
Delegation, Promotion, and Manager Selection

Benjamin Häusinger (LMU Munich)

Discussion Paper No. 209

December 9, 2019

Delegation, Promotion, and Manager Selection

Benjamin Häusinger*

University of Munich

This version: December 9, 2019

Abstract

Promotions serve two purposes. They ought to provide incentives for employees and to select the best employee for a management position. However, if non-contractible managerial decision rights give rise to private benefits and preference misalignment between managers and the firm, these two purposes are in conflict. This is because the worker with the largest private benefit as a manager has the strongest incentives to work hard to get promoted. This article shows how the interplay of managerial decision rights and performance-based promotions leads to a situation often referred to as the Peter principle: employees that create lower expected profits as managers have yet better promotion prospects. That finding still holds when the firm owner optimally chooses the promotion rule, the degree of delegation, and wage payments to both employees and managers. To optimize organizational design, the firm balances better worker incentivization but worse manager selection by using performance-based promotions and restricting managerial decision rights.

JEL classification codes: D2, L22, M51, M52

Keywords: Peter Principle, Promotion, Delegation of Decision Rights, Incentives, Manager Selection, Organizational Design.

***Correspondence:** Department of Economics, LMU Munich, Ludwigstr. 28 (Rg.), D-80539 München, Germany, email: benjamin.haeusinger@lmu.de.

Acknowledgments: I thank Dan Barron, Hungni Chen, Marvin Deversi, Florian Englmaier, Matthias Fahn, Yves Le Yaouanq, Steve Leider, Niko Matouschek, Mike Powell, Klaus Schmidt and Peter Schwardmann as well as participants at seminars at LMU Munich, the 11th RGS Doctoral Conference 2018, the 21st Colloquium for Personnel Economics 2018, the 33rd Annual Congress of the European Economic Association 2018, and the Annual Meeting of the Verein für Socialpolitik 2018 for valuable comments and suggestions. I gratefully acknowledge financial support from the *Elitenetzwerk Bayern* through their International Doctoral Program “Evidence-Based Economics” and the *Deutsche Forschungsgemeinschaft* through CRC TRR 190 (project number 280092119).

1 Introduction

Internal promotions are widely used to fill job vacancies.¹ They are ought to achieve two objectives at once, namely to create incentives for employees and to select the best managers (Milgrom and Roberts 1992). It is not obvious, however, that the two objectives are always in line. To motivate employees, promotions must be based on current employee performance. To select the best managers, they must be based on expected manager performance. If the two measures are negatively correlated, a conflict of objectives arises.

There is ample evidence that promotions are mostly based on past and current employee performance (Frederiksen et al. 2019); and the Peter principle states that this approach leads to worse management selection, in line with a conflict of objectives (Peter and Hull 1969). That claim, a trade-off between motivation and selection, is substantiated in Benson et al. (2018). They provide evidence that firms rather promote the best performing employees than the expectedly best managers. Indeed, firms could improve managerial performance by 30% if promotion decisions were based on other measures than employee performance.

This paper examines an arising trade-off between the two objectives of performance-based promotions, motivation and selection. Most importantly, it first illustrates that such trade-off emerges from misaligned incentives between managers and the firm and the delegation of decision rights. That approach differs from previous “skill-based” models such as Fairburn and Malcomson (2001), Lazear (2004), and Schöttner and Thiele (2010) and from the original intuition behind the Peter principle (Peter and Hull 1969). These models blame skill differences between employee and manager tasks for bad manager selection of performance-based promotions. In contrast, my theory offers another explanation, namely the exploitation of decision rights by promoted managers.

I show that employees who generate lower profits as managers are *incentivized more strongly* by the prospect of becoming manager; consequently they are also more likely to get promoted. The intuition stems from the fact that a management position, i.e. the promotion prize, entails more responsibility and more decision rights. Yet managerial decision-making is not contractible. Hence a manager can exploit decision rights at his own interest; only the preference alignment between manager and principal determines how private benefits and profits are influenced by the manager’s decisions. Consequently, lower preference alignment leads to higher private benefits for the manager and lower profits for the firm.

It follows that those employees who gain more from a promotion because of higher

¹For example, DeVaro et al. (2019) find that around 60% of new jobs are filled via internal promotions. Baker et al. (1988, p.600) state that “promotions are used as the primary incentive device in most organizations”. For a broader overview on empirical studies, see Gibbons and Roberts (2012).

private benefits generate lower profits. If promotions are based on employee performance, a trade-off between motivation and selection arises. First, employees work hard to get promoted as they are incentivized by future private benefits as managers. Furthermore, employees with lower preference alignment receive higher private benefits when promoted and thus they will work even harder than their competitors. Such behavior leads to a higher promotion probability and a negative selection effect arises: employees who generate lower profits as managers are more likely to get promoted.

The model further investigates the trade-off between motivation and selection from different angles. First it examines how a principal should optimally delegate decision rights when promotions are based on employee performance. Since managers receive private benefits from decision-making, the principal could limit their decision rights, and thus private benefits, by delegating fewer decisions. As a result, bad selection of managers is reduced at the expense of employee motivation. Consequently the optimal level of delegation balances both effects. One of the main findings shows that partial delegation can be optimal under performance-based promotions: the principal delegates only a limited number of decisions to the manager as more delegation would attract unprofitable managers too strongly.

Moreover, I analyze the optimality of performance-based promotions in light of the manager's decision rights. If a management position is equipped with many decision rights, the selection effect will be particularly pronounced. In this case the principal will not use performance-based promotions to select managers, to prevent those with strong preference misalignment from rising to the top. I further consider the optimal design of the organization, i.e. the joint choice of delegation and promotion. I show that (a) delegation and the simultaneous use of performance-based promotions lead to higher motivation of employees and (b) their joint use increases if preferences are sufficiently aligned and the selection effect is sufficiently weak.

The model also offers a new perspective to why we observe promotions at all when the Peter principle applies. Fairburn and Malcomson (2001, p.45) ask: "Why not (..) use promotions to assign employees to jobs and monetary bonuses to provide incentives?" I investigate this question in two extensions. I first analyze how worker wages and delegation interact. Workers are motivated by both wages and expected private benefits from delegation, implying a substitutability between the two incentive devices. Higher managerial discretion then comes with lower subordinates' wages. Overall, including monetary incentives decreases the number of decisions delegated, but, in general, partial delegation remains optimal. Secondly, I introduce further promotion-related wage increases (Baker et al. 1994). I show that if such a wage increase is optimal for the principal, the joint use of performance-based promotions and delegation of decision rights is profitable as well.

Related Literature. This work combines two distinct strands of literature, namely that on the (negative) selection effects of promotions and the optimal delegation of decision rights. First, it offers a new explanation for why performance-based promotion schemes induce inefficient selection of managers.² While theories differ in explaining the benefits of performance-based promotions³, they share the intuition behind its inefficiency: performance-based promotions result in managers with an insufficient skills. Accordingly, low manager performance arises because promoted workers lack the required skills to be good managers (Peter and Hull 1969; Bernhardt 1995; Fairburn and Malcomson 2001; Lazear 2004; Schöttner and Thiele 2010; Koch and Nafziger 2012). The current approach is different as it is fully independent of skill considerations. In my model, low manager performance arises because promoted workers exploit managerial decision rights; and they vary in the degree that they do so.

I also contribute to the literature on optimal delegation in which the principal delegates decision rights to the manager to make him, e.g., acquire information (Aghion and Tirole 1997), communicate truthfully (Dessein 2002), or exert effort (Bester and Krämer 2008). In contrast to these papers, I am concerned with the effects of delegation on the behavior of *employees*. My model focuses on the link between managerial benefits and employees' behavior via performance-based promotions. It is silent about potential sources of these benefits as well as the manager's decision problem. Empirical evidence regarding managerial private benefits can be, e.g., found in the literature on managerial empire building (Jensen 1986; Hope and Thomas 2008), short-termism (Bebchuk and Stole 1993; Edmans et al. 2017) and intrinsic valuation of decision rights (Fehr et al. 2013; Bartling et al. 2014). In general, managerial private benefits can arise from misaligned preferences as well as ill-designed incentive schemes (see the discussion in Dessein 2002, p.815).

There often exist interaction effects between different dimensions of organizational design (Roberts 2007), for example between job design, monitoring and incentives (so called "high performance work systems", see Ichniowski and Shaw 2003), or between the hierarchical structure of an organization and its use of promotions (Ke et al. 2018). In the current model, complementarities between delegation and the use of promotions arise because delegation increases private benefits of managers and thus the prize for winning a promotion.

Lastly, my work complements a literature in political economy on the delegation of

²Empirical evidence for that claim can be found in Grabner and Moers (2013) and Benson et al. (2018). They show promoting high-performing employees correlates negatively with manager quality. More specifically, Benson et al. (2018) find that firms could increase management performance by 30% if they based promotions on other measures than employee performance.

³For example, in Fairburn and Malcomson (2001) they prevent influence activities. In my model, in Schöttner and Thiele (2010) and Koch and Nafziger (2012) and a large literature on tournaments (Rosen 1986), performance-based promotions are used because they act as an incentive device.

authority and selection effects. Already Knight (1938) and Hayek (1944) discussed the influence of institutions and political systems on the selection of politicians. Besley (2005) points out that a political office’s “attractiveness” to candidates crucially depends on the rents they can extract while in office; these in turn depend on the office’s power. Such consideration will affect who is running for office, “egoistic” or “public-spirited” politicians.⁴ Similarly, the current model shows that decision rights, the “attractiveness” of a management position, attract employees who want to exploit those decision rights. Therefore, rents to promotion and power must be limited to mitigate potential selection effects. For political offices, this can be done, e.g., by setting up institutions to align a politician’s private interests (Barro 1973), his accountability, for instance via re-elections (Maskin and Tirole 2004; Acemoglu et al. 2010), the implementation of “checks and balances” (Persson et al. 1997), or power de-concentration (Grunewald et al. 2019). In organizations, the principal can simply restrict a manager’s decision rights.

The remainder of the paper is as follows. In Section 2 I introduce the model and then analyze optimal delegation and optimal promotion rules in Section 3. Section 4 presents two extensions of the model by introducing monetary incentives for the workers, via bonus schemes and via promotion-related wage increases. Section 5 concludes.

2 Model

Overview

The firm consists of a principal (female) and two workers (male). Both workers exert unobservable effort and compete for a promotion to a management position. The principal receives profits from both hierarchy levels, i.e. the workers’ work effort and from the decisions made by the manager who is a promoted worker. In order to maximize profits, the principal ex-ante chooses an organizational design that has two dimensions. Thereby she chooses a promotion rule and the degree of delegation of decision rights to the management position.

