}-\overline{PGR}*\overline{PLR}}{2*\overline{PLR}-\overline{PGR}*\overline{PLR}} (2 * \overline{PLR} - \overline{PGR} * \overline{PLR}) + \overline{PGR} * \overline{PLR} - 2\overline{PGR} \Big) \\
= & 0.
\end{array}$$

That is, if conventional violation propensities are positive, but below a certain threshold, they cannot imply a positive disposition measure.

(iii) Let $\kappa^L_{KQ} + \kappa^L_{KS} + \sigma^G_{SQ} + \lambda^G_{SQ} + \kappa^L_{SK} > 0$, i.e., at least one of the conventional violation propensities has to be strictly positive. Further, let all unconventional violation propensities be equal to zero, i.e., $\kappa^G_{KQ} = \kappa^G_{KS} = \sigma^L_{SQ} = \lambda^L_{SQ} = \kappa^G_{SK} = 0$. Set $\varepsilon := \overline{PLR} - \overline{PGR} > 0$ and let

$$\max\{\kappa^L_{KQ}, \kappa^L_{KS}, \sigma^G_{SQ}, \lambda^G_{SQ}, \kappa^L_{SK}\} \leq \frac{\varepsilon}{3\max\{PGV_{SQ}, \overline{PLR}\}}.$$

Then, for DM_1 our decomposition implies

$$\begin{aligned}
DM_1 =& PGV_{KQ} + PGV_{KS} + PGV_{SQ} * (\sigma^G_{SQ} + \lambda^G_{SQ}) + PGV_{SK} \\
& - PLV_{KQ} * (1 - \kappa^L_{KQ}) - PLV_{KS} * (1 - \kappa^L_{KS}) - PLV_{SK} * (1 - \kappa^L_{SK}) \\
\leq & \overline{PGR} + PGV_{SQ} * (\sigma^G_{SQ} + \lambda^G_{SQ}) \\
& - \overline{PLR} * (1 - \max\{\kappa^L_{KQ}, \kappa^L_{KS}, \kappa^L_{SK}\}) \\
=& \overline{PGR} - \overline{PLR} + \max\{\kappa^L_{KQ}, \kappa^L_{KS}, \kappa^L_{SK}\} * \overline{PLR} + PGV_{SQ} * (\sigma^G_{SQ} + \lambda^G_{SQ}) \\
\leq & \quad -\varepsilon \quad + \frac{\varepsilon}{3} \quad + \frac{2\varepsilon}{3} \\
\leq & 0.
\end{aligned}$$

For DM_2 our decomposition implies

$$\begin{aligned}
DM_2 &= \frac{PGV_{KQ} + PGV_{KS} + PGV_{SQ} * (\sigma^G_{SQ} + \lambda^G_{SQ}) + PGV_{SK}}{PLV_{KQ} * (1 - \kappa^L_{KQ}) + PLV_{KS} * (1 - \kappa^L_{KS}) + PLV_{SK} * (1 - \kappa^L_{SK})} - 1 \\
&\leq \frac{\overline{PGR} + PGV_{SQ} * (\sigma^G_{SQ} + \lambda^G_{SQ})}{\overline{PLR} * (1 - \max\{\kappa^L_{KQ}, \kappa^L_{KS}, \kappa^L_{SK}\})} - 1,
\end{aligned}$$

so that $DM_2 \leq 0$ if

$$\begin{aligned}
& \frac{\overline{PGR} + PGV_{SQ} * (\sigma^G_{SQ} + \lambda^G_{SQ})}{\overline{PLR} * (1 - \max\{\kappa^L_{KQ}, \kappa^L_{KS}, \kappa^L_{SK}\})} \leq 1 \\
\iff & \overline{PGR} - \overline{PLR} + \max\{\kappa^L_{KQ}, \kappa^L_{KS}, \kappa^L_{SK}\} * \overline{PLR} + PGV_{SQ} * (\sigma^G_{SQ} + \lambda^G_{SQ}) \leq 0,
\end{aligned}$$

which is fulfilled since

$$\overline{PGR} - \overline{PLR} + \max\{\kappa^L_{KQ}, \kappa^L_{KS}, \kappa^L_{SK}\} * \overline{PLR} + PGV_{SQ} * (\sigma^G_{SQ} + \lambda^G_{SQ}) \leq -\varepsilon + \frac{\varepsilon}{3} + \frac{2\varepsilon}{3} = 0.$$

That is, if conventional violation propensities are positive, but below a certain threshold, they cannot imply a positive disposition measure, not even in case of zero unconventional violation propensities.

■

Proof. [Proposition 3] Let $\kappa^D_{KQ} = \kappa^D_{KS} = \sigma^D_{SQ} = \lambda^D_{SQ} = \kappa^D_{SK} =: \mu \in [0, 1/2]$ for $D \in \{G, L\}$ and let $\varepsilon := \overline{PLR} - \overline{PGR} = \overline{PGK} - \overline{PLK}$ (since $\overline{PGK} = 1 - \overline{PGR}$ and $\overline{PLK} = 1 - \overline{PLR}$).

Then, our decomposition yields $DM_i > 0$ if and only if

$$\begin{aligned}
& (1-\mu)\overline{PGR} + 2\mu * \overline{PGK} > (1-\mu)\overline{PLR} + 2\mu * \overline{PLK} \\
\iff & \quad 2\mu * \varepsilon > (1-\mu) * \varepsilon \\
\iff & \quad \mu > 1/3
\end{aligned}$$

That is, a positive disposition measure can arise even if all violation propensities are identical as long as they are sufficiently large.

Furthermore, our decomposition yields $DM_1 > \overline{DM}_1$ if and only if

$$\begin{aligned}
& (1-\mu)*\overline{PGR}+2\mu*\overline{PGK}-\overline{PGR} > (1-\mu)*\overline{PLR}+2\mu*\overline{PLK}-\overline{PLR} \\
\iff & \mu*\varepsilon > -2\mu*\varepsilon \\
\iff & \mu > 0
\end{aligned}$$

and $DM_2 > \overline{DM}_2$ if and only if

$$\begin{aligned}
& \Big((1-\mu)\overline{PGR}+2\mu(1-\overline{PGR})\Big)\overline{PLR} > \Big((1-\mu)\overline{PLR}+2\mu(1-\overline{PLR})\Big)\overline{PGR} \\
\iff & \overline{PGR}*\overline{PLR}(1-\mu-1+\mu-2\mu+2\mu) > 2\mu(\overline{PGR}-\overline{PLR}) \\
\iff & 0 > 2\mu(-\varepsilon) \\
\iff & 0 < \mu
\end{aligned}$$

■

Proof. [Proposition 4] Let all unconventional violation propensities be equal to γ, so that $\kappa^G_{KQ} = \kappa^G_{KS} = \kappa^G_{SK} = \sigma^L_{SQ} = \lambda^L_{SQ} =: \gamma$, and let all conventional violation propensities be equal to $\gamma - \nu$, so that $\kappa^L_{KQ} = \kappa^L_{KS} = \kappa^L_{SK} = \sigma^G_{SQ} = \lambda^G_{SQ} =: \gamma - \nu$ with $\nu > 0$. Then, our decomposition yields

$$\begin{aligned}
PGR - PLR &= PGV_{KQ}(1-\gamma) + PGV_{KS}(1-\gamma) + PGV_{SQ}(2\gamma-2\nu) + PGV_{SK}(1-\gamma) \\
&\quad - \Big[PLV_{KQ}(1-\gamma+\nu) + PLV_{KS}(1-\gamma+\nu) + PLV_{SQ}(2\gamma) + PLV_{SK}(1-\gamma+\nu)\Big] \\
&= \overline{PGR}(1-\gamma) + \overline{PGK}(2\gamma-2\nu) - \Big[\overline{PLR}(1-\gamma+\nu) + \overline{PLK}(2\gamma)\Big] \\
&= (\overline{PGR}-\overline{PLR})(1-\gamma) - \nu\overline{PLR} - (\overline{PLK}-\overline{PGK})(2\gamma) - \overline{PGK}(2\nu) \\
&= (-\varepsilon)(1-\gamma) - \nu\overline{PLR} + \varepsilon(2\gamma) - \overline{PGK}(2\nu),
\end{aligned}$$

where the last equality follows by letting $\varepsilon := \overline{PLR} - \overline{PGR} = \overline{PGK} - \overline{PLK}$ (since $\overline{PGK} = 1 - \overline{PGR}$ and $\overline{PLK} = 1 - \overline{PLR}$). Since $DM_i > 0 \iff PGR - PLR > 0$, rearranging this last expression yields $PGR - PLR > 0$ if and only if

$$\nu < \frac{\varepsilon(3\gamma-1)}{2+\varepsilon-\overline{PGR}} = \frac{(\overline{PLR}-\overline{PGR})(3\gamma-1)}{2+\overline{PLR}-2\overline{PGR}}.$$

Since $3\gamma - 1 \leq 1/2$, we have

$$\begin{aligned}
\frac{(\overline{PLR}-\overline{PGR})(3\gamma-1)}{2+\overline{PLR}-2\overline{PGR}} &= \frac{(\overline{PLR}-\overline{PGR})(3\gamma-1)}{1+(1-\overline{PLR})+2(\overline{PLR}-\overline{PGR})} \\
&\leq \frac{(\overline{PLR}-\overline{PGR})}{2+2(1-\overline{PLR})+4(\overline{PLR}-\overline{PGR})} < \frac{1}{4} < \gamma,
\end{aligned}$$

where the second to last inequality follows since $\frac{x}{4x+y} < \frac{1}{4}$ with $x := \overline{PLR} - \overline{PGR}$ being positive (by Proposition 1) and $y := 2 + 2(1 - \overline{PLR})$ being positive as well. The last inequality follows by letting $\gamma \in (1/3, 1/2]$, so that $\nu < \gamma$ holds (the lower bound of γ assures that $v > 0$ and the upper bound is required as all action propensities of one benchmark state add up to one).

