The Bargaining Trap

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Abstract

I revisit the Rubinstein (1982) model for the classic problem of price haggling and show that bargaining can become a “trap,” where equilibrium leaves one party strictly worse off than if no transaction took place (e.g., the equilibrium price exceeds a buyer’s valuation). This arises when one party is impatient about capturing zero surplus (e.g., Rubinstein’s example of fixed bargaining costs). Augmenting the protocol with unilateral exit options for responding bargainers generally removes the trap.

Keywords: alternating offers, bargaining, time preferences, haggling costs, outside options

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1 Introduction

The seminal bargaining protocol proposed by Rubinstein (1982) has two parties alternate in offering shares \( x \in [0, 1] \) of the surplus until an offer is accepted. Considering the classic version of the problem of price haggling by a seller \( S \) and a buyer \( B \) with quasi-linear utility and monetary valuations \( v_i \geq 0 \) for the good to be traded, such that \( v_B > v_S \), the restriction to surplus shares translates into the restriction that price offers \( p \) are from the range \([v_S, v_B]\). Contrary to the intuition that this is without loss of generality, I show that there are preferences covered by Rubinstein’s assumptions for which equilibrium implies a price agreement outside of this surplus division range. Hence, for one of the two parties, bargaining constitutes a trap: Its outcome is not individually rational, and this party would be better off avoiding the transaction altogether.

The bargaining trap occurs for preferences such that some party is willing to pay to avoid delaying even a “zero-surplus agreement” (e.g., for the buyer this means price \( p = v_B \)), and it is then a consequence of the protocol’s assumption that the only way to get out of the bargaining is by reaching agreement.\(^1\) As I demonstrate, this potentially results in one party’s having extreme bargaining power (or rather “superpower”) over the other. Bargaining is then not just about the surplus but also about ending it. In this sense, restricting offers to surplus shares removes a key strategic element.

I illustrate the bargaining trap specifically for Rubinstein’s example of fixed bargaining costs, where my only modification to his model is to extend the range of price offers to \([v_S - l, v_B + l]\), for some \( l \geq 0 \) (for \( l = 0 \), the model is a special case of Rubinstein, 1982). I obtain the extreme result that if the seller has an arbitrarily small advantage in terms of bargaining costs and makes the first offer, then the unique equilibrium has immediate (hence efficient) agreement on the maximal possible price \( p = v_B + l \), for \( l \). As this maximal price becomes large (\( l \to \infty \)), the seller’s (resp., buyer’s) payoff approaches plus (resp., minus) infinity, regardless of who gets to make the initial offer.\(^2\) While the bargaining outcome is efficient, the buyer’s greater bargaining costs turn her into a money pump for the seller because she cannot get out of the bargaining without the latter’s agreement. This implies that no matter how large the surplus to be shared, the buyer would rather avoid the transaction altogether than to

\(^1\)Relatedly, Shaked (1994, p. 421) describes this assumption as follows: “(...) the two agents are doomed to continue bargaining forever unless they reach an agreement.”

\(^2\)Interestingly, in experimental implementations, bargaining with such fixed costs has produced results much better in line with equilibrium predictions than bargaining with discounting; see Rapoport, Weg, and Felsenthal (1990), and Weg and Zwick (1991), as well as the survey by Roth (1995).
bargain with the seller. Taking this selection into account, the bargaining mechanism becomes a source of strong inefficiency, despite perfect information. Indeed, when both parties have identical costs, the (stationary) equilibrium multiplicity shown in Rubinstein (1982, Conclusion I, p. 107) extends with the offer range, thus permitting an even starker bargaining trap: Non-stationary equilibria exist in which the agreement’s delay costs more than the entire surplus, so that both parties are strictly worse off than by not transacting at all.

I then clarify that fixed bargaining costs preferences are in fact merely an alternative representation of exponential discounting preferences (see also Fishburn and Rubinstein, 1982). The specific illustration of the bargaining trap is therefore not at all about how delay as such is discounted. Rather, as indicated above, it relies on (at least) one party’s strict preference to reach agreement on a “zero-surplus agreement” immediately over doing so with delay. In terms of exponentially discounted utility from agreed shares $\delta^t \cdot u(x)$, this means $u(0) > 0$, and I show how such preferences capture haggling costs, beyond “pure” time preferences.