The model is introduced step-by-step. First I present the workers’ effort choice and how it is shaped by promotion prospects. Then I continue by introducing the management stage and the delegation decision. The model setup is concluded by bringing both parts together. The incorporation of wage payments, either to the manager or to the employees, is relegated to the model extensions in Section 4. Section 5 concludes.

⁴This idea of political selection is prevalent even in science-fiction novels. To quote David Brin, the author of the post-apocalyptic novel “The Postman” (1985, p. 267): “It is said that power corrupts, but actually it’s more true that power attracts the corruptible.”

Promotions and Workers' Effort

In the firm there are two workers competing for a promotion to a management position. A worker i exerts unobservable effort $e_i \in [0, 1]$ on a project at convex effort costs $c(e_i) = \frac{c}{2}e_i^2$. Each worker's project can either be a success or a failure. In case of success, the principal receives $S > 0$, otherwise 0. A worker i 's project success probability, given by $pr(success) = e_i$, increases linearly in his effort and is independent from the other worker's effort.

Workers are incentivized purely by promotion prospects and do not receive any wage payments.⁵ The principal P (she) ex-ante commits to a promotion rule. The promotion rule is fully captured by a promotion probability $p_i(e_i, e_j)$ for a worker i , given i 's and his coworker j 's effort levels e_i and e_j .

If worker i gets promoted he receives private benefits as a manager. These are denoted by u_{m_i} and will depend on the delegation decision as introduced later. In general the risk-neutral worker i 's utility function is given by

$$u_i(e_i) = p_i(e_i, e_j) \cdot u_{m_i} - \frac{c}{2}e_i^2. \quad (1)$$

Promotion Rules

The principal can decide between two promotion rules $prom \in \{\mathcal{P}, \mathcal{R}\}$. The “performance-based promotion” \mathcal{P} is based on the workers' project outcomes. When only one project is successful the principal promotes the successful worker. Otherwise, he randomizes between the workers.⁶ In contrast, the “random promotion” \mathcal{R} is fully independent of the workers' work. In that case, the principal randomizes between the workers and chooses each of them with probability $p^{\mathcal{R}} = 0.5$.⁷

Delegation

In the firm, a finite divisible number of similar decisions, normalized to 1, need to be made. The principal P can delegate $k \in [0, 1]$ of these decisions to a manager M (he) who can then implement his favored choice. The payoffs for each decision depend on the decision-maker:

⁵In Section 4.1, workers receive additional performance-based wage payments. The main results remain unchanged.

⁶In Appendix A, I show that these two promotion rules are superior to any convex combination of the two. Therefore, if it is optimal not to randomize fully between the workers, it is optimal not to randomize at all.

⁷One famous example for purely random promotions in a slightly different setting is that of ancient Athens. There, political offices were filled via lots to ensure “pure” democracy (see Headlam 1891). I thank Mike Powell for bringing up this example.

1. If P makes a decision, total surplus from this decision is $\pi > 0$. The manager cannot extract any private benefits and thus his payoff is $u_M = 0$. The principal receives the full surplus, thus her profits are $\Pi = \pi$.
2. If M makes a decision, total surplus from this decision is $\pi^D > \pi$. But the manager extracts a share $\alpha \in [0, 1]$ of the surplus, and his payoff is $u_M = \alpha\pi^D$. The principal receives the remaining share, $\Pi = (1 - \alpha)\pi^D$.

The principal delegates decision rights over a fraction of k decisions to the manager, and keeps decision rights over a fraction of $(1 - k)$ decisions to herself. Thus overall payoffs from managerial decision-making are given by

$$\Pi(k) = k \cdot (1 - \alpha)\pi^D + (1 - k) \cdot \pi = \pi + k \cdot (\delta - \alpha)\pi^D \quad (2)$$

$$u_M(k) = k \cdot \alpha\pi^D. \quad (3)$$

Here, $\delta = \frac{\pi^D - \pi}{\pi^D} \in (0, 1)$ displays the relative surplus increase due to better managerial decision-making. The setup resembles the main trade-off of a standard delegation problem in a stylized way.⁸ A manager makes overall better decisions than the principal, captured by $\pi^D > \pi$.⁹ However, delegation also comes with a loss of control which stems from the decisions' non-contractibility and a preference misalignment between manager and principal. The degree of preference misalignment is measured by α . Increasing α implies a stronger preference misalignment. Profits fall and private benefits rise.

Unknown Managerial Types

There are two different types of managers that have distinct degrees of preference misalignment. Manager types are private information. Each worker i is one of the two manager types, denoted by $m_i \in \{A, B\}$, with $prob(m_i = B) = \mu \in (0, 1)$. Both types are equally skilled and generate the same total surplus when making a decision, given by π^D . However, the types' preference alignment differs. A -type managers' preferences are well aligned, B -type managers' are not. I assume that $\alpha_B > \delta > \alpha_A$. Thus the principal would delegate all decisions to an A -type, but none to a B -type manager *if she knew the type*.

From a worker's perspective, a promotion hence yields private benefits of $u_{m_i}(k)$ that depend on his private type m_i as well as the management position's amount of decision

⁸For example, it arises from Aghion and Tirole (1997) with the following parameter values: $\alpha^{AT} = (1 - \alpha)\frac{\pi^D}{\pi}$, $\beta^{AT} = 0$, $b^{AT} = \alpha\pi^D$, $B^{AT} = \pi$, effort levels of $e^{AT} = E^{AT} = 1$ and normalized costs of $g_P^{AT}(1) = g_A^{AT}(1) = 0$.

⁹Reasons for the superiority of managers' decision-making include local information that are available only to the manager (Hayek 1945) or that delegation increases the manager's initiative (Aghion and Tirole 1997).

rights k . Therefore, worker i 's private benefits from a promotion are given by $u_{m_i} = k\alpha_{m_i}\pi^D$.¹⁰ I can re-write worker i 's utility function as

$$u_i(e_i) = p_i \cdot \underbrace{k \cdot \alpha_{m_i} \pi^D}_{u_{m_i}} - \frac{c}{2} e_i^2. \quad (4)$$

Timeline

To conclude the model setup, the time structure is as follows.

1. The principal chooses the degree of delegation k and the promotion rule $prom$.
2. Workers are independently drawn from the population with respect to their managerial type.
3. Workers simultaneously exert unobservable effort, and each worker's project outcome is realized and observed.
4. The principal promotes one of the workers according to the promotion rule chosen in $t = 1$. The other worker leaves the firm and receives an outside utility of $\underline{u} = 0$.
5. Decision rights are delegated to the newly promoted manager, according to the choice in $t = 1$. Payoffs from decision-making are realized and the game ends.

3 Analysis

In this section I analyze the model presented above. First I examine the optimal choice of delegation for each promotion rule. Then I derive the optimal promotion rule, having fixed the degree of delegation. At the end of this section I analyze the optimal joint decision of delegation and promotion rule. All proofs can be found in Appendix B.

3.1 The Effects of Promotion Rules on Delegation

3.1.1 Random Promotion

Random promotion implies a fixed promotion probability of $p^{\mathcal{R}} = 0.5$ for each worker that is independent of effort. A worker cannot influence the probability of promotion and thus has no incentives to work. It follows that $e^{\mathcal{R}} = 0$ for both workers. Then the principal faces a B-type manager with probability μ , as if there was a random draw from the population. This is because workers are drawn independently from the population and

¹⁰For simplicity, I assume $u_B \leq 2c$. This assures interior solutions for the optimal effort levels, see Lemma 1.

are also promoted randomly.¹¹ Let $\bar{\alpha} = (1 - \mu)\alpha_A + \mu\alpha_B$ denote the expected preference misalignment. As $e^{\mathcal{R}} = 0$, projects are unsuccessful with certainty and the principal's profits only consist of the payoff from decision-making:

$$E\Pi^{\mathcal{R}} = \mu\Pi_B + (1 - \mu)\Pi_A = \pi + k(\delta - \bar{\alpha})\pi^D \quad (5)$$

where $\Pi_m = \pi + (\delta - \alpha_m)\pi^D$ are decision-making profits when facing manager type $m \in \{A, B\}$.

Consequently, optimal delegation under random promotion depends on the relative benefits from managerial decision-making and losses from *expected* preference misalignment. Note that the principal's profits are linear in k . Therefore, if and only if the benefits from managerial decision-making, given by δ , outweigh the expected loss of control due to preference misalignment, $\bar{\alpha}$, the principal delegates all decision rights to the manager, and none otherwise. This is summarized in Proposition 1.

Proposition 1.

Under random promotion, the principal either delegates all decisions, or none:

$$k^{\mathcal{R}} = \begin{cases} 1 & \text{if } \delta \geq \bar{\alpha} \\ 0 & \text{if } \delta < \bar{\alpha}. \end{cases} \quad (6)$$

3.1.2 Performance-Based Promotion

Under performance-based promotions each worker's promotion probability depends on the respective project success. It follows that workers face a strategic game in which their expected utility and thus their optimal strategy depend both on their own and their co-worker's exerted effort. However, workers observe neither their co-worker's managerial type nor their effort level. This game is simplified by its binary structure. A worker i 's expected promotion probability is given by

$$\begin{aligned}
p_i = & \underbrace{\mu \left[\underbrace{e_i(1 - e_{B_j})}_{i \text{ is successful}} + \underbrace{0.5e_i e_{B_j}}_{\text{both successful}} + \underbrace{0.5(1 - e_i)(1 - e_{B_j})}_{\text{both unsuccessful}} \right]}_{\text{The other worker is a B-type}} \\
& + (1 - \mu) \underbrace{\left[\underbrace{e_i(1 - e_{A_j})}_{i \text{ is successful}} + \underbrace{0.5e_i e_{A_j}}_{\text{both successful}} + \underbrace{0.5(1 - e_i)(1 - e_{A_j})}_{\text{both unsuccessful}} \right]}_{\text{The other worker is a A-type}} \quad (7)
\end{aligned}$$

¹¹Therefore, an alternative interpretation of random promotions in this model is the hiring of outsider managers. In this case, both workers do not exert effort, i.e. $e = 0$, leave the firm and receive their outside utility of $\underline{u} = 0$. A manager from outside the firm is hired as a random draw from the population. All results for random promotions thus also hold for the hiring of outsider managers.

which can be simplified to

$$p_i = 0.5 + 0.5(e_i - \bar{e}_j). \quad (8)$$

Here, $\bar{e}_j = (1 - \mu)e_{A_j} + \mu e_{B_j}$ denotes the ex-ante expected effort level of a worker with e_{m_j} defining the effort level of worker j with managerial type m . The resulting functional form of p_i is a Difference Contest Success Function with unknown contenders (Hirshleifer 1989). It follows that the marginal effect of i 's effort on his promotion probability is independent of his expected co-worker's effort. Therefore, each worker has unique dominant strategy that is derived by standard utility maximization. Lemma 1 states optimal effort provision in the resulting Nash equilibrium.

