Knowing Me, Imagining You:

Projection and Overbidding in Auctions

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Discussion Paper No. 36

June 6, 2017

Collaborative Research Center Transregio 190 | www.rationality-and-competition.de
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Knowing me, imagining you: Projection and overbidding in auctions

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Abstract

Overbidding in auctions has been attributed to e.g. risk aversion, loser regret, level-k, and cursedness, relying on varying identifying assumptions. I argue that "type projection" organizes these findings and largely captures observed behavior. Type projection formally models that people tend to believe others have object values similar to their own—a robust psychological phenomenon that naturally applies to auctions. First, I show that type projection generates the main behavioral phenomena observed in auctions, including increased sense of competition ("loser regret") and broken Bayesian updating ("cursedness"). Second, re-analyzing data from seven experiments, I show that type projection explains the stylized facts of behavior across private and common value auctions. Third, in a structural analysis relaxing the identifying assumptions made in earlier studies, type projection consistently captures behavior best, in-sample and out-of-sample. The results reconcile bidding patterns across conditions and have implications for behavioral and empirical analyses as well as policy.

JEL–Codes: C72, C91, D44
Keywords: auctions, overbidding, projection, risk aversion, cursed equilibrium, depth of reasoning

*I thank Friedel Bolle, Jana Friedrichsen, Paul Heidhues, Matthias Lang, Theo Offerman, Sebastian Schweighofer-Kodritsch, Jean-Robert Tyran, Felix Weinhardt, Georg Weizsäcker, seminar audiences in Berlin, Nottingham and Regensburg, and conference participants in Montreal (ESWC 2015) and Münster (VfS Jahrestagung, 2015) for helpful comments. Financial support of the DFG (BR 4648/1 and CRC TRR 190) is greatly appreciated. Address: Spandauer Str. 1, 10099 Berlin, Germany, email: yves.breitmoser@hu-berlin.de, phone: +49 30 2093 99408.

1 Introduction

The false consensus bias is the tendency to assume that one's own opinions, preferences and values are typical and shared by others. Following Ross et al. (1977), such "projection" has been confirmed in many experiments (Mullen et al., 1985) and shown to persist even if subjects are provided with factually contradicting information (Krueger and Clement, 1994). Thus, projection is of intuitive relevance in all choices under incomplete information—not just in the non-strategic environments on which the psychological literature traditionally focuses, but also in strategic interactions. Existing studies of projection in "games" focus on games with one-sided incomplete information. Loewenstein et al. (2003) study projection of utility onto future selves, finding that it explains anomalies in purchases of durable goods, and Madarász (2012) studies projection of information from an informed player to an uninformed one, which explains the hindsight bias in agency problems. The present paper provides a comprehensive analysis of projection in a class of games with two-sided incomplete information, auctions.

Auctions are widely analyzed games with two-sided incomplete information about individual object values. I introduce a model of type projection where players may overestimate the probability that their opponents share their type—i.e. their signal about the object value—ranging from zero projection (the original Bayesian case) to full projection (disregarding all prior information).[1] The degree of projection is denoted by $\rho \in [0,1]$. In equilibrium, players anticipate their opponent types' actual strategies, but overestimating the probability that opponents share their type, they perceive competition to be fiercer than it is and they wrongly update their estimate of the object value conditional on winning. This generates the behavioral phenomena observed in bidding across conditions, and based on my theoretical and econometric analysis, I argue that type projection, as predicted by a host of psychological evidence, captures bidding fairly comprehensively and substantially better than existing models.

The basic idea is simple. Type-projecting bidders project their signals about the object value. This builds on psychological evidence showing that object values indeed are projected, e.g. in bargaining (Bottom and Paese, 1999; Galinsky and Mussweiler, 2001) and in consumption decisions (Frederick, 2012; Kurt and Inman, 2013). As for auctions, consider bidding to buy a house. Projecting bidders neglect competitors whose values are vastly inferior, against whom they surely win, and competitors whose values are vastly superior, against whom they surely lose. They focus on competitors with similar values, trying to ensure winning against them. This focus increases the sense of competition and obscures the perceived value distribution. The former induces overbidding in any first-price auction, essentially to avoid "loser regret", and the latter weakens Bayesian updating in any common value auction (the Winner's Curse).

That is, the robust psychological finding of (type) projection already implies the main behavioral phenomena in auctions, and in addition, it correctly predicts a number of more subtle findings that are incompatible with existing models. For example, in private value auctions, projecting bidders overbid as they overestimate the share of opponents with similar values. They outbid them to increase the probability of winning. In contrast, risk

[1] Full projection is regularly considered in analyses of social preferences. The present paper considers the more intricate case of imperfect projection. Allowing for imperfect projection is critical, as full projection is neither observed in psychology nor fits bidding in auctions.

aversion emphasizes a trade-off between increasing winning probability and increasing conditional profit. Following Engelbrecht-Wiggans (1989), the former relates to loser regret (regret of losers if they could have won profitably) and the latter relates to winner regret (regret of winners if they could have won with lower bids). Filiz-Ozbay and Ozbay (2007) find that subjects do not trade off these regrets but focus on loser regret. This focus contradicts risk aversion and is implied by type projection. At the individual level, I find that subjects randomize consistently and use left-skewed mixed strategies, which also contradicts risk aversion and is predicted by type projection equilibrium.

Analyzing common value auctions, I similarly find that subjects randomize consistently and that they overbid more with common values than with private values. Again, both observations are implied by type projection and not implied by existing models such as risk aversion or cursed equilibrium (Eyster and Rabin, 2005).[2] This range of observations uniquely predicted by type projection, and considering that projection is a robust phenomenon known to affect behavior under incomplete information, raises the question to which degree projection can be considered a robust, potentially comprehensive explanation of bidding in auctions.[3] To answer this question, I conduct a structural analysis of data from seven experiments. The data set forms the union of the data sets analyzed in seminal structural analyses of bidding, which limits data selection effects in favor of type projection. In addition, merging multiple data sets allows me to assess whether models are precise (in-sample) and reliable (out-of-sample).

Both features are desirable in behavioral and empirical analysis, but reliability will be of particular relevance here. To clarify, let me briefly review existing results. Goeree et al. (2002b) and Bajari and Hortacsu (2005) show that risk aversion captures bidding in private value auctions, Filiz-Ozbay and Ozbay (2007) and Engelbrecht-Wiggans and Katok (2007) observe loser regret, Eyster and Rabin (2005) observe cursedness in common value auctions, and Crawford and Iriberri (2007) observe limited depth of reasoning in either condition. That is, the results vary enormously between studies. The main reason appears to relate to the identifying assumptions on strategic beliefs, which range from naive beliefs (level-1) over Nash beliefs (equilibrium without anticipating errors) to rational expectations. To reconcile these results, such specific assumptions on belief formation should therefore be avoided. I introduce a concept based on quantal response equilibrium (McKelvey and Palfrey, 1995) that nests the three belief models above and endogenizes the assumption on belief formation. While this solves one problem, Haile et al. (2008) suspect that generalized forms of QRE may overfit and lack robustness themselves. The data used here allow me to directly address this issue by evaluating robustness, i.e. the

[2] Cursed bidders believe their opponents get random signals with probability χ and the true signals with $1-\chi$. Type projecting bidders believe their opponents have signals similar to their own with probability ρ and the true signals with probability $1-\rho$. In both cases, bidders underestimate the informativeness of their opponents' bids about the object value and experience the Winner's Curse, which captures the intuition usually expressed by economists (e.g. Milgrom, 1989), but between the two approaches, only the belief perturbation underlying type projection is supported by independent psychological evidence. This evidence (on false consensus) draws from interactions with symmetric type sets, and in turn, cursed equilibrium appears more appropriate to model games with asymmetric type sets (e.g. buyer-seller interactions).