From above it also follows that

$$\begin{aligned}
DM_1 - \overline{DM}_1 &= (\overline{PGR}-\overline{PLR})(1-\gamma) - \nu\overline{PLR} - (\overline{PLK}-\overline{PGK})(2\gamma) - \overline{PGK}(2\nu) - \overline{PGR} + \overline{PLR} \\
&= \varepsilon\gamma - \nu\overline{PLR} + \varepsilon(2\gamma) - \overline{PGK}(2\nu)
\end{aligned}$$

Rearranging this last expression yields $DM_1 - \overline{DM}_1 > 0$ if and only if

$$\nu < \frac{\varepsilon\gamma + \varepsilon(2\gamma)}{\overline{PLR} + 2\overline{PGK}} = \frac{(\overline{PLR} - \overline{PGR})(3\gamma)}{2 + \overline{PLR} - 2\overline{PGR}} < \gamma,$$

where the last inequality follows since $\frac{(\overline{PLR}-\overline{PGR})(3\gamma)}{2+\overline{PLR}-2\overline{PGR}} = \gamma\frac{(\overline{PLR}-\overline{PGR})3}{1+(1-\overline{PLR})+2(\overline{PLR}-\overline{PGR})}$ with $\frac{3\varepsilon}{1+2\varepsilon+(1-\overline{PLR})} < 1$ because $0 < \varepsilon < \overline{PLR} < 1$. Now, we have $\nu < \gamma \in (0, 1/2]$ (again, the lower bound of γ assures that $v > 0$ and the upper bound is required as all action propensities of one benchmark state add up to one).

Finally, we have $DM_2 - \overline{DM}_2 > 0$ if and only if

$$\begin{aligned}
&\Big((1-\gamma)\overline{PGR} + (2\gamma - 2\nu)(1-\overline{PGR})\Big)\overline{PLR} > \Big((1-\gamma+\nu)\overline{PLR} + 2\gamma(1-\overline{PLR})\Big)\overline{PGR} \\
\iff\ & \overline{PGR} * \overline{PLR}(1 - \gamma - 2\gamma + 2\nu - 1 + \gamma - \nu + 2\gamma) - 2\nu * \overline{PLR} > 2\gamma(\overline{PGR} - \overline{PLR}) \\
\iff\ & \nu(\overline{PGR} * \overline{PLR} - 2 * \overline{PLR}) > 2\gamma(\overline{PGR} - \overline{PLR}).
\end{aligned}$$

Since $\overline{PGR} * \overline{PLR} - 2 * \overline{PLR} < 0$ (by Proposition 1), we have $DM_2 - \overline{DM}_2 > 0$ if and only if

$$\nu < \frac{(\overline{PLR} - \overline{PGR})2\gamma}{2 * \overline{PLR} - \overline{PGR} * \overline{PLR}} < \gamma,$$

where the last inequality follows since $\frac{(\overline{PLR}-\overline{PGR})2\gamma}{2*\overline{PLR}-\overline{PGR}*\overline{PLR}} = \gamma\frac{2\varepsilon}{\overline{PLR}(2-\overline{PGR})}$ with $\frac{2\varepsilon}{\overline{PLR}(2-\overline{PGR})} < 1 \iff 2 * \overline{PGR} > \overline{PLR} * \overline{PGR}$ which is fulfilled since $0 < \overline{PGR} < \overline{PLR} < 1$. Now, we have $\nu < \gamma \in (0, 1/2]$ (again, the lower bound of γ assures that $v > 0$ and the upper bound is required as all action propensities of one benchmark state add up to one). ■

Proof. [Proposition 5] Let $\Pr(K|V,D) = \Pr(K)$, $\Pr(S|V,D) = \Pr(S)$, and $\Pr(Q|V,D) = \Pr(Q)$ with $V \in \{V_{KQ}, V_{KS}, V_{SK}, V_{SQ}\}$ and $D \in \{G, L\}$. Then, by our decomposition we have

$$\begin{aligned}
PGR &= PGV_{KQ}(1-\Pr(K)) + PGV_{KS}(1-\Pr(K)) + PGV_{SQ}(\Pr(S)+\Pr(Q)) + PGV_{SK}(1-\Pr(K)) \\
&= \overline{PGR}(1-\Pr(K)) + \overline{PGK}(\Pr(S)+\Pr(Q)) \\
&= 1-\Pr(K), \\
PLR &= PLV_{KQ}(1-\Pr(K)) + PLV_{KS}(1-\Pr(K)) + PLV_{SQ}(\Pr(S)+\Pr(Q)) + PLV_{SK}(1-\Pr(K)) \\
&= \overline{PLR}(1-\Pr(K)) + \overline{PLK}(\Pr(S)+\Pr(Q)) \\
&= 1-\Pr(K),
\end{aligned}$$

where $\overline{PGR} + \overline{PGK} = \overline{PLR} + \overline{PLK} = 1$ is an implication of the fact that our benchmark specifies a unique appropriate action for each state of the world, and $\Pr(K) + \Pr(S) + \Pr(Q) = 1$ is an implication of the fact that the actions keep (K), switch (S), and liquidate (Q) cover the entire action space. It follows that $DM_1 = PGR - PLR = 0$ and $DM_2 = \frac{PGR}{PLR} - 1 = 0$. Since $\overline{DM}_i < 0$ (by Proposition 1), we also have that $DM_i = 0 > \overline{DM}_i$. ■

Proof. [Proposition 6] We prove each case of Proposition 6 separately.

(i) By our decomposition, PGR (PLR) is increasing (decreasing) in conventional and decreasing (increasing) in unconventional violation propensities. Thus, DM_i is increasing in conventional and decreasing in unconventional violation propensities.

(*ii*) Let $\kappa^D_{KQ} = \kappa^D_{KS} = \sigma^D_{SQ} = \lambda^D_{SQ} = \kappa^D_{SK} =: \mu \in [0, 1/2)$ for $D \in \{G, L\}$ and note that $\overline{PGK} = 1 - \overline{PGR}$ and $\overline{PLK} = 1 - \overline{PLR}$. Then, by our decomposition, we have $DM_1 = (\overline{PLR} - \overline{PGR})(3\mu - 1)$, so that

$$\frac{dDM_1}{d\mu} = (\overline{PLR} - \overline{PGR})3,$$

which is strictly positive by Proposition 1.

By our decomposition, we also have $DM_2 = \frac{\overline{PGR}+\mu(2-3\overline{PGR})}{\overline{PLR}+\mu(2-3\overline{PLR})} - 1$, so that

$$\begin{aligned}\frac{dDM_2}{d\mu} &= \frac{(2-3\overline{PGR})(\overline{PLR}+\mu(2-3\overline{PLR})) - (2-3\overline{PLR})(\overline{PGR}+\mu(2-3\overline{PGR}))}{(\overline{PLR}+\mu(2-3\overline{PLR}))^2} \\ &= \frac{(\overline{PLR}-\overline{PGR})2}{(\overline{PLR}+\mu(2-3\overline{PLR}))^2},\end{aligned}$$

where the denominator is strictly positive and the numerator is strictly positive by Proposition 1.