Lemma 1.

A worker's optimal equilibrium effort under performance-based promotions increases in the degree of delegation and is higher for B-type workers and is given by

$$e_m^{\mathcal{P}} = k \cdot \frac{\alpha_m \pi^D}{2c}. \quad (9)$$

Workers exert effort, i.e. $e_m^{\mathcal{P}} > 0$, only if there is a positive amount of delegation, i.e. $k > 0$. Moreover, because private benefits of a manager are increasing in both the degree of delegation and the preference misalignment, so are expected utility and effort provision. This is the *incentive effect* of performance-based promotions. Moreover, since the preference misalignment of a B-type manager is larger, i.e. $\alpha_B > \alpha_A$, B-type workers exert higher effort than A-types. This translates into a higher probability of promotion for the B-type. A *selection effect* arises, stated in Lemma 2a. Note that the selection effect only arises with a heterogeneous workforce, i.e. if there are one A-type as well as one B-type worker among the workers. In a homogeneous workforce, both workers are of the same type, exert the same effort and are promoted with the same probability, as described by Lemma 2b.

Lemma 2.

Under performance-based promotion,

(a) *with a heterogeneous workforce with one A-type and one B-type worker, the following statements hold if and only if there is delegation ($k > 0$):*

– *B-type workers are promoted with a higher probability than A-types:*

$$p_B^h - p_A^h = k \cdot \frac{(\alpha_B - \alpha_A)\pi^D}{2c} > 0 \quad (10)$$

where p_m^h denotes the probability of success for type m in a heterogeneous workforce.

- The types' difference in their probability of promotion is increasing in the degree of delegation: $\frac{\partial(p_B^h - p_A^h)}{\partial k} > 0$.

(b) with a homogeneous workforce, workers exert identical, but type-dependent effort and thus have the same probability of promotion of $p_m^{hom} = 0.5$.

Lemma 2a says that workers who make less profitable decisions are promoted with a higher probability. This is a mildly revised, more general version of the Peter principle as it implies that, in expectations, profits are reduced by promoting the worse manager. The mechanism in Lemma 2 also implies that a negative selection effect persists even when the principal bases the promotion decision on other measures than current performance, provided that workers still can influence their promotion probability, for instance by shifting effort towards the new promotion measure or by gaming.

Taking optimal worker behavior from Lemmas 1 and 2 as given, the principal maximizes his own expected profits $E\Pi^P$ over the degree of delegation k . Since managerial types are private, expected profits are given by

$$E\Pi^P = \overbrace{(1 - \mu)^2[\Pi_A + 2e_A^P S]}^{\text{only A-type workers}} + \overbrace{\mu(1 - \mu)[p_A^h \Pi_A + p_B^h \Pi_B + 2\bar{e}^P S]}^{\text{heterogenous workforce}} + \overbrace{\mu^2[\Pi_B + 2e_B^P S]}^{\text{only B-type workers}} \quad (11)$$

with $\bar{e}^P = (1 - \mu)e_A^P + \mu e_B^P$ and optimal effort levels given by Lemma 1. Expected profits can be displayed by considering each potential workforce composition. For each of these, profits depend on the (expected) payoff from managerial decision-making and workers' project success. Moreover, Equation (11) can be decomposed into three parts:

$$\begin{aligned} E\Pi^P &= \underbrace{(1 - \mu)\Pi_A + \mu\Pi_B}_{\text{Random Promotion}} + \underbrace{2 \cdot \bar{e}^P S}_{\text{Incentive Effect}} + \underbrace{2\mu(1 - \mu) \frac{(p_B^h - p_A^h)(\Pi_B - \Pi_A)}{2}}_{\text{Selection Effect}} \\ &= \pi + k(\delta - \bar{\alpha})\pi^D + \frac{k\bar{\alpha}\pi^D}{c} S - \mu(1 - \mu) \frac{(k(\alpha_B - \alpha_A)\pi^D)^2}{2c}. \end{aligned} \quad (12)$$

Compared to random promotions, a performance-based promotion induces two further effects on expected profits. It increases worker incentives and thus expected gains from successful projects (the incentive effect). On the other hand, it worsens management selection and lowers expected profits from managerial decision-making by promoting the “wrong kind of manager” with a higher probability (the selection effect). Profit maximization leads to the optimal degree of delegation, given in Proposition 2.

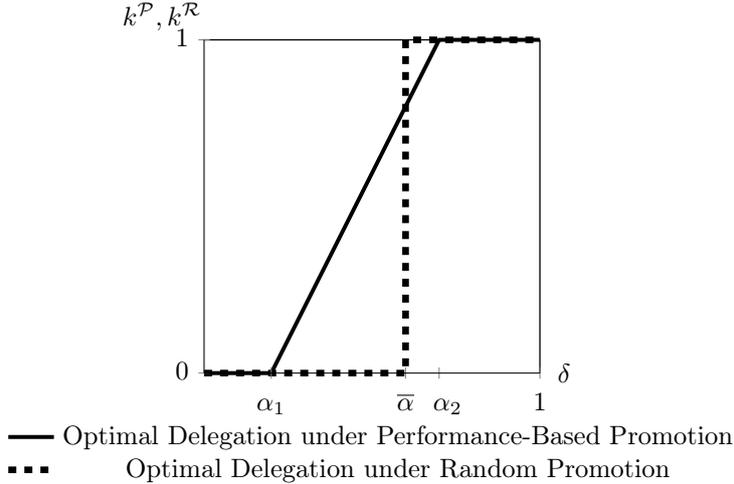


Figure 1: Optimal Delegation under Random and Performance-Based Promotions.

Note: This is an illustration of Propositions 1 and 2 for $\alpha_2 > \bar{\alpha}$.

Proposition 2.

Under performance-based promotion, optimal delegation is given by

$$k^P = \begin{cases} 1 & \text{if } \delta \geq \alpha_2 \\ \tilde{k} & \text{if } \delta \in [\alpha_1, \alpha_2) \\ 0 & \text{if } \delta < \alpha_1 \end{cases} \quad (13)$$

with $\tilde{k} = \frac{c(\delta - \bar{\alpha}) + \bar{\alpha}S}{\mu(1-\mu)(\alpha_B - \alpha_A)^2\pi^D}$, $\alpha_1 = (1 - \frac{S}{c}) \cdot \bar{\alpha}$ and $\alpha_2 = (1 - \frac{S}{c}) \cdot \bar{\alpha} + \frac{\mu(1-\mu)(\alpha_B - \alpha_A)^2\pi^D}{c}$.

Figure 1 gives a graphical illustration of that result. One can explain optimal delegation by focusing on the three effects of delegation on expected profits, as displayed in Equation (12). First, optimal delegation for random promotion is given by Proposition 1. If $\delta \geq \bar{\alpha}$, there is full delegation, and none otherwise. This is depicted in Figure 1 as the dotted line. Under performance-based promotions, two additional effects come into play. The incentive effect on worker behavior increases profits as more delegation makes both workers work harder and thus increases the probability of project success, as is shown in Lemma 1. Generally the incentive to delegate increases if the expected loss from preference misalignment is sufficiently low, i.e. $\alpha_1 < \delta$. The selection effect only arises in a heterogeneous workforce which occurs with probability $2\mu(1 - \mu)$, see Lemma 2. It is a combination of two distinct effects as delegation affects both workers' and the manager's behavior. Since delegation affects workers' effort differently, it increases the probability that a B-type worker gets promoted, $\frac{\partial(p_B^h - p_A^h)}{\partial k} > 0$. It also increases the relative loss in profits when a B-type manager takes decisions instead of an A-type manager, $\frac{\partial(\Pi_B - \Pi_A)}{\partial k} < 0$. Additionally, there exists an interaction between the two effects. The

severity of an increase in B-type's probability of getting promoted depends on the loss that is related to B-type managers, and vice versa.

To gain more intuition, suppose $k = 0$. Then, the relative loss and the difference in promotion probabilities are zero as well. Increase k marginally. The effect of a marginal increase on profits is given by the marginal increase in B-type promotion probability, *holding fixed the relative loss*, and the marginal increases in the relative loss, *holding fixed promotion probabilities*. A marginal increase in B-type promotion probability does not affect profits as the relative loss at $k = 0$ is still zero, and vice versa. Therefore, at $k = 0$, a marginal increase in delegation *does not affect* profits via the selection effect. On the other hand, suppose k is close to 1. Then, a marginal increase in k is severe as (a) the marginal effect on promotion probabilities is large because relative loss is high already and (b) the marginal effect on the relative loss matters because the difference in promotion probabilities is also high. Thus, even though profits from decision-making and optimal effort level are linearly increasing in k , expected profits are quadratic in k , due to the selection effect. This intuition is summarized in Lemma 3.

Lemma 3.

The selection effect of delegation under performance-based promotion is an increasing, quadratic function of k , being zero at $k = 0$. It decreases expected profits for any $k > 0$.

There are two implications of Lemma 3. First, because the selection effect does not exist at $k = 0$, the threshold α_1 is unaffected by selection concerns. Secondly, under a performance-based promotion the optimal degree of delegation is continuous for intermediate δ , as shown in Proposition 2. While the incentive effect makes delegation profitable for $\delta \in (\alpha, \bar{\alpha})$, the convexity of the selection effect makes full delegation too costly if $\delta \in [\bar{\alpha}, \alpha_2)$.

Comparative Statics. If the manager's relative advantage in decision-making is small (low δ) or the expected loss of control is large (high $\bar{\alpha}$), it is not worthwhile for the principal to delegate because profits from delegation will be low. The same applies if gains from worker incentivization are sufficiently small because of a low value of successful projects (low S) or very high effort costs (high c).

On the other hand, delegation is always positive if projects are sufficiently profitable, i.e. if $S > c$. In this case, the incentive effect is positive and outweighs potential losses due to worse decision-making.¹² Furthermore, for sufficiently high profits from managerial decision-making, the principal delegates all decisions if the project implications from bad selection are sufficiently harmless, for instance because a heterogeneous workforce is

¹²One can see that as, at $k = 0$, the marginal effect of increasing k on expected profits is given by $(\delta - \bar{\alpha} + \frac{\bar{\alpha}S}{c})\pi^D$ which is positive for $S > c$.

unlikely (μ close to 0 or 1). Selection is also a minor concern if the heterogeneity among managers is low (low $\alpha_B - \alpha_A$).

