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Discussion Paper No. 18

March 25, 2017

Collaborative Research Center Transregio 190 | www.rationality-and-competition.de
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Relative Consumption Preferences and Public Provision of Private Goods *

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February, 2017

Abstract

This paper shows that the public provision of private goods may be justified on pure efficiency grounds in an environment where individuals have relative consumption concerns. By providing private goods, governments directly intervene in the consumption structure, and thereby have an instrument to correct for the excessive consumption of positional goods. We identify sufficient conditions when the public provision of private goods is always Pareto-improving, even when (linear) consumption taxes are available. In fact, with the public provision of private goods, there are cases where first-best allocations can be achieved, and a luxury tax on the positional good is redundant.

JEL Codes: H42, D62, D63.

Keywords: Public Provision, Social Preferences, Status-Seeking.

*We thank Paul Heidhues, Steffen Huck, Dorothea Kuebler, Andreas Wagener, Georg Weizsaecker and seminar participants in Aarhus, Berlin, Gothenburg, Manchester, Munich, Lisbon and Taormina for their helpful comments and suggestions. Financial support from the German Research Association (DFG) through Collaborative Research Center Transregio CRC TRR 190 is gratefully acknowledged.

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To a substantial extent, countries provide goods to their citizens that are essentially private in nature, such as health care, child care, education, and nutritional assistance.[1] From a traditional economic perspective, the public provision of private goods is puzzling. Given that these goods and services are typically available in private markets, replacing public provision with equivalent cash payments should actually increase welfare, people can then choose the consumption bundle that best suits their interest. Several explanations for this public provision puzzle have been put forward in the normative economics literature, including paternalism or merit good arguments (Musgrave, 1959; Sandmo, 1983; Besley, 1988), motives of redistribution under informational constraints (Nichols and Zeckhauser, 1982; Blackorby and Donaldson, 1988; Besley and Coate, 1991), and notions of equality of opportunity (Gasparini and Pinto, 2006). In this paper, we provide an alternative rationale: We argue that the public provision of private goods can correct for the inefficiencies arising from individuals' concern for positional or relative consumption. While the traditional view of the inefficiency of in-kind spending assumes that individuals only derive utility from goods and services *per se*, there is growing evidence that is consistent with the idea that consumers' choices and satisfaction are also affected by the social comparisons they make: individuals seem to relate their absolute consumption levels to that of "referent" others and feel pleasure when they possess more than their social peers, and discomfort when they possess less (e.g., Alpizar et al., 2005; Solnick and Hemenway, 2005; Carlsson et al., 2007; Charles et al., 2009; Heffetz, 2011; Kuhn et al., 2011; Mechtel and Friehe, 2014; Roth, 2014). For instance, individuals enjoy their new car, but they feel even better when the car is bigger than that of their friends, colleagues or neighbors. Conversely, people may no longer be satisfied with their purchases if they see others enjoying a more lavish lifestyle – and they may engage in further spending races to try catch up (see, e.g., Dupor and Liu, 2003; Hopkins and Kornienko, 2004).

As has been argued by several classical writers (e.g., Veblen, 1899; Duesenberry, 1949; Leibenstein, 1950; Hirsch, 1976), and largely supported by the recent empirical status consumption literature (see, e.g., Charles et al. 2009), social comparisons are not equally important for all kinds of goods. For example, cars, jewelry, and electronic devices are more useful for demonstrating wealth or economic power than health insurance, nutrition or savings.[2] Following Hirsch (1976), Frank (1985a) denotes the goods whose value depends greatly on how they compare with the things owned by others as "positional goods"; goods that are less dependent on such comparisons, he refers to as "non-positional

[1] The fraction of GDP devoted to "in-kind" spending ranges between 5 and 20 percentage in OECD member states, and is still growing in many countries, both developed and developing (Currie and Gahvari, 2008).

[2] For further evidence, see, e.g., Solnick and Hemenway (2005), Heffetz (2011), Roth (2014).

goods."[3] Irrespective of the underlying motive as to why humans may care about the relative amounts of certain items they posses (status signaling, evolutionary reasons, envy etc.),[4] social comparisons give rise to a market inefficiency that can be described as an economic externality (Glazer and Konrad, 1996; Hopkins and Kornienko, 2004): individuals trying to improve their relative position, adjust their budget shares toward the positional goods, neglecting that their positional consumption imposes a harm onto others (if one person consumes more of a positional good, another persons' relative consumption will ceteris paribus decline). As a consequence, they over[under] consume the positional [non-positional] goods from a social point of view, calling for government intervention.

In this paper, we show that the public provision of private goods can serve as a policy device to induce Pareto-improvements when individuals are concerned about their relative position. In particular, by providing the by-and-large non-positional goods like health, nutrition, and basic education, government may encourage the consumption of goods that are underconsumed in the positional arms race and divert resources away from the positional goods, thus having a quantity-based instrument to correct for the positional externality. We identify the conditions on the nature of relative consumption preferences such that public provision can always induce Pareto-improvements in a simple production-exchange economy. We also demonstrate that Pareto-improvements can be achieved even when a positional tax, an often proposed price instrument to deal with the inefficiency of positional consumption, is available.

We present a model with two private goods and two types of individuals who differ in their exogenous gross incomes ("rich" and "poor"). One good is positional, the other non-positional. In addition to the absolute consumption of both goods, individuals care about how their consumption of the positional good compares to an endogenous reference level which they dislike being behind and enjoy being ahead of. Moreover, reference levels may differ across income groups and are modeled as a general, weakly increasing function of both groups' average positional spending. This formulation allows us to distinguish between the different types of social comparisons discussed in the social scientific literature, including upward, downward or within-group comparisons.[5] We start to consider

[3]Other terms for a positional good are Veblen good, a conspicuous good or status good. We use the term positional good, as it is neutral to the possible motives of why people have positional consumption concerns. Important for our analysis is that individuals have interdependent utilities in the sense that well-being is negatively affected by the consumption variables of others.

[4]For a discussion, see, e.g., Postlewaite (1998), Arrow and Dasgupta (2009), and Bilancini and Boncinelli (2012).

[5]While earlier contributions focus on comparisons with the economy-wide average positional consumption (see, e.g., Abel, 1990; Clark and Oswald, 1998; Ljungqvist and Uhlig, 2000; Dupor and Liu, 2003; Alonso-Carrera et al., 2004; Liu and Turnovsky, 2005), a growing body of work studies the policy implications of different comparison motives (see, e.g., Micheletto, 2008; Eckerstorfer and Wendner,

public provision of non-positional goods which is typically ascribed to health care, old-age provision, nutrition and basic education (Solnick and Hemenway, 2005; Charles et al. 2009; Heffetz, 2011). In particular, the government may provide a uniform amount of the non-positional good that is offered to all individuals free of charge and that can be topped up by additional purchases on a market. To finance public provision, (lump-sum) income taxes can be levied.

We find that public provision can always achieve Pareto-improvements over the laissez faire if at least one income group compares its positional good consumption with that of the poor – a condition satisfied by many specifications of reference functions such as average, within-group or downward comparisons. To get an intuition for this result, assume that the government sets the public provision level slightly above the amount consumed by the poor in the laissez faire, and raises income taxes by an equal amount. This policy change does not alter the material utilities of both income types: as the rich had purchased a larger amount of the non-positional good anyway, their consumption choice is not affected; they take the publicly provided level and top it up through additional purchases. The poor are forced to marginally reduce their spending on the positional and to increase the consumption of the non-positional good, which – by the envelope theorem – produces only negligible second-order effects. However, if at least one type of individuals socially compares herself with the poor, the enforced reduction in positional spending of the poor decreases (some) individuals' reference levels, and thus, has positive first-order welfare effects.

We extend our simple model into several directions. In our basic model, social comparisons have an effect on only utility but not on consumption behavior. However, an individual's relative position may also affect her marginal propensity to consume. While the presence of such peer effects in consumption considerably complicates the analysis and can give rise to multiple equilibria, we derive a tractable sufficient condition for public provision to be always Pareto-improving. This condition not only requires the existence of a social type comparing herself to the poor, but also that people consume more of the positional good if others' consumption levels rise, a scenario that is labeled "keeping-up with the Joneses" and considered the relevant case in the recent empirical literature on conspicuous consumption (see, e.g., Kuhn et al., 2011; Roth, 2014).

When government can make use of a uniform tax on the positional good, we show that there is a knife-edge preference scenario where public provision can never achieve a Pareto-improvement, namely whenever the social harms of positional consumption are the same across types. In fact, there are scenarios (pure-within group comparisons of the poor), in

2013).

which public provision alone can achieve full Pareto-efficiency such that a positional tax is redundant. Finally, we analyze the role of the public provision of the positional good, which may apply for higher education. In this case, the government must be given a stronger provision system in order to be able to induce Pareto-improvements: additional private purchases of the positional good must be restrictable.

The relevance of our analysis is supported by the growing evidence suggesting that concerns for relative standing play a crucial role in explaining individuals' consumption patterns. A robust finding in this literature is that the tendency for positional spending seems to be particularly pronounced for lower income or status groups. For instance, using US household consumption data, Charles et al. (2009) find that Blacks and Hispanics spend more on conspicuous goods (clothing, jewelry, cars) than do comparable Whites. These expenditure differences are associated with substantial diversions of resources from inconspicuous or non-positional goods such as education, health care and food. Similar patterns hold for developing countries, where the poor devote relatively large proportions of their incomes to lavish festivals, weddings or funerals (Banerjee and Duflo, 2007; Case et al., 2008). Such behavior is often interpreted as satisfying needs to signal a high relative standing (Moav and Neemann, 2010, 2012), and, in most cases, financed by borrowing against the future (e.g., reduced old-age provision) or by diverting resources away from basic education or health prevention like mosquito nets, preventive drugs or basic vaccination (Brown et al., 2011; Khamis et al., 2012; Moav and Neemann, 2012). Our paper gives direct policy recommendation related to these findings. By providing non-positional goods (health care, basic education, etc.) with a simple top-up system, the government can target the spending races of the poor separately, while leaving the consumption choices of richer individuals unaffected – an objective that a uniform consumption tax cannot achieve.

Our paper contributes to the general question on the role of status concerns for policy design, which has recently emerged in the public economics literature. This literature mainly focuses on price instruments (e.g., Frank, 1985; Ng, 1987; Corneo and Jeanne; 1997; Hopkins and Kornienko, 2004; Micheletto, 2008; Truyts, 2012; Eckerstorfer and Wendner, 2013) and income taxation (e.g., Boskin and Sheshinski, 1978; Blomquist, 1993; Ireland, 1998, 2001; Aronsson and Johansson-Stenman, 2010; Bilancini and Boncinelli, 2012). In contrast to these papers, we highlight in-kind spending and thus point to the general usefulness of quantity instruments to cope with the market inefficiency related to relative consumption concerns – a role that has been largely ignored in previous work. An important exception in this regard is Ireland (1994). Using a signaling framework, he first develops the idea that public provision or in-kind spending is an instrument to reduce people's consumption of conspicuous goods. We go beyond this study by not just emphasizing the behavioral consequences of public provision, but by also asking about when these behavioral consequences translate into Pareto-improvements, thus conducting a normative policy analysis (which he not does). Second, while Ireland uses quasi-linear examples, our results apply for more general preferences, which enables us to trace the role of social comparison direction for policy design. Finally, we provide a joint analysis of consumption taxes and public provision.

The paper is structured as follows. Section 1 introduces the theoretical framework and presents the economic problem. Section 1.2 illustrates the efficiency-enhancing potential of public provision. Peer effects in consumption are considered in section 2. Section 3 studies the case where consumption taxes are available. Section 4 analyzes whether Pareto-efficient allocations can be attained. Section 5 considers the public provision of positional goods, while the final section concludes. All proofs are relegated to Appendix 3.A.

1 Basic Model

1.1 Framework

General. Consider an economy populated by a large finite number of individuals. There are two private goods c and x. Individuals can be of two types $i = 1, 2$ who differ in their endowment income consisting of y^i units of good x, where $y^1 < y^2$. We henceforth label individuals of type 1 as "the poor" and of type 2 as "the rich." The number of each type is normalized to one. Good c is produced from good x by a competitive industry that uses one unit of good x to produce one unit of good c.

Preferences. Individuals enjoy both goods *per se*, i.e., they derive utility from the absolute consumption levels of these goods. In addition, they care about *relative* consumption, i.e., about how their own level consumption compares to that of referent others. There is evidence suggesting that relative consumption concerns are not equally important for all kinds of goods (see, e.g., Charles et al. 2009; Heffetz, 2011). For simplicity, we assume that one of the goods, good x, is entirely "non-positional": for this good only absolute consumption levels matter. In contrast, good c is a "positional good," by which we mean that *both* the absolute and relative consumption levels matter for individual well-being. Specifically, preferences of income type i are represented by the utility function $U^i : \mathbb{R}^3 \to \mathbb{R}$ with

$$U^i(c^i, x^i, \Delta^i) = u(c^i, x^i) + \Delta^i. \tag{1}$$

In (1), $u : \mathbb{R}^2_+ \to \mathbb{R}$, $u^i = u(c^i, x^i)$, is the utility from absolute consumption, which is increasing in each argument, is twice continuously differentiable, and strictly quasi-concave. The term Δ^i represents i's relative consumption, which we define as the difference between her own level of consumption of good c and some reference level r^i:[6]

$$\Delta^i := c^i - r^i. \tag{2}$$

We assume that type i's reference level is endogenous in that it negatively depends on the average amount of the positional good possessed by her own income group, $\bar{c}^i$, and the average positional consumption of the respective other income group, $\bar{c}^{3-i}$:

$$r^i := h^i(\bar{c}^i, \bar{c}^j) \quad \text{for} \quad i \neq j.$$

As individuals with the same gross incomes are identical, c^i will coincide with the average consumption of group i in equilibrium. Thus, with a slight abuse of notation, we define r^i as a non-decreasing, twice continuously differentiable function $h^i : \mathbb{R}^2_+ \to K^i \subset \mathbb{R}^1$, with

$$r^i := h^i(c^i, c^j), \quad \frac{\partial h^i}{\partial c^i} \geq 0 \quad \text{and} \quad \frac{\partial h^i}{\partial c^j} \geq 0 \quad \text{for} \quad i \neq j. \tag{3}$$

We restrict the range of h^i to $K^i := \{r^i \mid 0 \leq r^i \leq y^1 + y^2\}$, which means that reference levels cannot exceed the economy's aggregate resource endowment. In addition, for at least one income type, either of the derivatives $\partial h^i / \partial c^i$ or $\partial h^i / \partial c^j$ must be strictly greater than zero: if all reference levels were exogenous, a market inefficiency would not occur.
Substituting (2) and (3) into (1), we obtain

$$U^i_*(c^i, x^i, c^j) := u(c^i, x^i) + c^i - h^i(c^i, c^j) \quad \text{for} \quad i \neq j, \tag{4}$$

where $U^i_* : \mathbb{R}^3_+ \to \mathbb{R}$. We assume that U^i_* is strongly quasi-concave and that relative consumption concerns are not "too strong" in the sense that U^i_* strictly increases in c^i. This means that if positional consumption is simultaneously increased for all members of a given income group, the resulting negative social comparison effect can never outweigh the positive material effect on individual well-being.[7]

[6] We use this specific form of preferences to illustrate our main points as simply as possible. In section 2, we study more general preferences.

[7] We thus avoid the "pervert" case that an allocation where everybody has zero amounts of the positional good c is Pareto-efficient. A similar assumption is made in Dupor and Liu (2003) and Dufwenberg et al. (2011).