[3] There is evidence that preferences due to e.g. spite influence behavior in second-price private-value auctions (Cooper and Fang, 2008), as discussed below. In this sense, projection cannot be fully comprehensive, naturally. In my analysis, I focus on projection in relation to concepts used in structural analyses of bidding, namely risk aversion, cursedness and limited depth of reasoning.

accuracy of predictions across experiments.[4] In addition, this analysis verifies whether the models are applicable across data sets, e.g. in (future) analyses of different data.

The results corroborate the compatibility with psychological intuition and stylized facts. Type projection indeed captures behavior well, both descriptively (in-sample) and in particular predictively (out-of-sample). Further, inexperienced subjects tend to underestimate the rationality of others, though not in the way predicted by level-k. As subjects gain experience, their beliefs approach rational expectations, the precision in maximizing utility increases, subject heterogeneity becomes significant, and yet, the degree of projection remains largely constant (around 0.5). Thus, type projection appears to be a robust facet of behavior, and in the analyzed auctions, it is comprehensive in the sense that neither risk aversion nor cursedness capture facets of behavior incompatible with projection. The results have policy implications, as the projection bias is reduced when subjects are educated explicitly (Engelmann and Strobel, 2012), which enables efficiency gains, and they have implications for behavioral and empirical work. For, type projection intuitively factors in all symmetric Bayesian games, and thus needs to be controlled for in analyses of social preferences under anonymity (for related evidence, see Blanco et al., 2014), and as it fits robustly across private and common values, it promises to capture field auctions which tend to be hybrid (Haile, 2001; Goeree and Offerman, 2002).

Section 2 introduces the model of type projection and analyzes type projection in auctions. Section 3 introduces the data sets and evaluates type projection's basic predictions. Section 4 contains the structural analysis of bidding. Section 5 concludes. The appendix contains technical material, the supplementary material provides robustness checks.

2 Type projection in auctions

2.1 Definition

There are n symmetric bidders, denoted as $i \in N = \{1,2,\ldots,n\}$, and each bidder gets a signal $x \in [\underline{x},\overline{x}]$. Signals may be correlated. A bidder's expectation of the object value conditional on signal x is $v(x)$, the expectation conditional on both the own signal x and the highest opponent signal y is $v(x,y)$. The density of the highest opponent signal y conditional on the own signal x is $f_Y(y|x)$. A pure strategy $b_\star$ is a function mapping signals x to bids $b \in \mathbb{R}$. The expected payoff of bidding $b \in \mathbb{R}$, conditional on own signal x and in response to the opponents' bidding function $b_\star$, is

$$\Pi_0(b|b_\star,x) = E\left[(V_i - b)\, I_{b_\star(Y)<b} | X_i = x\right] = \int_{\underline{x}}^{b_\star^{-1}(b)} \big(v(x,y) - b\big) f_Y(y|x)\, dy. \qquad (1)$$

The symmetric, pure BNE satisfies $b_\star(x) \in \arg\max_b \Pi_0(b|b_\star,x)$ for all signals x. Note that I refer to the expected payoff in this case of *zero projection* as Π_0.

[4] Another issue with using the generalization of QRE is that the underlying QRE needs to be computed explicitly—the fixed point computation cannot be avoided using the insight of Bajari and Hortacsu (2005), by exploiting rational expectations, as relaxing rational expectations is exactly the point. The explicit computation of QREs is computationally demanding in standard auctions, due to the complexity of randomized bidding functions, but a novel observation allows me to reduce the strategy complexity by an order of magnitude and thus enables computation of QREs using massive parallelization (on GPUs).

Type-projecting bidders partially replace the objective information about the type distribution, weight $1-\rho$ with $\rho \in [0,1]$, by their projection that all bidders' types are equal to the own type, weight ρ. The parameter ρ is called *degree of projection*. First consider the case of full projection $\rho = 1$. In this case, the expected payoff of bidding b is

$$\Pi_1(b|b_\star,x) = s(b,b_\star,x) \cdot \int_{\underline{x}}^{\overline{x}} \big(v(x,y)-b\big) f_Y(y|x)\,dy, \tag{2}$$

where $s(b,b_\star,x)$ denotes the fully projecting bidder's share of the prize contingent on bidding b: $s=0$ if $b<b_\star(x)$, $s=1/n$ if $b=b_\star(x)$, and $s=1$ if $b>b_\star(x)$. Now, given the assumption that the projection has weight ρ, the expected payoff is simply the weighted mean of "objective" payoff $\Pi_0(b|b_\star,x)$ and "projected" payoff $\Pi_1(b|b_\star,x)$, i.e.

$$\Pi_\rho(b|b_\star,x) = (1-\rho)\Pi_0(b|b_\star,x) + \rho\,\Pi_1(b|b_\star,x). \tag{3}$$

The symmetric, pure ρ–Type Projection Equilibrium (ρ-TPE), with $\rho \in [0,1]$, satisfies $b_\star(x) \in \arg\max_b \Pi_\rho(b|b_\star,x)$ for all signals x.

This model provides a simple and tractable formulation of type projection in auctions. To clarify this, let me briefly describe how the model could be generalized or adapted. The tractability follows from two assumptions. On the one hand, bidders project their types onto *all* their opponents simultaneously. Alternatively, one might assume that bidders project their types independently onto their various opponents. The resulting model of projection appears to be qualitatively rather similar to the above model. Besides improving tractability, correlated projection captures the observation that individuals tend to believe their opponents make correlated choices.[5] On the other hand, bidders project their exact type. In Bayesian games with ordered type sets, bidders might instead believe that the opponents are of "similar" rather than "equal" types. Such a model of "fuzzy projection" may be more descriptive in specific circumstances, but the simpler model of exact projection applies to both ordered and unordered type sets, and it avoids the free parameters in defining distance functions. Its parsimony seems particularly desirable in the present analysis of the basic implications of type projection.

2.2 Related literature

Psychology Much evidence suggests that individuals assume the own opinions, preferences and values are shared by others. This phenomenon is labeled projection bias or false consensus. Ross et al. (1977) showed that subjects' beliefs about others' choices correlate with their own choices, and that their beliefs about others' characteristics correlate with their own. Projection biases behavior in relation to the rational benchmark, but it actually seems helpful in predicting characteristics of other individuals. Hoch (1987) finds that the majority of individuals tend not to use available information and would actually improve their predictions if they weighted their own positions even stronger.

A large number of studies subsequently showed that individuals project object values and preferences (both labeled "types" in Bayesian games) onto others, suggesting that

[5] See Camerer et al. (2004), Costa-Gomes et al. (2009), and Breitmoser (2012). Let me refer to the literature on "clustering illusion" in psychology for further discussion.

type projection likely matters in auctions and thus affects bidding. Individuals project values such as their willingness to pay (Frederick, 2012; Kurt and Inman, 2013) and their reservation prices (Bottom and Paese, 1999; Galinsky and Mussweiler, 2001). Individuals also project preferences and strategies onto opponents in strategic games such as the one-shot Prisoner's Dilemma (Messé and Sivacek, 1979), public goods games (Offerman et al., 1996), and distribution games including dictator, public goods, and ultimatum games (Blanco et al., 2011, 2014). Further, Iedema and Poppe (1995) and e.g. Aksoy and Weesie (2012) show that individuals project their social value orientation, and Bellemare et al. (2011) find preference projection with respect to guilt aversion.[6]

More generally, projection is strongest in relation to people similar to oneself (Clement and Krueger, 2002), and in auctions, bidders arguably consider each other similar, as they are interested in buying the same object. Mullen et al. (1985) show that projection occurs robustly, persisting even if subjects are provided with information factually contradicting their projection ("truly false consensus", Krueger and Clement, 1994). Engelmann and Strobel (2000, 2012) show that in order for the projection bias to disappear, the objective information must be handed to subjects on a "silver platter" and it must be usable at very low cognitive costs. Hence, the background information provided to subjects in laboratory auctions, about the abstract type distributions, is likely not obstructive to their projection of values and preferences.[7] In field auctions, no objective information about the opponents' values is provided at all, suggesting projection is likely even stronger, and in this sense, analyzing laboratory auctions, we will observe a lower bound for the relevance of projection in auctions in general.