Partial delegation arises whenever gains from worker incentives are sufficiently high ($\delta > \alpha_1$) but the selection effect is sufficiently strong ($\delta < \alpha_2$).

Does performance-based promotion always lead to higher delegation compared to random promotion? The answer again depends on the relative strength of the selection effect. It determines the size of α_2 , the “full delegation” threshold under performance-based promotion. If the selection effect is sufficiently strong, then $\alpha_2 > \bar{\alpha}$ and the principal uses partial delegation under performance-based promotion and full delegation under random promotion for $\delta \in [\bar{\alpha}, \alpha_2]$. However, if the selection effect is relatively weak compared to the incentive effect then optimal delegation is always higher under performance-based promotion. This finding is summarized in Corollary 1 and depicted in Figure 2.

Corollary 1.

Performance-based promotions induce higher delegation than random promotion if $\mu(1 - \mu)(\alpha_B - \alpha_A)^2 \pi^D < \bar{\alpha}S$, i.e. if the selection effect is sufficiently weak.

Corollary 1 shows a positive correlation between the use of performance-based promotions and the degree of delegation for a sufficiently weak selection effect. This is consistent with empirical evidence provided by Alfaro et al. (2016). They find a positive correlation between a plant’s incentive practices, such as performance-based promotions, and the degree of delegation from headquarters to that plant.¹³

3.2 The Effects of Delegation on Promotion Rules

Following Corollary 1, I further analyze the optimal choice of promotion rule holding fixed the degree of delegation. That mimics manifold situations in which the principal is bound to a positive degree of delegation, at least in the short term. Reasons include information overload, time constraints and hierarchical structures within the firm. In the model, I fix k at $\bar{k} > 0$. The principal can only decide on the promotion rule. Comparing the expected profits under the two promotions practices, gives by Equations (5) and (12), implies that the optimal promotion practice ultimately depends on the incentive and the selection effect:

$$E\Pi^P - E\Pi^R = \underbrace{2 \cdot \bar{e}^P S}_{\text{Incentive Effect}} + 2\mu(1 - \mu) \underbrace{\frac{(p_B^h - p_A^h)(\Pi_B - \Pi_A)}{2}}_{\text{Selection Effect}}. \quad (14)$$

¹³Thereby, they use data on management policies from the World Management Survey. Its index on incentive practices includes two measures on the use of performance-based promotions, bonuses and talent management.

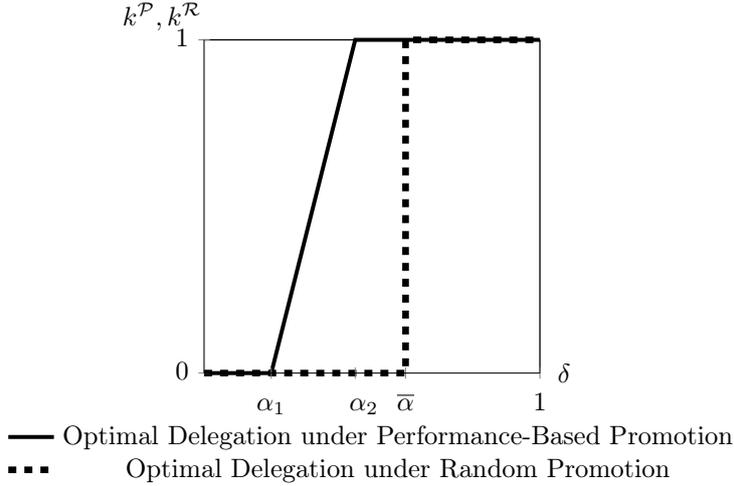


Figure 2: Higher Delegation under Performance-Based Promotions.

Note: This is an illustration of Corollary 1. If $\alpha_2 < \bar{\alpha}$, delegation is always weakly higher under performance-based promotion.

Thus, performance-based promotions are optimal if and only if the selection effect is sufficiently small. This in turn depends on the degree of delegation. The selection effect leads to a disproportionate reduction in profits with increasing delegation. Therefore performance-based promotions outperform random promotions for sufficiently low levels of \bar{k} , as stated in Corollary 2.

Corollary 2.

For a fixed degree of delegation \bar{k} , performance-based promotions outperform random promotions if and only if $\bar{k} < \bar{k}^{\mathcal{P}} = \frac{2\bar{\alpha}S}{\mu(1-\mu)(\alpha_B - \alpha_A)^2\pi^D}$, i.e. the degree of delegation is sufficiently low .

Note that $\bar{k}^{\mathcal{P}}$ is decreasing in the size of the selection effect, $\mu(1-\mu)(\alpha_B - \alpha_A)^2\pi^D$, and increasing in the expected profits from motivating workers, $2\bar{\alpha}S$. Furthermore, performance-based promotion is optimal for any degree of delegation if the selection effect is sufficiently weak because in that case $\bar{k}^{\mathcal{P}} \geq 1$. If the selection effect is strong, and the degree of delegation is sufficiently high ($\bar{k} > \bar{k}^{\mathcal{P}}$), the principal may refrain from performance-based promotion. This is related to a finding by Benson et al. (2018). They show that the use of performance-based promotion is decreasing in the manager’s team size. Under the assumption of a positive correlation between team size and the team manager’s decision rights, Corollary 2 provides an explanation. The negative selection effect of performance-based promotions outweighs the expected benefits from worker motivation.

3.3 Optimal Organizational Design

When designing an organization, the principal must jointly consider all dimensions of organizational design. Only then she accounts for potential complementarities between different dimensions of organizational design as argued by Roberts (2007). Otherwise, the implementation of a certain policy on one dimension can well be ineffective, or even harmful, without complementary changes in another.¹⁴ In the current model the principal jointly decides over the optimal degree of delegation and the promotion rule, i.e. she chooses $(k^*, prom^*) \in \{(k^{\mathcal{P}}, \mathcal{P}); (k^{\mathcal{R}}, \mathcal{R})\}$ with

$$(k^*, prom^*) \arg \max \{E\Pi^{\mathcal{P}}(k^{\mathcal{P}}); E\Pi^{\mathcal{R}}(k^{\mathcal{R}})\}. \quad (15)$$

Indeed, the optimal organizational design as described in Proposition 3 does make use of complementarities between delegation and promotion rules. Profitability of delegation is higher under performance-based promotion practices unless the selection effect is too strong, and vice versa.

Proposition 3.

The optimal organizational design is as follows.

- If $\delta \leq \alpha_1$, there is no delegation and the promotion rules are equivalent:

$$(k^*, prom^*) = (0, \mathcal{P}) = (0, \mathcal{R}). \quad (16)$$

- If $\delta \in (\alpha_1, \bar{\alpha}]$, performance-based promotion with delegation is optimal:

$$(k^*, prom^*) = (k^{\mathcal{P}}, \mathcal{P}). \quad (17)$$

- If $\delta > \bar{\alpha}$, performance-based promotion with delegation is optimal if and only if the negative selection effect is sufficiently small:

$$(k^*, prom^*) = \begin{cases} (k^{\mathcal{P}}, \mathcal{P}) & \text{iff } \mu(1 - \mu)(\alpha_B - \alpha_A)^2 \pi^D \leq \frac{[c(\delta - \bar{\alpha}) + \bar{\alpha}S]^2}{c(\delta - \bar{\alpha})} \\ (1, \mathcal{R}) & \text{iff } \mu(1 - \mu)(\alpha_B - \alpha_A)^2 \pi^D > \frac{[c(\delta - \bar{\alpha}) + \bar{\alpha}S]^2}{c(\delta - \bar{\alpha})}. \end{cases} \quad (18)$$

The intuition is as follows. If benefits from managerial decision-making are sufficiently low, i.e. $\delta \leq \alpha_1 < \bar{\alpha}$, the principal refrains from delegating any decision rights. Yet if there is no delegation the private benefits of a manager are zero and both promotion rules

¹⁴One example for a failure of organizational design due to missing complementarities are GM's investments in automation and flexibility in its production processes in the 1980s. As GM did not make complementary changes in other dimensions of internal organization, the investment eventually led to large losses (Roberts 2007, p.40).

become equivalent. Thus, the principal is indifferent between the two. If $\delta \in (\alpha_1, \bar{\alpha}]$, the principal optimally chooses either partial or full delegation ($k^P = \{\tilde{k}, 1\} > 0$) under a performance-based promotion and no delegation under a random promotion. But because $k^P = \{\tilde{k}, 1\} > 0$, zero delegation is dominated under performance-based promotion and so is zero delegation under random promotion, i.e. $E\Pi^P(\tilde{k}^P) > E\Pi^P(k^P = 0) = E\Pi^R(k^R = 0)$. If $\delta > \bar{\alpha}$, the principal fully delegates under a random promotion. Thus, if she also fully delegates under a performance-based promotion (i.e. when $\delta > \alpha_2$) the trade-off between motivation and selection determines the optimal promotion rule. A performance-based promotion is optimal if the selection effect is sufficiently weak. For partial delegation under a performance-based promotion (i.e. when $\bar{\alpha} \leq \delta < \alpha_2$), a random promotion gives higher payoffs from managerial decision-making, but lacks incentives and selection. Consequently, a sufficiently low selection effect implies that the gains from increasing worker incentives may outweigh lower managerial profits due to lower delegation.

4 Extensions

This section discusses the effects of monetary incentives for workers and managers on optimal delegation under performance-based promotions.¹⁵

4.1 Worker Wages

Additionally to their promotion prospects, workers receive wage payments dependent on the project outcome, namely w_S in case of success and w_F in case of failure. Workers remain risk-neutral. Furthermore, wages are constrained to be non-negative (e.g. because of workers' limited liability). Worker i 's utility is then given by $u_i = p_i \cdot u_{m_i} + e_i \cdot w_S + (1 - e_i) \cdot w_F - \frac{c}{2} e_i^2 \geq 0$, and consequently optimal effort provision is given by $e_i^w = \frac{2(w_S - w_F) + u_{m_i}}{2c}$. The principal chooses jointly chooses $\{k, w_F, w_S\}$ to maximize expected profits of

$$E\Pi^w = \pi + k(\delta - \bar{\alpha})\pi^D + 2\bar{e}^w(S - w_S) + 2\mu(1 - \mu) \frac{(p_B^w - p_A^w)(\Pi_B - \Pi_A)}{2} - w_F. \quad (19)$$

Introducing the bonus scheme has two effects. First, it motivates workers by giving additional incentives. But it also decreases potential profits from a worker's project success as the principal has to pay $w_S > 0$ to the worker in case of success. Both effects influence optimal delegation that is determined by Proposition 4.