Types of social comparisons. The general formulation of h^i includes comparisons with the average consumption of the entire economy $\bar{c} := (c^1 + c^2)/2$ as a special case. A key feature of our formulation is, however, to allow different individuals to have different reference levels, which captures ideas from social psychology that the notion of what constitutes a "referent other" may considerably vary across social groups (Suls and Wills, 1991).

According to the similarity hypothesis, individuals are concerned about the consumption levels of individuals who are similar to them, rather than about those who are socially distant (see, e.g., Festinger, 1954; Runciman, 1966; Frank, 1984). In this case, an individual feels envious when another person has a bigger car, but only so if the other person is a colleague, friend, family member or other social neighbor. In our framework with two income types, we can represent such local comparisons by letting individuals' reference levels be exclusively sensitive to the consumption of the members of their own income group: $\partial h^1/\partial c^1 > 0$, $\partial h^2/\partial c^2 > 0$, and $\partial h^1/\partial c^2 = \partial h^2/\partial c^1 = 0$ (*within-group comparisons*).

In contrast to this, Duesenberry (1949) suggests that people only care about the consumption of individuals who are ranked socially higher: $\partial h^1/\partial c^2 > 0$, $\partial h^1/\partial c^1 = \partial h^2/\partial c^2 = \partial h^2/\partial c^1 = 0$ (*upward comparisons*). The opposite polar case is when individuals relate to those below them in the income ranking: $\partial h^2/\partial c^1 > 0$ and $\partial h^2/\partial c^2 = \partial h^1/\partial c^1 = \partial h^1/\partial c^2 = 0$ (*downward comparisons*). This reference specification applies when peoples' consumption choices reflect a desire to distance themselves from the poor (Bowles and Park, 2005).[8]

Generally, there may be several reasons as to why individuals may care about their relative consumption. To the extent that a favorable consumption position conveys a superior position on some underlying status scale (such as wealth or income), relative consumption preferences are compatible with a desire for a high social standing or status. Likewise, as utility decreases the more others consume, our modeling also entails elements of jealousy, envy or relative deprivation. As is common in the literature on relative consumption, we assume that consumption variables of other individuals directly enter utility, without presupposing one or another candidate motive. What is crucial for our analysis is the hypothesis that an individual suffers utility losses when referent others' consumption rise, because her relative consumption declines.[9]

[8] As advocated by self-enhancement theory, individuals compare with others in order to make themselves feel better and they therefore tend to compare downward as a means to enhance self-esteem (Wood and Taylor, 1991).

[9] For further discussion of the different motives compatible with relative consumption preferences, see, e.g., Arrow and Dasgupta (2009); Heffetz and Frank (2011).

Individual maximization and equilibrium. When making consumption choices, each individual treats her reference level as exogenous.[10] Utility for a given reference level is represented by the function $\tilde{U} : \mathbb{R}^3_+ \to \mathbb{R}$, with

$$\tilde{U}(c^i, x^i, r^i) := u(c^i, x^i) + c^i - r^i. \tag{5}$$

We require $\tilde{U}$ to be strongly quasi-concave in x^i and c^i, i.e., given a reference level, preferences over the positional and non-positional good are strictly convex. In addition, let $\tilde{U}\left(c^i, x^i, r^i\right) > \tilde{U}\left(0, \hat{x}^i, r^i\right)$ and $\tilde{U}\left(c^i, x^i, r^i\right) > \tilde{U}\left(\hat{c}^i, 0, r^i\right)$ for all r^i, $c^i > 0, x^i > 0$, $\hat{c}^i \geq 0$ and $\hat{x}^i \geq 0$, which ensures strictly positive demands for both goods for all positive incomes.

Denote the set of consumption bundles available to individual i by $B^i \subset \mathbb{R}^2_+$. In all scenarios we consider, this budget set will be compact and convex.[11] The individual maximization problem can be written as

$$\max_{c^i, x^i} \tilde{U}(c^i, x^i, r^i) \quad \text{s.t.} \quad (c^i, x^i) \in B^i. \tag{6}$$

Since $\tilde{U}$ is strongly quasi-concave, problem (6) has a unique solution, defining type i's demands for goods c and x. By additive separability of $\tilde{U}$, demands do not depend on the reference level r^i: social comparisons shift utility, without having any consequences for behavior.[12]

In the course of the paper, we will repeatedly make use of i's "ordinary" demand functions, denoted by $c^i_d\left(I^i\right)$ and $x^i_d\left(I^i\right)$. By this we mean the solution of problem (6), when the budget set takes the form $B^i_u := \{(c^i, x^i) : c^i + x^i \leq I^i\}$, where I^i denotes disposable income. Throughout the paper, we assume that both the positional and the non-positional good are normal, i.e., $\partial c^i_d(I^i)/\partial I^i > 0$ and $\partial x^i_d(I^i)/\partial I^i > 0$.

We define an equilibrium as follows:

Definition 1 *An allocation $C := (c^1, x^1, c^2, x^2)$ and a corresponding pair of reference levels (r^1, r^2) constitute an equilibrium if*

(i) for every i, (c^i, x^i) solves (6) contingent on the budget set B^i,

[10] This assumption is standard in the literature on relative consumption concerns and analogue to price-taking behavior of atomistic individuals. Intuitively, as the number of individuals is large, they regard their own contribution to the reference level as negligible. Since reference levels are a function of the consumption of others, we could equivalently say that individuals take others' consumption levels as exogenously given.

[11] The general formulation of B^i simplifies the exposition as it allows us to express the definition of an economic equilibrium as generally as possible and includes each policy scenario as a special case.

[12] This assumption is relaxed in section 2.

(ii) $\sum_{i=1}^{2}\left(c^i + x^i - y^i\right) = 0$,

(iii) for every $i \neq j$, $r^i = h^i\left(c^i, c^j\right)$.

According to items (i) and (ii), an equilibrium allocation must maximize individuals' utility given their budgets and satisfy the economy's resource constraint. Item (iii) requires reference levels to be consistent with actual behavior (or, alternatively, requires individuals to foresee others' behavior correctly). With preferences as in (1) and (2), a unique equilibrium always exists. When preferences are more general and reference levels do affect behavior, multiple equilibria can emerge. We will study this case in section 2.

Inefficiency of the laissez-faire. In the following, we characterize the laissez-faire, the benchmark of our model. Without state intervention, the individual budget set is $B^i_{LF} := \{(c^i, x^i) : c^i + x^i \leq y^i\}$. Hence, the laissez-faire allocation is given by $C_{LF} := (c^1_{LF}, x^1_{LF}, c^2_{LF}, x^2_{LF})$ with $c^i_{LF} := c_d\left(y^i\right)$ and $x^i_{LF} := x_d\left(y^i\right)$. From the first-order conditions of problem (6), C_{LF} must satisfy

$$MRS(c^i, x^i) = 1 \quad \text{for} \quad i = 1, 2, \tag{7}$$

where $MRS\left(c^i, x^i\right) := (u^i_c + 1)/u^i_x)$ is the marginal rate of substitution between goods c and x, i.e., type i's willingness to pay for the positional good measured in units of the non-positional good.[13]

Denote the set of Pareto-efficient allocations by $\mathcal{P}$. As shown in Appendix A.1, any Pareto-efficient allocation $C_* := (c^1_*, x^1_*, c^2_*, x^2_*) \in \mathcal{P}$ must satisfy

$$MRS(c^i, x^i) - \Gamma^i = 1 \quad \text{for} \quad i = 1, 2, \tag{8}$$

where

$$\Gamma^i := \left[\frac{1}{u^i_x}\frac{\partial h^i}{\partial c^i} + \frac{1}{u^j_x}\frac{\partial h^j}{\partial c^i}\right] \geq 0 \quad \text{for} \quad i \neq j. \tag{9}$$

The term Γ^i measures the aggregate willingness to pay to avoid an increase in reference levels, induced by a marginal increase in c^i. We will refer to Γ^i as the marginal "social harm" of i's positional good consumption. The left-hand side of (8) thus gives the aggregate or willingness to pay for i's consumption of the positional good. At a Pareto-efficient allocation, this expression must be equal to the marginal rate of transformation between the two private goods, which is 1.

Under relative consumption preferences with endogenous reference levels, Γ^i is greater than zero for at least one type. As a consequence, condition (7) does not coincide with

[13] In the following, we abbreviate $u^i_c := \partial u(c^i, x^i)/\partial c$ and $u^i_x := \partial u(c^i, x^i)/\partial x$.

(8) for at least one i: C_{LF} is Pareto-inefficient. Intuitively, individuals neglect that their consumption of the positional good increases others' reference levels in their private optimization, such that there is divergence between the private and the social evaluation of the positional good.
In addition, all individuals for whom Γ^i is non-zero overconsume [underconsume] the positional [non-positional good] in the laissez-faire: by (7), the aggregate willingness to pay for c^i equals $1 - \Gamma^i$ at C_{LF}, and thus falls short of the social cost. Since the utility function U_*^i is strongly quasi-concave, there always exists a consumption bundle containing a slightly lower [higher] level of c^i $[x^i]$ that is Pareto-superior to (c_{LF}^i, x_{LF}^i).
In the course of the paper, we will assume that for any element in $\mathcal{P}$, $c_*^1 < c_*^2$ and $x_*^1 < x_*^2$.[14]

1.2 The efficiency role of public provision

Many goods and services that governments provide play a subordinate role for social comparisons. For example, health care and old-age savings have relatively little effect on gaining high social status compared to smartphones, clothes and cars (see Alpizar et al., 2005; Solnick and Hemenway, 2005; Charles et al. 2009; Heffetz, 2011). We now lay out the basic argument for why publicly providing non-positional goods can achieve Pareto-improvements over the laissez-faire.
Consider a simple provision scheme, where the government may provide a uniform level g of the non-positional good, which is offered to all individuals free of charge and which can be topped up with private purchases, but not resold in the market $(x^i \geq g)$.[15] To finance public provision, the government has access to non-linear income taxes $T^i := T(y^i)$. We denote type i's income after taxes by $b^i := y^i - T^i$. It will be convenient to use b^i as a government choice variable, rather than T^i itself. We call $P = (b^1, b^2, g) \in \mathbb{R}_+^3$ a policy, which is a vector assigning the uniform provision level and a net income to each individual. Any policy must be feasible in the sense that it balances the government budget:

$$G := y^1 - b^1 + y^2 - b^2 - 2g = 0. \tag{10}$$

We restrict the set of available policies to those where $b^2 > b^1$. We further require $b^i > 0$ for $i = 1, 2$, i.e., that income taxation is not exhaustive.

Individual behavior under public provision. For a given policy, individuals decide how to allocate their net income on market purchases of the two private goods. Formally,

[14] The assumption serves to simplify the exposition and is not essential for any of our results.

[15] If individuals can resell the amounts of the non-positional good they receive from the government, public provision would be equivalent to a cash transfer.

each individual solves (6), given her budget

$$B_g^i := \left\{(c^i, x^i) : c^i + x^i - g \leq b^i, x^i \geq g\right\}.$$

The unique solution to this problem must satisfy

$$MRS(c^i, x^i) - 1 \geq 0 \quad \text{and} \quad \left(x^i - g\right)\left[MRS(c^i, x^i) - 1\right] = 0. \tag{11}$$

The demand functions can be shown to be given by[16]

$$c^i(b^i, g) = \begin{cases} c_d\left(b^i + g\right) & \text{if } g < x_d\left(b^i + g\right), \\ b^i & \text{if } g \geq x_d\left(b^i + g\right), \end{cases} \tag{12}$$

$$x^i(b^i, g) = \begin{cases} x_d\left(b^i + g\right) & \text{if } g < x_d\left(b^i + g\right), \\ g & \text{if } g \geq x_d\left(b^i + g\right), \end{cases} \tag{13}$$

where $c_d(\cdot)$ and $x_d(\cdot)$ are the "ordinary" or "unconstrained" demand functions.[17] Observe that $x_d\left(b^i + g\right)$ gives the amount of the non-positional good that an individual with income b^i would buy if she received g in cash. If governments provides a lower amount of the non-positional good than this critical level ($g < x_d\left(b^i + g\right)$), public provision is equivalent to a cash transfer: the individual takes the publicly provided good and purchases additional units of the non-positional good in the market so that her total consumption of good x equals $x_d\left(b^i + g\right)$. If, in contrast, the provision level is large enough such that the individual would buy less than the publicly provided amount ($g \geq x_d\left(b^i + g\right)$), she reduces her private purchases of the non-positional to zero and spends her entire net income on the positional good. We say that the individual is "crowded out." As good x is normal, the poor are crowded out at a lower level of public provision than the rich.[18]
Let

$$V^i(b^i, b^j, g) := u^i(c^i(b^i, g), x^i(b^i, g)) + c^i(b^i, g) - h^i(c^i(b^i, g), c^j(b^j, g)) \tag{14}$$

be the equilibrium indirect utility of type i. Generally, from the definition of h^i, V^i also

[16]The formal derivation of the demand functions is equivalent to Epple and Romano (1996b) and therefore omitted.

[17]Demand functions are continuous, but only one-sided differentiable at policies where $g = x_d\left(b^i + g\right)$. One-sided differentiability is sufficient for our proofs to follow.

[18]It might be that crowding out levels do not exist. Crucial for our analysis will be that type i does reduce her private purchases to zero when faced with a policy where $g = T^i$.

depends on the net income of the respective other type. Indirect utilities are continuous and are one-sided differentiable at points where $g = x_d\left(b^i + g\right)$.

Conditions for public provision to be Pareto-improving. Our first proposition establishes that, under relatively mild assumptions on the reference functions h^i, public provision of the non-positional good can *always* achieve Pareto-improvements over the laissez-faire:

Proposition 1 *If $\partial h^1/\partial c^1 > 0$ or $\partial h^2/\partial c^1 > 0$ for all c^1 and c^2, there exists a policy $P = (b^1, b^2, g)$ with $g > 0$ that achieves a Pareto-improvement over the laissez-faire.*

The intuition for Proposition 1 is as follows. Consider a policy scheme where any public provision level is financed by an equal-sized reduction in both types' net incomes, $b^i = y^i - g$. Under this scheme, an individual is crowded out by public provision if $g \geq x_d\left(b^i + g\right) = x_d\left(y^i\right) = x^i_{LF}$. By demand functions (12) and (13), setting the public provision level equal to $g = x^1_{LF}$ then leads to the same consumption allocation – and hence, utilities levels – as in the laissez-faire. For this policy, the poor are "just" constrained by public provision and spend their entire net income on the positional good, whereas public provision is cash-equivalent for the rich.

Now, consider a marginal increase in the provision level slightly above x^1_{LF}. This policy change would have no effect on both types' utilities if reference levels were constant. By the continuity of individual demand functions, the rich still consume the same consumption bundle as in the laissez-faire. The poor, in contrast, are forced to consume slightly more of the non-positional good and less of the positional good, which has a negative effect on $u\left(c^1, x^1\right) + c^1$. But, as the poor are in their private optimum at $g = x^1_{LF}$, this effect is of second-order and therefore negligible. However, if $\partial h^1/\partial c^1 > 0$ or $\partial h^2/\partial c^1 > 0$, the forced reduction in c^1 has a positive first-order welfare effect, as it then lowers the reference level of at least one income group. Since the proposed policy changes are always feasible, $\partial h^1/\partial c^1 > 0$ or $\partial h^2/\partial c^1 > 0$ is sufficient for the existence of a Pareto-improving policy with $g > 0$, as stated in the proposition.