Related models Type projection distinctly differs from existing models of projection. Let me start with Loewenstein et al. (2003), who consider a decision maker predicting his own utility in future states of the world. Given consumption c and current state s, the decision maker predicts the utility to be $(1-\alpha)\,u(c,s') + \alpha u(c,s)$, $\alpha \in [0,1]$, in alternative state s'. The main difference between type projection and such "utility projection" materializes in strategic games (such as auctions). Since each type plays a distinct strategy, a type-projecting player associates each list of opponents' types t_{-i} with *mixed strategies*—with probability $1-\rho$ the true types t_{-i} play and with probability ρ the projected types play. In turn, utility projection implies that players believe their opponents' types have "averaged" utilities and thus play pure strategies each.[8]

[6]Additional evidence shows projection of preferences and beliefs onto future selves. Gilbert et al. (1998) show that individuals overestimate the duration of affective reactions to negative events, Read and Van Leeuwen (1998) show that individuals project their current state of appetite when ordering meals in advance, Conlin et al. (2007) show that individuals project their current preferences analyzing catalog orders, Simonsohn (2010) observe preference projection in college enrollment decisions, via the cloud cover observed on visiting day, Grable et al. (2004), Grable et al. (2006), and Kliger and Levy (2008) show that projection in reaction to stock market price changes explain investment decisions.

[7]Engelmann and Strobel (2012) suggest the information about the opponents' signals contained in winning the auction is implicit and therefore likely neglected by bidders, not being handed on a silver platter. They argue that this may help explain imperfect Bayesian updating and thus the Winner's curse in CV auctions. My model of projection indeed implies (and in this sense explains) such negligence and imperfect updating in CV auctions, and additionally it explains overbidding (loser regret) in private value auctions which does not relate to negligence of the information contained in winning.

[8]Also note the difference to the notion of "strategy" projection: A type projecting player believes opponents share his type but keep their individual incentives. Both utility projecting and strategy projecting

An information projecting player (Madarász, 2012) believes his opponents know all he knows, in addition to their existing knowledge. In auctions, information projection implies I believe the opponents know my values, in addition to knowing their own values. Type projection assumes instead that the opponents share i's value. Information projection is appealing in cases of one-sided "missing" information, and it provides an intriguing explanation of the hindsight bias, but it appears less appealing in auctions—where there is no objectively "missing" information, but simply differences in types (values).[9]

Cursed equilibrium (Eyster and Rabin, 2005) is related in that it also assumes players correctly anticipate their opponent types' strategies but misperceive the type distribution. Type projection explicitly implements the projection bias as defined in psychology, that people project their own traits or opinions, which captures evidence from interactions with ex-ante symmetric type sets (such as auctions). In turn, cursed equilibrium appears more appropriate to capture beliefs if type sets are clearly asymmetric, as result of which projection of the own type appears less intuitive. Market interactions with one-sided incomplete information as analyzed in Eyster and Rabin (2005) appear to be a prototypical example of a Bayesian game that is more intuitively captured by cursed equilibrium. A concept inverting the idea of cursed equilibrium is the level-k model as applied to auctions by Crawford and Iriberri (2007). Contrary to cursed equilibrium, where strategies are correct but perceived types are random, level-k assumes types are correct but perceived strategies are random. The predictions are rather similar (see Crawford and Iriberri, 2007). Finally, analogy-based expectation equilibrium (Jehiel and Koessler, 2008) also captures the idea that the perceived type distribution is wrong, in that types are bundled into analogy classes and thus perceived to be coarser than they actually are. This biases Bayesian updating in common value auctions, without affecting behavior in (typical) private value auctions, similarly to level-k and cursed equilibrium.

2.3 Theoretical framework

Type projection induces a form of loser regret in first-price auctions and conservatism in belief revision about common values. The former induces overbidding in relation to the Bayesian benchmark, while the latter may induce over- or underbidding, depending on signal structure.[10] In order to provide a unified treatment of both private and common value auctions, i.e. to clarify the main predictions most transparently, I will focus on cases where these two forces point into the same direction. Alternative cases can be analyzed similarly, as illustrated in the supplementary material.

Both loser regret and conservatism induce overbidding if winning constitutes "bad news" and the environment exhibits strategic complementarity. "Bad news" are implied if the object value conditional on just winning is smaller than the unconditional object

players implicitly assume the opponents neglect their original incentives and adopt his utilities or strategies.

[9] Madarász (2015) generalizes the concept by including "ignorance projection" and applies it to games with two-sided incomplete information. The differences still appear major, as information projection appears to predict pure equilibria in auctions (since payoffs are continuous), but precise comparisons are impossible, as the shape of equilibrium strategies under information projection in auctions (which are not the main application) is not characterized (see Example 2.1.2 in Madarász, 2015).

[10] An example for underbidding in CV auctions is the "wallet game" (Avery and Kagel, 1997) where the object value is the sum of the bidders' signals. Conservatively updating bidders underbid the Bayesian Nash equilibrium if they have got high signals. An analysis of such auctions is provided in the supplement.

value. Strategic complementarity obtains if the more aggressive my opponents bid, the more aggressive I should bid, which in common value auctions depends on how "bad" the bad news are. For, the more aggressive the opponents bid, the worse are their signals in the case I am winning, and the lower is the object value conditional on winning. As a result of this "information effect", the best response declines in the opponents' bids. This information effect may dominate the general incentive to match the aggressiveness of the opponents' bids, and then strategic complementarity is violated (Klemperer, 1998).

For the purpose of a unified analysis, I analyze an environment exhibiting a weak form of strategic complementarity: one's best response is not declining in the opponents' bids. Relaxing this assumption would leave most statements made below unchanged, but the lower bound of equilibrium bids cannot be characterized tightly, as overbidding is not generally predicted (the wallet game is the pathological example where over- and underbidding may result, depending on one's signal). Weak strategic complementarity obtains in the canonical auctions with either affiliated private values (APV) or common values (CV) analyzed in experiments (Kagel and Levin, 1986; Kagel et al., 1987). I analyze a generalized information structure containing these two auctions as special cases.

Definition 1 (Hybrid first-price auctions)**.** X_0 is uniform on $[\underline{X}_0, \overline{X}_0]$, and for all bidders i, the private signals X_i conditional on X_0 are uniform on $[X_0 - \varepsilon, X_0 + \varepsilon]$ with $\varepsilon > 0$. The bidder's object values are $v_i = \delta\, X_i + (1-\delta)\, X_0$, with $\delta \in [0,1]$, and $\delta = 1$ in APV auctions and $\delta = 0$ in CV auctions. The winners pay their bids.

The pure common value case ($\delta = 1$) is the borderline case where one's best response is independent of the opponents' strategy, i.e. the two effects exactly cancel out. The case of independent private values (IPV) is qualitatively similar to APV in many ways, but the notation of mixed strategies needs to be modified, obfuscating a joint discussion.

We will find that all ρ-TPEs are mixed in these auctions, which is relevant, as mixed equilibria must be analyzed under a curse of dimensionality. In contrast to pure strategy equilibria, it is not sufficient to focus on the opponent with the highest signal anymore. Bidders with lower signals may also place the winning bid and thus must be considered explicitly in the analysis. Accounting for this is in principle straightforward, but tedious without offering any obvious additional insights. For ease of exposition, I therefore focus on auctions with two bidders, which suffices to the clarify the main insights.