¹⁵One may also wonder about a fully-integrated model with worker wages, manager wages and delegation. Yet Proposition 5 shows that manager wages are independent of delegation and consequently also of worker wages. Thus, results for a fully integrated model would not differ from the results presented in this section.

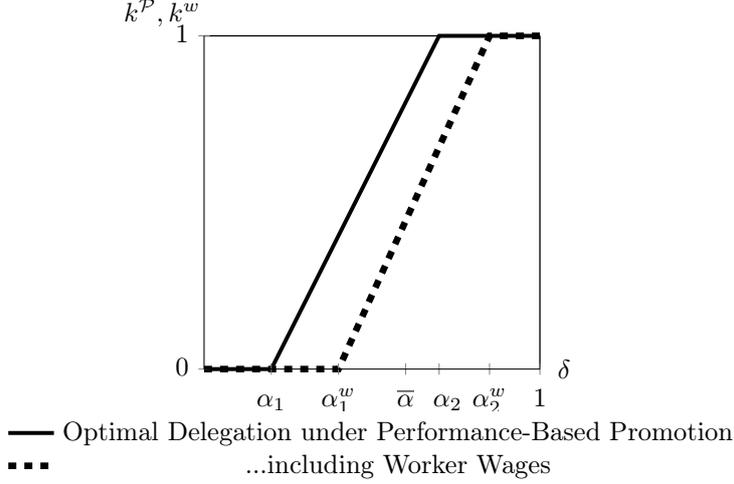


Figure 3: Optimal Delegation with and without Worker Wages.

Note: This is an illustration of Proposition 4. For any given preference misalignment δ , optimal delegation under performance-based promotion with additional wage payments to the workers is weakly lower than without.

Proposition 4.

The optimal organizational design with delegation and bonus pay, defined by $\{k^w, w_F^*, w_S^*\}$, is given by

$$(w_F^*, w_S^*) = \left(0, \frac{2S - k^w \cdot \bar{\alpha} \pi^D}{4}\right) \text{ and } k^w = \begin{cases} 1 & \text{if } \delta \geq \alpha_2^w \\ \tilde{k}^w & \text{if } \delta \in [\alpha_1^w, \alpha_2^w) \\ 0 & \text{if } \delta < \alpha_1^w \end{cases} \quad (20)$$

with $\tilde{k}^w = \frac{c(\delta - \bar{\alpha}) + \frac{\bar{\alpha} S}{2}}{\mu(1-\mu)(\alpha_B - \alpha_A)^2 \pi^D + \frac{\bar{\alpha}^2 \pi^D}{4}}$, $\alpha_1^w = (1 - \frac{S}{2c})\bar{\alpha}$, and $\alpha_2^w = (1 - \frac{S}{2c})\bar{\alpha} + (\mu(1 - \mu)(\alpha_B - \alpha_A)^2 \pi^D + \frac{\bar{\alpha}^2 \pi^D}{4})/c$.

There is a two-fold interaction between wages and delegation that is summarized by Corollaries 3 and 4 below. First, wages decrease profits in case of success which in turn lowers the optimal use of delegation. This is illustrated in Figure 3. Optimal delegation with additional wages (displayed by the dashed line) is weakly lower than optimal delegation without (straight line).

Corollary 3.

Introducing a bonus scheme lowers the use of delegation if $\delta \in (\alpha_1, \alpha_2^w)$.

Secondly, due to their joint use as worker incentive bonuses and delegation are substitutes as described by Corollary 4. For the intuition suppose there is a mean-preserving spread in the workforce heterogeneity, i.e. $\alpha_B - \alpha_A$ increases while $\bar{\alpha}$ remains constant.

Then, the selection effect worsens and delegation becomes more costly. Consequently, the principal lowers delegation but increases bonuses to provide sufficient incentives to the workers.

Corollary 4.

Higher managerial discretion implies lower wages for subordinates, and vice versa.

4.2 Manager Wages

There is strong empirical evidence that “promotions are associated with large wage increases” (Waldman 2012, p.523). In contrast to private benefits from decision-making, the value of money is homogeneous for all workers. Thus, it prevents a negative selection effect. If that is the case, why not incentivize workers with wage increases instead of private benefits from decision-making? The answer is given below. Similar to a wage increase, delegation in this model is essentially a linear transfer of utility from principal to manager. The effects of both incentive devices are mainly similar. But delegation additionally comes with better decision-making by managers, thus increasing total surplus from management. Because costs from delegation, given by the selection effect, are negligible for low values of k , delegation becomes more profitable than similar increases in the manager’s wage.

The principal offers an additional monetary prize of $\hat{w} \in [0, \hat{w}_{max}]$ to the promoted worker. A worker i ’s utility function is given by $\hat{u}_i = p_i(e_i, e_j) \cdot (\hat{w} + u_{m_i}) - \frac{c}{2}\hat{e}_i^2$. Note that the wage increase and private benefits are perfect substitutes in the worker’s utility function. Given the resulting optimal effort level of $\hat{e}_i = \frac{\hat{w} + u_{m_i}}{2c}$, the principal maximizes his expected profits of

$$E\hat{\Pi} = \pi + k(\delta - \bar{\alpha})\pi^D + \frac{(s\hat{w} + k\bar{\alpha}\pi^D)S}{c} + 2\mu(1 - \mu)\frac{(p_B - p_A)(\Pi_B - \Pi_A)}{2} - \hat{w}. \quad (21)$$

Profit maximization over \hat{w} and k gives the optimal organizational design with management wages. It is stated in Proposition 5.

Proposition 5.

The optimal organizational design with delegation and manager wages under performance-based promotions, defined by $\{\hat{w}^, \hat{k}^*\}$, is given by*

$$\{\hat{w}^*, \hat{k}^*\} = \begin{cases} \{0, 0\} & \text{if } S < c \cdot (1 - \frac{\delta}{\alpha}) \\ \{0, k^P\} & \text{if } c \cdot (1 - \frac{\delta}{\alpha}) < S < c \\ \{\hat{w}_{max}, k^P\} & \text{if } S > c \end{cases} \quad (22)$$

where k^P is given in Proposition 2.

There are three insights from Proposition 5. First, expected profits are linear in \hat{w} and thus there exists a corner solution. Secondly, there is no interdependence between manager wages and delegation. Thus, the optimal amount of delegation is the same as without manager wages and equivalent to optimal delegation in Proposition 2.

Thirdly, there exist an interesting relationship between manager wages and delegation on the extensive margin. If wages increase when switching from a working position to management (i.e. when $S > c$), the principal will also delegate. For the intuition, consider the marginal effects of wage increases and delegation on profits at zero. Remember from Section 3.1 that the selection effect is zero at $k = 0$. Thus, the marginal effect of delegation at zero is given by higher incentives and better managerial decision-making at the costs of loss of control: $\frac{\bar{\alpha}\pi^D \cdot S}{c} + \delta\pi^D - \bar{\alpha}\pi^D = \bar{\alpha}\pi^D(\frac{S}{c} - 1) + \delta\pi^D$. Compare this to the marginal effects from a wage increase on profits that stems from higher incentives and wage costs, i.e. $\frac{S}{c} - 1$.

Note the similarities between the two marginal effects. Delegation is essentially a utility transfer from principal to manager. By delegating a decision the principal gives, in expectation, $\bar{\alpha}\pi^D$ to the manager and thereby incentivizes workers. This translates into higher probabilities of success and the principal's expected profits are increased by $\frac{\bar{\alpha}\pi^D \cdot S}{c}$. This multiplier is equivalent for wage increases. Wages induce a marginal transfer of 1 from the principal to the worker, to receive higher expected profits of $\frac{S}{c}$. After accounting for the different "dimensions of utility" (monetary payment vs. private benefits from decision-making) the marginal effects are essentially the same. However delegation additionally increases total surplus as managers make better decisions, captured by $\delta\pi^D = \pi^D - \pi$. When delegating the principal keeps some share of that surplus increase. This gives additional incentives to delegate. Consequently, even when wage increases are not profitable, i.e. $\frac{S}{c} < 1$, delegation may still be. Proposition 5 therefore relies on the efficiency of delegation in this model. In contrast, if delegation decreases total surplus ($\pi^D < \pi$) the principal only delegates if gains from worker incentives are sufficiently large. Hence, the relationship between wage increases and delegation on the extensive margin would be reversed.

5 Conclusion

Many organizations rely on internal promotions to fill management positions, often based on employees' performance. Yet this wide-spread business practice can lead to suboptimal promotion decisions (Benson et al. 2018). Traditionally such findings were explained via differences in skill sets - employees who are good in worker tasks may not have the proper skills for management tasks. In this paper I pursue a different approach, showing that

suboptimal promotions can optimally arise even without considering skill effects. This is due to the non-contractibility of management decisions. It gives rise to managerial benefits that in turn affects workers behavior differently under performance-based promotions. Consequently, workers who make less profitable decisions as managers are promoted more likely.

I show that such interaction between managerial decision rights and worker behavior has various implications for organizational design. It affects how many decision rights should be delegated to a management position (Proposition 2), how promotions should be designed optimally (Corollary 2) and the joint decision of the two (Proposition 3). Moreover, optimal incentive schemes for both managers and workers can be linked to delegation, promotion and management selection (Propositions 4 and 5).

In the current model, workers want to become managers due to a non-contractibility of decision-making and subsequent private benefits. A recent literature has emphasized another reason for why individuals value decision rights. Fehr et al. (2013) and Bartling et al. (2014) find an *intrinsic* motivation for decision-making. In their experiments, individuals forgo money to make decisions themselves, without any instrumental or informational advantage. In an organizational context, these “power-hungry” individuals will influence for instance the optimal hierarchy of firms (Dessein and Holden 2019). Such preferences for power can also be put into the present context. Workers with higher intrinsic valuation of decision-rights have an higher incentive to work hard and thus will have a higher probability of getting promoted. Furthermore, as managers such individuals may try to hoard even more decision rights, for example by acquiring inefficiently many firms and becoming an empire-builder (Jensen 1986). In this case a trade-off between selection and incentives arises, similar to the current model. It will be interesting to further examine how the intrinsic valuation of decision rights interacts with different choices of organizational design.