(*iii*) Let $\varepsilon \in [0, \bar{\varepsilon})$ with $\bar{\varepsilon} = 1/2 - \max\{\kappa^D_{KQ}, \kappa^D_{KS}, \kappa^D_{SK}, \sigma^D_{SQ}, \lambda^D_{SQ}\}$ for $D \in \{G, L\}$ be added to all violation propensities and note that $\overline{PGK} = 1 - \overline{PGR}$ and $\overline{PLK} = 1 - \overline{PLR}$. Then, by our decomposition, we have

$$\begin{aligned}PGR &= PGV_{KQ}(1-\kappa^G_{KQ}-\varepsilon) + PGV_{KS}(1-\kappa^G_{KS}-\varepsilon) + PGV_{SK}(1-\kappa^G_{SK}-\varepsilon) \\ &\quad + PGV_{SQ}(\sigma^G_{SQ}+\varepsilon+\lambda^G_{SQ}+\varepsilon), \\ PLR &= PLV_{KQ}(1-\kappa^L_{KQ}-\varepsilon) + PLV_{KS}(1-\kappa^L_{KS}-\varepsilon) + PLV_{SK}(1-\kappa^L_{SK}-\varepsilon) \\ &\quad + PLV_{SQ}(\sigma^L_{SQ}+\varepsilon+\lambda^L_{SQ}+\varepsilon).\end{aligned}$$

Thus,

$$\begin{aligned}\frac{dDM_1}{d\varepsilon} &= \Big(-\overline{PGR} + (1-\overline{PGR})2\Big) - \Big((-\overline{PLR} + (1-\overline{PLR})2\Big) \\ &= (\overline{PLR} - \overline{PGR})3,\end{aligned}$$

which is strictly positive by Proposition 1, and

$$\frac{dDM_2}{d\varepsilon} = \frac{(2-3\overline{PGR})PLR - (2-3\overline{PLR})PGR}{(\overline{PLR})^2},$$

where the denominator is strictly positive and the numerator is strictly positive by Proposition 1 if $\overline{PLR} > 2/3$ and $PGR > PLR$ or if $\overline{PLR} < 2/3$ and $PGR < PLR$.

■

Proof. [Lemma 1] This proof builds upon Appendix B and the proof of Proposition 1. In this proof, we investigate the effect that an increase of g by 1 has on $\overline{PGR}$ and $\overline{PLR}$. The effect of increasing it by more than 1 (up to $n-g$) evolves accordingly, as it is a strictly monotone effect. The proof works similarly both for $\theta > 0$ and $\theta = 0$. For brevity, in the following we focus on $\theta > 0$ only. The case of $\theta = 0$ can be shown analogously, but requires more notation due to the initial randomization for uninformative priors at $t = \tau$.

We start with the proof of $\overline{PGR}$, which relies on the following lemma:

Lemma 5 *Let $(\alpha_i)_{i\in\{0,\dots,n\}}$ and $(\beta_i)_{i\in\{0,\dots,n\}}$ be positive sequences such that $\frac{\alpha_i}{\beta_i}$ is strictly decreasing in i. Then,*

$$\forall\, g \in \{1,\dots,n-1\} : \frac{\sum_{i=k+g}^{k+n}\alpha_i}{\sum_{i=k+g}^{k+n}\beta_i} > \frac{\sum_{i=k+g+1}^{k+n}\alpha_i}{\sum_{i=k+g+1}^{k+n}\beta_i}.$$

Proof. [Lemma 5] Since $\frac{\alpha_i}{\beta_i}$ is strictly decreasing in i, we know that

$$\frac{\alpha_{k+g}}{\beta_{k+g}} > \frac{\alpha_i}{\beta_i}\ \forall\, i \in \{k+g+1,\dots,k+n\}. \tag{6}$$

Since (6) is equivalent to $\beta_i\alpha_{k+g} > \alpha_i\beta_{k+g}\ \forall\, i \in \{k+g+1,\dots,k+n\}$, it implies that

$$\begin{aligned}\sum_{i=k+g+1}^{k+n}\beta_i\alpha_{k+g} &> \sum_{i=k+g+1}^{k+n}\alpha_i\beta_{k+g}\\ \iff \frac{\alpha_{k+g}}{\beta_{k+g}} &> \frac{\sum_{i=k+g+1}^{k+n}\alpha_i}{\sum_{i=k+g+1}^{k+n}\beta_i}.\end{aligned} \tag{7}$$

Also, note that

$$\frac{\sum_{i=k+g}^{k+n}\alpha_i}{\sum_{i=k+g}^{k+n}\beta_i} = \frac{\sum_{i=k+g+1}^{k+n}\alpha_i + \alpha_{k+g}}{\sum_{i=k+g+1}^{k+n}\beta_i + \beta_{k+g}}. \tag{8}$$

We want to prove that

$$\frac{\sum_{i=k+g}^{k+n}\alpha_i}{\sum_{i=k+g}^{k+n}\beta_i} > \frac{\sum_{i=k+g+1}^{k+n}\alpha_i}{\sum_{i=k+g+1}^{k+n}\beta_i} \tag{9}$$

$$\iff \frac{\sum_{i=k+g+1}^{k+n}\alpha_i + \alpha_{k+g}}{\sum_{i=k+g+1}^{k+n}\beta_i + \beta_{k+g}} > \frac{\sum_{i=k+g+1}^{k+n}\alpha_i}{\sum_{i=k+g+1}^{k+n}\beta_i}, \tag{10}$$

where the equivalence follows by plugging (8) into (9). Now, (10) is equivalent to

$$\begin{aligned}\left(\sum_{i=k+g+1}^{k+n}\alpha_i + \alpha_{k+g}\right)\left(\sum_{i=k+g+1}^{k+n}\beta_i\right) &> \left(\sum_{i=k+g+1}^{k+n}\alpha_i\right)\left(\sum_{i=k+g+1}^{k+n}\beta_i + \beta_{k+g}\right)\\ \iff \alpha_{k+g}\sum_{i=k+g+1}^{k+n}\beta_i &> \beta_{k+g}\sum_{i=k+g+1}^{k+n}\alpha_i\\ \iff \frac{\alpha_{k+g}}{\beta_{k+g}} &> \frac{\sum_{i=k+g+1}^{k+n}\alpha_i}{\sum_{i=k+g+1}^{k+n}\beta_i},\end{aligned}$$

which is equivalent to (7). Note that for the strict inequality result in Lemma 5 to hold it is actually sufficient that sequence is decreasing with only one strict inequality. ■

Note first that we can rewrite $\overline{PGR}$ as

$$\begin{aligned}\overline{PGR} = &\frac{\sum_{k=\theta}^{\tau}\sum_{l=0}^{k-\theta}\Pr(a=k,b=l)(\overline{PGR}|a=k,b=l)}{\sum_{k=\theta}^{\tau}\sum_{l=0}^{k-\theta}\Pr(a=k,b=l)+\sum_{k=\theta}^{\tau}\sum_{l=0}^{k-\theta}\Pr(b=k,a=l)}\\ &+\frac{\sum_{k=\theta}^{\tau}\sum_{l=0}^{k-\theta}\Pr(b=k,a=l)(\overline{PGR}|b=k,a=l)}{\sum_{k=\theta}^{\tau}\sum_{l=0}^{k-\theta}\Pr(a=k,b=l)+\sum_{k=\theta}^{\tau}\sum_{l=0}^{k-\theta}\Pr(b=k,a=l)}\end{aligned}$$

Thus, to prove that $\overline{PGR}$ is decreasing in g, for fixed θ it is sufficient to prove that $(\overline{PGR}|a = k, b = l)$ and $(\overline{PGR}|b = k, a = l)$ are decreasing in g for all values of k and l satisfying $|k-l| \geq \theta$. In the following, we will prove that $(\overline{PGR}|a = k, b = l)$ is decreasing in g. By "swapping" places of a and b, the proof for $(\overline{PGR}|b = k, a = l)$ decreasing in g works similarly and is therefore omitted. Note that

$$(\overline{PGR}|a = k, b = l) = \frac{(\overline{NPGR}|a = k, b = l)}{(\overline{DPGR}|a = k, b = l)},$$

where $(\overline{NPGR}|a = k, b = l)$ and $(\overline{DPGR}|a = k, b = l)$ are already defined in the proof of Proposition 1. Similar to the proof of Proposition 1, we again use the fact that $\frac{(\alpha_m|a=k,b=l)}{(\beta_m|a=k,b=l)} \leq 1$ is decreasing in m, where $(\alpha_m|a = k, b = l)$ and $(\beta_m|a = k, b = l)$ are defined in (3) and (4) of the proof of Proposition 1.