Finally, I show how to get rid of the bargaining trap and ensure that the restriction of offers to surplus shares is without loss for determining the set of equilibrium outcomes. This obtains by augmenting the protocol to allow any party responding to an offer to also irreversibly exit and thereby end the bargaining, i.e., by adding a unilateral exit option. One way of interpreting the bargaining trap and its “fix” is therefore as pointing out an omission in Rubinstein (1982). However, and potentially more importantly, the results of this short paper shall open the door to further theoretical developments in bargaining theory with haggling costs, which have long been considered a key element to the theory of the firm (see Hart, 2008).

\footnote{Both delay-cost formulations are considered in the literature on search and matching; e.g., see Chade (2001) or Atakan (2006) for fixed costs, and Burdett and Coles (1997), Shimer and Smith (2000), or Atakan and Ekmecki (2014) for discounting. The equivalence pointed out here relies on the absence of randomization, however.}

\footnote{A recent experimental study by Gago (2019) establishes the existence of significant psychological haggling costs, and his finding that women perceive greater such costs than men is well in line with the literature relating gender inequality to wage bargaining (see, e.g., the book by Babcock and Laschever, 2003).}
2 The Bargaining Trap, and How to Get Rid of It

2.1 Basic Model

Seller $S$ and buyer $B$ bargain over the price $p$ at which $S$ sells an indivisible good to $B$. The good is worth $v_i$ to individual $i \in \{S, B\}$, where $v_B - v_S \equiv K > 0$, so there are social gains from trade. Bargaining takes place according to the standard alternating-offers protocol: In any of possibly infinitely many discrete bargaining periods, one party $i$ is the proposer and offers a price $p$, and the other party $j \neq i$ is the respondent, who then either accepts the offer (the game ends with this price agreement) or rejects it; in this case, bargaining continues to the next period, where $j$ becomes the proposer and $i$ becomes the respondent, and so on, until agreement. (We will consider both cases for who gets to make the initial offer.)

Preferences over agreements $(p, t)$ on a price $p$ and with a delay of $t = 0, 1, 2, \ldots$ bargaining periods are represented by the utility functions

$$U_S(p, t) = (p - v_S) - c_S \cdot t$$

and

$$U_B(p, t) = (v_B - p) - c_B \cdot t;$$

i.e., the two individuals perceive a fixed cost $c_i > 0$ per bargaining period (more precisely, per disagreement period), and we will focus on the case where $c_S < c_B < K$ (inviting the reader to think of them as small). Note that for preferences to be well-defined on the space of all possible bargaining outcomes, including perpetual disagreement as the least preferred outcome, we need to extend the utility range by $\{-\infty\}$.

If we were to restrict price offers $p$ to $[v_S, v_B]$, we would obtain a special case of Rubinstein (1982). However, since $p$ is simply a monetary transfer, we relax this (and only this) restriction here and allow for any $p \in [v_S - l, v_B + l]$, where $l \geq 0$ is a parameter of the game. We consider pure subgame-perfect equilibrium—in what follows, simply “equilibrium”—and we call any subgame starting with individual $i$’s offer, all of which are formally identical due to the stationarity of both the protocol and preferences, “$i$-game.”

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5Divide $U_i$ by $K$ to obtain $x_i(p) - (c_i/K) \cdot t$, which maintains fixed bargaining costs, but where $x_i : [v_S, v_B] \to [0, 1]$ translates any price offer into $i$’s surplus share; $x_S$ is linearly increasing in $p$, and $x_S(p) + x_B(p) = 1$ for any $p \in [v_S, v_B]$. 
2.2 The Bargaining Trap

**Proposition 1.** For any value \( l \geq 0 \), equilibrium of both the \( S \)- and the \( B \)-game is unique, with the following outcome: In the \( S \)-game, \( S \) offers the maximal possible price \( v_B + l = p_S(l) \), and \( B \) accepts; in the \( B \)-game, \( B \) offers the price \( p_S(l) - c_S = p_B(l) \) and \( S \) accepts. As \( l \to \infty \), in both the \( S \)- and the \( B \)-game the equilibrium payoffs \((U^*_S, U^*_B)\) approach \((+\infty, -\infty)\).