According to Proposition 1, public provision is always desirable from an efficiency perspective if the reference level of at least one type depends on the positional consumption of the poor. This dependence is a rather weak assumption about the nature of relative preferences: no information about the strength of relative consumption concerns is needed; the mere existence of consumption comparisons can suffice for public provision to be Pareto-improving.

One scenario where the condition in Proposition 1 is fulfilled is when there is social competition among the poor, i.e., when the poor engage in conspicuous consumption (at least partly) to impress the other poor ($\partial h^1/\partial c^1 \neq 0$). The recently growing empirical literature on status consumption provides evidence that, in many countries (including developing ones), the poor devote a relatively large share of their income to positional spending items, suggesting that relative consumption concerns are indeed relevant for this income class (Charles et al. 2009; Brown et al. 2011; Heffetz, 2011). Taken together with evidence that social comparisons are particularly strong among own social class members (individuals compare themselves with their immediate social peers, see, e.g., Luttmer, 2005; Clark and Oswald, 1996; Kuhn et al. 2011; Roth, 2014), this suggests that $\partial h^1/\partial c^1 \neq 0$ is likely to hold for many societies. In these cases, forcing the poor to slightly overconsume the non-positional good through public provision is always desirable from an efficiency perspective.[19]

Another scenario where the condition in Proposition 1 holds is when individuals (to whatever small extent) are downward-oriented, i.e., when individuals try to socially separate from the poor. Evidence for the existence of downward comparisons is provided in Falk and Knell (2004). In these cases, the rich would benefit from crowding out the poor, without presuming paternalism.

The condition provided in Proposition 1 is sufficient, but is not necessary for the existence of Pareto-improving public provision. To see this, let the provision scheme again be given by $b^i = y^i - g$. Further suppose that the poor are purely upward-looking and that the rich are entirely in-group-oriented. Then, providing the non-positional good in the interval $g \in (x^1_{LF}, x^2_{LF}]$ only has negative efficiency effects: public provision distorts the poor away from their preferred consumption bundle without affecting people's reference levels. But if the provision level is set slightly above x^2_{LF}, it forces the rich to reduce their positional consumption – which *ceteris paribus* has positive welfare effects since $\partial h^1/\partial c^2 > 0$ and $\partial h^2/\partial c^2 > 0$. If this effect is strong enough, a Pareto-improvement may occur for some $g > x^2_{LF}$. In the following example, we demonstrate that this can indeed happen:

Example 1: Assume that material utility is Cobb-Douglas with $u(c^i, x^i) = c^i x^i$. Individuals then maximize $\tilde{U}(c^i, x^i, r^i) = c^i x^i + c^i - r^i$. Further assume (pure) upward comparisons of the poor and (pure) within-group comparisons of the rich, i.e., $h^1(c^1, c^2) = h^2(c^2, c^1) = c^2$. Set the parameters of the model to $y^1 = 10$, and $y^2 = 10.4$. In this case, the laissez-faire allocation is $C_{LF} = (c^1_{LF}, x^1_{LF}, c^2_{LF}, x^2_{LF}) = (5.5, 4.5, 5.75, 4.75)$. Now, introducing public provision with $g = 5$ and $T^1 = T^2 = 5$ yields $(c^1, x^1, c^2, x^2) = (5, 5, 5.4, 5)$ as the equilibrium consumption allocation. The resulting utility differential is

[19] Notice that in our model such a policy is in the very self-interest of the poor; there is no paternalism of the rich.

$V^1(5, 5.4, 5) - V^1(10, 10.4, 0) = 0.05 > 0$ and $V^2(5, 5.4, 5) - V^2(10, 10.4, 0) = 0.21 > 0$, i.e., public provision makes both types strictly better off.

We considered a simple provision system where all individuals receive a uniform amount of the non-positional good. Since the poor are first crowded out by public provision, the government can correct the externality of their positional spending without causing any negative efficiency effects for the rich. As Example 1 shows, this does not hold vice versa; when the public provision level is uniform, reducing the positional spending of the rich necessarily distorts the consumption choices of the poor. If we allowed for income-dependent provision levels, the case for public provision would be even stronger: the government could then separately target each individual's social harm and always achieve Pareto-improvements for any type of relative consumption preferences. The appeal of our analysis is that we provide sufficient conditions when public provision is Pareto-improving under a weak public provision mechanism.

2 Peer effects

In our basic model, social comparisons have well-being effects alone: reference levels enter utility without having any effect on consumption behavior. While this modeling is consistent with a large body of research showing that a favorable social rank positively correlates with measures of life satisfaction (see, e.g., Luttmer, 2005, and the references cited therein), the recent empirical conspicuous consumption literature provides evidence suggesting that an individual's relative position may also affect her marginal propensity to consume. In this section, we therefore allow reference levels to reflect in behavior.

Preferences and behavior. Let the preferences of individual i be represented by a strictly increasing, twice continuously differentiable function $U : \mathbb{R}^3 \to \mathbb{R}$, where

$$U^i := U(c^i, x^i, \Delta^i). \tag{15}$$

Unlike in section 1, preferences no longer need to be separable in relative consumption: $U_{c\Delta}, U_{x\Delta} \geq 0$. In addition, Δ^i is a general function $\Delta^i : \mathbb{R}^2_+ \to \mathbb{R}$, with

$$\Delta^i := \Delta(c^i, r^i), \quad \frac{\partial \Delta}{\partial c} > 0, \quad \frac{\partial \Delta}{\partial r} < 0. \tag{16}$$

Reference levels continue to be given by (3).[20]

[20]Notice that in addition to difference comparisons used so far ($\Delta^i = c^i - r^i$), our general formulation of preferences also allows for ratio-comparisons, where relative consumption is given by (or some monotone

Substituting (16) into (15) gives utility for constant reference levels:

$$\tilde{U}(c^i, x^i, r^i) := U(c^i, x^i, \Delta(c^i, r^i)). \tag{17}$$

As in the previous section, we assume that $\tilde{U}^i$ is strongly quasi-concave in c^i and x^i and that its level curves do not intersect the axis in the c/x plane.
For simplicity, and without loss of generality, we suppose that government is restricted to a public provision scheme where $T^1 = T^2 = g$, rendering the policy space one-dimensional. Given g and r^i, individuals maximize (17) subject to $(c^i, x^i) \in \tilde{B}_g$, with

$$\tilde{B}_g^i := \left\{(c^i, x^i) : c^i + x^i \leq y^i, x^i \geq g\right\}.$$

The solution to this problem is

$$c^i(g, r^i) = \begin{cases} c_d^i\left(y^i, r^i\right) & \text{if } g < x_d^i\left(y^i, r^i\right), \\ y^i - g & \text{if } g \geq x_d^i\left(y^i, r^i\right), \end{cases} \tag{18}$$

$$x^i(g, r^i) = \begin{cases} x_d^i\left(y^i, r^i\right) & \text{if } g < x_d^i\left(y^i, r^i\right), \\ g & \text{if } g \geq x_d^i\left(y^i, r^i\right). \end{cases} \tag{19}$$

The crucial difference to the previous chapter is that an individuals' demands now depend on her reference level when not being crowded out. In this case, the comparative statics of c^i with respect to r^i are generally unclear in sign and depend on how the marginal rate of substitution between the positional and non-positional good, $MRS := \tilde{U}_c^i(c^i, x^i, r^i)/\tilde{U}_x^i(c^i, x^i, r^i)$, varies with r^i.[21] We consider two polar cases: one where the MRS is globally increasing and one where it is globally decreasing in the reference level for each type. In Appendix A.3, we show that

$$\frac{\partial MRS\left(c^i, x^i, r^i\right)}{\partial r^i} > (<)\, 0 \quad \Longleftrightarrow \quad \frac{\partial c_d^i}{\partial r^i} > (<)\, 0.$$

Hence, with an increasing MRS, an individual who experiences an increase in the positional consumption of persons she compares herself to will respond by increasing her own positional consumption, too. Following Dupor and Liu (2003), we say that preferences ex-

transformation of) $\Delta^i(c^i, r^i) = c^i/r^i$. Ratio comparisons have been used in, e.g., Clark and Oswald (1998).

[21] Whenever an individual is crowded out, the reference level has no effect on consumption as in our basic model, i.e., $\partial c^i/\partial r^i = 0$ and $\partial x^i/\partial r^i = 0$.

hibit "keeping up with the Joneses" (KUJ). In the reverse case, where $MRS\left(c^i, x^i, r^i\right)/\partial r^i <$ 0, an individual responds by cutting back her positional consumption whenever that of her referent others increases ($\partial c^i_d/\partial r^i < 0$); we say that preferences exhibit "running away from the Joneses" (RAJ).

Equilibria. Again, an equilibrium is defined as a consumption allocation and pair of reference levels that satisfy the conditions in Definition 1. When preferences are separable in r^i, there exists a unique equilibrium for a given policy. With non-separable preferences as in (15), multiple equilibria can occur. To see this, note that condition (iii) of Definition 1 requires consistency of reference levels and actual behavior, i.e.,

$$\begin{aligned} r^1 - h^1(c^1(g, r^1), c^2(g, r^2)) = 0, \\ r^2 - h^2(c^2(g, r^2), c^1(g, r^1)) = 0. \end{aligned} \tag{20}$$

For every g, system (20) is a fixed-point equation in $\mathbb{R}^2$. By applying Brouwer's fixed-point theorem, there exists a pair of reference levels $(\hat{r}^1, \hat{r}^2)$ that solves (20).[22] However, as demands for the positional good depend on r^i, several pairs of reference levels may satisfy (20). Since each reference level pair corresponds to a unique allocation, multiple equilibrium allocations can emerge. Given a public provision level, we collect the solutions to (20) in the set $\mathcal{E}_g$.

In the following, we make an assumption that ensures equilibria to be at least *locally* unique – in the sense that for every $(\hat{r}^1, \hat{r}^2) \in \mathcal{E}_g$, there is no other reference pair solving (20) sufficiently close to it (see Appendix A.3). The assumption also guarantees that ordinary demands c^i_d and x^i_d remain normal in the presence of peer effects. Specifically, we impose:

Assumption 1 *For all r^1, r^2 and g:*

$$\begin{aligned} &(i)\, A := \left(1 - \frac{\partial h^1}{\partial c^1}\frac{\partial c^1_d}{\partial r^1}\right)\left(1 - \frac{\partial h^2}{\partial c^2}\frac{\partial c^2_d}{\partial r^2}\right) - \frac{\partial h^1}{\partial c^2}\frac{\partial c^2_d}{\partial r^2}\frac{\partial h^2}{\partial c^1}\frac{\partial c^1_d}{\partial r^1} > 0. \\ &(ii)\, 1 - \frac{\partial h^i}{\partial c^i}\frac{\partial c^i_d}{\partial r^i} > 0 \quad for \quad i = 1, 2. \end{aligned}$$

Assumption 1 allows us to apply an implicit function theorem for non-differentiable mappings such that we can express any solution to (20) as a function of the public provision

[22]To see this, rewrite (20) as $\mathbf{r} = (r^1, r^2)' = \mathbf{h}(\mathbf{r})$, where $\mathbf{h} : \mathbb{R}^2 \to \mathbb{R}^2$ is given by $\mathbf{h}(r) := (\tilde{h}^1, \tilde{h}^2)'$ with $\tilde{h}^i(r^1, r^2) := h^i(c^i(g, r^i), c^j(g, r^j))$. Since demand functions are continuous in r^i, $\mathbf{h}$ is a continuous function mapping each point (r^1, r^2) of the convex and compact set $K^1 \times K^2 \in \mathbb{R}^2$ into itself, which ensures the existence of a $(\hat{r}^1, \hat{r}^2)$ that solves (20) for any g.

level, $\hat{\mathbf{r}}(g) = (\hat{r}^1(g), \hat{r}^2(g))$.[23] For every g, we can therefore write individual i's indirect utility as

$$V^i(g) = u^i(c^i(g, \hat{r}^i(g)), x^i(g, \hat{r}^i(g)), \Delta^i(c^i(g, \hat{r}^i(g)), \hat{r}^i(g))). \tag{21}$$

Denote the set of equilibrium reference levels in the laissez-faire ($g = 0$) by $\mathcal{E}_{LF}$. Again, individuals overconsume the positional and underconsume the non-positional good at every $(\hat{r}^1, \hat{r}^2) \in \mathcal{E}_{LF}$. The reason is that, irrespective of whether preferences satisfy KUJ or RAJ, the source of inefficiency is not removed by social peer effects: individuals still neglect that their own amount of the positional good negatively affects the well-being of others.

In what follows, we assume that the rich consume more of both goods than the poor in any laissez-faire equilibrium.[24] Furthermore, the government can select between different laissez-faire equilibria in the sense that, for a given $(\hat{r}^1, \hat{r}^2) \in \mathcal{E}_{LF}$, reference levels remain constant whenever the public provision level is set equal to $g = x^1_{LF}$. This ensures that any given laissez-faire equilibrium can be replicated by public provision.

Sufficient condition. The following proposition identifies when public provision can always induce Pareto-improvements in the presence of peer effects:

Proposition 2 *For every $(\hat{r}^1, \hat{r}^2) \in \mathcal{E}_{LF}$, there exists a Pareto-improving policy with $g > 0$ if*

(i) $\partial h^1/\partial c^1 > 0$ or $\partial h^2/\partial c^1 > 0$ for all c^1 and c^2.

(ii) preferences exhibit KUJ ($\partial c^i_d/\partial r^i > 0$.

According to Proposition 2, the existence of (at least) one income type who feels in social competition with the poor is no longer sufficient for the existence of a Pareto-improving provision scheme: preferences must also satisfy KUJ (positional consumption choices are strategic complements). The logic behind this result is as follows. When the government sets the provision level slightly above a given x^1_{LF}, the poor are forced to slightly reduce their positional spending to the advantage of the non-positional good, which, as in the basic model, *ceteris paribus* has strictly positive welfare effects whenever $\partial h^1/\partial c^1 > 0$ or $\partial h^2/\partial c^1 > 0$. However, the lower positional consumption of the poor may now evoke a behavioral change for the rich who are not constrained at $g = x^1_{LF} < x^2_{LF} = c^2_d(y^2, r^2)$. If preferences exhibit RAJ ($\partial c^i_d/\partial r^i < 0$) and $\partial h^2/\partial c^1 \neq 0$, the rich will react to the decrease

[23] See Appendix A.3.

[24] As the ordinary demands are normal, this is generally satisfied if both types have the same reference level. However, as we allow reference levels to vary across types, it may happen that the rich have a lower demand for one of the two goods. For simplicity, we exclude such cases.

in c^1 with an increase of their positional spending, which *ceteris paribus* exerts an upward pressure on reference levels. If this effect is strong enough, forcing the poor away from the positional good will increase reference levels in equilibrium and lower indirect utilities such that publicly providing the non-positional good is harmful.

This is different under KUJ, $\partial c_d^i/\partial r^i > 0$, where c^1 and c^2 tend to move in the same direction. Then, crowding out the positional consumption of the poor will also reduce the positional consumption of the rich and will thus reinforce the initial positive welfare effect. Therefore, under KUJ, setting the provision level slightly above x_{LF}^1 will unambiguously provide a Pareto-improvement, as long as somebody compares herself with the poor.