Then, Lemma 5 implies that

$$\frac{\sum_{m=k+g}^{k+n}(\alpha_m|a = k, b = l)}{\sum_{m=k+g}^{k+n}(\beta_m|a = k, b = l)} > \frac{\sum_{m=k+g+1}^{k+n}(\alpha_m|a = k, b = l)}{\sum_{m=k+g+1}^{k+n}(\beta_m|a = k, b = l)}. \tag{11}$$

Plugging (3) and (4) into (11) yields

$$\begin{aligned}
& \frac{\sum_{m=k+g}^{k+n} \Pr(a' = m|a = k, b = l) \sum_{j=m-\theta+1}^{l+n} \Pr(b' = j|a = k, b = l)}{\sum_{m=k+g}^{k+n} \Pr(a' = m|a = k, b = l) \sum_{j=l}^{l+n} \Pr(b' = j|a = k, b = l)} \\
> \quad & \frac{\sum_{m=k+g+1}^{k+n} \Pr(a' = m|a = k, b = l) \sum_{j=m-\theta+1}^{l+n} \Pr(b' = j|a = k, b = l)}{\sum_{m=k+g+1}^{k+n} \Pr(a' = m|a = k, b = l) \sum_{j=l}^{l+n} \Pr(b' = j|a = k, b = l)} \\
\iff \quad & \left.\frac{(\overline{NPGR}|a = k, b = l)}{(\overline{DPGR}|a = k, b = l)}\right|_g > \left.\frac{(\overline{NPGR}|a = k, b = l)}{(\overline{DPGR}|a = k, b = l)}\right|_{g+1} \\
\iff \quad & (\overline{PGR}|a = k, b = l)|_g > (\overline{PGR}|a = k, b = l)|_{g+1}
\end{aligned}$$

Hence $\overline{PGR}$ is decreasing in g.

Next, we prove $\overline{PLR}$, which relies on the following lemma:

Lemma 6 *Let $(\alpha_i)_{i\in\{0,\ldots,n\}}$ and $(\beta_i)_{i\in\{0,\ldots,n\}}$ be positive sequences such that $\frac{\alpha_i}{\beta_i}$ is strictly decreasing in i. Then,*

$$\forall\, g \in \{1, \ldots, n-1\} : \frac{\sum_{i=k}^{k+g-1} \alpha_i}{\sum_{i=k}^{k+g-1} \beta_i} > \frac{\sum_{i=k}^{k+g} \alpha_i}{\sum_{i=k}^{k+g} \beta_i}.$$

Proof. [Lemma 6] Since $\frac{\alpha_i}{\beta_i}$ is strictly decreasing in i, we know that

$$\frac{\alpha_{k+g}}{\beta_{k+g}} < \frac{\alpha_i}{\beta_i}\ \forall\, i \in \{k, \ldots, k+g-1\}. \tag{12}$$

Since (12) is equivalent to $\beta_i\alpha_{k+g} < \alpha_i\beta_{k+g}\ \forall\, i \in \{k, \ldots, k+g-1\}$, it implies that

$$\begin{aligned}
\sum_{i=k}^{k+g-1} \beta_i\alpha_{k+g} \quad &< \quad \sum_{i=k}^{k+g-1} \alpha_i\beta_{k+g} \\
\iff \frac{\alpha_{k+g}}{\beta_{k+g}} \quad &< \quad \frac{\sum_{i=k}^{k+g-1} \alpha_i}{\sum_{i=k}^{k+g-1} \beta_i}.
\end{aligned} \tag{13}$$

Also, note that

$$\frac{\sum_{i=k}^{k+g} \alpha_i}{\sum_{i=k}^{k+g} \beta_i} = \frac{\sum_{i=k}^{k+g-1} \alpha_i + \alpha_{k+g}}{\sum_{i=k}^{k+g-1} \beta_i + \beta_{k+g}}. \tag{14}$$

We want to prove that

$$\frac{\sum_{i=k}^{k+g-1} \alpha_i}{\sum_{i=k}^{k+g-1} \beta_i} > \frac{\sum_{i=k}^{k+g} \alpha_i}{\sum_{i=k}^{k+g} \beta_i} \tag{15}$$

$$\iff \frac{\sum_{i=k}^{k+g-1} \alpha_i}{\sum_{i=k}^{k+g-1} \beta_i} > \frac{\sum_{i=k}^{k+g-1} \alpha_i + \alpha_{k+g}}{\sum_{i=k}^{k+g-1} \beta_i + \beta_{k+g}}, \tag{16}$$

where the equivalence follows by plugging (14) into (15). Now, (16) is equivalent to

$$\begin{aligned}
\left(\sum_{i=k}^{k+g-1} \alpha_i + \alpha_{k+g}\right)\left(\sum_{i=k}^{k+g-1} \beta_i\right) &< \left(\sum_{i=k}^{k+g-1} \alpha_i\right)\left(\sum_{i=k}^{k+g-1} \beta_i + \beta_{k+g}\right) \\
\iff \alpha_{k+g} \sum_{i=k}^{k+g-1} \beta_i &< \beta_{k+g} \sum_{i=k}^{k+g-1} \alpha_i \\
\iff \frac{\alpha_{k+g}}{\beta_{k+g}} &< \frac{\sum_{i=k}^{k+g-1} \alpha_i}{\sum_{i=k}^{k+g-1} \beta_i},
\end{aligned}$$

which is equivalent to (13). Note that for the strict inequality result in Lemma 5 to hold it is actually sufficient that sequence is decreasing with only one strict inequality. ■

Note first that we can rewrite $\overline{PLR}$ as

$$\begin{aligned}
\overline{PLR} = &\frac{\sum_{k=\theta}^{\tau} \sum_{l=0}^{k-\theta} \Pr(a = k, b = l)(\overline{PLR}|a = k, b = l)}{\sum_{k=\theta}^{\tau} \sum_{l=0}^{k-\theta} \Pr(a = k, b = l) + \sum_{k=\theta}^{\tau} \sum_{l=0}^{k-\theta} \Pr(b = k, a = l)} \\
&+ \frac{\sum_{k=\theta}^{\tau} \sum_{l=0}^{k-\theta} \Pr(b = k, a = l)(\overline{PLR}|b = k, a = l)}{\sum_{k=\theta}^{\tau} \sum_{l=0}^{k-\theta} \Pr(a = k, b = l) + \sum_{k=\theta}^{\tau} \sum_{l=0}^{k-\theta} \Pr(b = k, a = l)}
\end{aligned}$$

Again, to prove that $\overline{PLR}$ is decreasing in g, for fixed θ it is sufficient to prove that $(\overline{PLR}|a = k, b = l)$ and $(\overline{PLR}|b = k, a = l)$ are decreasing in g for all values of k and l satisfying $|k - l| \geq \theta$. In the following, we will prove that $(\overline{PLR}|a = k, b = l)$ is decreasing in g. By "swapping" places of a and b, the proof for $(\overline{PLR}|b = k, a = l)$ decreasing in g works similarly and is therefore omitted. Note that

$$(\overline{PLR}|a = k, b = l) = \frac{(\overline{NPLR}|a = k, b = l)}{(\overline{DPLR}|a = k, b = l)},$$

where $(\overline{NPLR}|a = k, b = l)$ and $(\overline{DPLR}|a = k, b = l)$ are already defined in the proof of Proposition 1. As before, we use the fact that $\frac{(\alpha_m|a=k,b=l)}{(\beta_m|a=k,b=l)} \leq 1$ is decreasing in m, where $(\alpha_m|a = k, b = l)$ and $(\beta_m|a = k, b = l)$ are defined in (3) and (4) of the proof of Proposition 1.

Then, Lemma 6 implies that

$$\frac{\sum_{m=k}^{k+g-1}(\alpha_m|a = k, b = l)}{\sum_{m=k}^{k+g-1}(\beta_m|a = k, b = l)} > \frac{\sum_{m=k}^{k+g}(\alpha_m|a = k, b = l)}{\sum_{m=k}^{k+g}(\beta_m|a = k, b = l)}. \tag{17}$$

Plugging (3) and (4) into (17) yields

$$
\begin{aligned}
& \frac{\sum_{m=k}^{k+g-1} \Pr\left(a' = m | a = k, b = l\right) \sum_{j=m-\theta+1}^{l+n} \Pr\left(b' = j | a = k, b = l\right)}{\sum_{m=k}^{k+g-1} \Pr\left(a' = m | a = k, b = l\right) \sum_{j=l}^{l+n} \Pr\left(b' = j | a = k, b = l\right)} \\
> \quad & \frac{\sum_{m=k}^{k+g} \Pr\left(a' = m | a = k, b = l\right) \sum_{j=m-\theta+1}^{l+n} \Pr\left(b' = j | a = k, b = l\right)}{\sum_{m=k}^{k+g} \Pr\left(a' = m | a = k, b = l\right) \sum_{j=l}^{l+n} \Pr\left(b' = j | a = k, b = l\right)} \\
\iff \quad & \left. \frac{(\overline{NPLR} | a = k, b = l)}{(\overline{DPLR} | a = k, b = l)} \right|_{g-1} > \left. \frac{(\overline{NPLR} | a = k, b = l)}{(\overline{DPLR} | a = k, b = l)} \right|_{g} \\
\iff \quad & (\overline{PLR} | a = k, b = l)|_{g-1} > (\overline{PLR} | a = k, b = l)|_{g}
\end{aligned}
$$

Hence $\overline{PLR}$ is decreasing in g. Therefore, $\overline{PGR}$ and $\overline{PLR}$ are both decreasing in g. ■