Without projection, the approximate BNE bids are $b_\star \approx x - \varepsilon$ in CV auctions ($\delta = 0$) and $b_\star \approx x - \varepsilon \cdot 2/n$ in APV auctions ($\delta = 1$). There are small distortions if x is close to the bounds of the signal space, but as in most structural analysis of auctions, I will abstract from these distortions.[11] Theoretically, this is adequate only if the difference between the bidders' signals $D = X - Y$ is independent of one's signal X.

Assumption 1. Consider a two bidder auction with the signals x and y of the bidders. The distribution of $D = X - Y$ is independent of X, has density f_D and support $[\underline{d}, \overline{d}]$.

Example 1. *In the hybrid first-price auctions, $D = X - Y$ is triangular on $[\underline{d}, \overline{d}] = [-2\varepsilon, 2\varepsilon]$ and independent of X if $X \in [\underline{X}_0 + \varepsilon, \overline{X}_0 - \varepsilon]$.*

[11] For example, in CV auctions, the exact BNE strategy is $b(x_i) = x_i - \varepsilon + Y$ with $Y = \frac{2\varepsilon}{n+1} \times \exp\big\{ - n(x_i - \underline{x} - \varepsilon)/2\varepsilon \big\}$, but $Y \approx 0$ if the signal x_i is not very close to the bounds of the signal space.

For example, in the common value auction of Kagel and Levin (1986) where the common value X_0 is uniform on $[50,500]$ and individual signals X_i are independently uniform on $[X_0-10, X_0+10]$, independence obtains in the eyes of i if $60 \leq X_i \leq 490$, i.e. with a probability near 1. Independence is thus valid in the interior of the signal space and abstracts from distortions induced by the signal space bounds—which in turn allows us to focus on the strategic aspects in bidding.[12] Specifically, Assumption 1 allows us to focus on "normalized" bids without loss of generality, i.e. on bids normalized in relation to the signal. Given signal x, the own normalized bid is $r = b(x) - x$, and correspondingly, the opponent's normalized bid is $r_\star = b_\star(y) - y$. Normalized bids express the "degree of bid shading", i.e. the amount by which the bidders undercut their signals x and y, respectively. Similarly, the normalized expected object value is $\tilde{v}(d) := v(x, x+d) - x$ and the normalized unconditional object value is $\tilde{V} = \int_{\underline{d}}^{\overline{d}} \tilde{v}(d) f_D(d)\, dd$, both with $d = y - x$ and f_D as defined in Assumption 1. The normalized values express the difference between expected object value and signal. Given these normalizations, the normalized expected payoff of bidding r in response to the pure strategy $r_\star$ is (without projection)

$$\tilde{\Pi}_0(r \,|\, r_\star) = \int_{\underline{d}}^{r - r_\star} \big(\tilde{v}(d) - r\big) f_D(d)\, dd. \tag{4}$$

The auction is fully characterized by the duple $\langle \tilde{v}, f_D \rangle$, and thus, if Assumption 1 is satisfied and the opponent's normalized bid $r_\star$ is independent of x, then one's best response is also independent of x. Hence, any equilibrium in normalized bids that are independent of x must correspond with an equilibrium of the original auction, and using uniqueness of BNE, this implies that we can focus on normalized bids that are independent of x without loss of generality. This theoretical prediction, that normalized strategies are independent of x, will also be tested (and confirmed) econometrically below.

Example 2. *In the hybrid first-price auctions, $\tilde{v}(d) = \delta \cdot 0 + (1-\delta) \cdot d/2$ for all $\delta \in [0,1]$ and all $d \in [-2\varepsilon, 2\varepsilon]$, the unconditional value is $\tilde{V} = 0$, and the normalized BNE without projection is $r_\star = -\varepsilon$ (independently of δ) in two-bidder auctions.*

As illustration, recall that the BNE bids are $b_\star = x - \varepsilon$ and $b_\star = x - \varepsilon \cdot 2/n$ in case of $\delta = 1$ and $\delta = 0$, respectively, implying that the normalized BNE bids are $r_\star = -\varepsilon$ and $b_\star = -\varepsilon \cdot 2/n$, respectively. In equilibrium, the normalized bid $r_\star$ is negative and the expected normalized payoff is positive. Also note that the unconditional object value is normalized to zero and BNE strategy normalized to $r_\star = -\varepsilon$ for all instances of the hybrid value structure in the two-bidder case. This facilitates the unified analysis of APV and CV auctions. Finally, the normalization reduces the dimensionality of the strategy space, from analyzing equilibria in bidding functions to analyzing equilibria in scalar degrees of bid shading. This enables econometric analyses of mixed strategies as discussed below.

2.4 Analysis of projection in auctions

Let $R \subset \mathbb{R}$ denote the set of normalized strategies r. A mixed strategy $\sigma \in \Delta R$ is the density of a distribution on R. The expected (normalized) payoff of bidding r in response

[12] "Independence" in this sense is violated for independent private values, which prevents a joint analysis.

to the mixed strategy σ is (without projection)

$$\tilde{\Pi}_0(r\,|\,\sigma) = \int_R \sigma(r_\star)\,\tilde{\Pi}_0(r|r_\star)\,dr_\star = \int_R \sigma(r_\star)\int_{\overline{d}}^{r-r_\star}\big(\tilde{v}(d)-r\big)f_D(d)\,dd\,dr_\star. \tag{5}$$

Under full projection, bidders assume that their opponent's signal equates with theirs, implying they do not learn anything new when winning the auction. Hence, the object value conditional on winning equates with the unconditional object value, $\tilde{V}$, and the probability of winning in response to σ equates with σ's cumulative density $F_\sigma(r) = \int_{r_\star<r}\sigma(r_\star)dr_\star$.

$$\tilde{\Pi}_1(r\,|\,\sigma) = \int_{\underline{d}}^{\overline{d}}\big(\tilde{v}(d)-r\big)f_D(d)\cdot F_\sigma(r)\,dd = (\tilde{V}-r)\cdot F_\sigma(r) \tag{6}$$

Under ρ-projection, the expected payoff is again the weighted average of zero projection payoffs $\tilde{\Pi}_0$ and full projection payoffs $\tilde{\Pi}_1$.

$$\tilde{\Pi}_\rho(r|\sigma) = (1-\rho)\,\tilde{\Pi}_0(r|\sigma)+\rho\,(\tilde{V}-r)F_\sigma(r) \tag{7}$$

A mixed ρ-TPE σ satisfies $r\in\arg\max_{r'}\tilde{\Pi}_\rho(r'|\sigma)$ if and only if $\sigma(r)>0$, for almost all $r\in R$. To characterize these equilibria, let $S_\sigma=\{r\in R|\sigma(r)>0\}$ denote the support of strategy σ, with bounds $\underline{r}=\inf S_\sigma$ and $\overline{r}=\sup S_\sigma$. The following proposition establishes mixedness, overbidding, and skewness in all hybrid first-price auctions, assuming $\rho\in(0,1)$. In the limiting cases $\rho=0$ and $\rho=1$, pure equilibria obtain.