References

- ACEMOGLU, D., G. EGOROV, and K. SONIN** (2010). “Political Selection and Persistence of Bad Governments”. *Quarterly Journal of Economics*, vol. 125 (4), pp. 1511–1575.
- AGHION, P. and J. TIROLE** (1997). “Formal and Real Authority in Organizations”. *Journal of Political Economy*, vol. 105 (1), pp. 1–29.
- ALFARO, L., N. BLOOM, P. CONCONI, H. FADINGER, P. LEGROS, A. F. NEWMAN, R. SADUN, and J. VAN REENEN** (2016). “All Together Now: Integration, Delegation and Management”. Unpublished.
- BAKER, G., M. GIBBS, and B. HOLMSTROM** (1994). “The Wage Policy of a Firm”. *Quarterly Journal of Economics*, vol. 109 (4), pp. 921–955.
- BAKER, G., M. JENSEN, and K. MURPHY** (1988). “Compensation and Incentives: Practice vs. Theory”. *Journal of Finance*, vol. 43 (3), pp. 593–616.
- BARRO, R. J.** (1973). “The Control of Politicians: An Economic Model”. *Public Choice*, vol. 14, pp. 19–42.
- BARTLING, B., E. FEHR, and H. HERZ** (2014). “The Intrinsic Value of Decision Rights”. *Econometrica*, vol. 82 (6), pp. 2005–2039.
- BEBCHUK, L. A. and L. A. STOLE** (1993). “Do Short-Term Objectives Lead to Under- or Overinvestment in Long-Term Projects?” *Journal of Finance*, vol. 48 (2), pp. 719–729.
- BENSON, A., D. LI, and K. SHUE** (2018). *Promotions and the Peter Principle*. Working Paper 24343. National Bureau of Economic Research.
- BERNHARDT, D.** (1995). “Strategic Promotion and Compensation”. *Review of Economic Studies*, vol. 62 (2), pp. 315–339.
- BESLEY, T.** (2005). “Political Selection”. *Journal of Economic Perspectives*, vol. 19 (3), pp. 43–60.
- BESTER, H. and D. KRÄHMER** (2008). “Delegation and Incentives”. *RAND Journal of Economics*, vol. 39 (3), pp. 664–682.
- BRIN, D.** (1985). *The Postman*. New York City: Bantam Books.
- DESSEIN, W.** (2002). “Authority and Communication in Organizations”. *Review of Economic Studies*, vol. 69 (4), pp. 811–838.
- DESSEIN, W. and R. HOLDEN** (2019). *Organizations with Power-Hungry Agents*. Working Paper 13526. CEPR Discussion Paper.
- DEVARO, J., A. KAUKANEN, and N. VALMARI** (2019). “Internal and External Hiring”. *ILR Review*, vol. 72 (4), pp. 981–1008.
- EDMANS, A., V. W. FANG, and K. A. LEWELLEN** (2017). “Equity Vesting and Investment”. *Review of Financial Studies*, vol. 30 (7), pp. 2229–2271.

- FAIRBURN, J. A.** and **J. M. MALCOMSON** (2001). “Performance, Promotion, and the Peter Principle”. *Review of Economic Studies*, vol. 68 (1), pp. 45–66.
- FEHR, E., H. HERZ,** and **T. WILKENING** (2013). “The Lure of Authority: Motivation and Incentive Effects of Power”. *American Economic Review*, vol. 103 (4), pp. 1325–59.
- FREDERIKSEN, A., L. B. KAHN,** and **F. LANGE** (2019). *Supervisors and Performance Management Systems*. Working Paper 23351. National Bureau of Economic Research.
- GIBBONS, R.** and **J. ROBERTS** (2012). *The Handbook of Organizational Economics*. Princeton: Princeton University Press.
- GRABNER, I.** and **F. MOERS** (2013). “Managers’ Choices of Performance Measures in Promotion Decisions: An Analysis of Alternative Job Assignments”. *Journal of Accounting Research*, vol. 51 (5), pp. 1187–1220.
- GRUNEWALD, A., E. HANSEN,** and **G. PÖNITZSCH** (2019). “Political Selection and the Optimal Concentration of Political Power”. Working Paper.
- HAYEK, F. A.** (1944). *The Road to Serfdom*. Chicago: University of Chicago Press.
- (1945). “The Use of Knowledge in Society”. *American Economic Review*, vol. 35 (4), pp. 519–530.
- HEADLAM, J. W.** (1891). *Election by Lot at Athens*. Cambridge, UK: Cambridge University Press.
- HIRSHLEIFER, J.** (1989). “Conflict and Rent-Seeking Success Functions: Ratio vs. Difference Models of Relative Success”. *Public Choice*, vol. 63 (2), pp. 101–112.
- HOPE, O.-K.** and **W. B. THOMAS** (2008). “Managerial Empire Building and Firm Disclosure”. *Journal of Accounting Research*, vol. 46 (3), pp. 591–626.
- ICHNIEWSKI, C.** and **K. SHAW** (2003). “Beyond Incentive Pay: Insiders’ Estimates of the Value of Complementary Human Resource Management Practices”. *Journal of Economic Perspectives*, vol. 17 (1), pp. 155–180.
- JENSEN, M. C.** (1986). “Agency Costs of Free Cash Flow, Corporate Finance, and Takeovers”. *American Economic Review*, vol. 76 (2), pp. 323–329.
- KE, R., J. LI,** and **M. POWELL** (2018). “Managing Careers in Organizations”. *Journal of Labor Economics*, vol. 36 (1), pp. 197–252.
- KNIGHT, F. H.** (1938). “Lippmann’s The Good Society”. *Journal of Political Economy*, vol. 46 (6), pp. 864–872.
- KOCH, A. K.** and **J. NAFZIGER** (2012). “Job Assignments under Moral Hazard: The Peter Principle Revisited”. *Journal of Economics & Management Strategy*, vol. 21 (4), pp. 1029–1059.
- LAZEAR, E. P.** (2004). “The Peter Principle. A Theory of Decline”. *Journal of Political Economy*, vol. 112 (S1), S141–S163.

- MASKIN, E.** and **J. TIROLE** (2004). “The Politician and the Judge: Accountability in Government”. *American Economic Review*, vol. 94 (4), pp. 1034–1054.
- MILGROM, P.** and **J. ROBERTS** (1992). *Economics, Organization and Management*. Englewood Cliffs, N.J.: Prentice-Hall.
- PERSSON, T., G. ROLAND,** and **G. TABELLINI** (1997). “Separation of Powers and Political Accountability”. *Quarterly Journal of Economics*, vol. 112 (4), pp. 1163–1202.
- PETER, L. J.** and **R. HULL** (1969). *The Peter Principle*. New York City: William Morrow & Company, Inc.
- ROBERTS, J.** (2007). *The Modern Firm*. Oxford: Oxford University Press.
- ROSEN, S.** (1986). “Prizes and Incentives in Elimination Tournaments”. *American Economic Review*, vol. 76 (4), pp. 701–715.
- SCHÖTTNER, A.** and **V. THIELE** (2010). “Promotion Tournaments and Individual Performance Pay”. *Journal of Economics & Management Strategy*, vol. 19 (3), pp. 699–731.
- WALDMAN, M.** (2012). “Theory and Evidence in Internal Labor Markets”. *The Handbook of Organizational Economics*. Princeton: Princeton University Press, pp. 520–571.

A Optimal Promotion Tournaments

This appendix shortly discusses promotion rules that are a convex combination of random and performance-based promotions. For example, a firm could probabilistically switch its promotions practices between promoting the most successful worker or promoting based on other, performance-independent measures such as tenure.

For simplicity, fix k . Suppose the principal additionally chooses a probability $\rho \in [0, 1]$ that determines the likelihood of a successful worker's promotion when his co-worker's project was a failure. $\rho = 1$ is equivalent to a (fully) performance-based promotion rule, $\rho = 0.5$ to random promotion. This gives a general promotion probability of $p_i^n = 0.5 + (\rho - 0.5)(e_i - \bar{e}_j)$ for worker i , resulting in optimal effort provision of $e_i^n = \frac{(\rho - 0.5)u_{m_i}}{c}$. Note that for any $\rho \leq 0.5$, $e_i^n = 0$. Then, the principal's maximization problem is given by

$$\max_{\rho} E\Pi^n = \mu\Pi_B + (1 - \mu)\Pi_A + 2\bar{e}^n(\rho) \cdot S + 2\mu(1 - \mu) \frac{(p_B^n(\rho) - p_A^n(\rho))(\Pi_B - \Pi_A)}{2}. \quad (23)$$

Note that the effect of ρ only comes via workers' behavior. ρ increases average effort provision, but also increases the spread in promotion probabilities. However, due to the linearity of effort in ρ and the linearity of promotion probability in effort expected profits are linear in ρ . Thus, a "binary" solution arises. The principal either uses random promotion or performance-based promotion. A convex combination between the two is never optimal, stated in Proposition 6. For the proof, see Appendix B.

Proposition 6.

Either a random or a performance-based promotion is optimal, i.e. $\rho^n = \{0.5, 1\}$.

B Proofs

Proposition 1

Note that expected profits (Equation (5)) are linear in k . They are increasing if $\delta > \bar{\alpha}$, constant if $\delta = \bar{\alpha}$ and decreasing if $\delta < \bar{\alpha}$. Proposition 1 follows immediately. \square

Lemma 1

First note that expected utility is concave in e_i , and thus there is a unique maximum. The first order condition is given by $0.5 \cdot u_{m_i} - ce_i \stackrel{!}{=} 0$. Solving for the optimal effort level gives $e_i^{\mathcal{P}} = \frac{u_{m_i}}{2c} = k \cdot \frac{\alpha_{m_i} \pi^D}{2c}$. Note that because $u_B < 2c$, $e_i^{\mathcal{P}} \in (0, 1) \forall i$. \square

Lemma 2

Under a heterogeneous workforce, as $p_i^h = 0.5 + 0.5(e_i - e_j)$, see Equation (8), we get

$$p_B^h - p_A^h = 0.5(e_B^{\mathcal{P}} - e_A^{\mathcal{P}}) - 0.5(e_A^{\mathcal{P}} - e_B^{\mathcal{P}}) = e_B^{\mathcal{P}} - e_A^{\mathcal{P}} = \frac{u_{m_B} - u_{m_A}}{2c} = k \frac{(\alpha_B - \alpha_A)\pi^D}{2c}. \quad (24)$$

Under a homogeneous workforce, $j = i$, and thus $p_i^{hom} = p_j^{hom} = 0.5$. \square

Proposition 2

First, I show the simplification of the profit function to Equation (12), before maximizing Equation (12) over k .