Evidence of positive interaction effects can be found in the social networks literature, which reports of conformity or bandwagon effects for various social contexts, including risky behavior, recreational activities or labor supply decisions (for a survey, see Durlauf and Ioannides, 2010). Recently, positive peer effects have also been confirmed for positional consumption choices. For instance, Kuhn et al. (2011) show that neighbors of lottery winners (who received cash and a new BMW) increase spending on cars and exterior home renovation. Similarly, analyzing data from a randomized conditional cash transfer program in Indonesia, Roth (2014) finds that the expenditure share of visible goods rises for (untreated) households whose reference group's visible consumption is exogenously increased. In fact, we are not aware of any paper that reports of negative correlations between peer decision variables – neither in the conspicuous consumption, nor in the broader social network literature. We therefore argue that condition (ii) of Proposition 2 is likely to hold such that, if anything, peer effects are likely to reinforce the effectiveness of public provision to correct consumers' distortions related to the concern for relative consumption.

3 Taxation of the positional good

In this chapter, we study the case where taxation of the positional good is possible. For simplicity, we will abstract from peer effects in consumption and return to the basic model of section 1.

We assume that government can make use of linear or uniform commodity taxation, where every income type faces the same tax rate per unit of positional spending. We make this assumption since personalized consumption taxation is difficult to implement in practice. For instance, this would mean that every customer can be charged with a different price at the cash register or that government can directly observe the identity of the purchaser (i.e., who consumes how much of a given good), which is administratively and/or politically infeasible (see, e.g., Diamond, 1973; Sandmo, 1975; Green and Sheshinski, 1976; Micheletto, 2008; Eckerstorfer and Wendner, 2013).

3.1 Available policies and benchmark situation

In addition to income taxation and uniform public provision, the government can now levy a uniform per unit tax t on good c. Each individual then faces a consumer price of $p(t) = 1 + t$. A policy is described by the vector $P = (b^1, b^2, g, t)$. Again, policy must be feasible and should balance the government budget:

$$G := y^1 - b^1 + tc^1 + y^2 - b^2 + tc^2 - 2g = 0. \tag{22}$$

The individual budget set is given by

$$B_t^i = \left\{(c^i, x^i) : (1+t)c^i + x^i \leq b^i + g, x^i \geq g\right\}. \tag{23}$$

Preferences are the same as in the basic model and are described by (1) to (3).

To see whether there is a case for public provision when a positional tax is available, we proceed as follows. We start from a situation where the government can make use of the positional tax and lump-sum income taxes only and sets these instruments optimally, in the sense that no further Pareto-improvements can be achieved, given a minimum utility requirement for one income type. We then analyze whether allowing for positive levels of public provision can induce a Pareto-improvement over this situation, the optimal positional tax solution.

The policy problem in the absence of public provision can be stated so as to find the policy (b^1, b^2, g, t) that maximizes the indirect utility of the poor, $V^1(b^1, b^2, g, p(t))$, given that the indirect utility of the rich $V^2(b^2, b^1, g, p(t))$ does not fall below a minimum level $\bar{U}^2$, the government budget is balanced, and $g = 0$. Solving this problem for varying levels of $\bar{U}^2$ gives the set of optimal tax policies $\mathcal{S}$. We assume that there exists a unique interior solution for every given $\bar{U}^2$. Denote this solution $P_0 = (b_0^1, b_0^2, 0, t_0)$ and the corresponding consumption allocation $C_0 = (c_0^1, x_0^1, c_0^2, x_0^2)$. We restrict $\mathcal{S}$ to policies where $b^2 > b^1$ such that type 2 is also richer than type 1 in terms of net income. In Appendix A.4, we show that for any $P_0 \in \mathcal{S}$, the optimal tax on the positional good can be (implicitly) written as a weighted average of both groups' social harms of positional consumption

$$t_0 = [\alpha\Gamma^1 + (1-\alpha)\Gamma^2] > 0, \tag{24}$$

where the weights $\alpha \in (0, 1)$ and $(1-\alpha) \in (0, 1)$ are defined in Appendix A.4. This formula is familiar from the recent literature on positional externalities and their implications for optimal commodity taxation (see, e.g., Micheletto, 2008; Eckerstorfer and Wendner, 2013).

3.2 When public provision can achieve Pareto-improvements

If the optimal positional tax suffices to implement Pareto-efficient allocations, public provision – or any other policy instrument – would be redundant. The next proposition identifies when this is the case:

Proposition 3 *An optimal positional tax $P_0 \in \mathcal{S}$ induces a Pareto-efficient allocation $C_* \in \mathcal{P}$ if and only if $\Gamma^1|_{C=C_0} = \Gamma^2|_{C=C_0}$.*

When both income groups impose identical marginal social harms, a uniform tax on the positional good – combined with appropriate income taxes – is sufficient to restore efficiency: as individuals choose both goods according to $MRS(c^i, x^i) = 1 + t$, a tax of $t = \Gamma^1 = \Gamma^2$ implies $MRS(c^i, x^i) - \Gamma^i = 1$, which coincides with the efficiency condition (8). Conversely, if $\Gamma^1 \neq \Gamma^2$, both income groups exert different marginal externalities, and uniform consumption taxes can never implement efficient allocations. In such cases, public provision as an additional policy instrument can be valuable.

The question is, which types of social comparisons imply identical social harms? Using the definition in (9), $\Gamma^1 = \Gamma^2$ is equivalent to

$$\frac{1}{u_x^1}\left(\frac{\partial h^1}{\partial c^1} - \frac{\partial h^1}{\partial c^2}\right) = \frac{1}{u_x^2}\left(\frac{\partial h^2}{\partial c^2} - \frac{\partial h^2}{\partial c^1}\right). \tag{25}$$

It is straightforward to check that condition (25) holds if every individual compares her own consumption with the economy's average $\bar{c}$. More generally, there is no role for public provision if, for both income types, the identity of the individual who purchases the positional good is irrelevant, i.e., if $\partial h^i / \partial c^i = \partial h^i / \partial c^j$ for all $i \neq j$. In this case, it would be immaterial to an individual whether it is her neighbor who buys a new car or a socially more distant member of society. However, recent empirical evidence suggests that social distance does matter for social comparison: individuals are more concerned about the possessions of others if these others are members of their own social class or if they are individuals they have direct social interaction with (for evidence of the local comparison hypothesis, see, e.g., Luttmer, 2005; Clark and Oswald, 1996; Roth, 2014.) In fact, most of the examples we discussed in section 1 imply different marginal social harms. This clearly holds for pure upward and downward comparisons (where one of Γ^1 or Γ^2 is equal to zero), but is generally also true for pure within-group comparisons and all intermediate types with $\partial h^i/\partial c^i \neq \partial h^i/\partial c^j$.[25] Hence, in general, there is room for the public provision of private goods even if an optimal uniform tax on the positional good is in place.

The next proposition demonstrates that if the poor impose the larger social harm than the rich, public provision of the non-positional good can always be Pareto-improving compared to policies P_0:

Proposition 4 *For every $P_0 \in \mathcal{S}$, if $\Gamma^1|_{C=C_0} > \Gamma^2|_{C=C_0}$, there exists a policy P with $g > 0$ which is Pareto-superior to P_0.*

To get an intuition for Proposition 4, consider a policy $P_0 \in \mathcal{S}$ and the corresponding allocation C_0. By setting the public provision level slightly above x_0^1 and lowering the net income of each type by an equal-valued amount, the government can force the poor to reduce their positional spending, which has a social benefit given by Γ^1. In contrast to the case without consumption taxes, however, an additional effect emerges: the lower positional consumption of the poor creates a tax revenue loss equal to t_0. If the marginal benefit from crowding out the poor outweighs the revenue loss (i.e., if $\Gamma^1 > t_0$), one can always find a feasible policy that induces a Pareto-improvement over P_0. Using formula (24) for the optimal tax t_0, the requirement $\Gamma^1 > t_0$ is equivalent to $\Gamma^1 > \Gamma^2$ – the condition stated in Proposition 4.

From the definitions in (9), $\Gamma^1 > \Gamma^2$ is satisfied if

$$\frac{1}{u_x^1}\left(\frac{\partial h^1}{\partial c^1} + \frac{\partial h^2}{\partial c^1}\right) > \frac{1}{u_x^2}\left(\frac{\partial h^2}{\partial c^2} + \frac{\partial h^1}{\partial c^2}\right). \tag{26}$$

As opposed to the case where positional good consumption cannot be taxed, condition (26) involves information on the relative strength of $\partial h^i/\partial c^i$ and $\partial h^i/\partial c^j$. In particular, public provision is always desirable when richer individuals have sufficiently strong needs to separate themselves from the poor or when the poor are sufficiently concerned with the consumption levels of the other poor (ceteris paribus, $\partial h^2/\partial c^1$ or $\partial h^1/\partial c^1$ must be large enough).[26]

Public provision of non-positional goods might be desirable even if $\Gamma^2 > \Gamma^1$. We demonstrate this in a numerical example in Appendix A.7, where we assume pure upward comparisons of the poor. As in the case without consumption taxes, public provision then

[25] An positional externality with $\partial h^i/\partial c^i \neq \partial h^i/\partial c^j$ for at least one i is sometimes called non-atmospheric. The implications of non-atmospheric externalities for the theory of optimal commodity taxation have recently been studied in, e.g., Micheletto (2008) and Eckerstorfer and Wendner (2013).

[26] Condition (26) globally holds in the polar cases of pure downward comparisons of the rich or pure within-group comparisons of the poor.

needs to constrain both the rich and the poor. In sum, for a wide range of social comparison types, public provision can achieve Pareto-improvements even if governments can implement optimal uniform taxes on the positional good.

4 Implementation of Pareto-efficient allocations

As shown in previous sections, public provision can be a valuable instrument to attain Pareto-improvements in the presence of relative consumption concerns. We derived our results under mild assumptions on the type of the public provision system: the provision level was uniform and individuals were allowed to purchase additional units of the non-positional good on the market. In this section, we ask whether such a weak provision scheme can also implement Pareto-efficient allocations. We abstract from peer effects and study the cases with and without a consumption tax separately. When the government has access to a positional tax, we only consider scenarios where $\Gamma^1 \neq \Gamma^2$, as public provision is redundant otherwise.

Proposition 5 *For any $C_* \in \mathcal{P}$,*

(i) if a tax on the positional good is not available, there exists a policy $P = (b^1, b^2, g)$ with $g > 0$ that implements C_* if and only if $\partial h^1/\partial c^2 = \partial h^2/\partial c^2 = 0$.

(ii) if a tax on the positional good is available, there exists a policy $P = (b^1, b^2, g, t)$ with $g > 0$ that implements C_* if and only if $\Gamma^1|_{C=C_*} > \Gamma^2|_{C=C_*}$. This policy entails $g = x^1_*$ and $t = \Gamma^2|_{C=C_*}$.

Item (i) of Proposition 5 shows that there are preference scenarios where public provision alone can support efficient allocations, namely whenever $\partial h^1/\partial c^2 = \partial h^2/\partial c^2 = 0$. The intuition is that when reference levels do not depend on c^2, the rich do not exert a positional externality (Γ^2 is globally zero) such that one needs to correct the consumption choices of the poor only. With our simple public provision system this is possible: as both goods are normal and individuals are allowed to top up, by setting the provision level to $g = x^1_*$ and reducing gross incomes by an equal-valued amount, the government can force the poor to choose the first-best bundle (c^1_*, x^1_*), without constraining the rich.

Preference settings that fulfill the condition of item (i) are pure within-group comparisons of the poor or pure downward comparisons of the rich. Observe that a linear positional tax alone can never achieve the first-best in such cases. The reason is that a uniform positional tax reduces the incentive to consume the positional good for every income type; it does not allow for targeting the poor separately. Hence, there are preference scenarios where

public provision strictly dominates a price instrument and where positional externalities are better addressed by publicly providing health care or basic education, but not taxing positional goods like smartphones, cars or jewelry.
If we allow for a consumption tax as an additional instrument, Pareto-efficient allocations can be achieved even if $\Gamma^2 > 0$. As stated in item (ii) of Proposition 5, this requires a sufficiently strong social harm of the poor in the sense that $\Gamma^1 > \Gamma^2$, evaluated at the first-best. To get an intuition, assume for a moment that public provision is not feasible. Government can induce the rich to choose her first-best bundle by setting the positional tax to $t = \Gamma^2$ and adjusting income taxes appropriately. If $\Gamma^1 > \Gamma^2$, the poor will overconsume [underconsume] the positional [non-positional] good at this policy, as the tax rate required to fully internalize their externality must be higher (this required a tax rate of $t = \Gamma^1$). The poor's consumption bundle can, however, be adjusted to the first-best by introducing public provision and setting the provision level equal to $g = x^1_*$. If, in contrast, the poor impose a smaller social harm than the rich ($\Gamma^1 < \Gamma^2$), the poor *under*consume the positional good at the externality-internalizing tax rate of the rich, which would call for a reduction of x^1. This, however, cannot be achieved by providing the non-positional good with a top-up system. Therefore, when a positional tax is available, $\Gamma^1 > \Gamma^2$ is both sufficient and necessary to decentralize a given first-best allocation.
When $\Gamma^2 > \Gamma^1$, we need stronger public provision systems. One possibility would be to allow for income-dependent public provision. Any efficient allocation could then be directly implemented by setting the provision levels equal to x^1_* and x^2_*. However, income-specific provision might be infeasible due to high administrative costs or political economy considerations. Alternatively, governments could keep with uniform provision, but restrict additional purchases of the non-positional good in the market. Individuals would then face a choice between accepting the publicly provided level or opting out of the public system and purchasing the non-positional good entirely in the market. Educational services are often available via such a "dual" provision system, where parents can send their children to either a public or a private school (but not both). With public provision of this type, Pareto-efficient allocations are attainable even if the marginal social harm is stronger among the rich. This requires, however, the poor [the rich] to indeed stay in [out of] the public system at policies with $g = x^1_*$ and $t = \Gamma^2$.

5 Public provision of the positional good

In the previous sections, the publicly provided good was *non-positional*. In line with empirical evidence, this characteristic can be reasonably attributed to goods like health care, health insurance or old-age savings. The case of education is perhaps less clear-cut, even though the overwhelming majority of studies in the empirical literature on conspicuous consumption classifies spending on education as non-positional (see, e.g., Charles et al., 2009; Heffetz, 2011; Khamis et al., 2012; Friehe and Mechtel, 2014).[27] For instance, Frank (1985) argues that education expenditure (e.g., tuition fees for private schools) may serve as a conspicuous signal to demonstrate wealth, and that it is precisely this positional aspect of education that may justify government intervention in the educational sector in the form of providing public schools.

Generally, what makes a good positional good may depend on the social and cultural environment. In this section, we analyze the public provision of positional goods, which may apply for higher education. We identify a condition when publicly providing the positional good can always produce Pareto-improvements. This condition can be related to social comparisons direction and income inequality.[28]

Framework. Preferences are as in section 1. Let the uniform provision level of the positional good be denoted by e. As individuals tend to overconsume the positional good in the laissez-faire, the consumption of good c must be distorted *downwards*. But this is impossible with a top-up provision: whenever the government provided a lower amount of the positional good than an individual demands for a given net income, the individual would purchase additional units in the market until she reaches her desired quantity. Public provision of the positional good must therefore be through an opt-out system, where individuals must decide whether to accept the publicly provided level e or to buy the positional good entirely in the market. The government furthermore has access to income taxes $T^i = T(y^i)$ such that net income is given by $b^i = y^i - T^i$.

The chronology of events is as follows. In the first stage, the government specifies a policy $P = (b^1, b^2, e)$. In the second stage, individuals decide whether or not to stay in the public system. In the third stage, individuals spend their net incomes to maximize utility. We assume that in stages 2 and 3, individuals take their reference levels as exogenously given.