Proposition 1. *Consider a two-bidder hybrid first-price auction. For any $\rho\in(0,1)$, any symmetric ρ-TPE strategy is mixed, its support satisfies $r^{BNE}\le\underline{r}<\overline{r}\le\tilde{V}$ (overbidding), and the density is monotonically increasing on its support (left-skewness).*

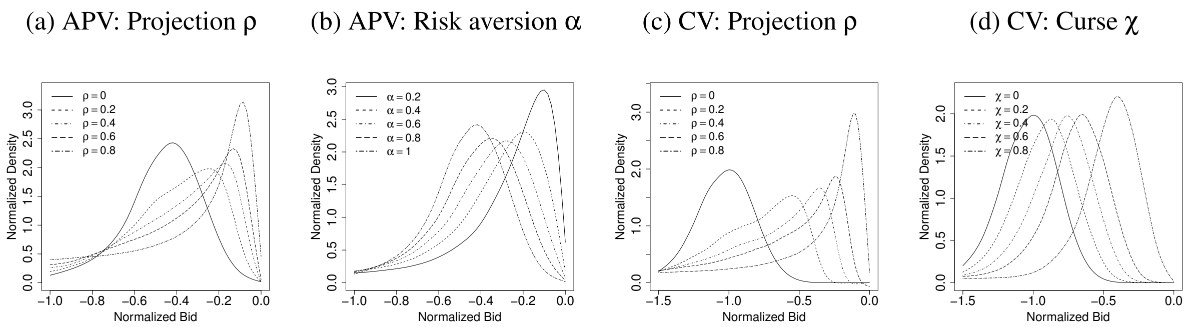
The proof is relegated to the appendix. In the following, I focus on the underlying intuition.[13] Figure 1 plots the predictions of type projection in APV and CV auctions, alongside those of risk aversion in APV auctions and cursedness in CV auctions.[14] The predictions are plotted for logit equilibria as analyzed in the econometric analysis below, which illustrates that the predicted bounds and shape of the equilibrium strategies are robust to (small) logit errors. Both risk aversion and cursedness predict symmetric distributions, while type projection predicts left-skewed strategies.

Overbidding to avoid loser regret Let $w_0(r|r_\star)$ denote the probability of winning without projection, and $w_\rho(r|r_\star)$ denote the respective probability with ρ-projection. If the projecting bidder bids less than opponents with the same signal, $r<r_\star$, he underestimates the probability of winning, as $w_\rho(r|r_\star)=(1-\rho)\,w_0(r|r_\star)+\rho\cdot 0$ is then less than the objective probability $w_0(r|r_\star)$. If he bids more than opponents with the same signal, he overestimates the probability of winning, as $w_\rho(r|r_\star)=(1-\rho)\,w_0(r|r_\star)+\rho\cdot 1>w_0(r|r_\star)$

[13]Throughout, I abstract from discussing existence, as existence can be established straightforwardly for finite games, and symmetric auctions seem to be representable as the limit of finite games similarly to symmetric Bertrand competition (as opposed to asymmetric Bertrand competition).

[14]Risk aversion does not affect equilibrium predictions in common value auctions and cursedness does not affect predictions in private value auctions. Hence, the corresponding plots are skipped.

Figure 1: Projection predicts skewed overbidding in both APV and CV auctions. Risk aversion and cursedness predict symmetric overbidding in APV and CV, respectively



results then. This induces an incentive to outbid opponents with the same signal, in all information conditions. These incentives resemble *loser regret* (Filiz-Ozbay and Ozbay, 2007), i.e. to feel regret if a higher bid would have won the auction profitably. Projecting bidders act as if they felt "conditional loser regret", i.e. regret if a higher bid would have won the auction against opponents with the same valuation. The differences are minor, as loser regret materializes only if the opponents' values are similar. Thus, I will say that projection induces loser regret as observed by Filiz-Ozbay and Ozbay (2007).[15]

Cursed value perception If one outbids opponents with the same signal, i.e. if $r > r_\star$, the expected object value under projection is equal to the expectation under cursedness (assuming $\rho = \chi$), a weighted average of conditional and unconditional value. Alternatively, if $r < r_\star$, the projected expectation equates with the Bayesian expectation. That is, the projected expectation is biased only if one outbids opponents with the same value. In standard common value auctions, the bias is an upward bias, i.e. the object value is overestimated, and the projected expectation exhibits an upward jump at $b = b_\star(x)$. Besides inducing cursed object valuations, this increment of the expectation adds to the loser regret. Thus, the incentives of projecting bidders to outbid opponents with the same signal are particularly strong in common value auctions. On a qualitative basis, type projection therefore predicts that if we hold the degree of projection constant, overbidding occurs in both information conditions, but the normalized degree of overbidding (suitably defined) is larger in common value auctions than in private value auctions.

Equilibrium strategies are mixed Overtaking the opponents, by bidding some $r = r_\star + \varepsilon$, induces upward jumps in both perceived winning probability and perceived object value, at infinitesimally small costs. If the expected payoff after bid increment is positive, the projecting bidder therefore prefers outbidding the opponents to matching their bids. In turn, a symmetric, pure strategy profile can be an equilibrium only if it induces zero expected payoffs. Then, however, projecting bidders can realize positive profits by deviating to bids $r < r_\star$. They lose against bidders with similar valuations (probability

[15]Note that the projected probability of winning is discontinuous in r if the opponents play a pure strategy. It jumps at $r = r_\star$ where one "overtakes" opponents with the same signal. The discontinuity will disappear once we allow for mixed strategies, but the incentive to slightly outbid opponents with similar values is robust to allowing for mixed strategies.

$\rho < 1$), but they win profitably against bidders with lower valuations. This yields positive profits with positive probability, and in turn, pure symmetric equilibria do not exist. As a result, the type projection equilibria must be mixed if $0 < \rho < 1$.

Support and skewness The lower bound of the support exceeds the BNE bid due to loser regret and cursed value perception, given the weak strategic complementarity, and the upper bound does not exceed the unconditional value as expected payoffs there are negative (even for projecting bidders). Now, taking the derivative of the payoff with respect to r (in response to σ) and solving for $\sigma(r)$, we obtain for interior $r \in (\underline{r}, \overline{r})$

$$\sigma(r) = \frac{1}{\tilde{V} - r} \cdot \left(F_\sigma(r) - \frac{1-\rho}{\rho} \cdot \tilde{\Pi}_0'(r|\sigma) \right),$$

which implies that $\sigma(r)$ is increasing in r, since $F_\sigma(r)$ is increasing and $\tilde{\Pi}_0'(r|\sigma)$ is decreasing (in the relevant range). The equilibrium strategy is thus left-skewed, i.e. the mean is left to the median. The upper bound $\overline{r}$ of the support converges to r^{BNE} as $\rho \to 0$ and the lower bound converges to the unconditional value $\tilde{V}$ as $\rho \to 1$.

Second-price auctions Finally, let us briefly look at second-price auctions. Due to the second-price payment rule, the loser-regret component of projection vanishes, but the cursed value perception continues to affect behavior. The projected expectation exhibits a jump discontinuity at $r = r_\star$, where one overtakes the opponent, if the object value has a common component ($\delta > 0$). In such cases, bidders again perceive to benefit from overtaking opponents, which rules out optimality of bidding one's value and the existence of pure equilibria. The resulting mixed equilibria are similar to their first-price counterparts, as illustrated in the supplementary material. In second-price private-value auctions ($\delta = 0$), projection equilibria are pure and bidders bid their values, as predicted by all belief-based concepts. Since overbidding is a systematic phenomenon also in second-price private-value auctions, even if related to the bidders' beliefs about their opponents' values (Cooper and Fang, 2008), this suggests that preferences relating to spite, inequity aversion, or joy of winning are likely also behaviorally relevant in auctions.

3 Testing the qualitative predictions

I re-analyze seven experiments. Pooling data from multiple experiments reduces the risk of misinterpreting model adequacy due to data selection and the fallacy to overfitting by assessing predictive adequacy across experiments. Evaluating predictive adequacy additionally clarifies to which degree the results obtained here may be helpful in (future) analyses of different data sets. Finally, pooling auctions under varying information conditions (IPV, APV and CV) allows me to examine robustness to real-world conditions which tend to be hybrid (Haile, 2001; Goeree and Offerman, 2002).

The data sources are listed in Table 1. These data sets form exactly the union of the data sets analyzed in the most influential studies of bidding behavior, Goeree et al. (2002b), Bajari and Hortacsu (2005), Eyster and Rabin (2005), and Crawford and Iriberri

(2007).[16] The repetitive re-analysis of these data sets indicates consensus on their adequacy to study bidding behavior, and re-analyzing these very data sets implies that if data selection influences the results, it would be in favor of existing theories.