Proof. [Proposition 7] Let the unconventional violation propensities all be equal to γ, so that $\kappa_{KQ}^G = \kappa_{KS}^G = \kappa_{SK}^G = \sigma_{SQ}^L = \lambda_{SQ}^L =: \gamma \in (0, 1/2]$. Further, let the conventional violation propensities all be equal to $\gamma - \nu$, so that $\kappa_{KQ}^L = \kappa_{KS}^L = \kappa_{SK}^L = \sigma_{SQ}^G = \lambda_{SQ}^G =: \gamma - \nu \in (0, 1/2]$. Then, our decomposition yields

$$
\begin{aligned}
PGR &= PGV_{KQ}(1-\gamma) + PGV_{KS}(1-\gamma) + PGV_{SQ}(2\gamma - 2\nu) + PGV_{SK}(1-\gamma) \\
&= 2\gamma - 2\nu + \overline{PGR}(1 - 3\gamma + 2\nu), \\
PLR &= PLV_{KQ}(1-\gamma+\nu) + PLV_{KS}(1-\gamma+\nu) + PLV_{SQ}(2\gamma) + PLV_{SK}(1-\gamma+\nu) \\
&= 2\gamma + \overline{PLR}(1 - 3\gamma + \nu),
\end{aligned}
$$

because $PGV_{KQ} + PGV_{KS} + PGV_{SK} = \overline{PGR}$ and $PGV_{SQ} = \overline{PGK} = 1 - \overline{PGR}$ in gains as well as $PLV_{KQ} + PLV_{KS} + PLV_{SK} = \overline{PLR}$ and $PLV_{SQ} = \overline{PLK} = 1 - \overline{PLR}$ in losses. In this proof, we denote $\frac{\Delta X(g)}{\Delta g}$ the effect that a change of g has on $X(g)$ (whereas in the remaining paper Δ denotes the difference in the number of price appreciations between the two risky assets until period τ). Then,

$$
\begin{aligned}
\frac{\Delta DM_1(g)}{\Delta g} &= \frac{\Delta \overline{PGR}(g)}{\Delta g}(1 - 3\gamma + 2\nu) - \frac{\Delta \overline{PLR}(g)}{\Delta g}(1 - 3\gamma + \nu), \\
\frac{\Delta DM_2(g)}{\Delta g} &= \frac{\frac{\Delta \overline{PGR}(g)}{\Delta g}(1 - 3\gamma + 2\nu) PLR(g) - \frac{\Delta \overline{PLR}(g)}{\Delta g}(1 - 3\gamma + \nu) PGR(g)}{(PLR(g))^2}.
\end{aligned}
$$

Note that $PGR > 0$ since $\gamma - \nu > 0$ and $PLR > 0$ since $\gamma > 0$. Suppose first that $k = 1$, so that $\nu = 3\gamma - 1 \iff \gamma = \frac{\nu+1}{3}$. By Lemma 1 we know that $\frac{\Delta \overline{PGR}(g)}{\Delta g} < 0$. Thus, $\frac{\Delta DM_i(g)}{\Delta g} > (<)\ 0$ if and only if $1 - 3(\frac{\nu+1}{3}) + 2\nu < (>)\ 0 \iff \nu < (>)\ 0$. Suppose next that $k = 2$, so that $2\nu = 3\gamma - 1 \iff \gamma = \frac{2\nu+1}{3}$. By Lemma 1 we also know that $\frac{\Delta \overline{PLR}(g)}{\Delta g} < 0$. Thus, $\frac{\Delta DM_i(g)}{\Delta g} > (<)\ 0$ if and only if $1 - 3(\frac{2\nu+1}{3}) + \nu > (<)\ 0 \iff \nu < (>)\ 0$. ■

Proof. [Lemma 2] This proof applies the *de Moivre-Laplace theorem*, which is a special case of the central limit theorem and states that the distribution of total appreciations of a binomial process, where $p \in (0, 1)$ is the probability to appreciate and x the number of trials, converges to the normal distribution with mean xp and standard deviation $\sqrt{xp(1-p)}$ as x grows large.

In this proof, we investigate the effect of τ growing large. By the de Moivre-Laplace theorem, the distribution of $|\Delta|$ converges to the normal distribution with mean $\tau|p_h - p_l|$. Thus, the expected difference in appreciations, i.e., $\mathbb{E}(|\Delta|)$, increases in τ. For sufficiently large τ

and fixed n, the value of $|\Delta' - \Delta|$ is irrelevant as $|\Delta|$ is way above the investment threshold θ (see Appendix B). These "extreme" values of $|\Delta|$ become more likely for larger τ. This is because until period τ, the high (low) process F_h (F_l) converges to the normal distribution with mean τp_h (τp_l) and standard deviation $\sqrt{\tau p_h(1-p_h)}$ ($\sqrt{\tau p_l(1-p_l)}$) as τ grows large, and dividing by $\sqrt{\tau}$ yields the normal distribution with mean $\sqrt{\tau}p_h$ ($\sqrt{\tau}p_l$) and standard deviation $\sqrt{p_h(1-p_h)}$ ($\sqrt{p_l(1-p_l)}$). As a result, the means of both processes increase with the rate $\sqrt{\tau}$ and the standard deviations remain constant as τ grows large. Thus, the probability of $|\Delta|$ to be weakly below $\theta + n$ is converging to 0 as τ grows large, so that the investor should never realize her asset. Therefore, $\overline{PGR}$ and $\overline{PLR}$ converge to 0 as τ grows large. ■

Proof. [Proposition 8] Let $\kappa^D_{KQ} = \kappa^D_{KS} = \kappa^D_{SK} =: \kappa^D \in [0,1]$ and $\sigma^D_{SQ} + \lambda^D_{SQ} =: \rho^D \in (0,1]$ in domain $D \in \{G, L\}$. Then, for DM_1 our decomposition yields

$$\begin{aligned} DM_1 &= PGV_{KQ}(1-\kappa^G) + PGV_{KS}(1-\kappa^G) + PGV_{SQ}(\rho^G) + PGV_{SK}(1-\kappa^G) \\ &- \Big[PLV_{KQ}(1-\kappa^L) + PLV_{KS}(1-\kappa^L) + PLV_{SQ}(\rho^L) + PLV_{SK}(1-\kappa^L)\Big] \\ &= \rho^G - \rho^L + \overline{PGR}(1-\kappa^G-\rho^G) - \overline{PLR}(1-\kappa^L-\rho^L) \end{aligned}$$

because $PGV_{KQ} + PGV_{KS} + PGV_{SK} = \overline{PGR}$ and $PGV_{SQ} = \overline{PGK} = 1 - \overline{PGR}$ in gains as well as $PLV_{KQ} + PLV_{KS} + PLV_{SK} = \overline{PLR}$ and $PLV_{SQ} = \overline{PLK} = 1 - \overline{PLR}$ in losses. By Lemma 2, both $\overline{PGR}$ and $\overline{PLR}$ converge to 0 for "large τ," so that DM_1 converges to $\rho^G - \rho^L$ for "large τ." This limit value of DM_1 is approached from below (above) if and only if

$$\overline{PGR}(1-\kappa^G-\rho^G) < (>)\ \overline{PLR}(1-\kappa^L-\rho^L).$$

By Proposition 1, we know that $0 < \overline{PGR} < \overline{PLR} < 1$ outside the limit. Thus, if $\kappa^L + \rho^L \geq 1 \wedge \kappa^G + \rho^G \leq 1$ (with at least one strict inequality), the limit value of DM_1 is approached from above. If $\kappa^L + \rho^L \leq 1 \wedge \kappa^G + \rho^G \geq 1$ (with at least one strict inequality), the limit value of DM_1 is approached from below.