[27] The only exception in this literature is Roth (2014). In his study using Indonesian household survey data, education is placed second on a visibility ranking scale, and is thus considered a positional good.

[28] Our analysis can be seen as a formalization of Frank's arguments. Interestingly, though Frank (1985a) advocates taxes on other luxury items such as yachts and jewelry, he does not suggest this price instrument to regulate overspending in education, maybe since this involves the contra-intuitive result that education should be taxed rather than subsidized.

Solving the model backwards, consider first stage 3. If an individual stays in the public system for a given policy, she consumes e units of good c and spends her entire net income on the non-positional good. Hence, her utility is given by $V_{in}^i := u(e, b^i) + e - r^i$. If an individual opts out of the public system, she chooses (c^i, x^i) as to maximize $u(c^i, x^i) + c^i - r^i$ subject to $c^i + x^i = b^i$. The solution to this problem gives the demand functions $c_d(b^i)$ and $x_d(b^i)$, which – as opposed to the top-up case – do not depend on the provision level e. The utility from opting out is therefore $V_{out}^i := u(c_d(b^i), x_d(b^i)) + c_d(b^i) - r^i$.
In the second stage, an individual stays in the public system if $V_{in}^i \geq V_{out}^i$, defining a cut-off rule. It can be shown that for every net income, there is a unique provision level $\hat{e}^i$ such that individual i stays in (opts out of) the public system if $e \geq (<)\, \hat{e}^i$.[29] The critical level $\hat{e}(b^i)$ is determined by

$$u(\hat{e}^i, b^i) + \hat{e}^i - r^i = u(c_d(b^i), x_d(b^i)) + c_d(b^i) - r^i. \tag{27}$$

Intuitively, $\hat{e}$ is the minimum provision level that individual i must be given in order to prevent her from consuming the positional good in the market. This level is lower than the amount of the positional good an individual would like to buy for a given net income, i.e., $\hat{e}^i < c_d(b^i)$. The reason is that public provision is provided free of charge; therefore, an individual is willing to accept a discount on her desired amount of the positional good when attracted by the public system. Due to this feature of opt-out provision, government may induce individuals to reduce their positional consumption in favor of the non-positional good. Second, as demands are normal, the critical level $\hat{e}$ is increasing in net income, so that the government has some scope to target different types differently.[30]
Denote the number of individuals who choose public provision by $N \in [0, 2]$. As unique critical levels $\hat{e}^i$ exist, N is uniquely determined for every given policy, and we can write it as a function of policy variables, i.e., $N = N(b^1, b^2, e)$. We define an equilibrium of the economy as follows:

Definition 2 *An allocation $C = (c^1, c^2, x^1, x^2)$, a corresponding pair of reference levels (r^1, r^2) and a policy $P = (b^1, b^2, e)$ constitute an equilibrium if*

(i) for every i, (c^i, x^i) solves the individual maximization problems at stages 2 and 3

[29]To see this, note that at $e = 0$, $V_{out}^i > V_{in}^i$ by the assumption that both goods are essential. This inequality is reversed when e is sufficiently large: if e is set equal to $c_d(b^i)$, $V_{out}^i < V_{in}^i$ as $b^i > x_d(b^i)$. By the continuity of V_{out}^i and V_{in}^i, $\hat{e}^i$ exists. Since V_{out}^i is independent of e while V_{in}^i strictly increases, $\hat{e}^i$ is unique.

[30]This is seen by implicitly differentiating (27) with respect to b^i and using the fact that, along an indifference curve, the marginal utility of good c is declining when both goods are normal (for a detailed proof in the case without relative consumption, which applies here as well as when preferences are separable, see Epple and Romano, 1996a).

for a given P,

(ii) $\sum_{i=1}^{2}(x^i + c^i - y^i) = 0$,

(iii) for every $i \neq j$, $r^i = h^i(c^i, c^j)$,

(iv) $y^1 - b^1 + c^1 + y^2 - b^2 + c^2 - N(b^1, b^2, e)\, e = 0$.

A sufficient condition.

Proposition 6 *There always exist a Pareto-improving policy with $e > 0$ if*

(i) $\partial h^1/\partial c^1 > 0$ *or* $\partial h^2/\partial c^1 > 0$ *and*

(ii) $c^1_{LF} < \hat{e}^2(y^2)$.

The logic behind Proposition 6 is as follows. Consider a policy regime where the provision level is set to the amount of the positional good which the poor consume in the laissez-faire. In addition, the poor's income is reduced by an equal amount and the rich remain untaxed. Under this policy, the poor receive the same consumption bundle as in the laissez-faire, with the difference being that they now obtain the positional good from the public system. If then the government marginally reduces the provision level and, at the same time, decreases the income tax for the poor, the poor will adjust their consumption bundle toward the non-positional good. This policy shift yields a Pareto-improvement, provided that the poor's consumption matters to someone (condition (i)) and the rich are not attracted by the public system. The latter is ensured if the rich stay out of public provision at the initial provision level, i.e., $c^1_{LF}(y^1) < \hat{e}^2(y^2)$ (condition (ii)).

Condition (ii) of Proposition 6 cannot be directly linked to relative consumption concerns, as the decision as to whether or not to stay in the public system depends on the total utility differential $V^i_{in} - V^i_{out}$, and hence, on absolute consumption utility terms as well. Nevertheless, it provides insights when the public provision of the positional good is particularly useful. Both $c^1_{LF}(y^1)$ and $\hat{e}^2(y^2)$ are increasing functions of income. Hence, condition (ii) will hold if the income differential $y^2 - y^1$ is sufficiently large, which can be interpreted as a measure of income inequality. As condition (i) is met for virtually all types of relative consumption concerns (the only exception is a situation where all reference function derivatives but $\partial h^2/\partial c^1$ are zero), Proposition 6 suggests that the public provision of the positional good is particularly effective when income disparities are strong.

6 Conclusion

The public provision of specific private goods is often justified by paternalistic or merit good arguments: from the perspective of some "outside observer" (e.g., the government, an altruistic donor or even a person's own future self) consumers spend insufficient amounts of money on goods like education, health care or old-age consumption when left to their own devices. In this paper, we offered a rationale for public provision that fully respects individual preferences. People care about their relative standing and therefore devote inefficiently high shares of their budgets to positional or status goods. By publicly providing the goods and services that are underconsumed in the positional arms race, governments can correct for the market inefficiency related to relative consumption concerns and therefore induce Pareto-improvements. In contrast to paternalistic or merit goods arguments, individuals would then agree to restrict consumer sovereignty by public provision.

In addition to providing a new rationale for public provision, our paper contributes to addressing the more general question of policy design in the presence of consumer distortions due to concern for relative standing. The recent empirical literature on conspicuous consumption provides evidence that is consistent with the idea that positional spending and status-seeking is particularly pronounced at the bottom of the income distribution: low-income individuals devote a relatively large share of their income to positional spending items at the expense of the less positional goods. Our analysis provides policy guidance for such cases: then, publicly providing non-positional goods like basic education or health insurance dominates a positional tax on luxury goods, a price instrument to counter the positional externality. The reason is that public provision allows to target the poor separately, provided that the demand for non-positional goods is increasing in income. Our paper can therefore be seen as contributing to the general question of the relative merits of price vs. quantity instruments, which has recently received considerable attention in the broader field of behavioral economics (see, e.g., Farhi and Gabaix, 2015).

It should be noted that our results are derived from a stylized model, where apart from social comparisons no further source of market or policy inefficiency exists. If, for example, we allowed for endogenous labor supply, then our sufficient conditions would have to be adjusted by the distortionary effects of the income taxation needed to finance public provision. The same applies when having more than two goods. In this case, the scope for public provision is somewhat reduced, as only those of the non-positional goods that are not too complementary to the consumption of positional goods are able to induce Pareto-improvements. Our main insights, however, that public provision can be Pareto-improving under relative consumption concerns and that public provision is a particularly effective policy tool whenever there are behavioral "deficiencies" at the bottom of the income distributions generalizes to richer frameworks.

Finally, though our study is normative in nature, we think that it may also provide a new perspective on individuals' voting incentives over public provision. In particular, forcing the poor away from positional goods allows the rich to reduce the economic resources they have to invest in maintaining their relative consumption position. Concerns for relative standing may thus endow richer individuals with a motive to politically support social benefits in kind, even when they are highly redistributive. Our analysis therefore suggests that social comparisons may also affect the political economy of public provision – a topic left for future research.

References

Abel, A. B. (1990). Asset prices under habit formation and catching up with the Joneses. American Economic Review Papers and Proceeding 80, 38-42.

Alonso-Carrera, J., Caball J. and X. Raurich (2004). Consumption externalities, habit formation, and equilibrium efficiency. Scandinavian Journal of Economics 106, 231.251.

Alpizar, F., Carlsson, F. and O. Johansson-Stenman (2005). How much do we care about absolute versus relative income and consumption? Journal of Economic Behavior and Organization 56, 405-421.

Aronsson, T. and O. Johansson-Stenman (2010). Positional concerns in an OLG model: optimal labor and capital income taxation. International Economic Review 51, 1071-1095.

Arrow, K. J. and P. S. Dasgupta (2009). Conspicuous consumption, inconspicuous leisure. Economic Journal 119, 497-516.

Banerjee, A. V. and E. Duflo (2007). The economic lives of the poor. Journal of Economic Perspectives 21, 141-168.

Barten, A. P. and V. Böhm (1982). Consumer theory. In: Arrow, K. J. and M. D. Intriligator (eds.). Handbook of Mathematical Economics 2, 381-429.

Besley, T. (1988). A simple model for merit good arguments. Journal of Public Economics 35, 371-383.

Besley, T. and S. Coate (1991). Public provision of private goods and the redistribution of income. American Economic Review 81, 979-984.

Bilancini, E. and L. Boncinelli (2012). Redistribution and the notion of social status. Journal of Public Economics 96, 651-657.

Blackorby, C. and D. Donaldson (1988). Cash versus kind, self-selection, and efficient transfers. American economic Review 78, 691-700.

Blomquist, S. (1993). Interdependent behavior and the effect of taxes. Journal of Public Economics 51, 211-218.

Boskin, M. J. and E. Sheshinski (1978). Individual welfare depends upon relative income. Quarterly Journal of Economics 92, 589-601.

Bowles, S. and Y. Park (2005). Emulation, inequality, and work hours: was Thorstein Veblen right? Economic Journal 115, 397-412.

Brown, P. H., Bulte, E. and X. Zhang (2011). Positional spending and status seeking in rural China. Journal of Development Economics 96, 139-149.

Carlsson, F., Johansson-Stenman O. and P. Martinsson (2007). Do you enjoy having more than others? Survey evidence of positional goods. Economica 74, 586-598.

Case, A., Garrib, A., Menendez, A. and Olgiati, A. (2008). Paying the piper: the high cost of funerals in South Africa. NBER Working Paper No. 14456.

Charles, K. K., Hurst, E. and N. Roussanov (2009). Conspicuous consumption and race. Quarterly Journal of Economics 124, 425-467.

Clark, A. E. and A. J. Oswald (1996). Satisfaction and comparison income. Journal of Public Economics 61, 359-381.

Clark, A. E. and A. J. Oswald (1998). Comparison-concave utility and following behavior in social and economic settings. Journal of Public Economics 70, 133-155.

Corneo, G. and O. Jeanne (1997). Conspicuous consumption, snobbism and conformism. Journal of Public Economics 66, 55-71.

Currie, J. and F. Gahvari (2008). Transfers in cash and in-kind: theory meets the data. Journal of Economic Literature 46, 333-383.

Diamond, P. A. (1973). Consumption externalities and imperfect corrective pricing. Bell Journal of Economic and Management Science 4, 526-538.

Duesenberry, J. (1949). Income, saving, and the theory of consumer behavior. Cambridge: Harvard University Press.

Dufwenberg, M., Heidhues, P., Kirchsteiger, G., Riedel F. and J. Sobel (2011). Other-regarding preferences in general equilibrium. Review of Economic Studies 78, 613-639.

Dupor, B. and W.-F. Liu (2003). Jealousy and equilibrium overconsumption. American Economic Review 93, 423-428.

Durlauf, S. N. and Y. M. Ioannides (2010). Social interactions. Annual Review of Economics 2, 451-478.

Eckerstorfer, P. and R. Wendner (2013). Asymmetric and non-atmospheric consumption externalities, and efficient consumption taxation. Journal of Public Economics 106, 42-56.

Epple, D. and R. E. Romano (1996a). Public provision of private goods. Journal of Political Economy 104, 57-84.

Epple, D. and R. E. Romano (1996b). Ends against the middle: determining public service provision when there are private alternatives. Journal of Public Economics 62, 297-325.

Falk, A. and M. Knell (2004). Choosing the Joneses: endogenous goals and reference standards. Scandinavian Journal of Economics 106, 417-435.

Festinger, L. (1954). A theory of social comparison processes. Human Relations 7, 117-140.

Farhi, E. and X. Gabaix (2015). Optimal taxation with behavioral agents. NBER Working Paper 21524.

Frank, R. H. (1984). Interdependent preferences and the competitive wage structure. RAND Journal of Economics 15, 510-520.

Frank, R. H. (1985a). The demand for unobservable and other non-positional goods. American Economic Review 75, 101-116.

Frank, R. H. (1985). Choosing the right pond. Human behavior and the quest for status. Oxford: Oxford University Press.

Friehe, T. and M. Mechtel (2014). Conspicuous consumption and political regimes: evidence from East and West Germany. European Economic Review 67, 62-81.

Gasparini, L. C. and S. M. Pinto (2006). Equality of opportunity and optimal cash and in-kind policies. Journal of Public Economics 90, 143-169.

Glazer, A. and K. A. Konrad (1996). A signaling explanation for charity. American Economic Review 86, 1019-1028.

Green, J. and E. Sheshinski (1976). Direct versus indirect remedies for externalities. Journal of Political Economy 84, 797-808.

Heffetz, O. (2011). A test of conspicuous consumption: visibility and income elasticities. Review of Economics and Statistics 93, 1101-1117.

Heffetz, O. and R. H. Frank (2011). Preferences for status: evidence and economic implications. In: Benhabib, J., Bisin, A. and M. O. Jackson (eds.). Handbook of Socio Economics 1, 69-91.

Hirsch, F. (1976). Social limits to growth. Cambridge: Harvard University Press.

Hopkins E. and T. Konienko (2004). Running to keep in the same place: consumer choice as a game of status. American Economic Review 94, 1085-1107.

Ireland, N. J. (1994). On limiting the market for status signals. Journal of Public Economics 53, 91-110.

Ireland, N. J. (1998). Status-seeking, income taxation and efficiency. Journal of Public Economics 70, 99-113.

Ireland, N. J. (2001). Optimal income tax in the presence of status effects. Journal of Public Economics 81, 193-212.

Khamis, M., Prakash, N. and Z. Siddique (2012). Consumption and social identity: evidence from India. Journal of Economic Behavior and Organization 83, 353-371.

Kuhn, P., Kooreman, P., Soetevent, A. and A. Kapteyn (2011). The effects of lottery prizes on winners and their neighbors: evidence from the Dutch postcode lottery. American Economic Review 101, 2226-2247.

Kumagai, S. (1980). An implicit function theorem: comment. Journal of Optimization Theory and Applications 31, 285-288.

Leibenstein, H. (1950). Bandwagon, snob, and Veblen effects in the theory of consumers' demand. Quarterly Journal of Economics 64, 183-207.