**Proof.** Consider the following strategy profile (defined for both games): \( S \) always offers \( p_S(l) \) and accepts an offer \( p \) if and only if \( p \geq p_B(l) \); \( B \) always offers \( p_B(l) \) and accepts an offer \( p \) if and only if \( p \leq p_S(l) \), i.e., \( B \) accepts any possible offer. Clearly, \( S \) has no profitable deviation. Note then that \( B \)'s rejection as respondent results in payoff
\[
v_B - p_B(l) - c_B = -l + c_S - c_B < -l,
\]
and since \(-l \leq v_B - p\) for any \( p \in [v_S - l, v_B + l] \), she will indeed accept any offer; given that, \( B \)'s offer of \( p_B(l) \) is optimal. The associated equilibrium payoffs \((U^*_S, U^*_B)\) are \((K + l, -l)\) in the \( S \)-game and \((K + l - c_S, -l + c_S)\) in the \( B \)-game, and both converge to \((+\infty, -\infty)\) as \( l \to \infty \).

It only remains to show that the above is the unique equilibrium. First, note that perpetual disagreement cannot be an equilibrium: Taking the \( i \)-game, the maximal possible rejection payoff of respondent \( j \neq i \) is bounded, so there are prices that she will always accept (irrespective of whatever complicated continuation outcome may arise) and that yield proposer \( i \) a payoff that is bounded from below and hence greater than that of \(-\infty\) under perpetual disagreement. (For instance, if \( j = S \) then her maximal possible rejection payoff equals \( p_S(l) - v_S - c_S \), so she will always accept offers \( p > p_B(l) \), implying that \( i = B \)'s payoff cannot be less than \( v_B - p_B(l) = -l + c_S \).)

Second, define \( A_i \) as the set of pairs \((p, t)\) that are equilibrium (agreement) outcomes of the \( i \)-game, and let \( U_i = \sup \{ U_i(p, t) : (p, t) \in A_i \} \) and \( L_i = \inf \{ U_i(p, t) : (p, t) \in A_i \} \). Given the above equilibrium, the latter are well-defined, because \( A_i \neq \emptyset \); moreover, \( U_S = K + l \) and \( U_B = -l + c_S \). Now define \( q \) and \( r \) such that \( U_B \equiv v_B - q \) and \( U_S \equiv r - v_S \), respectively, so respondent \( B \) accepts any price \( p < q + c_B \), in any equilibrium, and respondent \( S \) rejects any price \( p < r - c_S \), in any equilibrium. Accounting for \( v_S - l \leq p \leq v_B + l \),
\[
r \geq \min \{q + c_B, v_B + l\}
\]
and
\[
q \geq \max \{r - c_S, v_S - l\}.
\]

\[\text{\textsuperscript{6}}\text{For the second inequality, additionally note that the buyer's supremum value over delayed equilibrium agreements cannot exceed that over immediate equilibrium agreements.}\]
If \( q + c_B \leq v_B + l \), then \( r \geq q + c_B \geq \max \{ r - c_S + c_B, v_S - l + c_B \} \), which implies that \( r \geq r - c_S + c_B \). Since \( c_B > c_S \), this is impossible. Hence, \( q + c_B > v_B + l \) must hold true. This, however, implies that \( U_S = U_S = K + l \), from which uniqueness of the above equilibrium follows.

As soon as the buyer’s bargaining costs are greater than the seller’s, even by the tiniest amount, the buyer will be exploited to the maximal possible degree by the seller. Bargaining, though efficient (see Coase, 1960), becomes a trap for the buyer. Indeed, upon anticipating this outcome, the buyer will choose to avoid the transaction to begin with, regardless of how much surplus is thereby forgone, which is strongly inefficient.\(^7\)

Intuitively, for every bargaining period that \( i \) proposes she gets to extract \( j \)'s cost of rejecting \( c_j \), and given \( c_S < c_B \), \( S \) ends up extracting \( c_B - c_S \) for every two periods, which becomes arbitrarily large with an infinite horizon. The problem the buyer faces is that the only way for her to get out of the bargaining is to accept the seller’s offer, or make an offer the seller will accept. The seller with her lower bargaining costs will strategically exploit this dependence and make the buyer’s way out maximally costly. Though extreme, it captures why some people steer clear of the bazaar.