For DM_2 our decomposition yields

$$\begin{aligned} DM_2 &= \frac{PGV_{KQ}(1-\kappa^G) + PGV_{KS}(1-\kappa^G) + PGV_{SQ}(\rho^G) + PGV_{SK}(1-\kappa^G)}{PLV_{KQ}(1-\kappa^L) + PLV_{KS}(1-\kappa^L) + PLV_{SQ}(\rho^L) + PLV_{SK}(1-\kappa^L)} - 1 \\ &= \frac{\rho^G + \overline{PGR}(1-\kappa^G-\rho^G)}{\rho^L + \overline{PLR}(1-\kappa^L-\rho^L)} - 1 \end{aligned}$$

because $PGV_{KQ} + PGV_{KS} + PGV_{SK} = \overline{PGR}$ and $PGV_{SQ} = \overline{PGK} = 1 - \overline{PGR}$ in gains as well as $PLV_{KQ} + PLV_{KS} + PLV_{SK} = \overline{PLR}$ and $PLV_{SQ} = \overline{PLK} = 1 - \overline{PLR}$ in losses. By Lemma 2, both $\overline{PGR}$ and $\overline{PLR}$ converge to 0 for "large τ," so that DM_2 converges to $\frac{\rho^G}{\rho^L} - 1$ for "large τ." This limit value of DM_2 is approached from below (above) if and only if

$$\overline{PGR}(1-\kappa^G-\rho^G)\rho^L < (>)\ \overline{PLR}(1-\kappa^L-\rho^L)\rho^G.$$

By Proposition 1, we know that $0 < \overline{PGR} < \overline{PLR} < 1$ outside the limit. Further, note that $\rho^D > 0$. Thus, if $\kappa^L + \rho^L \geq 1 \wedge \kappa^G + \rho^G \leq 1$ (with at least one strict inequality), the limit value of DM_2 is approached from above. If $\kappa^L + \rho^L \leq 1 \wedge \kappa^G + \rho^G \geq 1$ (with at least one strict inequality), the limit value of DM_2 is approached from below. ■

Proof. [Lemma 3] Similar to the proof of Lemma 2, this proof applies the *de Moivre-Laplace theorem*. In this proof, we investigate the effect of n growing large. First, note that by the de Moivre-Laplace theorem, between periods τ and τ' the high (low) process F_h (F_l) converges to the normal distribution with mean np_h (np_l) and standard deviation $\sqrt{np_h(1-p_h)}$ ($\sqrt{np_l(1-p_l)}$) as

n grows large, and dividing by n yields the normal distribution with mean p_h (p_l) and standard deviation $\sqrt{\frac{1}{n}p_h(1-p_h)}$ ($\sqrt{\frac{1}{n}p_l(1-p_l)}$). Thus, both standard deviations are decreasing in n and the condition $p_l < \frac{g}{n} < p_h$ assures that the high process yields only gains and the low process only losses when n grows large.

Second, by the de Moivre-Laplace theorem, the distribution of $|\Delta'|$ converges to the normal distribution with mean $(\tau+n)|p_h - p_l|$. Thus, the expected difference in appreciations, i.e., $\mathbb{E}(|\Delta'|)$, increases in n. For sufficiently large n and fixed τ, the value of $|\Delta|$ is irrelevant as $|\Delta'|$ is way above the investment threshold θ (see Appendix B). These "extreme" values of $|\Delta'|$ become more likely for larger n. Thus, the probability of $|\Delta'|$ to be weakly below θ is converging to 0 as n grows large. Together with the argument above, this implies that the investor should never realize a gain and always realize a loss as n grows large. Therefore, $\overline{PGR}$ converges to 0, $\overline{PLR}$ converges to 1, and $\overline{DM_i}$ converges to -1 as n grows large. ■

Proof. [Proposition 9] Let $\kappa^D_{KQ} = \kappa^D_{KS} = \kappa^D_{SK} = \kappa^D \in [0,1)$ and $\sigma^D_{SQ} + \lambda^D_{SQ} = \rho^D \in (0,1]$ in domain $D \in \{G, L\}$. Then, for DM_1 our decomposition yields

$$\begin{aligned} DM_1 &= PGV_{KQ}(1-\kappa^G) + PGV_{KS}(1-\kappa^G) + PGV_{SQ}(\rho^G) + PGV_{SK}(1-\kappa^G) \\ &- \Big[PLV_{KQ}(1-\kappa^L) + PLV_{KS}(1-\kappa^L) + PLV_{SQ}(\rho^L) + PLV_{SK}(1-\kappa^L)\Big] \\ &= \rho^G - \rho^L + \overline{PGR}(1-\kappa^G-\rho^G) - \overline{PLR}(1-\kappa^L-\rho^L) \end{aligned}$$

because $PGV_{KQ} + PGV_{KS} + PGV_{SK} = \overline{PGR}$ and $PGV_{SQ} = \overline{PGK} = 1 - \overline{PGR}$ in gains as well as $PLV_{KQ} + PLV_{KS} + PLV_{SK} = \overline{PLR}$ and $PLV_{SQ} = \overline{PLK} = 1 - \overline{PLR}$ in losses. By Lemma 3, $\overline{PGR}$ converges to 0 and $\overline{PLR}$ converges to 1 for "large n," so that DM_1 converges to $\kappa^L + \rho^G - 1$ for "large n." This limit value of DM_1 is approached from below (above) if and only if

$$\overline{PGR}(1-\kappa^G-\rho^G) < (>)\ (\overline{PLR}-1)(1-\kappa^L-\rho^L).$$

By Proposition 1, we know that $0 < \overline{PGR} < \overline{PLR} < 1$ outside the limit. Thus, if $\kappa^L + \rho^L \leq 1 \wedge \kappa^G + \rho^G \leq 1$ (with at least one strict inequality), the limit value of DM_1 is approached from above. If $\kappa^L + \rho^L \geq 1 \wedge \kappa^G + \rho^G \geq 1$ (with at least one strict inequality), the limit value of DM_1 is approached from below.

For DM_2 our decomposition yields

$$\begin{aligned} DM_2 &= \frac{PGV_{KQ}(1-\kappa^G) + PGV_{KS}(1-\kappa^G) + PGV_{SQ}(\rho^G) + PGV_{SK}(1-\kappa^G)}{PLV_{KQ}(1-\kappa^L) + PLV_{KS}(1-\kappa^L) + PLV_{SQ}(\rho^L) + PLV_{SK}(1-\kappa^L)} - 1 \\ &= \frac{\rho^G + \overline{PGR}(1-\kappa^G-\rho^G)}{\rho^L + \overline{PLR}(1-\kappa^L-\rho^L)} - 1 \end{aligned}$$

because $PGV_{KQ} + PGV_{KS} + PGV_{SK} = \overline{PGR}$ and $PGV_{SQ} = \overline{PGK} = 1 - \overline{PGR}$ in gains as well as $PLV_{KQ} + PLV_{KS} + PLV_{SK} = \overline{PLR}$ and $PLV_{SQ} = \overline{PLK} = 1 - \overline{PLR}$ in losses. By Lemma 3, $\overline{PGR}$ converges to 0 and $\overline{PLR}$ converges to 1 for "large n," so that DM_2 converges to $\frac{\rho^G}{1-\kappa^L} - 1$ for "large n." This limit value of DM_2 is approached from below (above) if and only if

$$\overline{PGR}(1-\kappa^G-\rho^G)(1-\kappa^L) < (>)\ (\overline{PLR}-1)(1-\kappa^L-\rho^L)\rho^G.$$

By Proposition 1, we know that $0 < \overline{PGR} < \overline{PLR} < 1$ outside the limit. Further, note that $\rho^D > 0$ and $\kappa^L < 1$. Thus, if $\kappa^L + \rho^L \leq 1 \wedge \kappa^G + \rho^G \leq 1$ (with at least one strict inequality), the limit value of DM_2 is approached from above. If $\kappa^L + \rho^L \geq 1 \wedge \kappa^G + \rho^G \geq 1$ (with at least one strict inequality), the limit value of DM_2 is approached from below. ■

B Appendix: Benchmark Disposition Measure

In this appendix, we show how $\overline{PGR}$ and $\overline{PLR}$ (and hence $\overline{DM_i}$) can be explicitly computed for any parameterization of the stochastic environment and for any risk attitude of the investor. Precisely,

$$\begin{aligned}
\overline{PGR} &= PGV_{KQ} + PGV_{KS} + PGV_{SK} = \frac{\overline{NPGR}(\theta)}{\overline{DPGR}(\theta)}, \\
\overline{PLR} &= PLV_{KQ} + PLV_{KS} + PLV_{SK} \quad = \frac{\overline{NPLR}(\theta)}{\overline{DPLR}(\theta)}.
\end{aligned}$$

Our monotonicity assumption on the choice rule in period τ (see Section 2.3) implies that

$$\exists\, \theta \geq 0 \quad \text{with either} \qquad \theta > 0 \;\text{ s.t. }\; f(\Delta) = \begin{cases} A & \text{if } \Delta \geq \theta \\ O & \text{if } |\Delta| < \theta \\ B & \text{if } \Delta \leq -\theta, \end{cases}$$

$$\text{or} \qquad \theta = 0 \;\text{ s.t. }\; f(\Delta) = \begin{cases} A & \text{if } \Delta > 0 \\ (0.5 \circ A, 0.5 \circ B) & \text{if } \Delta = 0 \\ B & \text{if } \Delta < 0. \end{cases}$$

Note that for such a θ to exist, the expected utility of the better process needs to exceed that of the outside option.