$$\begin{aligned}
E\Pi &= (1 - \mu)^2 (\Pi_A + 2e_A S) + 2\mu(1 - \mu) (p_A^h \Pi_A + p_B^h \Pi_B + 2\bar{e} S) + \mu^2 (\Pi_B + 2e_B S) \\
&= \Pi_A \cdot \left((1 - \mu)^2 + 2\mu(1 - \mu)p_A^h \right) + \Pi_B \cdot \left(\mu^2 + 2\mu(1 - \mu)p_B^h \right) \\
&\quad + 2S \cdot \left((1 - \mu)^2 e_A + 2\mu(1 - \mu)\bar{e} + \mu^2 e_B \right) \\
&= \Pi_A \cdot (1 - \mu) + 2\mu(1 - \mu) \frac{(e_A - e_B)}{2} \Pi_A + \Pi_B \cdot \mu + 2\mu(1 - \mu) \frac{(e_B - e_A)}{2} \Pi_B \\
&\quad + 2S \cdot \left((1 - \mu)^2 e_A + 2\mu(1 - \mu)(\mu e_A + (1 - \mu)e_B) + \mu^2 e_B \right) \\
&= (1 - \mu)\Pi_A + \mu\Pi_B + 2S \cdot (\mu e_A + (1 - \mu)e_B) + 2\mu(1 - \mu) \frac{(e_B - e_A)}{2} (\Pi_B - \Pi_A) \\
&= (1 - \mu)\Pi_A + \mu\Pi_B + 2\bar{e} S + 2\mu(1 - \mu) \frac{(p_B^h - p_A^h)(\Pi_B - \Pi_A)}{2}. \tag{25}
\end{aligned}$$

Plugging in $(p_B - p_A)(\Pi_B - \Pi_A) = \frac{(k\bar{\alpha}\pi^D)^2}{2c}$ and $\bar{e} = \frac{k\bar{\alpha}\pi^D}{2c}$ gives Equation (12). The maximization problem is then

$$\max_k E\Pi^P = \pi + k(\delta - \bar{\alpha})\pi^D + \frac{k\bar{\alpha}\pi^D}{c} S - \mu(1 - \mu) \frac{(k(\alpha_B - \alpha_A)\pi^D)^2}{2c}. \tag{26}$$

The first- and second-order derivatives are given by

$$FD_k = (\delta - \bar{\alpha})\pi^D + \frac{\bar{\alpha}\pi^D}{c} S - k\mu(1 - \mu) \frac{((\alpha_B - \alpha_A)\pi^D)^2}{c} \tag{27}$$

$$SD_k = -\mu(1 - \mu) \frac{((\alpha_B - \alpha_A)\pi^D)^2}{c} < 0. \tag{28}$$

Thus, the profit function is concave in k . Note that due to the concavity of $E\Pi^P$, if FD_k is negative at zero, $k = 0$ is optimal. Moreover, if FD_k is positive at 1, $k = 1$ is optimal. Furthermore, any $\tilde{k} \in [0, 1]$ with $FD_k(\tilde{k}) \stackrel{!}{=} 0$ is the unique interior solution to the maximization problem above.

1. FD_k is negative at zero if $(\delta - \bar{\alpha})\pi^D + \frac{\bar{\alpha}\pi^D}{c} S < 0$, or $\delta < \bar{\alpha} - \frac{S}{c}\bar{\alpha}$. Thus, $k^P = 0$ if $\delta < \alpha_1 = (1 - \frac{S}{c})\bar{\alpha}$.
2. FD_k is non-negative at 1 if $(\delta - \bar{\alpha})\pi^D + \frac{\bar{\alpha}\pi^D}{c} S - \mu(1 - \mu) \frac{((\alpha_B - \alpha_A)\pi^D)^2}{c} \geq 0$, or $\delta \geq (1 - \frac{S}{c})\bar{\alpha} + \mu(1 - \mu) \frac{(\alpha_B - \alpha_A)^2 \pi^D}{c} = \alpha_2$. Thus, $k^P = 1$ if $\delta \geq \alpha_2$.
3. In any other case, we have an interior solution, implicitly given by $FD_k(\tilde{k}) \stackrel{!}{=} 0$ which gives $\tilde{k} = \frac{c(\delta - \bar{\alpha}) + \bar{\alpha} S}{\mu(1 - \mu)(\alpha_B - \alpha_A)^2 \pi^D}$. \square

Lemma 3

The selection effect is given by $-2\mu(1 - \mu) \frac{(k(\alpha_B - \alpha_A)\pi^D)^2}{4c}$. First note that it is zero at $k = 0$ and negative for $k > 0$. Taking the first-order derivative w.r.t k gives $-\mu(1 - \mu)k \cdot \frac{((\alpha_B - \alpha_A)\pi^D)^2}{c}$ which is negative for $k > 0$ and zero for $k = 0$. \square

Corollary 1

Suppose $\alpha_2 \leq \bar{\alpha}$. Then, for any $\bar{\alpha} > \delta \geq \alpha_2$, $k^{\mathcal{P}} = 1 > 0 = k^{\mathcal{R}}$. This implies

$$\begin{aligned} k^{\mathcal{P}} &= k^{\mathcal{R}} \text{ if } \delta \leq \alpha_1 \\ k^{\mathcal{P}} &> k^{\mathcal{R}} \text{ if } \delta \in (\alpha_1, \alpha_2) \\ k^{\mathcal{P}} &> k^{\mathcal{R}} \text{ if } \delta \in [\alpha_2, \bar{\alpha}) \\ k^{\mathcal{P}} &= k^{\mathcal{R}} \text{ if } \delta \geq \bar{\alpha}. \end{aligned}$$

$\alpha_2 \leq \bar{\alpha}$ holds if $\mu(1-\mu)(\alpha_B - \alpha_A)^2 \pi^D \leq \bar{\alpha}S < S$. □

Corollary 2

The difference in expected profits, given by Equation (14), is

$$\begin{aligned} E\Pi^{\mathcal{P}} - E\Pi^{\mathcal{R}} &= 2\bar{e}S + 2\mu(1-\mu)(p_B^h - p_A^h)(\Pi_B - \Pi_A) \\ &= \frac{k\bar{\alpha}\pi^D S}{c} - \mu(1-\mu) \frac{(k(\alpha_B - \alpha_A)\pi^D)^2}{2c} \\ &= \frac{k\pi^D}{2c} \cdot (2\bar{\alpha}S - \mu(1-\mu)k(\alpha_B - \alpha_A)^2 \pi^D), \end{aligned} \quad (29)$$

which is positive whenever $k < \bar{k}^{\mathcal{P}} = \frac{2\bar{\alpha}S}{\mu(1-\mu)(\alpha_B - \alpha_A)^2 \pi^D}$. □

Proposition 3

The proof for Proposition 3 is completed in several steps. Generally, we need to compare optimal expected profits under performance-based promotion against optimal expected profits under random promotion. As in both cases, the optimal degree of delegation is piecewise, we continue case by case. First note that $\alpha_1 < \min\{\bar{\alpha}, \alpha_2\}$.

1. Suppose $\delta \leq \alpha_1 < \min\{\bar{\alpha}, \alpha_2\}$.
Then, $k^{\mathcal{P}} = k^{\mathcal{R}} = 0$ and thus $E\Pi^{\mathcal{P}} = E\Pi^{\mathcal{R}} = \pi$.
2. Suppose $\alpha_1 \leq \delta < \min\{\bar{\alpha}, \alpha_2\}$.
Then, $k^{\mathcal{P}} = \tilde{k}$ and $k^{\mathcal{R}} = 0$ and thus the difference in expected payoffs is given by

$$\begin{aligned} E\Pi^{\mathcal{P}}(\tilde{k}) - E\Pi^{\mathcal{R}}(0) &= \tilde{k} \cdot (\delta - \bar{\alpha})\pi^D + \tilde{k} \cdot \frac{\bar{\alpha}\pi^D}{c} S - \mu(1-\mu) \frac{(\tilde{k}(\alpha_B - \alpha_A)\pi^D)^2}{2c} \\ &= \tilde{k}\pi^D \left[\delta - \left(1 - \frac{S}{c}\right)\bar{\alpha} - \mu(1-\mu)\tilde{k} \frac{(\alpha_B - \alpha_A)^2 \pi^D}{2} \right] \\ &= \tilde{k}\pi^D \left[\delta - \left(1 - \frac{S}{c}\right)\bar{\alpha} - \mu(1-\mu) \frac{c(\delta - \bar{\alpha}) + \bar{\alpha}S}{\mu(1-\mu)(\alpha_B - \alpha_A)^2 \pi^D} \frac{(\alpha_B - \alpha_A)^2 \pi^D}{2} \right] \\ &= \tilde{k}\pi^D \left[\delta - \left(1 - \frac{S}{c}\right)\bar{\alpha} - \frac{\delta - \left(1 - \frac{S}{c}\right)\bar{\alpha}}{2} \right] \\ &= \tilde{k}\pi^D \left[\frac{\delta - \left(1 - \frac{S}{c}\right)\bar{\alpha}}{2} \right] = \frac{\tilde{k}\pi^D}{2} [\delta - \alpha_1] > 0 \end{aligned} \quad (30)$$

3. Suppose $\alpha_2 < \bar{\alpha}$ and $\alpha_2 \leq \delta < \bar{\alpha}$.

Then, $k^{\mathcal{P}} = 1$ and $k^{\mathcal{R}} = 0$ and thus the difference in expected payoffs is given by

$$\begin{aligned} E\Pi^{\mathcal{P}}(1) - E\Pi^{\mathcal{R}}(0) &= (\delta - \bar{\alpha})\pi^D + \frac{\bar{\alpha}\pi^D}{c}S - \mu(1 - \mu)\frac{(\alpha_B - \alpha_A)\pi^D)^2}{2c} \\ &= \pi^D \cdot \left[\delta - \left(\bar{\alpha}\left(1 - \frac{S}{c}\right) + \mu(1 - \mu)\frac{(\alpha_B - \alpha_A)^2\pi^D}{2c} \right) \right] \\ &= \pi^D \cdot [\delta - \alpha_2] > 0. \end{aligned} \quad (31)$$

4. Suppose $\alpha_2 > \bar{\alpha}$ and $\bar{\alpha} < \delta < \alpha_2$.
Then, $k^{\mathcal{P}} = \tilde{k}$ and $k^{\mathcal{R}} = 1$ and thus

$$\begin{aligned} E\Pi^{\mathcal{P}}(\tilde{k}) - E\Pi^{\mathcal{R}}(1) &= (\delta - \bar{\alpha})\pi^D(\tilde{k} - 1) + \tilde{k}\frac{\bar{\alpha}\pi^D}{c}S - \mu(1 - \mu)\frac{(\tilde{k}(\alpha_B - \alpha_A)\pi^D)^2}{2c} \\ &= \frac{\pi^D}{c} \cdot \left[\frac{c(\delta - \bar{\alpha}) + \bar{\alpha}S}{2} \cdot \tilde{k} - (\delta - \bar{\alpha})c \right] \\ &= \frac{\pi^D}{c} \cdot \left[\frac{c(\delta - \bar{\alpha}) + \bar{\alpha}S}{2} \cdot \frac{c(\delta - \bar{\alpha}) + \bar{\alpha}S}{\mu(1 - \mu)(\alpha_B - \alpha_A)^2\pi^D} - (\delta - \bar{\alpha})c \right], \end{aligned} \quad (32)$$

which is non-negative if and only if $\mu(1 - \mu)(\alpha_B - \alpha_A)^2\pi^D \leq \frac{[c(\delta - \bar{\alpha}) + \bar{\alpha}S]^2}{2c(\delta - \bar{\alpha})}$. Note that for $\alpha_2 > \bar{\alpha}$, it must hold that $\mu(1 - \mu)(\alpha_B - \alpha_A)^2\pi^D > \bar{\alpha}S$, but the two conditions are consistent.