### 2.2.1 A Collective Bargaining Trap

The case of \( c_B < c_S < K \) adds nothing new, of course: Equilibrium is then simply the mirror image of the case dealt with above, so that \( B \) gets to maximally exploit \( S \). However, the case of equal costs \( c_S = c_B \equiv c < K \) adds another stark result about the bargaining trap, namely that there exists an (non-stationary and non-unique) equilibrium in which both parties are worse off than by not transacting at all!

To begin with, and hardly surprisingly, the well-known multiplicity of stationary equilibria from Rubinstein (1982, Conclusion I), all of which are efficient, extends to the larger price range: Taking any price \( p_S \in [v_S - l + c, v_B + l] \), the strategy profile such that \( S \) always offers \( p_S \), \( B \) accepts an offer \( p \) if and only if \( p \leq p_S \), \( B \) always offers \( p_B = p_S - c \) (note that \( p_B \in [v_S - l, v_B + l - c] \)), and \( S \) accepts an offer \( p \) if and only if \( p \geq p_B \) constitutes a stationary equilibrium. (The set of such equilibria is obtained by varying \( p_S \) in the range.) Based on this multiplicity, we also obtain non-stationary equilibria in which agreement is inefficiently delayed. Allowing for the

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\(^7\)While reminiscent of the hold-up problem—see, e.g., Che and Sákovics (2008)—here the investment is to engage in bargaining itself, and the seller would benefit from committing to a “fair” price. Relatedly, see Anderlini and Felli (2006).
larger price range, delay is supported to an extent that not only destroys the entire surplus but, on top of that, makes both individuals pay for the bargaining. The key incentive constraint—following the parsimonious construction of Schweighofer-Kodritsch (2018a) with extreme threats—is that, when proposing earliest on the path of such an equilibrium, following it (by making a rejected offer) must yield $i$ at least

$$U_i \equiv -l + c$$

for both individuals. We can then show that the maximal delay (for which a price exists so that both incentive constraints can be satisfied) equals the largest integer weakly less than $t^* (l) \equiv \frac{1}{c} \cdot \left( l + \frac{1}{2} \cdot (K - c) \right)$, and this approaches $+\infty$ as $l$ does so (cf. Schweighofer-Kodritsch, 2018b). Since the minimal delay required for total delay costs to (weakly) exceed the entire surplus is the smallest integer weakly above $\hat{t} \equiv \frac{1}{c} \cdot \frac{1}{2} \cdot K$, this stark possibility of “haggling away” all surplus in equilibrium arises whenever $t^* (l) \geq \hat{t} \iff l \geq 2c$. As $c$ becomes small, this is satisfied for any $l > 0$.

### 2.2.2 Time Preferences vs. Haggling Costs

What is the key characteristic of preferences that gives rise to the bargaining trap? To begin with, it is worthwhile pointing out that the preferences studied here actually have an exponential-discounting representation, namely

$$U_S (p, t) = \delta_S^t \cdot \exp (p - v_S) \quad \text{and} \quad U_B (p, t) = \delta_B^t \cdot \exp (v_B - p),$$

where $\delta_i \equiv \exp (-c_i) \in (0, 1)$. In fact, this is true for any preferences of the form $u (m) - c \cdot t$, since any positive monotonic transformation preserves ordinal preferences (in particular, taking the exponential, or the natural logarithm as its inverse). Hence, we have here no departure from the benchmark model of “rational” delay discounting.

Instead, the key characteristic of preferences is their departure from “pure” time preferences: Our bargainers are not indifferent as to the timing of getting nothing, whereby merely waiting for a monetary reward is different from bargaining to get it. A strict preference for getting nothing sooner rather than later, together with the dependence on the other’s agreement in order even to get nothing is what gives rise to the bargaining trap. With preferences of the form $\delta^t \cdot u (m)$, this means that $u (0) > 0$. A decision-maker with $u (0) = 0$ would be perfectly patient about getting nothing of the surplus; given an opponent who is impatient about getting something, she must at least

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8In order to be well-defined, the logarithmic transformation that yields the bargaining-costs representation requires even more, namely that $\inf_{m \in M} u (m) \geq 0$ for $M$ the possible range of monetary rewards, which may include very low (negative) amounts.