In the case where $\theta > 0$, we then have

$$\begin{aligned}
\overline{NPGR}(\theta > 0) :=& \sum_{k=\theta}^{\tau} \sum_{l=0}^{k-\theta} \sum_{m=k+g}^{k+n} \sum_{j=m-\theta+1}^{l+n} \Pr\left(a = k,\, b = l,\, a' = m,\, b' = j\right) \\
&+ \sum_{k=\theta}^{\tau} \sum_{l=0}^{k-\theta} \sum_{m=k+g}^{k+n} \sum_{j=m-\theta+1}^{l+n} \Pr\left(b = k,\, a = l,\, b' = m,\, a' = j\right), \\
\overline{DPGR}(\theta > 0) :=& \sum_{k=\theta}^{\tau} \sum_{l=0}^{k-\theta} \sum_{m=k+g}^{k+n} \sum_{j=l}^{l+n} \Pr\left(a = k,\, b = l,\, a' = m,\, b' = j\right) \\
&+ \sum_{k=\theta}^{\tau} \sum_{l=0}^{k-\theta} \sum_{m=k+g}^{k+n} \sum_{j=l}^{l+n} \Pr\left(b = k,\, a = l,\, b' = m,\, a' = j\right), \\
\overline{NPLR}(\theta > 0) :=& \sum_{k=\theta}^{\tau} \sum_{l=0}^{k-\theta} \sum_{m=k}^{k+g-1} \sum_{j=m-\theta+1}^{l+n} \Pr\left(a = k,\, b = l,\, a' = m,\, b' = j\right) \\
&+ \sum_{k=\theta}^{\tau} \sum_{l=0}^{k-\theta} \sum_{m=k}^{k+g-1} \sum_{j=m-\theta+1}^{l+n} \Pr\left(b = k,\, a = l,\, b' = m,\, a' = j\right), \\
\text{and} \quad \overline{DPLR}(\theta > 0) :=& \sum_{k=\theta}^{\tau} \sum_{l=0}^{k-\theta} \sum_{m=k}^{k+g-1} \sum_{j=l}^{l+n} \Pr\left(a = k,\, b = l,\, a' = m,\, b' = j\right) \\
&+ \sum_{k=\theta}^{\tau} \sum_{l=0}^{k-\theta} \sum_{m=k}^{k+g-1} \sum_{j=l}^{l+n} \Pr\left(b = k,\, a = l,\, b' = m,\, a' = j\right).
\end{aligned}$$

These multi-sums of price-path realization-combinations of both assets are the numerators and denominators of the proportions of gains respectively losses that should be realized, according to the full benchmark. The first two sums over k and l specify an asset price combination at the initial purchase decision at $t = \tau$, and the boundaries are chosen such that only those combinations are considered where a risky asset is purchased, given the risk preference represented by θ. The third sum pins down whether the own asset is in gains or losses, i.e., it specifies the number of price increases of the own asset between periods τ and τ' as bigger or smaller than g (recall that g is the minimum number of "ups" between τ and τ' for an asset to be in gains). Last but not least, the inner-most sum collects the states where the own asset should be realized in the numerators, and all possible realizations of "the other" asset in the denominators. An asset should be realized, i.e., sold, whenever it is first- or second-order dominated, i.e., whenever the overall difference in the number of ups of the own and the other asset is negative or below the investment threshold θ.

Two remarks are in order. First, the fact that A and B may "swap" places does not imply that both sums together are equal to two times one of the sums, because even though the bounds of the sums are identical, the arguments, i.e., the probabilities, are not symmetric in A and B since

$$\begin{aligned}&\Pr\left(a = k, b = l, a' = m, b' = j\right)\\&= \binom{\tau}{k}\binom{\tau}{l}\binom{n}{m-k}\binom{n}{j-l} p_A^m (1-p_A)^{\tau+n-m} p_B^j (1-p_B)^{\tau+n-j}\end{aligned}$$

and

$$\begin{aligned}&\Pr\left(b = k, a = l, b' = m, a' = j\right)\\&= \binom{\tau}{k}\binom{\tau}{l}\binom{n}{m-k}\binom{n}{j-l} p_B^m (1-p_B)^{\tau+n-m} p_A^j (1-p_A)^{\tau+n-j},\end{aligned}$$

where $p_A = p_h$, $p_B = p_l$ if A follows F_h and $p_A = p_l$, $p_B = p_h$ if B follows F_h.

Second, the outer three sums are always non-empty, whereas the inner-most sum is empty for "big" m. Precisely, $\theta \leq \tau$ is needed for the investor to buy an asset in some contingency, which is of course necessary for the disposition measure to exist; $k - \theta \geq 0$ is implied by $k \geq \theta$ by definition of the first sum; $k + n \geq k + g$ follows from $n \geq g$; $m - \theta + 1 > l + n$ for $m = k + n$ (upper bound in gains) as $k - \theta \geq l$ by definition of the second sum, and $m - \theta + 1 \leq l + n$ for $m = k$ (lower bound in losses) as $k \leq \tau < n$.

In the other case where $\theta = 0$, due to the initial randomization for uninformative priors at $t = \tau$, we have

$$\begin{aligned}\overline{NPGR}(\theta = 0) := &\sum_{k=1}^{\tau}\sum_{l=0}^{k-1}\sum_{m=k+g}^{k+n}\sum_{j=m+1}^{l+n} \Pr\left(a = k,\, b = l,\, a' = m,\, b' = j\right)\\&+ \sum_{k=1}^{\tau}\sum_{l=0}^{k-1}\sum_{m=k+g}^{k+n}\sum_{j=m+1}^{l+n} \Pr\left(b = k,\, a = l,\, b' = m,\, a' = j\right)\\&+ \frac{1}{2}\sum_{k=0}^{\tau}\sum_{m=k+g}^{k+n}\sum_{j=m+1}^{k+n} \Pr\left(a = k,\, b = k,\, a' = m,\, b' = j\right)\\&+ \frac{1}{2}\sum_{k=0}^{\tau}\sum_{m=k+g}^{k+n}\sum_{j=m+1}^{k+n} \Pr\left(b = k,\, a = k,\, b' = m,\, a' = j\right),\end{aligned}$$

$$
\begin{aligned}
\overline{DPGR}(\theta=0) := & \sum_{k=1}^{\tau}\sum_{l=0}^{k-1}\sum_{m=k+g}^{k+n}\sum_{j=l}^{l+n}\Pr\left(a=k,\, b=l,\, a'=m,\, b'=j\right) \\
& + \sum_{k=1}^{\tau}\sum_{l=0}^{k-1}\sum_{m=k+g}^{k+n}\sum_{j=l}^{l+n}\Pr\left(b=k,\, a=l,\, b'=m,\, a'=j\right) \\
& + \frac{1}{2}\sum_{k=0}^{\tau}\sum_{m=k+g}^{k+n}\sum_{j=k}^{k+n}\Pr\left(a=k,\, b=k,\, a'=m,\, b'=j\right) \\
& + \frac{1}{2}\sum_{k=0}^{\tau}\sum_{m=k+g}^{k+n}\sum_{j=k}^{k+n}\Pr\left(b=k,\, a=k,\, b'=m,\, a'=j\right), \\
\overline{NPLR}(\theta=0) := & \sum_{k=1}^{\tau}\sum_{l=0}^{k-1}\sum_{m=k}^{k+g-1}\sum_{j=m+1}^{l+n}\Pr\left(a=k,\, b=l,\, a'=m,\, b'=j\right) \\
& + \sum_{k=1}^{\tau}\sum_{l=0}^{k-1}\sum_{m=k}^{k+g-1}\sum_{j=m+1}^{l+n}\Pr\left(b=k,\, a=l,\, b'=m,\, a'=j\right) \\
& + \frac{1}{2}\sum_{k=0}^{\tau}\sum_{m=k}^{k+g-1}\sum_{j=m+1}^{k+n}\Pr\left(a=k,\, b=k,\, a'=m,\, b'=j\right) \\
& + \frac{1}{2}\sum_{k=0}^{\tau}\sum_{m=k}^{k+g-1}\sum_{j=m+1}^{k+n}\Pr\left(b=k,\, a=k,\, b'=m,\, a'=j\right), \\
\text{and}\quad \overline{DPLR}(\theta=0) := & \sum_{k=1}^{\tau}\sum_{l=0}^{k-1}\sum_{m=k}^{k+g-1}\sum_{j=l}^{l+n}\Pr\left(a=k,\, b=l,\, a'=m,\, b'=j\right) \\
& + \sum_{k=1}^{\tau}\sum_{l=0}^{k-1}\sum_{m=k}^{k+g-1}\sum_{j=l}^{l+n}\Pr\left(b=k,\, a=l,\, b'=m,\, a'=j\right) \\
& + \frac{1}{2}\sum_{k=0}^{\tau}\sum_{m=k}^{k+g-1}\sum_{j=k}^{k+n}\Pr\left(a=k,\, b=k,\, a'=m,\, b'=j\right) \\
& + \frac{1}{2}\sum_{k=0}^{\tau}\sum_{m=k}^{k+g-1}\sum_{j=k}^{k+n}\Pr\left(b=k,\, a=k,\, b'=m,\, a'=j\right).
\end{aligned}
$$

Thus, for any risk attitude of the investor, $\overline{PGR}$, $\overline{PLR}$, and hence $\overline{DM}_i$ can be explicitly computed.