As all of these data sets are well known and frequently analyzed, I skip an overly detailed discussion. The purpose of this section is to provide an overview of behavior in relation to the novel predictions derived above, based on estimates of the bidding functions and of the first three moments of the normalized bids. Specifically, I test the predicted independence of normalized strategies of signals x and the predictions of type projection about overbidding, mixedness and skewness. Throughout, I distinguish experienced subjects and inexperienced subjects. This comparison complements existing studies, which analyze either inexperienced subjects (Crawford and Iriberri, 2007) or experienced ones (most other studies). In particular, depth of reasoning and rationality of expectations are argued to vary with experience (e.g. Crawford and Iriberri, 2007): initial behavior (inexperienced subjects) is intuitively closer to level-k and converged behavior (experienced subjects) is intuitively closer to rational expectations and equilibrium. Following Crawford and Iriberri (2007), a subject is called "inexperienced" during the first five auctions, and by inversion, "experienced" during the last five auctions (of some 20 auctions in a session).[17]

The APV and CV auctions are exactly as defined above, and the IPV auctions are based on private values $v = X_i$ distributed as $X_i \sim U[0,30]$ or $X_i \sim U[0,28.3]$. In the APV and CV auctions analyzed, the BNE bids are $b(x_i) \approx x_i - \varepsilon$ and $b(x_i) \approx x_i - 2\varepsilon/n$, i.e. bidders are predicted to shade bids by *absolute* amounts in relation to signals (as discussed above). To evaluate this prediction, I estimate bidding functions $b = \alpha \cdot \varepsilon + \beta \cdot x$, testing the nulls $\alpha < 0$ and $\beta = 1$. The "signal bandwidth" ε is constant within treatments, i.e. its inclusion is econometrically irrelevant, but controlling for ε facilitates comparisons across treatments. In the IPV auctions, both BNE and cursed equilibrium predict bids $b(x) = (n-1)/n \cdot x$ and CRRA predicts $b(x) = (n-1)/(n-1+\alpha) \cdot x$ for $\alpha \in (0,1]$, see Cox et al. (1985). That is, equilibrium bids are fixed fractions of signals, to which I refer as *relative* bid shading. Here, I estimate $b = \alpha + \beta \cdot x$ to test the predictions that $\alpha = 0$ and $\beta < 1$. In all cases, I include subject-level random effects, bootstrap p-values,[18] and report significance at two levels: 0.05 and 0.005. The former is standard, and the latter implements the Bonferroni correction assuming 10 simultaneous tests across treatments and models, which is about adequate per level of experience.

Table 2, column "Bidding function", provides the estimated bidding functions. In

[16] Two of data sets analyzed in some of these studies, namely Goeree et al. (2002b) and Avery and Kagel (1997), are examined in the supplementary material, as they are "non-standard" (exhibiting either discrete signals or signals conditional on object value are not independent) and hence they cannot be discussed in a unified manner alongside the other auction experiments listed in Table 1.

[17] In common value auctions, in particular, behavior has not converged after five auctions, which precludes me from using all observations from the sixth auction on in the analysis of experienced subjects. In turn, behavior is independent of time during the first five auctions and during the last five auctions, respectively (as shown in the supplementary material), indicating that these partitions of the data set meet the time invariance assumed in the analysis.

[18] The bootstrap accounts for the panel structure of the data. Specifically, the data set is resampled $R = 10.000$ times at the subject level (reflecting the panel structure of the data). To define the p-value of the null hypothesis that some statistic s is zero, let s_b denote its value in sample b and let s_0 denote its original value. The p-value of the two-sided test is $\frac{1}{2R}\#\{b : |s_b - \bar{s}| > |s_0|\} + \frac{1}{2R}\#\{b : |s_b - \bar{s}| \geq |s_0|\}$, where $\bar{s}$ is the mean of (s_b) and R the number of samples. Other p-values are defined analogously.

Table 1: Data sources

Format	Source	Values	Signals	Inexperienced #Subj	Inexperienced #Obs	Experienced #Subj	Experienced #Obs
First price, common value	Kagel and Levin (2002)	$v = X_0$	$X_i\|X_0 \sim U[s \pm \varepsilon]$	51	255		
	Kagel and Levin (1986)	$v = X_0$	$X_i\|X_0 \sim U[s \pm \varepsilon]$			49	237
Second price, common value	Garvin and Kagel (1994)	$v = X_0$	$X_i\|X_0 \sim U[s \pm \varepsilon]$	28	140		
First price, affiliated private	Kagel et al. (1987)	$v = X_i$	$X_i\|X_0 \sim U[x_0 \pm \varepsilon]$	42	210	42	210
First price,	Dyer et al. (1989)	$v = X_i$	$X_i \sim U[0,30]$	18	180	18	180
Independ. private	Kagel and Levin (1993)	$v = X_i$	$X_i \sim U[0,28.3]$	10	50	10	100
Experiments on non-standard auctions (see supplementary material)							
First price, Independ. private	Goeree et al. (2002b)	$v = X_i$	X_i discrete	80	400	80	400
Second price, Common value	Avery and Kagel (1997)	$v = X_1 + X_2$	$X_i \sim U[1,4]$	23	115	23	115

Note: The data for inexperienced subjects are mostly from Crawford and Iriberri (2007). In most rounds of Dyer et al. (1989) and Kagel and Levin (1993), the subjects played two auction markets simultaneously. Focusing on the first and last five rounds they played, we mostly have ten observations per subject. Due to bankruptcies in CV auctions, there are not always five observations per subject.

APV and CV auctions, the coefficient of signal x differs significantly from 1 in only one of the twelve treatments (at $\alpha = .05$), which is well within the limits of chance. In IPV auctions, intercept α is insignificantly different from zero in all cases, suggesting that subjects indeed make relative reductions. The estimated parameters are also economically insignificant, i.e. small in relation to the range of signals.

Result 1 (Independence of x)**.** *In APV and CV auctions normalized bids $r := (b-x)/\varepsilon$ are independent of x, and in IPV auctions normalized bids $r := b/x$ are independent of x.*

From now on, I focus on analyzing these *normalized bids*, i.e. $r = (b-x)/\varepsilon$ in APV and CV auctions and $r = b/x$ in IPV auctions. As above, r represents the inverted degree of bid shading.[19] Values close to 0 in APV and CV auctions, or close to 1 in IPV auctions, indicate zero bid shading. Figure 2 illustrates the distributions across conditions.

Next I test if the normalized bid distributions are unimodal. The level-k theory predicts multiple modes, which would require finite mixture modeling in the econometric analysis, as opposed to random effects models here and mixed logit models below. The kernel density estimates in Figure 2 suggest unimodality, and as econometric test I estimate finite mixture models with up to three components. Each component is characterized by a mean normalized strategy, a between-subject variance regarding the subjects making up the component, and a within-subject variance to capture individual randomization. The details are relegated to the supplementary material, as the impression given by the histograms is confirmed: the bid distributions are unimodal, in the sense that secondary components (modes) are significant in only 2 of 18 treatments.

[19] The lower the normalized bid, the higher the degree of bid shading. For example, in APV and CV auctions, with $r = -0.4$, subjects bid $0.4 \cdot \varepsilon$ less than their signal, $r = 0$ indicates bidding one's signal, $r = -2/n$ is the BNE strategy in APV auctions, and $r = -1$ is the BNE strategy in CV auctions.