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get that. With \( u(0) > 0 \), however, she is impatient about ending the bargaining for its own sake, i.e., she experiences haggling costs. Letting \( \epsilon \equiv u(0) \) and \( \tilde{u}(m) \equiv u(m) - u(0) \), we can write \( \delta^t \cdot u(m) \equiv \delta^t \cdot (\tilde{u}(m) + \epsilon) \), with \( \tilde{u}(0) = 0 \). Hence, we can interpret these preferences as consisting of a standard “pure” time preferences component \( \delta^t \cdot \tilde{u}(m) \) and an additional psychological component \( \delta^t \cdot \epsilon \). This latter component depends solely on how long it takes to reach agreement, it is positive and declines exponentially with delay. What appears as fixed bargaining costs under the logarithmic transform, is here a delay discounted fixed benefit, a “joy” of agreement.

We conclude this discussion with a simple alternative example of the bargaining trap, based on the above separation. It shows that the bargaining trap is bound to arise for \( \epsilon > 0 \) against a sufficiently patient opponent. Suppose that \( U_S(p,t) = \delta^t_S \cdot (p - v_S) \) and \( U_B(p,t) = \delta^t_B \cdot ((v_B - p) + \epsilon) \), for some \( \epsilon > 0 \), to be thought of as small. These are preferences as in the basic textbook version of the Rubinstein (1982) model, where instantaneous utility is linear in money and discounted exponentially (e.g., Muthoo, 1999), except that here the buyer additionally experiences a (small) joy of agreement \( \epsilon \). This impatience about agreeing in addition to impatience about the material agreement introduces bargaining costs. Standard arguments deliver the unique (and stationary) equilibrium, which, for \( l > 0 \) sufficiently large, has the seller initially offer the price

\[
p_S(\epsilon) = \frac{(1 - \delta_B) \cdot v_B + \delta_B \cdot (1 - \delta_S) \cdot v_S}{1 - \delta_B \delta_S} + \frac{(1 - \delta_B) \cdot \epsilon}{1 - \delta_B \delta_S},
\]

and the buyer accept this offer, as the largest acceptable price. The first term on the right-hand side is the usual equilibrium agreement \( p_S(0) \): In terms of shares \( x_i \) of the material surplus \( K \) for each individual, it equals the division such that \( x_S = 1 - x_B = \frac{1 - \delta_B}{1 - \delta_B \delta_S} \). The second term that adds to the seller’s price is due to the buyer’s additional cost of disagreement. Overall, the price \( p_S(\epsilon) \) strictly exceeds \( v_B \) if and only if \( \frac{\epsilon}{K} > \frac{\delta_B(1 - \delta_S)}{1 - \delta_B} \). Hence, however close \( \epsilon > 0 \) and \( \delta_B < 1 \) are to zero and one, respectively, as \( \delta_S \to 1 \), the buyer finds herself in the bargaining trap, paying dearly for her joy of agreement.\(^9\) In this sense, the basic model’s predictions are not robust to introducing small costs of bargaining/disagreement.

\(^9\)Formally, this example is identical to having the buyer value the good at \( v_B + \epsilon \) and the surplus therefore equal to \( K + \epsilon \). Since \( p_S(\epsilon) \leq v_B + \epsilon \Leftrightarrow K + \epsilon \geq 0 \) holds true for any \( (\delta_S, \delta_B) \in (0,1)^2 \), there is no bargaining trap under this alternative interpretation. However, this is not a matter of mere interpretation: Eliciting the buyer’s willingness to pay for the good via a “market” mechanism, such as the widely applied Becker, DeGroot, and Marschak (1964) mechanism, would yield \( v_B \) rather than \( v_B + \epsilon \).
2.3 How to Get Rid of the Bargaining Trap

In view of the intuition given to Proposition 1, it seems clear that exit options during the bargaining would remove the bargaining trap and restore that it will be solely about the surplus. Especially if one considers the bargaining trap a mere theoretical curiosity pointing towards an omission from the Rubinstein (1982) model, adding such an exit option should result in the set of equilibrium outcomes being identical to that when exogenously restricting offers to surplus shares. However, as shown by Ponsati and Sákovics (1998), for the basic version of Rubinstein’s model, equilibrium outcomes are rather sensitive to the way exit options are being introduced (see also Shaked, 1994, regarding general outside options, and the related work by Avery and Zemsky, 1994).