References

Barber, Brad M., Yi-Tsung Lee, Yu-Jane Liu, and Terrance Odean, "Is the Aggregate Investor Reluctant to Realise Losses? Evidence From Taiwan," *European Financial Management*, 2007, *13* (3), 423–447.

Barberis, Nicholas and Wei Xiong, "What Drives the Disposition Effect? An Analysis of a Long-Standing Preference-Based Explanation," *Journal of Finance*, 2009, *64* (2), 751–784.

— **and** — , "Realization Utility," *Journal of Financial Economics*, 2012, *104* (2), 251–271.

Barberis, Nicholas C., "Thirty Years of Prospect Theory in Economics: A Review and Assessment," *Journal of Economic Perspectives*, 2013, *27* (1), 173–196.

— , "Psychology-Based Models of Asset Prices and Trading Volume," *in Handbook of Behavioral Economics*, 2019, *edited by Doug Bernheim, Stefano DellaVigna, and David Laibson*, Elsevier Press.

Brown, Philip, Nick Chappel, Ray da Silva Rosa, and Terry Walter, "The Reach of the Disposition Effect: Large Sample Evidence Across Investor Classes," *International Review of Finance*, 2006, *6* (1-2), 43–78.

Calvet, Laurent E., John Y. Campbell, and Paolo Sodini, "Fight or Flight? Portfolio Rebalancing By Individual Investors," *Quarterly Journal of Economics*, 2009, *124* (1), 301–348.

Camerer, Colin F., "Prospect Theory in the Wild: Evidence from the Field," *Chapter 16 in "Choices, Values, and Frames," edited by Daniel Kahneman and Amos Tversky*, 2000, *Cambridge University Press*, 288–300.

Chen, Gongmeng, Kenneth A. Kim, John R. Nofsinger, and Oliver M. Rui, "Trading Performance, Disposition Effect, Overconfidence, Representativeness Bias, and Experience of Emerging Market Investors," *Journal of Behavioral Decision Making*, 2007, *20* (4), 425–451.

Choe, Hyuk and Yunsung Eom, "The Disposition Effect and Investment Performance in the Futures Market," *Journal of Futures Markets*, 2009, *29* (6), 496–522.

Constantinides, George M., "Capital Market Equilibrium with Personal Tax," *Econometrica*, 1983, pp. 611–636.

— , "Optimal Stock Trading with Personal Taxes: Implications for Prices and the Abnormal January Returns," *Journal of Financial Economics*, 1984, *13* (1), 65–89.

Coval, Joshua D. and Tyler Shumway, "Do Behavioral Biases Affect Prices?," *Journal of Finance*, 2005, *60* (1), 1–34.

Crane, Alan D. and Jay C. Hartzell, "Is There a Disposition Effect in Corporate Investment Decisions? Evidence from Real Estate Investment Trusts," *Working Paper*, 2010.

DellaVigna, Stefano, "Psychology and Economics: Evidence from the Field," *Journal of Economic Literature*, 2009, *47* (2), 315–372.

Dhar, Ravi and Ning Zhu, "Up Close and Personal: Investor Sophistication and the Disposition Effect," *Management Science*, 2006, *52* (5), 726–740.

Dierick, Nicolas, Dries Heyman, Koen Inghelbrecht, and Hannes Stieperaere, "Financial Attention and the Disposition Effect," *Journal of Economic Behavior and Organization*, 2019, *163*, 190–217.

Feng, Lei and Mark S. Seasholes, "Do Investor Sophistication and Trading Experience Eliminate Behavioral Biases in Financial Markets?," *Review of Finance*, 2005, *9* (3), 305–351.

Fischbacher, Urs, Gerson Hoffmann, and Simeon Schudy, "The Causal Effect of Stop-Loss and Take-Gain Orders on the Disposition Effect," *Review of Financial Studies*, 2017, *30* (6), 2110–2129.

Frazzini, Andrea, "The Disposition Effect and Underreaction to News," *Journal of Finance*, 2006, *61* (4), 2017–2046.

Frydman, Cary, Nicholas Barberis, Colin Camerer, Peter Bossaerts, and Antonio Rangel, "Using Neural Data to Test a Theory of Investor Behavior: An Application to Realization Utility," *Journal of Finance*, 2014, *69* (2), 907–946.

— , Samuel M. Hartzmark, and David H. Solomon, "Rolling Mental Accounts," *Review of Financial Studies*, 2017, *31* (1), 362–397.

Genesove, David and Christopher Mayer, "Loss Aversion and Seller Behavior: Evidence From the Housing Market," *Quarterly Journal of Economics*, 2001, *116* (4), 1233–1260.

Gherzi, Svetlana, Daniel Egan, Neil Stewart, Emily Haisley, and Peter Ayton, "The Meerkat Effect: Personality and Market Returns Affect Investors' Portfolio Monitoring Behaviour," *Journal of Economic Behavior & Organization*, 2014, *107*, 512–526.

Grinblatt, Mark and Bing Han, "Prospect Theory, Mental Accounting, and Momentum," *Journal of Financial Economics*, 2005, *78*, 311–339.

— and Matti Keloharju, "What Makes Investors Trade?," *Journal of Finance*, 2001, *56* (2), 589–616.

— , — , and Juhani T. Linnainmaa, "IQ, Trading Behavior, and Performance," *Journal of Financial Economics*, 2012, *104* (2), 339–362.

Hartzmark, Samuel M. and David H. Solomon, "Efficiency and the Disposition Effect in NFL Prediction Markets," *Quarterly Journal of Finance*, 2012, *2* (3).

— , Samuel Hirshman, and Alex Imas, "Ownership, Learning, and Beliefs," *Working Paper*, 2020.

Heath, Chip, Steven Huddart, and Mark Lang, "Psychological Factors and Stock Option Exercise," *Quarterly Journal of Economics*, 1999, *114* (2), 601–627.

Heisler, Jeffrey, "Loss Aversion in a Futures Market: An Empirical Test," *Review of Futures Markets*, 1994, *13* (3), 793–826.

Henderson, Vicky, "Prospect Theory, Liquidation, and the Disposition Effect," *Management Science*, 2012, *58* (2), 445–460.

Hens, Thorsten and Martin Vlcek, "Does Prospect Theory Explain the Disposition Effect?," *Journal of Behavioral Finance*, 2011, *12* (3), 141–157.

Ingersoll, Jonathan E. and Lawrence J. Jin, "Realization Utility with Reference-Dependent Preferences," *Review of Financial Studies*, 2012, *26* (3), 723–767.

Karlsson, Niklas, George Loewenstein, and Duane Seppi, "The Ostrich Effect: Selective Attention to Information," *Journal of Risk and Uncertainty*, 2009, *38* (2), 95–115.

Kaustia, Markku, "Prospect Theory and the Disposition Effect," *Journal of Financial and Quantitative Analysis*, 2010, *45* (3), 791–812.

Locke, Peter R. and Steven C. Mann, "Professional Trader Discipline and Trade Disposition," *Journal of Financial Economics*, 2005, *76* (2), 401–444.

Magnani, Jacopo, "Testing for the Disposition Effect on Optimal Stopping Decisions," *American Economic Review: Papers and Proceedings*, 2015, *105* (5), 371–375.

Maier, Johannes K. and Dominik S. Fischer, "Behavioral Biases in Financial Decision Making," *Mimeo*, 2021.

Meng, Juanjuan and Xi Weng, "Can Prospect Theory Explain the Disposition Effect? A New Perspective on Reference Points," *Management Science*, 2017.

Odean, Terrance, "Are Investors Reluctant to Realize Their Losses?," *Journal of Finance*, 1998, *53* (5), 1775–1798.

Olafsson, Arna and Michaela Pagel, "The Ostrich in Us: Selective Attention to Personal Finances," *Working Paper*, 2018.

Poteshman, Allen M and Vitaly Serbin, "Clearly Irrational Financial Market Behavior: Evidence From the Early Exercise of Exchange Traded Stock Options," *Journal of Finance*, 2003, *58* (1), 37–70.

Shapira, Zur and Itzhak Venezia, "Patterns of Behavior of Professionally Managed and Independent Investors," *Journal of Banking & Finance*, 2001, *25* (8), 1573–1587.

Shefrin, Hersh and Meir Statman, "The Disposition to Sell Winners Too Early and Ride Losers Too Long: Theory and Evidence," *Journal of Finance*, 1985, *40* (3), 777–790.

Sicherman, Nachum, George Loewenstein, Duane J. Seppi, and Stephen P. Utkus, "Financial Attention," *Review of Financial Studies*, 2016, *29* (4), 863–897.

Weber, Martin and Colin F. Camerer, "The Disposition Effect in Securities Trading: An Experimental Analysis," *Journal of Economic Behavior & Organization*, 1998, *33* (2), 167–184.