5. $\delta \geq \max\{\bar{\alpha}, \alpha_2\}$.
Then, $k^{\mathcal{P}} = k^{\mathcal{R}} = 1$ and thus

$$E\Pi^{\mathcal{P}}(1) - E\Pi^{\mathcal{R}}(1) = \frac{\bar{\alpha}\pi^D}{c}S - \mu(1 - \mu)\frac{(\alpha_B - \alpha_A)\pi^D)^2}{2c}, \quad (33)$$

which is non-negative if and only if $\mu(1 - \mu)(\alpha_B - \alpha_A)^2\pi^D \leq 2\bar{\alpha}S$.

The first three cases can be summarized as:

$$\text{If } \delta \leq \alpha_1 : \quad (k^*, \text{prom}^*) = (0, \mathcal{P}) = (0, \mathcal{R}), \quad (34)$$

$$\text{If } \delta \in (\alpha_1, \bar{\alpha}] : \quad (k^*, \text{prom}^*) = (k^{\mathcal{P}}, \mathcal{P}). \quad (35)$$

First note that $\bar{\alpha}S < 2\bar{\alpha}S < \frac{[c(\delta - \bar{\alpha}) + \bar{\alpha}S]^2}{2c(\delta - \bar{\alpha})}$. Also for the two cases it holds that $\delta > \bar{\alpha}$. Further they can be summarized as follows.

1. $\mu(1 - \mu)(\alpha_B - \alpha_A)^2\pi^D < \bar{\alpha}S < 2\bar{\alpha}S$: $\alpha_2 < \bar{\alpha} < \delta$ and $(k^*, \text{prom}^*) = (1, \mathcal{P})$.
2. $\bar{\alpha}S \leq \mu(1 - \mu)(\alpha_B - \alpha_A)^2\pi^D \leq 2\bar{\alpha}S$: $\alpha_2 \geq \bar{\alpha}$ and $(k^*, \text{prom}^*) = (1, \mathcal{P})$.
3. $2\bar{\alpha}S < \mu(1 - \mu)(\alpha_B - \alpha_A)^2\pi^D < \frac{[c(\delta - \bar{\alpha}) + \bar{\alpha}S]^2}{2c(\delta - \bar{\alpha})}$:
 $\alpha_2 \geq \bar{\alpha}$ and $(k^*, \text{prom}^*) = (\tilde{k}_1, \mathcal{P})$ where $\tilde{k}_1 = 1$ if $\delta \geq \alpha_2$ and \tilde{k} otherwise.
4. $\mu(1 - \mu)(\alpha_B - \alpha_A)^2\pi^D > \frac{[c(\delta - \bar{\alpha}) + \bar{\alpha}S]^2}{2c(\delta - \bar{\alpha})}$: $\alpha_2 \geq \bar{\alpha}$ and $(k^*, \text{prom}^*) = (1, \mathcal{R})$.

Proposition 3 is then a re-formulation of the above stated. \square

Proposition 4

The profit maximization problem is given by

$$\max_{\{k, w_s, w_F\}} E\Pi^w = \pi + k(\delta - \bar{\alpha})\pi^D + 2\bar{e}^w(S - w_S) + \mu(1 - \mu)(p_B^w - p_A^w)(\Pi_B - \Pi_A) - w_F, \quad (36)$$

which can be re-written (analogously as in Proposition 2) as

$$\begin{aligned} \max_{\{k, w_S, w_F\}} E\Pi^w = & \pi + k(\delta - \bar{\alpha})\pi^D + \frac{2(w_S - w_F) + k\bar{\alpha}\pi^D}{c}(S - w_S) \\ & - \mu(1 - \mu)\frac{(k(\alpha_B - \alpha_A)\pi^D)^2}{2c} - w_F. \end{aligned} \quad (37)$$

Due to the workers' limited liability, $w_F = 0$ is optimal. Then, first- and second-order derivatives are then given by

$$FD_k = (\delta - \bar{\alpha})\pi^D + \frac{\bar{\alpha}\pi^D}{c}(S - w_S) - \mu(1 - \mu)k\frac{((\alpha_B - \alpha_A)\pi^D)^2}{c} \quad (38)$$

$$FD_{w_S} = \frac{(2(S - w_S) - 2w_S + k\bar{\alpha}\pi^D)}{c} \quad (39)$$

$$SD_k = -\mu(1 - \mu)\frac{((\alpha_B - \alpha_A)\pi^D)^2}{c} < 0 \quad (40)$$

$$SD_{w_S} = -2 < 0. \quad (41)$$

First, $w_S = \frac{2S - k\bar{\alpha}\pi^D}{4}$ is optimal, independent of k due to the independent concavity of profits in both parameters. Using this, we get

$$\begin{aligned} FD_k &= (\delta - \bar{\alpha})\pi^D + \frac{\bar{\alpha}\pi^D}{c}\left(S - \frac{2S - k\bar{\alpha}\pi^D}{4}\right) - \mu(1 - \mu)k\frac{((\alpha_B - \alpha_A)\pi^D)^2}{c} \\ &= \delta - \bar{\alpha}\left(1 - \frac{S}{2c}\right) - k\left(\frac{\bar{\alpha}^2\pi^D}{4c} + \mu(1 - \mu)\frac{(\alpha_B - \alpha_A)^2\pi^D}{c}\right) \end{aligned} \quad (42)$$

To find the optimal k I proceed as in the proof for Proposition 2, but the full procedure is omitted. Proposition 4 follows. \square

Corollary 3

We need to show that, on the extensive margin, the degree of delegation is lower with bonus schemes, thus for $k = 0$, we have that $\alpha_{w1} > \alpha_1$ and for $k = 1$, we have that $\alpha_{w2} > \alpha_2$. On the intensive margin, we need to show that $\tilde{k} > \tilde{k}^w$.

1. $\alpha_{w1} > \alpha_1$ holds as $\bar{\alpha} - \frac{\bar{\alpha}S}{2cx} > \bar{\alpha} - \frac{\bar{\alpha}S}{c}$.
2. $\alpha_{w2} > \alpha_2$ holds as $\alpha_{w2} = \alpha_2 + \frac{\bar{\alpha}S}{2c} + \frac{(\bar{\alpha}\pi^D)^2}{4c}$.
3. $\tilde{k} > \tilde{k}^w$ holds as

$$\tilde{k} = \frac{c(\delta - \bar{\alpha}) + \bar{\alpha}S}{\mu(1 - \mu)(\alpha_B - \alpha_A)^2\pi^D} > \frac{(c(\delta - \bar{\alpha}) + \frac{\bar{\alpha}S}{2})}{\mu(1 - \mu)(\alpha_B - \alpha_A)^2\pi^D + \frac{\bar{\alpha}^2\pi^D}{4}} = \tilde{k}^w, \quad (43)$$

as the RHS's numerator is smaller and denominator is larger. \square

Corollary 4

This follows directly from Proposition 4 as $\frac{\partial w^S}{\partial k^w} = -\frac{\bar{\alpha}\pi^D}{4}$ for $\delta \in [\alpha_1^w, \alpha_2^w]$ (and otherwise k^w is constant). \square

Proposition 5

The first-order conditions with respect to \hat{w} and k are given by

$$\frac{\partial \hat{E}\hat{\Pi}}{\partial \hat{w}} = \frac{S}{c} - 1 \stackrel{!}{=} 0 \quad (44)$$

$$\frac{\partial \hat{E}\hat{\Pi}}{\partial k} = (\delta - \bar{\alpha})\pi^D + \frac{\bar{\alpha}\pi^D}{c}S - k\mu(1 - \mu)\frac{((\alpha_B - \alpha_A)\pi^D)^2}{c} \stackrel{!}{=} 0 \quad (45)$$

First note that the FOCs are independent. Secondly, $FOC_{\hat{k}}$ is the same as in Proposition 2 and the optimal amount of delegation is given by k^P . Thirdly, to analyze when a positive amount of \hat{w} or \hat{k} is optimal, look at the behavior at $\hat{w} = 0$, and $k = 0$.

$$\frac{\partial \hat{E}\hat{\Pi}}{\partial w} \Big|_{\hat{w}=0} = \frac{S}{c} - 1 \quad (46)$$

$$\frac{\partial \hat{E}\hat{\Pi}}{\partial k} \Big|_{k=0} = \delta\pi^D + \bar{\alpha}\pi^D \cdot \left(\frac{S}{c} - 1\right). \quad (47)$$

Thus, $\frac{\partial \hat{E}\hat{\Pi}}{\partial \hat{w}} \Big|_{\hat{w}=0} > 0$ if and only if $S > c$ which in turn implies that $\frac{\partial \hat{E}\hat{\Pi}}{\partial k} \Big|_{k=0} > 0$. Also, $\frac{\partial \hat{E}\hat{\Pi}}{\partial k} \Big|_{k=0} > 0$ if and only if $S > c(1 - \frac{\delta}{\bar{\alpha}})$. Taken together, the three cases stated in Proposition 5 arise. \square

Proposition 6

First note $\bar{e}^n(\rho) = \frac{(\rho-0.5)k\bar{\alpha}\pi^D}{2c}$ and $p_B^n(\rho) - p_A^n(\rho) = \frac{(\rho-0.5)k(\alpha_B - \alpha_A)\pi^D}{2c}$. Then the first-order derivative of expected profits is $\frac{\partial E\Pi^n}{\partial \rho} = \frac{k\bar{\alpha}\pi^D S}{c} - \mu(1 - \mu)\frac{(k(\alpha_B - \alpha_A)\pi^D)^2}{2c}$, which is independent of ρ . Thus a binary solution is optimal. Since $\rho < \frac{1}{2}$ would imply $e < 0$ we can restrict the possible set of solutions to $\rho^n \in \{\frac{1}{2}, 1\}$. Thus either random or performance-based promotions optimally emerge. \square