Liu, W.-F. and S. J. Turnovsky (2005). Consumption externalities, production externalities, and long-run macroeconomic Efficiency. Journal of Public Economics 89, 1097-129.

Ljungqvist, L. and H. Uhlig (2000). Tax policy and aggregate demand management under catching up with the Joneses. American Economic Review 90, 356-366.

Luttmer, E. F. P. (2005). Neighbors as negatives: relative earnings and well-being. Quarterly Journal of Economics 120, 963-1002.

Micheletto, L. (2008). Redistribution and optimal mixed taxation in the presence of consumption externalities. Journal of Public Economics 92, 2262-2274.

Moav, O. and Z. Neeman (2010). Status and poverty. Journal of the European Economic Association 8, 413-420.

Moav, O. and Z. Neeman (2012). Savings rates and poverty: the role of conspicuous consumption and human capital. Economic Journal 122, 933-956.

Musgrave, R. A. (1959). The theory of public finance; a study in public economy. New York: McGraw-Hill.

Friehe, T. and M. Mechtel (2014). Conspicuous consumption and political regimes: evidence from east and west Germany. European Economic Review 67, 62-81.

Ng, Y. K. (1987). Diamonds are a government's best friend: burden-free taxes on goods valued for their values. American Economic Review 77, 186-191.

Nichols, A. L. and R. J. Zeckhauser (1982). Targeting transfers through restrictions on recipients. American Economic Review 72, 372-377.

Postlewaite, A. (1998). Social status, norms and economic performances: the social basis of interdependent preferences. European Economic Review 42, 779-800.

Roth, C. (2014). Conspicuous consumption and peer effects among the poor: evidence from a field experiment. CSAE Working Paper WPS/2014-29.

Runciman, W. G. (1966). Relative deprivation and social justice. London: Routledge and Kegan Paul.

Sandmo, A. (1975). Optimal taxation in the presence of externalities. Scandinavian Journal of economics 77, 86-98.

Sandmo, A. (1983). Ex post welfare economics and the theory of merit goods. Economica 50, 19-33.

Solnick, S. J. and D. Hemenway (2005). Are positional concerns stronger in some domains than in others? American Economic Review 95, 147-151.

Suls, J. and T. A. Wills (1991). Social comparison: contemporary theory and research. Hillsdale, NJ: Lawrence Erlbaum Associates, Inc..

Truyts, T. (2012). Signaling and indirect taxation. Journal of Public Economics 96, 331-340.

Veblen, T. (1899). The theory of the leisure class: an economic study of institutions (reprinted 1994). New York: Dover Publications.

Wood, J. V. and K. L. Taylor (1991). Serving self-relevant goals through social comparison. In: Suls, J. and T. A. Wills (eds.). Social comparison: contemporary theory and research. Hillsdale, NJ: Lawrence Erlbaum Associates, Inc..

Appendix

A.1 Pareto-efficient allocations

We can identify Pareto-efficient allocations C_* by maximizing U_*^1 given that (i) U_*^2 does not fall below a level $\bar{U}^2$ and (ii) the economy's resource constraint holds. Varying $\bar{U}^2$ then gives the whole set $\mathcal{P}$ of Pareto-efficient allocation. Formally, any C_* solves

$$\max_{c^1,x^1,c^2,x^2} U_*^1(c^1,x^1,c^2) \qquad \text{s.t.} \tag{28}$$

$$(\text{i}): \quad U^2(c^2, x^2, c^1) \geq \bar{U}^2,$$
$$(\text{ii}): \sum_{i=1}^{I}(y^i - c^i - x^i) \geq 0.$$

Since U_*^1 is strongly quasi-concave and the constraint set is convex, closed, and bounded, there exists a unique Pareto-efficient allocations for every given $\bar{U}^2$. Define $\mathcal{P}$ as the set of allocations C_* such that there exists $\bar{U}^2$ and the allocation solves (28).
The Lagrangian for problem (28) is

$$\begin{aligned} L = U_*^1(c^1, x^1, c^2) + \mu \left[U_*^2(c^2, x^2, c^1) - \bar{U}^2\right] \\ + \lambda \sum_{i=1}^{I}(y^i - c^i - x^i), \end{aligned} \tag{29}$$

where μ and λ denote the Lagrange multipliers associated with the utility and resource constraint, respectively. An interior solution $(c_*^1, x_*^1, c_*^2, x_*^2) \gg 0$ must satisfy the first-order conditions

$$\frac{\partial L}{\partial c^1} = u_c^1 + 1 - \frac{\partial h^1}{\partial c^1} - \mu^* \frac{\partial h^2}{\partial c^1} - \lambda^* = 0, \tag{30}$$
$$\frac{\partial L}{\partial x^1} = u_x^1 - \lambda^* = 0, \tag{31}$$
$$\frac{\partial L}{\partial c^2} = -\frac{\partial h^1}{\partial c^2} + \mu^* \left(u_c^2 + 1 - \frac{\partial h^2}{\partial c^2}\right) - \lambda^* = 0, \tag{32}$$
$$\frac{\partial L}{\partial x^2} = \mu^* u_x^2 - \lambda^* = 0. \tag{33}$$

From (31) and (33), we have $\lambda^* = u_x^1$ and $\mu^* = \lambda^*/u_x^2$. Plugging these expressions into (30) and (32) gives condition (8).

A.2 Proof of Proposition 1

We have to prove that under the conditions stated in Proposition 1, there always exists a policy $P = (b^1, b^2, g)$ with $g > 0$ that achieves a Pareto-improvement over the laissez-faire with $P_{LF} = (y^1, y^2, 0)$. Consider the policy scheme where $b^i = y^i - g$ for $i = 1, 2$. We will show that under this scheme, setting the public provision level g slightly above x_{LF}^1 raises the utility of at least one type compared to P_{LF} if $\partial h^1/\partial c^1 > 0$ or $\partial h^2/\partial c^1 > 0$.
To see this, note that with $b^i = y^i - g$, we have $x_d(b^i + g) = x_d(y^i) = x_{LF}^i$. Hence, an individual of type i is crowded out by public provision if $g \geq x_{LF}^i$. Consider a change from P_{LF} to policy $P_r = (b_r^1, b_r^2, g_r) = (y^1 - x_{LF}^1, y^2 - x_{LF}^1, x_{LF}^1)$. At P_r, both types

choose the same consumption bundle as in the laissez-faire: since $g_r = x^1_{LF}$, by demand functions (12) and (13), individuals of type 1 are just crowded out by public provision and choose $x^1(b^1_r, g_r) = g_r = x^1_{LF}$ and $c^1(b^1_r, g_r) = b^1_r = y^1 - x^1_{LF} = c^1_{LF}$. As $y^2 > y^1$, the normality of good x implies $g_r < x_d(y^2)$. Thus, demanded quantities of type 2 individuals are $c^2(b^2_r, g_r) = c_d(y^2) = c^2_{LF}$ and $x^2(b^2_r, g_r) = x_d(y^2) = x^2_{LF}$. As a consequence, at P_r each type's utility is the same as in the laissez-faire. Substitution of policy P_r into the government budget constraint yields $y^1 - c^1_{LF} - x^1_{LF} + y^2 - c^2_{LF} - x^2_{LF} = 0$. Therefore, P_r is feasible.

Now, consider a change from P_r to policy $P_p = (b^1_p, b^2_p, g_p) = (b^1_r + db^1, b^2_r + db^2, g_r + dg)$, where $dg > 0$ and $dg \to 0$. As $db^i = -dg$ for both types, policy P_p is feasible. Since $g_p > x^1_{LF}$, under the one-to-one policy scheme, individuals of type 1 remain crowded out after the policy change. By the continuity of demand functions (12) and (13), individuals of type 2 are not crowded out by public provision at P_p. Hence, at the two policies P_r and P_p, both types' demands are, respectively, given by $c^1(b^1, g) = b^1$, $x^1(b^1, g) = g$, $c^2(b^2, g) = x_d(b^2 + g)$ and $x^2(b^2, g) = x_d(b^2 + g)$. Inserting these demands into (14) gives indirect utilities

$$V^1(b^1, b^2, g) = u(b^1, g) + b^1 - h^1(b^1, c_d(b^2 + g)), \tag{34}$$
$$V^2(b^2, b^1, g) = u(c_d(b^2 + g), x_d(b^2 + g)) + c_d(b^2 + g) - h^2(c_d(b^2 + g), b^1). \tag{35}$$

Using $b^i = y^i - g$, the change in indirect utilities induced by the switch from policy P_r to P_p can be represented by differentiating (34) and (35) with respect to g at P_r:

$$\frac{dV^1}{dg} = -(u^1_c + 1) + x^1_x + \frac{\partial h^1}{\partial c^1}, \tag{36}$$
$$\frac{dV^2}{dg} = \frac{\partial h^2}{\partial c^1}. \tag{37}$$

Since $-(u^i_c + 1) + u^i_x = 0$ at P_r (see the first-order condition (11)) and $\partial h^1/\partial c^1 \geq 0$ and $\partial h^2/\partial c^1 \geq 0$, $dV^1/dg \geq 0$ and $dV^2/dg \geq 0$. Hence, no income type is worse off when g is raised slightly above x^1_{LF}. If $\partial h^1/\partial c^1 > 0$ or $\partial h^2/\partial c^1 > 0$, $dV^i/dg > 0$ for at least one type, and P_p achieves a Pareto-improvement over P_{LF}. This proves the proposition. ■

A.3 Proofs for section 2

A.3.1 Derivations for KUJ and RAJ

We will show that the sign of $\partial MRS(c^i, x^i, r^i)/\partial r^i$ is equivalent to the sign of $\partial c^i_d/\partial r^i$ if an individual is not constrained by public provision, i.e., if $x^i > g$. To see this, consider individuals' maximization problem for a given g:

$$\max_{c^i, x^i} \quad \tilde{U}(c^i, x^i, r^i) \quad \text{s.t.} \quad (c^i, x^i) \in \tilde{B}_g^i. \tag{38}$$

A solution to (38) with $x^i > g$ must satisfy

$$-\tilde{U}_c^i + \tilde{U}_x^i = 0 \quad \text{and} \quad c^i + x^i = y^i. \tag{39}$$

From (39), we can obtain

$$\frac{\partial c^i}{\partial r^i} = \frac{\tilde{U}_{cr}^i - \tilde{U}_{xr}^i}{-\tilde{U}_{cc}^i + 2\tilde{U}_{cx}^i - \tilde{U}_{xx}^i}, \tag{40}$$

where we abbreviated $\tilde{U}_{cc}^i := \partial^2 \tilde{U}(c^i, y^i - c^i, r^i)/\partial c^2$, $\tilde{U}_{cx}^i := \partial^2 \tilde{U}(c^i, y^i - c^i, r^i)/\partial c \partial x$, $\tilde{U}_{xx}^i := \partial^2 \tilde{U}(c^i, y^i - c^i, r^i)/\partial x^2$, $\tilde{U}_{cr}^i := \partial^2 \tilde{U}(c^i, y^i - c^i, r^i)/\partial c \partial r$, and
$\tilde{U}_{xr}^i := \partial^2 \tilde{U}(c^i, y^i - c^i, r^i)/\partial x \partial r$. Since $\tilde{U}$ is strongly quasi-concave, the denominator in (40) is positive (Barten and Böhm, 1982). The sign of $\partial c^i/\partial r^i$ is therefore determined by the sign of $\tilde{U}_{cr}^i - \tilde{U}_{xr}^i$. It remains to be shown that $\partial MRS^i\,(c^i, x^i, r^i)\,/\partial r^i > (<)\, 0 \iff \tilde{U}_{cr}^i - \tilde{U}_{xr}^i > (<)\, 0$. To see this, differentiate $MRS^i\,(c^i, x^i, r^i) = \tilde{U}_c^i/\tilde{U}_x^i$ with respect to r^i:

$$\frac{\partial MRS^i}{\partial r^i} = \frac{\tilde{U}_{cr}^i \tilde{U}_x^i - \tilde{U}_c^i \tilde{U}_{xr}^i}{(\tilde{U}_x^i)^2} = \frac{1}{\tilde{U}_x^i}\left[\tilde{U}_{cr}^i - \frac{\tilde{U}_c^i}{\tilde{U}_x^i}\tilde{U}_{xr}^i\right]. \tag{41}$$

Since, $\tilde{U}_c^i/\tilde{U}_x^i = 1$ at a solution to (38), $\partial MRS^i\,(c^i, x^i, r^i)\,/\partial r^i > (<)\, 0 \iff \partial c^i/\partial r^i > (<\,)\, 0$.

A.3.2 Proof of local uniqueness

To prove that Assumption 1 implies local uniqueness of equilibria, rewrite (20) as $F(\mathbf{r}, g) = 0$, where $F : \mathbb{R}^2 \times \mathbb{R}^1 \to \mathbb{R}^2$, $(\mathbf{r}, g) \mapsto F(\mathbf{r}, g) = (F_1(\mathbf{r}, g), F_2(\mathbf{r}, g))$ with $F_i(\mathbf{r}, g) = r^i - h^i(c^i(g, r^i), c^j(g, r^j))$. By the continuity of demand functions and h^i, F is continuous. However, as demand functions are not differentiable at points where $g = x_d^i(y^i, r^i)$ for one or both types, the standard implicit function theorem does not apply. However, local uniqueness can be ensured by applying an implicit function theorem for non-differentiable mappings by Kumagai (1980).
To see this, let $(\hat{r}^1, \hat{r}^2, \hat{g})$ be a point such that $F(\hat{r}^1, \hat{r}^2, \hat{g}) = 0$. If there exist open neighborhoods of $(\hat{r}^1, \hat{r}^2)$ and $\hat{g}$ on which $F(\cdot, g)$ is locally one-to-one, the theorem ensures that we can express references levels as an implicit function of g. When no individual is crowded out at $(\hat{r}^1, \hat{r}^2, \hat{g})$ (which includes the laissez-faire with $g = 0$), $F(\cdot, g)$ is locally

one-to-one: by Assumption 1, the determinant of the Jacobian of F with respect to $\mathbf{r}$, given by A, is non-zero such that F is invertible. Now consider a solution to $F(\mathbf{r}, g) = 0$ where one type i is just crowded out ($g = x^i_d(y^i, r^i)$). If we go into an ϵ-environment of $\hat{g}$ and keep $\mathbf{r}$ fixed at $(\hat{r}^1, \hat{r}^2)$, type i will either be strictly crowded out ($g > x_d(y^i, r^i)$) or she will top up ($g < x^i_d(y^i, r^i)$). This will remain true for a sufficiently close ball around $(\hat{r}^1, \hat{r}^2)$. Since in either of these cases, the determinant of the Jacobian of F with respect to $\mathbf{r}$ exists and has the same sign, we can conclude that F is also one-to-one around a solution where $F(\mathbf{r}, g) = 0$ is non-differentiable. Hence, Kumagai's theorem applies, and the equilibria are locally unique. ■