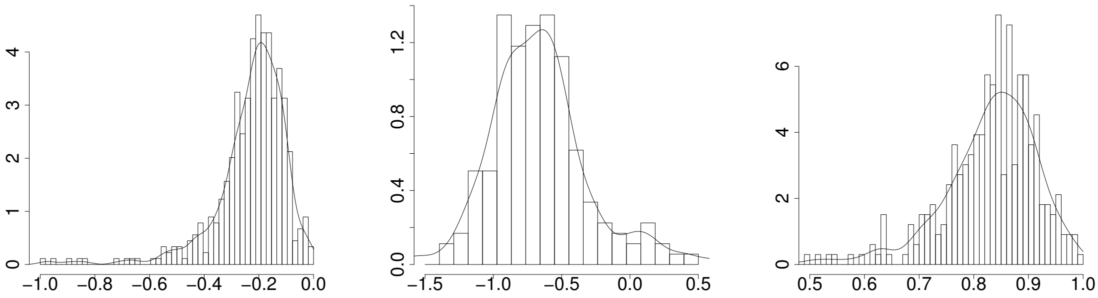
Table 2: Bidding functions and moments of normalized bids in first-price auctions (with bootstrapped standard errors in parentheses)

	Inexperienced subjects					Experienced subjects				
		Degree of	Standard Deviation				Degree of	Standard Deviation		
Condition	Bidding function	Overbidding	within Ss	between Ss	Skewness	Bidding function	Overbidding	within Ss	between Ss	Skewness
Independent private values, First price (DKL89, KL93)										
$N=3$	$b=-0.031+0.803^{\star\star}\cdot x$ (0.234) (0.018)	$0.104^{\star\star}$ (0.017)	0.161 (0.031)	0.025	$-3.17^{\star\star}$	$b=0.028+0.822^{\star\star}\cdot x$ (0.149) (0.01)	$0.143^{\star\star}$ (0.014)	0.126 (0.018)	0.054	$-3.26^{\star\star}$
$N=6$	$b=-0.039+0.849^{\star\star}\cdot x$ (0.196) (0.013)	-0.021 (0.02)	0.162 (0.034)	0.053	$-3.35^{\star\star}$	$b=0.037+0.875^{\star\star}\cdot x$ (0.182) (0.009)	$0.034^{\star\star}$ (0.012)	0.108 (0.018)	0.044	$-4.49^{\star\star}$
$N=5$	$b=0.195+0.886^{\star\star}\cdot x$ (0.241) (0.01)	$0.08^{\star\star}$ (0.021)	0.145 (0.053)	0.028	$-4.34^{\star\star}$	$b=-0.873+0.896^{\star\star}\cdot x$ (0.496) (0.017)	-0.021 (0.042)	0.264 (0.036)	0.129	$-1.65^{\star}$
Affiliated private values, First price (KHL87)										
$N=6,\varepsilon=6$	$b=0.986^{\star}\cdot x-0.284^{\star\star}\cdot\varepsilon$ (0.006) (0.079)	$-0.127^{\star\star}$ (0.038)	0.366 (0.085)	0.148	$-3.83^{\star\star}$					
$N=6,\varepsilon=12$	$b=0.992\cdot x-0.172^{\star\star}\cdot\varepsilon$ (0.006) (0.048)	$0.104^{\star\star}$ (0.026)	0.052 (0.006)	0.15	0.1	$b=1\cdot x-0.247^{\star\star}\cdot\varepsilon$ (0.005) (0.037)	$0.088^{\star\star}$ (0.021)	0.168 (0.044)	0.129	$-1.52^{\star}$
$N=6,w=24$						$b=1\cdot x-0.168^{\star\star}\cdot\varepsilon$ (0.006) (0.02)	$0.164^{\star\star}$ (0.022)	0.09 (0.015)	0.135	-0.24
Common value auctions, First price (KL86)										
$N\leq 4,\varepsilon=6$	$b=0.996\cdot x-0.22\cdot\varepsilon$ (0.003) (0.139)	$0.657^{\star\star}$ (0.066)	0.341 (0.033)	0.28	$0.57^{\star\star}$					
$N\leq 4,\varepsilon\leq 18$	$b=1.014\cdot x-0.676^{\star\star}\cdot\varepsilon$ (0.013) (0.172)	$0.551^{\star\star}$ (0.099)	0.43 (0.163)	0.095	$0.54^{\star}$	$b=1.002\cdot x-0.905^{\star\star}\cdot\varepsilon$ (0.016) (0.576)	$0.106^{\star}$ (0.038)	0.276 (0.087)	0.011	-1.77
$N\leq 4,\varepsilon\geq 24$						$b=1\cdot x-0.63^{\star\star}\cdot\varepsilon$ (0.021) (0.091)	$0.373^{\star\star}$ (0.05)	0.31 (0.058)	0.178	$0.7^{\star}$
$N=7,\varepsilon=6$	$b=0.999\cdot x-0.322^{\star\star}\cdot\varepsilon$ (0.002) (0.088)	$0.629^{\star\star}$ (0.067)	0.333 (0.036)	0.313	$0.54^{\star}$					
$N\geq 5,\varepsilon=12$						$b=0.99\cdot x-0.575^{\star\star}\cdot\varepsilon$ (0.007) (0.084)	$0.338^{\star\star}$ (0.051)	0.151 (0.025)	0.225	$1.02^{\star}$
$N\geq 5,\varepsilon=18$						$b=1\cdot x-0.654^{\star\star}\cdot\varepsilon$ (0.008) (0.082)	$0.348^{\star\star}$ (0.045)	0.296 (0.025)	0.206	$1.09^{\star\star}$
$N\geq 5,\varepsilon\geq 24$						$b=0.999\cdot x-0.714^{\star\star}\cdot\varepsilon$ (0.012) (0.085)	$0.279^{\star\star}$ (0.046)	0.231 (0.025)	0.201	$1.33^{\star}$

Notation: b is the bid, x is the signal, ε is the signal bandwidth in APV and CV auctions
Normalized bids: The normalized bids are $r=(b-x)/\varepsilon$ in APV and CV auction and $r=b/x$ in IPV auctions
Degree of overbidding is the difference between the mean normalized bid and the normalized equilibrium bid (BNE without risk aversion), it is estimated controlling for subject-level random effects ("between-subject standard deviation"). The within- and between-subject standard deviations refer to the distribution of normalized bids
Skewness: Skewness of the normalized bids after controlling for subject-level random effects (i.e. skewness of the errors in the regressions of normalized bids on intercept).
Experience: Subjects are "inexperienced" in their first five auctions and "experienced" in their last five auctions.
Asterisks indicate the bootstrapped p-values (see Footnote 18) of the null hypotheses that the respective parameters are either 1 (in case of the coefficients of x in APV and CV auctions, which are predicted to be 1) or 0 (in all other cases). "$\star\star$" indicates p-values less than .005, and "$\star$" indicates p-values between .005 and .05. The lower threshold .005 implements the Bonferroni correction for multiple testing across treatments (for around 10 treatments per level of experience).

Figure 2: Distribution of normalized bids across information conditions (experienced bidders; distributions for inexperienced bidders are similar and provided in the appendix)

(a) APV: $N = 6, w = 12$ (b) CV: $N = 5-7, w = 12, 18$ (c) IPV (DKL89): $N = 3$



Result 2 (Unimodality)**.** *Across information conditions and experience levels, bid distributions are unimodal (falsifying level-k). Secondary components are either insignificant (16 of 18 treatments) or contain less than 10 percent of the subjects (2 of 18 treatments).*

In the first-price auctions, normalized BNE bids are $r = -1$ in CV auctions, $r = -2/n$ in APV auctions, and $r = (n-1)/n$ in IPV auctions. In Table 2, the difference between normalized observed bid and normalized BNE bid is called "degree of overbidding". The degree of overbidding is significantly positive in 15 out of 18 cases (at $\alpha = .005$), i.e. subjects overbid consistently. Projection also predicts that subjects overbid more in CV auctions than in APV auctions. This can be studied by comparing KL86's CV auctions and KHL87's APV auctions, as they implement common values and affiliated private values in otherwise equivalent conditions: signal bandwidths ε are similar, numbers of bidders n are similar, and even experimental instructions and logistics are similar. The econometric approach is to regress the degree of overbidding on the information condition (APV or CV), controlling for subject-level random effects and bootstrapping p-values. Table 2 already shows that the degrees of overbidding are always below 0.2 in APV auctions and mostly above 0.3 in CV auctions, i.e. substantially higher. The regression results strongly confirm this impression (see Table 9 in the appendix): Across conditions and experience levels, the degree of overbidding is higher in CV auctions (5 of 6 times at $\alpha = .005$).