As the final result, I therefore propose a concrete such “fix” to the protocol. For simplicity, I formulate and prove it for the model with fixed bargaining costs studied above, though it shall become clear that it “works” in generality. The concrete fix is to augment the protocol with unilateral exit options of irreversibly quitting and thereby ending the bargaining with no further transaction for both parties (S retains the good, and no monetary transfers are made), as an action available to a respondent, in addition to accepting or rejecting. Formally, if individual $i$ takes her exit option in period $t + 1$ after $t$ disagreement periods, then her utility equals $0 - c_i \cdot t$, and that of $j \neq i$ equals $0 - c_j \cdot t$. At the time of exiting, the cost of previous disagreement is sunk, and taking the exit option is equivalent to not transacting at all. I term this an exit rather than an outside option, because it has this specific nature, whereas outside options may refer to various opportunities outside the relationship.

**Proposition 2.** Suppose that both individuals $i \in \{S, B\}$, whenever responding to an offer, may also irreversibly exit the bargaining and thereby end it without any further transaction. Then, for any value $l \geq 0$, equilibrium of both the S- and the B-game is unique, with the following outcome: In the S-game, S offers the price $p_S \equiv v_B$, and B accepts; in the B-game, B offers the price $p_B \equiv p_S - c_S$, and S accepts.

**Proof.** Consider the following strategy profile, defined for both games: S always offers $p_S$ and responds to an offer $p$ with acceptance if $p \geq p_B$ and rejection if $p < p_B$; B always offers $p_B$ and responds to an offer $p$ with acceptance if $p \leq p_S$ and exits if $p > p_S$. Clearly, S has no profitable deviation. Note then that B’s rejection as respondent results in payoff $v_B - p_B - c_B = c_S - c_B < 0$ and is therefore less than the zero payoff from exit; since $v_B - p \geq 0$ for any $p \leq p_S$, she will indeed accept any such offer; given that, B’s offer of $p_B$ is optimal.
To prove that this is the unique equilibrium, observe that the availability of exit options implies $U_i \leq K$, which in turn implies $U_i \geq c_j$ (recall here that $c_j < K$ for either $j$). The above equilibrium then yields that $U_S = K$ and $U_B = c_S$. Following the line of proof of Proposition 1, we now obtain

$$r \geq \min \{q + c_B, v_B\} \quad \text{and} \quad q \geq \max \{r - c_S, v_S\}$$

instead of the inequalities in (1). A similar argument then yields uniqueness of the above equilibrium.

Introducing this exit option, the unique equilibrium is essentially the same regardless of the value of $l$, and its outcome is that $S$ captures the entire surplus $K$ as proposer, but no more. (Accordingly, $B$ captures only $S$’s cost of disagreement $c_S < K$ as proposer.) The bargaining trap is removed, and the restriction of bargaining offers to surplus divisions becomes without loss, so that the same equilibrium outcome as in Rubinstein (1982) obtains.

Of course, it would also be sufficient to only add an exit option for the buyer, and only the first time that the seller makes her an offer. It should be clear, however, that the proposed “fix” is general, beyond the basic model for which is formulated. Not only does it maintain symmetry and stationarity of the protocol, but the fact that every equilibrium outcome is now guaranteed to be individually rational together with the outside option principle (Binmore, Shaked, and Sutton, 1989) imply that, given very general preferences, the sets of equilibrium outcomes under the bargaining protocol where (i) offers are restricted to surplus shares and there are no exit options, and under the alternative one where (ii) offers are unrestricted and there are exit options as above, will coincide.
References