A.3.3 Proof of Proposition 2

The proof follows the same logic as the proof of Proposition 1. Consider a given $(\hat{r}^1, \hat{r}^2) \in \mathcal{E}$ in the laissez-faire ($g = 0$) and the corresponding consumption allocation $C_{LF} = (c^1_{LF}, x^1_{LF}, c^2_{LF}, x^2_{LF})$, where $c^i_{LF} := c^i_d(y^i, \hat{r}^i)$ and $x^i_{LF} := x^i_d(y^i, \hat{r}^i)$.
Set the public provision level to $g_r = x^1_{LF}$. By the assumption that reference levels do not jump, policy g_r leads to the same consumption allocation C_{LF} as in the laissez-faire; as $g_r = x^1_{LF}$, by demand functions (18) and (19), individuals of type 1 choose $c^1(g_r, \hat{r}^1) = y^1 - g_r = c^1_{LF}$ and $x^1(g_r, \hat{r}^1) = g_r = x^1_{LF}$. Since $x^1_d(y^1, r^1) < x^2_d(y^2, r^2)$ by assumption, we have $g_r < x^2_{LF}$. Hence, individuals of type 2 choose $c^2(g_r, \hat{r}^2) = c^2_d(y^2, \hat{r}^2) = c^2_{LF}$ and $x^2(g_r, \hat{r}^2) = c^2_d(y^2, \hat{r}^2) = x^2_{LF}$. Therefore, at g_r, individuals of type 1 [type 2] are [not] crowded out by public provision, and system (20) reads

$$\begin{aligned} r^1 - h^1(y^1 - g, c^2_{LF}) &= 0 \\ r^2 - h^2(c^2_{LF}, y^1 - g) &= 0. \end{aligned} \tag{42}$$

Given $(\hat{r}^1, \hat{r}^2)$, indirect utilities can be expressed as

$$V^1(g) = U(y^1 - g, g, \Delta^1(y^1 - g, \hat{r}^1(g))), \tag{43}$$

$$V^2(g) = U(c^2_d(y^2, \hat{r}(g)), x^2_d(y^2, \hat{r}(g)), \Delta^2(c^2_d(y^2, \hat{r}(g)), \hat{r}^2(g))). \tag{44}$$

Consider a change in the public provision level from g_r to $g_p = g_r + dg$, where $dg > 0$ and $dg \to 0$. Using (42), the effect of this policy change on both types' reference levels is

$$\frac{d\hat{r}^1}{dg} = -\frac{\partial h^1}{\partial c^1} - \frac{1}{D_1}\frac{\partial h^2}{\partial c^1}\frac{\partial h^1}{\partial c^2}\frac{\partial c^2_d}{\partial r^2} \tag{45}$$

$$\frac{d\hat{r}^2}{dg} = -\frac{1}{D_1}\frac{\partial h^2}{\partial c^1}, \tag{46}$$

where

$$D_1 = 1 - \frac{\partial h^2}{\partial c^2}\frac{\partial c_d^2}{\partial r^2} > 0$$

by Assumption 1. Using (45), (46) and the first-order conditions of the individual maximization problems (38), the change in indirect utilities induced by the change from g_r to g_p can be calculated to

$$\frac{dV^1}{dg} = -U_\Delta^1 \frac{\partial \Delta^1}{\partial r^1}\left[\frac{\partial h^1}{\partial c^1} + \frac{1}{D_1}\frac{\partial h^2}{\partial c^1}\frac{\partial h^1}{\partial c^2}\frac{\partial c_d^2}{\partial r^2}\right], \tag{47}$$

$$\frac{dV^2}{dg} = -U_\Delta^2 \frac{1}{D_2}\frac{\partial h^2}{\partial c^1}, \tag{48}$$

where $U_\Delta^i := \partial U(c^i, x^i, \Delta^i)/\partial\Delta$ and (47) and (48) are evaluated at g_r. Under KUJ, $\partial c_d^2/\partial r^2 > 0$. Hence, dV^1/dg and dV^2/dg are non-negative at g_r. If $\partial h^1/\partial c^1 > 0$ or $\partial h^2/\partial c^1 > 0$, we have $dV^1 > 0$ or $dV^2 > 0$.

To complete the proof, we have to show that individuals of type 1 [type 2] are still [not] crowded out by public provision after the policy change from g_r to g_p. As demand functions are continuous and we require $x_{LF}^2 > x_{LF}^1$, individuals of type 2 are not crowded out by public provision at g_p. Type 1 individuals remain crowded out if the difference $\Omega^1(g, r^1) := g - x_d^1(y^1, r^1)$ is greater or equal to zero at g_p (see demand functions (18) and (19)). This holds if

$$d\Omega^1(g, r^1) = dg - \frac{\partial x_d^1}{\partial r^1}dr^1 \geq 0, \tag{49}$$

at g_r. Dividing by dg and inserting (45), this is equivalent to

$$-\frac{\partial x_d^1}{\partial r^1}\frac{\partial h^1}{\partial c^1} - \frac{1}{D_1}\frac{\partial h^2}{\partial c^1}\frac{\partial h^1}{\partial c^2}\frac{\partial c^2}{\partial r^2}\frac{\partial x_d^1}{\partial r^1} \leq 1. \tag{50}$$

From individuals' budget constraints, KUJ implies $\partial x^i/\partial r^i = -\partial c^i/\partial r^i < 0$. Hence, (50) can be rewritten

$$\frac{\partial h^1}{\partial c^1}\frac{\partial c_d^1}{\partial r^1} + \frac{1}{D_1}\frac{\partial h^2}{\partial c^1}\frac{\partial h^1}{\partial c^2}\frac{\partial c_d^2}{\partial r^2}\frac{\partial c_d^1}{\partial r^1} \leq 1. \tag{51}$$

By item (i) of Assumption 1,

$$\left(1 - \frac{\partial h^1}{\partial c^1}\frac{\partial c_d^1}{\partial r^1}\right)\left(1 - \frac{\partial h^2}{\partial c^2}\frac{\partial c_d^2}{\partial r^2}\right) - \frac{\partial h^1}{\partial c^2}\frac{\partial c_d^2}{\partial r^2}\frac{\partial h^2}{\partial c^1}\frac{\partial c_d^1}{\partial r^1} > 0. \tag{52}$$

Dividing by D_1 and rearranging leads to (51). Hence, individuals of type 1 stay crowded out after the change to policy P_p. This proves Proposition 2. ■

A.4 Derivation of optimal consumption taxes in the absence of public provision

The solution to the individual maximization problem (6) with each individual's budget set given by (23) must satisfy the first-order conditions

$$MRS(c^i, x^i) - p \geq 0 \quad \text{and} \quad \big(x^i - g\big)\big[MRS(c^i, x^i) - p\big] = 0. \tag{53}$$

Demand functions for goods c and x can be expressed as follows:

$$c^i(b^i, g, p) = \begin{cases} c_d(b^i + g, p) & \text{if } g < x_d(b^i + g, p), \\ b^i/p & \text{if } g \geq x_d(b^i + g, p), \end{cases} \tag{54}$$

$$x^i(b^i, g, p) = \begin{cases} x_d(b^i + g, p) & \text{if } g < x_d(b^i + g, p), \\ g & \text{if } g \geq x_d(b^i + g, p). \end{cases} \tag{55}$$

The functions $c_d(I, p)$ and $x_d(I, p)$ again give individuals' ordinary or unconstrained demands, and solve problem (6) for $B_u := \{(c, x) : pc + x \leq I\}$. Using that $p = p(t)$, we define the indirect utility of type i as

$$\begin{aligned} V^i(b^i, b^j, g, p(t)) :=\; & u^i(c^i(b^i, g, p(t)), x^i(b^i, g, p(t))) + c^i(b^i, g, p(t)) \\ & - h^i(c^i(b^i, g, p(t)), c^j(b^j, g, p(t))). \end{aligned} \tag{56}$$

Optimal policies in the absence of public provision solve

$$\max_{b^1, b^2, t} \quad V^1(b^1, b^2, 0, p(t)) \quad \text{s.t.} \tag{57}$$

$$\begin{aligned} \text{(i)}: \;& V^2(b^1, b^2, 0, p(t)) \geq \bar{U}^2, \\ \text{(ii)}: \;& y^1 - b^1 + tc^1 + y^2 - b^2 + tc^2 \geq 0. \end{aligned}$$

We define $\mathcal{S}$ as the set of policies $P_0 = (b_0^1, b_0^2, 0, t_0)$ such that there exists a level $\bar{U}^2$ and the policy solves (57).

Denote the Lagrangian to problem (57) by

$$
\begin{aligned}
L = {} & V^1(b^1, b^2, 0, p(t)) + \mu \left[V^2(b^1, b^2, 0, p(t)) - \bar{U}^2 \right] \\
& + \lambda \left[y^1 - b^1 + tc^1 + y^2 - b^2 + tc^2 \right],
\end{aligned} \tag{58}
$$

where μ and λ are the Lagrange multipliers associated with constraints (i) and (ii) of problem (57). Any interior solution $P_0 = (b_0^1, b_0^2, 0, t_0)$ in $\mathcal{S}$ must satisfy the first-order conditions:

$$
\frac{\partial L}{\partial b^1} = \frac{1}{p_0}(u_c^1 + 1) - \frac{\partial h^1}{\partial c^1}\frac{\partial c^1}{\partial b^1} - \mu^0 \frac{\partial h^2}{\partial c^1}\frac{\partial c^1}{\partial b^1} + \lambda^0 \left(-1 + t_0 \frac{\partial c^1}{\partial b^1} \right) = 0 \tag{59}
$$

$$
\frac{\partial L}{\partial b^2} = -\frac{\partial h^1}{\partial c^2}\frac{\partial c^2}{\partial b^2} + \mu^0 \left[\frac{1}{p_0}(u_c^2 + 1) - \frac{\partial h^2}{\partial c^2}\frac{\partial c^2}{\partial b^2} \right] + \lambda^0 \left(-1 + t_0 \frac{\partial c^2}{\partial b^2} \right) = 0 \tag{60}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial t} = {} & -\frac{1}{p_0}(u_c^1 + 1)\, c^1 - \frac{\partial h^1}{\partial c^1}\frac{\partial c^1}{\partial p} - \frac{\partial h^1}{\partial c^2}\frac{\partial c^2}{\partial p} \\
& + \mu^0 \left[-\frac{1}{p_0}(u_c^2 + 1)\, c^2 - \frac{\partial h^2}{\partial c^1}\frac{\partial c^1}{\partial p} - \frac{\partial h^2}{\partial c^2}\frac{\partial c^2}{\partial p} \right] \\
& + \lambda^0 \left[c^1 + t_0 \frac{\partial c^1}{\partial p} + c^2 + t_0 \frac{\partial c^2}{\partial p} \right] = 0,
\end{aligned} \tag{61}
$$

where $p_0 = 1 + t_0$. Solving (59) and (60) for μ^0 and λ^0 yields:

$$
\mu^0 = \frac{1}{D_2} \left[\frac{\left(-1 + t_0 \partial c^2 / \partial b^2 \right)}{-1 + t_0 \partial c^1 / \partial b^1} \left[\frac{1}{p_0}(u_c^1 + 1) - \frac{\partial h^1}{\partial c^1}\frac{\partial c^1}{\partial b^1} \right] + \frac{\partial h^1}{\partial c^2}\frac{\partial c^2}{\partial b^2} \right] \tag{62}
$$

$$
\begin{aligned}
\lambda^0 = {} & -\frac{1}{(-1 + t_0 \partial c^1 / \partial b^1)} \frac{1}{D_2} \left[\frac{1}{p_0}(u_c^2 + 1) \left[\frac{1}{p_0}(u_c^1 + 1) - \frac{\partial h^1}{\partial c^1}\frac{\partial c^1}{\partial b^1} \right] \right. \\
& \left. - \frac{1}{p_0}(u_c^1 + 1) \frac{\partial h^2}{\partial c^2}\frac{\partial c^2}{\partial b^2} + \frac{\partial c^1}{\partial b^1}\frac{\partial c^2}{\partial b^2} \left(\frac{\partial h^1}{\partial c^1}\frac{\partial h^2}{\partial c^2} - \frac{\partial h^2}{\partial c^1}\frac{\partial h^1}{\partial c^2} \right) \right],
\end{aligned} \tag{63}
$$

where

$$
D_2 := \left[\frac{1}{p_0}(u_c^2 + 1) - \frac{\partial h^2}{\partial c^2}\frac{\partial c^2}{\partial b^2} \right] + \frac{\left(-1 + t_0 \partial c^2 / \partial b^2 \right)}{-1 + t_0 \partial c^1 / \partial b^1} \frac{\partial h^2}{\partial c^1}\frac{\partial c^1}{\partial b^1} \tag{64}
$$

Combining (59), (60) with (61), substituting for λ^0 and μ^0, and rearranging gives

$$t_0 = \frac{\Gamma^1\beta + \Gamma^2\gamma}{\beta + \gamma}, \tag{65}$$

where

$$\beta := \left[\frac{\partial \tilde{c}^{\,1}}{\partial p}\left(1 - \frac{1}{u_x^2}\frac{\partial h^2}{\partial c^2}\frac{\partial c^2}{\partial b^2}\right) + \frac{\partial \tilde{c}^{\,2}}{\partial p}\frac{1}{u_x^1}\frac{\partial h^1}{\partial c^2}\frac{\partial c^1}{\partial b^1}\right] < 0, \tag{66}$$

$$\gamma := \left[\frac{\partial \tilde{c}^{\,2}}{\partial p}\left(1 - \frac{1}{u_x^1}\frac{\partial h^1}{\partial c^1}\frac{\partial c^1}{\partial b^1}\right) + \frac{\partial \tilde{c}^{\,1}}{\partial p}\frac{1}{u_x^2}\frac{\partial h^2}{\partial c^1}\frac{\partial c^2}{\partial b^2}\right] < 0. \tag{67}$$

In (66) and (67), $\tilde{c}^{\,i}\,(p, g, \bar{v}^i)$ denotes individual i's Hicksian or compensated demand function for good c for a given utility level $\bar{v}^i$.[31] The assumption that U_*^i increases in c^i implies that the terms in parentheses in (66) and (67) are, respectively, positive. Since $\partial\tilde{c}^{\,i}/\partial p < 0$, we have $\beta < 0$ and $\gamma < 0$. As $\Gamma^i < 0$ for at least one income type, $t_0 > 0$. Defining

$$\alpha := \frac{\beta}{\beta + \gamma} > 0$$

(65) can be written as in (24), the optimal positional tax formula.

A.5 Proof of Proposition 3

Consider an optimal policy $P_0 \in \mathcal{S}$. If $\Gamma^1|_{C=C_0} = \Gamma^2|_{C=C_0}$ at P_0, (24) implies $t_0 = \Gamma^1|_{C=C_0} = \Gamma^2|_{C=C_0}$. By (53), individuals choose c and x such that $MRS(c_0^i, x_0^i) = 1 + t_0$. Substituting for t_0 gives $MRS(c_0^i, x_0^i) - \Gamma^i|_{C=C_0} = 1$ for $i = 1, 2$, which coincides with (8). As the utility function U_*^i is strongly quasi-concave, any allocation that satisfies (8) is Pareto-efficient. Conversely, if $\Gamma^1|_{C=C_0} \neq \Gamma^2|_{C=C_0}$ at P_0, we have $MRS(c^i, x^i) - \Gamma^i \neq 1$ for at least one type, and condition (8) does not hold. Hence, policy P_0 implements an efficient allocation if and only if $\Gamma^1|_{C=C_0} = \Gamma^2|_{C=C_0}$. ■

A.6 Proof of Proposition 4

Consider the policy $P_r = (b_r^1, b_r^2, g_r, t_r)$ with

$$b_r^1 = b_0^1 - g_r, \quad b_r^2 = b_0^2 - g_r, \quad g_r = x_0^1, \quad t_r = t_0. \tag{69}$$

[31]Formally, $\tilde{c}^{\,i}\,(p, g, \bar{v})$ are obtained from the expenditure minimization problem

$$\min_{c^i, x^i} \quad p \cdot c^i + x^i - g \quad \text{s.t.} \quad u^i(c^i, x^i) + c^i - r^i \geq \bar{v}^i, \tag{68}$$
$$x^i \geq g, \quad c^i \geq 0.$$

Given our assumptions on preferences, there exists a unique $\tilde{c}^{\,i}(p, g, \bar{v}^i)$ for every $(p, g, \bar{v}^i)$.

At P_r, we have $x_d(b_r^1 + g_r, p_r) = x_d(b_0^1, p_0) = x_0^1$ and $x_d(b_r^2 + g_r, p_r) = x_d(b_0^2, p_0) = x_0^2$. Since $g_r = x_0^1 < x_0^2$, by (54) and (55), it follows that types 1 and 2 respectively choose $c^1(b_r^1, g_r, p_r) = b_r/p_r = c_0^1$, $x^1(b_r^1, g_r, p_r) = g_r = x_0^1$, $c^2(b_r^2, g_r, p_r) = c_d(b_0^2, p_0) = c_0^2$ and $x^2(b_r^2, g_r, p_r) = x_d(b_0^2, p_0) = x_0^2$. Thus, consumption allocations and individuals' indirect utilities at P_0 and P_r coincide. Substitution of policy P_r into the government budget constraint (22) shows that policy P_r is feasible. Since individuals of type 1 [type 2] are [not] constrained by public provision at P_r, indirect utilities and the government budget at P_r are

$$\begin{aligned} V^1(b^1, b^2, g, p) =& u(b^1/p, g) + b^1/p - h^1(b^1/p, c_d(b^2 + g, p)), && (70)\\ V^2(b^2, b^1, g, p) =& u(c_d(b^2 + g, p), x_d(b^2 + g, p)) && (71)\\ & + c_d(b^2 + g, p) - h^2(c_d(b^2 + g, p), b^1/p), \\ G =& y^1 - b^1 + tb^1/p + y^2 - b^2 + tc_d(b^2 + g, p) - 2g. && (72) \end{aligned}$$

Now, consider a change from P_r to policy $P_p = (b_r^1 + db^1, b_r^2 + db^2, g_r + dg, p_0)$, where $dg > 0$ and $dg \to 0$. Net incomes are adjusted such that (i) the government budget G remains balanced and (ii) the indirect utility of type 2 does not change. The consumption tax is held constant at $t = t_0$. Using (71) and (72), requirements (i) and (ii) can be represented by

$$dV^2 = -\frac{\partial h^2}{\partial c^1}\frac{1}{p}db^1 + \left(u_x^2 - \frac{\partial h^2}{\partial c^2}\frac{\partial c_d^2}{\partial I}\right)db^2 + \left(u_x^2 - \frac{\partial h^2}{\partial c^2}\frac{\partial c_d^2}{\partial I}\right)dg = 0, \quad (73)$$

$$dG = -\frac{1}{p}db^1 + \left(-1 + t\frac{\partial c_d^2}{\partial I}\right)db^2 + \left[-1 + \left(-1 + t\frac{\partial c_d^2}{\partial I}\right)\right]dg = 0, \quad (74)$$

where we abbreviated $\partial c_d^i/\partial I := \partial c_d(b^i + g, p)/\partial I$ and used that $(u_c^2 + 1)/u_x^2 = p$ at policy P_r. Solving for db^1 and db^2 gives

$$db^1 = -\left[1 - \frac{1}{u_x^2}\frac{\partial h^2}{\partial c^2}\frac{\partial c_d^2}{\partial I}\right](1 + t)\frac{1}{D_3}dg \quad (75)$$

$$db^2 = -\left[1 - \frac{1}{u_x^2}\frac{\partial h^2}{\partial c^2}\frac{\partial c_d^2}{\partial I} + \frac{1}{u_x^2}\frac{\partial h^2}{\partial c^1} - \frac{1}{u_x^2}\frac{\partial h^2}{\partial c^1}\left(-1 + t\frac{\partial c_d^2}{\partial I}\right)\right]\frac{1}{D_3}dg, \quad (76)$$

with

$$D_3 := \left[\underbrace{1 - \frac{1}{u_x^2}\frac{\partial h^2}{\partial c^2}\frac{\partial c_d^2}{\partial I}}_{>0} \underbrace{- \frac{1}{u_x^2}\frac{\partial h^2}{\partial c^1}\left(-1 + t\frac{\partial c_d^2}{\partial I}\right)}_{>0}\right] > 0. \tag{77}$$

The first term in (77) is positive as U_*^i increases in c^i. The same holds for the second term since $(-1 + t\partial c_d^2/\partial I) < 0$.[32]

The policy change from P_r to P_p achieves a Pareto-improvement if the indirect utility of type 1 increases. Totally differentiating (70), using (70), (75), (76), the definitions of Γ^1 and Γ^2, the expression for the optimal tax (24), while taking into account that consumption allocations at P_0 and P_r coincide, we obtain

$$dV^1 = -u_x^1\left[\underbrace{\frac{\partial \tilde{c}^2}{\partial p}\frac{1}{\beta + \gamma}}_{>0}\left[\Gamma^1 - \Gamma^2\right]D_4\right]\frac{1}{D_3}dg, \tag{78}$$

where

$$D_4 := \left[-1 + \frac{1}{u_x^1}\frac{\partial h^1}{\partial c^1}\frac{\partial c_d^1}{\partial I} + \frac{1}{u_x^2}\frac{\partial h^2}{\partial c^2}\frac{\partial c_d^2}{\partial I}\left(1 - \frac{1}{u_x^1}\frac{\partial h^1}{\partial c^1}\frac{\partial c_d^1}{\partial I}\right) + \frac{1}{u_x^1}\frac{\partial h^1}{\partial c^2}\frac{\partial c_d^1}{\partial I}\frac{1}{u_x^2}\frac{\partial h^2}{\partial c^1}\frac{\partial c_d^2}{\partial I}\right] < 0$$

The sign of D_4 follows since, at the optimal policy P_0, the Lagrange multiplier associated with the government budget constraint, λ^0, must be larger than zero (see equation 63). Thus, $dV^1 > 0$ if $\Gamma^1 > \Gamma^2$ at P_0.

It remains to show that individuals of type 1 [type 2] are [not] crowded out by public provision if we change the policy from P_r to P_p. By the continuity of demand functions, this is satisfied for type 2. Individuals of type 1 are crowded out at P_p if the difference $\Omega^1(b^1, g, p) := g - x_d^1(b^1 + g, p)$ is greater or equal to zero after the change to policy P_p. Formally, at P_r, this requires

$$\frac{\partial \Omega^1(b^1, g, p)}{\partial b^1}db^1 + \frac{\partial \Omega^1(b^1, g, p)}{\partial g}dg = \underbrace{\left[1 - \frac{\partial x_d^1}{\partial I}\right]dg}_{>0} \underbrace{-\frac{\partial x_d^1}{\partial I}db^1}_{>0} > 0. \tag{79}$$

[32]From individuals' budget constraint, $c_d(I^2, p) = 1/p(I^2 - x_d(I^2, p))$. Differentiating with respect to I and rearranging gives

$$-1 + t\frac{\partial c_d^2}{\partial I} + \frac{\partial c_d^2}{\partial I} + \frac{\partial x_d^2}{\partial I} = 0.$$

As both goods are normal, we must have $(-1 + t\partial c_d^2/\partial I) < 0$.

The first term of the right-hand side of (79) is positive as both goods are normal. The sign of the second term follows since $db^1 < 0$ by (75). Hence, type 1 individuals remain crowded out at P_p. As similar arguments apply for all $P_0 \in \mathcal{S}$, this finishes the proof of Proposition 4. ■

A.7 Example 2

Assume that the sub-utility function u and reference functions h^i are respectively given by

$$\tilde{U}^i(c, x, r^i) = \frac{1}{1-\sigma}\left(c^{1-\sigma} + \delta x^{1-\sigma}\right) - r^i, \quad h^1(c^1, c^2) = c^2, \quad h^2(c^2, c^1) = 0.$$

Hence, $\Gamma^2 > \Gamma^1 = 0$. We choose parameters $y^1 = 10$, $y^2 = 15$, $\sigma = 0.8$ and $\delta = 0.4$. Setting $\bar{U}^2 = 10.7189$ to laissez-faire level enjoyed by the rich, the optimal tax in the absence of public provision is $t_0 = 1.04$, and the poor obtain indirect utility $V^1 = -0.35$. Now, consider a switch to a policy $P' = (b^1, b^2, g, 0)$, where $b^1 = y^1 - 1.1g$, $b^2 = y^2 - 0.9g$ and $g = 6.05$. Under this policy, both types are constrained by public provision and obtain utilities $V^1 = -0.32$ and $V^2 = 10.7193$. Hence, under policy P', the poor and the rich are better off compared to P_0.

A.8 Proof of Proposition 5

Consider a given Pareto-efficient allocation $C_* \in \mathcal{P}$.

Item (i): To prove the "if"-part, consider the policy $P_* = (b^1_*, b^2_*, g_*)$, with

$$b^1_* = c^1_*, \quad b^2_* = c^2_* + x^2_* - g_*, \quad g_* = x^1_*. \tag{80}$$

At policy P_*, individuals of type 1 choose the bundle (c^1_*, x^1_*) if they are (just) constrained by public provision and do not buy additional units of x on the market. This happens if and only if

$$MRS(c^1_*, x^1_*) \geq 1. \tag{81}$$

From condition (8) we know that $MRS(c^1_*, x^1_*) = 1 + \Gamma^1$. Hence, as $\Gamma^1 > 0$, condition (81) holds. Individuals of type 2 can afford their intended bundle: by (80), their disposable income is effectively $I^2_* = b^2_* + g_*$ which is sufficient to buy (c^2_*, x^2_*). Moreover, since we consider only $C_* \in \mathcal{P}$ where $x^1_* < x^2_*$, by demand functions (12) and (13), we have $g < x_d(I^2_*) = x^2_*$ such that individuals of type 2 are not constrained at P_*. Hence, (c^2_*, x^2_*)

is the optimal choice: utility-maximization requires $MRS(c^2, x^2) = 1$. By (8) and the strict convexity of preferences, this condition can only hold at (c_*^2, x_*^2).
Substituting P_* into the government budget constraint (10) gives

$$c_*^1 + x_*^1 + c_*^2 + x_*^2 - y^1 - y^2 = 0, \tag{82}$$

which must hold for any C_* and therefore proves feasibility.
To prove the "only if" part, note that if $\Gamma^2 > 0$ policy P_* can never implement C_*: as individuals of type 2 choose goods c and x according to $MRS(c^2, x^2) = 1$ and $MRS(c_*^2, x_*^2) = 1 + \Gamma^2$ by (8), they would always choose a consumption bundle different from (c_*^2, x_*^2) when income is b_*^2.
Furthermore, there exist no other policies that might lead to C_*. Implementation with policies where $g > x_*^1$ is impossible since, by monotonicity, individuals never forego the publicly provided amount g. Hence, at least one income type would not consume the efficient level of good x. We can also rule out policies where $g < x_*^1$: if one or both types are constrained, Pareto-efficiency cannot be attained when $x_*^1 < x_*^2$; if both individuals are not crowded out, they would top up to a bundle that is not Pareto-efficient, since both choose goods c and x such that $MRS\left(c^i, x^i\right) = 1$.

Item (ii): Consider the policy $P_* = (b_*^1, b_*^2, g_*, t_*)$ with

$$b_*^1 = (1 + t_*) c_*^1, \quad b_*^2 = (1 + t_*)\, c_*^2 + x_*^2 - g_*, \quad g_* = x_*^1, \quad t_* = \Gamma^2\big|_{C = C_*}. \tag{83}$$

We first show that individuals choose C_* under P_* if and only if $\Gamma^1 > \Gamma^2$. We then verify that P_* is feasible if individuals were to choose C_* under this policy. In a last step, we prove that P_* is the only policy that can achieve C_*.
To see that C_* is consistent with utility maximization, consider first the decision problem of type 1 individuals. The bundle (c_*^1, x_*^1) is affordable since $g = x_*^1$ and the net income b_*^1 is designed such that c_*^1 can be just attained if $x^1 = g$. It is optimal for type 1 to choose (c_*^1, x_*^1) if

$$MRS(c_*^1, x_*^1) \geq p = 1 + t_*. \tag{84}$$

Substituting for t_* in (84) and rearranging gives $MRS(c_*^1, x_*^1) - \Gamma^2 \geq 1$. From condition (8) we know that $MRS(c_*^1, x_*^1) = 1 + \Gamma^1$. Thus, (c_*^1, x_*^1) is optimal for individuals of type 1 if and only if $\Gamma^1 \geq \Gamma^2$.
The rich can also afford their intended bundle: by (83), their disposable income is effec-

tively $I^2_* = b^2_* + g_*$ which is sufficient to buy (c^2_*, x^2_*). Moreover, since we consider only $C_* \in \mathcal{P}$ where $x^1_* < x^2_*$, we have $g < x^2_*$, and the rich are not constrained at P_*. Hence, (c^2_*, x^2_*) is the optimal choice: utility-maximization requires $MRS(c^2, x^2) = p = 1 + t_*$. As $t_* = \Gamma^2$, by (8) this condition can only hold at (c^2_*, x^2_*).
Inserting P_* into the government budget constraint (22) gives

$$c^1_* + x^1_* + c^2_* + x^2_* - y^1 - y^2 = 0, \tag{85}$$

which must hold for any C_* and therefore proves feasibility.
P^* is the only policy that may support C_*. Implementation with policies where $g > x^1_*$ is impossible since, by monotonicity, individuals never forego the publicly provided amount g. Hence, at least one income type would not consume the efficient level of good x. We can also rule out policies where $g < x^1_*$: if one or both types are constraint, Pareto-efficiency cannot be attained when $x^1_* < x^2_*$; if both individuals are not crowded out, they would top-up to a bundle that is not Pareto-efficient, since in the case where $\Gamma^1 \neq \Gamma^2$, the uniform tax rate can never be set such that private optimization yield $MRS + \Gamma^i = 1$ for $i = 1, 2$ for any given net incomes. ■

A.9 Proof of Proposition 6

Consider a switch from the laissez-faire to the following policy P_r:

$$e_r = c^1_{LF}; \quad b^1_r = y^1 - e_r; \quad b^2_r = y^2. \tag{86}$$

This policy induces individuals to choose the same consumption bundles as in the laissez-faire. Since $\hat{e}(b^1_r, p) < c_d(b^1_r, p) < c_d(y^1, p) = c^1_{LF}$, type 1 individuals choose public provision at P_r. Hence, $c^1(b^1_r, p, e_r) = e_r = c^1_{LF}$ and $x^1(b^1_r, p, e_r) = b^1_r = x^1_{LF}$. Assume that $c^1_{LF} < \hat{e}^2(y^2, p)$. Then, individuals of type 2 opt out and, as $b^2_r = y^2$, select $c^2(b^2_r, p, e_r) = c^2_{LF}$ and $x^2(b^2_r, p, e_r) = x^2_{LF}$, respectively.
Now, marginally decrease e at P_r and finance it by $de = -db^1$. By the continuity of utility functions, type 2 stays out of the public system after this policy change. The effect on both types' indirect utilities is

$$d\tilde{V}^1_{in} = -\frac{\partial h^1}{\partial c^1} de, \tag{87}$$

$$d\tilde{V}^2_{out} = -\frac{\partial h^2}{\partial c^1} de. \tag{88}$$

Hence, decreasing e benefits at least one type if $\partial h^1/\partial c^1 > 0$ or $\partial h^2/\partial c^1 > 0$.