Result 3 (Overbidding)**.** *Subjects overbid and more so in CV auctions than in APV auctions, confirming the predictions of* ρ*-TPE.*

Regarding the second moments of bids, projection predicts mixed equilibria, i.e. positive within-subject variances. I test this by verifying if within-subject variance is constant as subjects gain experience, using regression models with different within-subject variances for the two levels of experience. I test the null that variance is constant in multiple ways, either holding the conditions such as number of bidders N or signal bandwidth w constant, or pooling the data and then controlling for N or w, but the impression of Table 2 (column "Standard deviation within Ss") is very robust: the within-subject variance does not change as subjects gain experience. This holds true both in treatment-wise comparisons when they are possible, noting that treatment parameters in some experiments are changed as subjects gain experience, and after pooling treatments. Between the 13 tests I

Table 3: Stylized facts in relation to the models' predictions

	Empirical Observation	Theoretical prediction				
		Exp payoff	Risk aversion	Cursed Eq	Level-k	Type projection
Distribution	Unimodal	×	×	×		×
Overbidding PV	Yes		×			×
Overbidding CV	Yes			×	(×)	×
Rel. Overbidding	CV>PV			×	(×)	×
Randomization	Yes					×
Skewness PV	Left					×
Skewness CV	Right					

Note: The predictions refer to level-k for expected payoffs or to the BNE assuming either expected payoffs, risk aversion (CRRA), cursedness, or type projection. Level-k predictions reflect the standard assumption that level-0 bidders randomize uniformly given their actual knowledge (Stahl and Wilson, 1995; Nagel, 1995). Crawford and Iriberri (2007) analyze a model where level-0 bidders randomize uniformly around their opponents' values. This model predicts overbidding in CV auctions, as I indicate using parentheses.

made (Table 10 in appendix), there is exactly one significant relation for either direction at the .05 level, and none at the .005 level suggested by the Bonferroni correction.

Result 4 (Randomization)**.** *The within-subject variance is highly robust to experience, suggesting subjects randomize strategically and corroborating the prediction of* ρ*-TPE.*

Regarding the third moments of bids, the histograms in Figure 2 suggest that the overall distributions are left-skewed in private value auctions (both IPV and APV), while skewness may be inverted in CV auctions. To test the hypothesized skewness of (individual) bidding functions, I estimate the skewness of the errors when regressing the normalized bids on the intercept controlling for subject-level random effects. These estimates, reported in Table 2 in column "Skewness", confirm the impression of Figure 2: Skewness is mostly significant, at least at $p = .05$, and if it is significant, then toward left-skewness in PV auctions and toward right-skewness in CV auctions.

Result 5 (Skewness)**.** *Bids are left-skewed in private value auctions (confirming the prediction of* ρ*-TPE) and right-skewed in common-value auctions (contradicting all models).*

Table 3 summarizes the relation of the predictions of the best-known models of bidding to the (empirical) Results 2–5. The predictions of the existing concepts are well-known and therefore not explicitly derived.[20] Type projection explains all observations that existing concepts explain, and in addition it explains observations that existing concepts do not explain. Specifically, type projections explains overbidding as well as the existing concepts in conjunction, and it uniquely explains most observations on the higher moments (randomization and skewness).

[20] BNE for expected payoffs by definition does not predict overbidding, risk aversion (CRRA) predicts overbidding in private value auctions, and cursed equilibrium predicts overbidding in common value auctions. All these concepts predict pure equilibria, which explains neither randomization nor skewness. For detailed discussion on the existing concepts and on level-k, let me refer to Crawford and Iriberri (2007).

4 Structural analysis

As shown in the previous section, type projection equilibrium captures bidding in auctions substantially better than existing models. This yields the joint hypothesis that (i) type projection captures biases in computation of expected payoffs and (ii) equilibrium captures the beliefs of subjects. I will refer to (i) as a statement about the *payoff structure* and to (ii) as a statement about the *belief structure*. A structural analysis allows me to disentangle these statements and thus to clarify whether the apparent adequacy of type projection simply follows from an actually inadequate assumption of equilibrium beliefs. In this analysis, I allow for all of the standard payoff structures (expected payoff, CRRA, cursedness, projection) and all standard belief structures.

Obviously, it is possible to combine any belief structure with any payoff structure. Indeed, previous analyses examined a fairly large variety of combinations, but unfortunately with little overlap between studies,[21] showing mainly that the identified payoff structure depends on the assumed belief structure and vice versa. This relates to analyses of choice under risk, where identification of utility functions and probability weighting depends on the assumed model of stochastic choice, see e.g. Hey (2005), Blavatskyy and Pogrebna (2010), and Wilcox (2011). Thus, to reliably analyze the payoff structure, we have to relax the assumptions on belief formation as far as possible—to let the data speak for itself. Next, I describe how I achieve this generalization, using a novel belief model, and how I address potential concerns associated with using a general belief model.

4.1 Econometric specification

All concepts but type projection equilibrium predict pure strategies and thus fit the observations only if we allow for stochastic choice (i.e. "errors"). To not rule out these models right away, I allow for errors due to "logistic" perturbations of utilities. Given strategic beliefs $\tilde{\sigma}_{-i}$ and payoff structure $\tilde{\Pi}$, i.e. some function mapping actions $r \in R$ and beliefs $\tilde{\sigma}_{-i}$ to expected payoffs, subjects choose the logit response with precision λ if

$$Logit_i(\tilde{\sigma}_{-i}|\tilde{\Pi},\lambda) = \left\{\sigma^l(r)\right\}_{r\in R} \quad \text{with} \quad \sigma^l(r) = \frac{\exp\{\lambda\, \tilde{\Pi}\big(r|\tilde{\sigma}_{-i}\big)\}}{\sum_{r'\in R}\exp\{\lambda\, \tilde{\Pi}\big(r|\tilde{\sigma}_{-i}\big)\}}. \tag{8}$$

Logit implies that the higher the expected payoff of an action, the higher its probability, with precision ranging from $\lambda = 0$ (uniform randomization) to infinity (best response).

As for belief structures $\tilde{\sigma}_{-i}$, the existing literature distinguishes mainly between "equilibrium beliefs" (rational expectations), "level-k beliefs" (Crawford and Iriberri, 2007), and "Nash beliefs". By Nash belief, I refer to the belief that opponents play the BNE strategies for the respective payoff structure, as usually assumed in structural analyses of

[21] Goeree et al. (2002b) examine equilibrium beliefs in conjunction with risk aversion (and logit errors). Eyster and Rabin (2005) examine cursed equilibrium, i.e. Nash beliefs in conjunction with cursed payoffs (and behavioral errors). Crawford and Iriberri (2007) show that level-k beliefs fit better than Nash beliefs in private and common value auctions, assuming either expected or cursed payoffs and logit errors. Bajari and Hortacsu (2005) show that Nash beliefs with behavioral errors fit about as well as equilibrium beliefs with logit errors (in the sense that the differences are insignificant). That is, the only model considered by two studies is equilibrium beliefs with logit errors (Goeree et al., 2002b; Bajari and Hortacsu, 2005).

empirical auctions. Note the subtle difference between "equilibrium beliefs" and "Nash beliefs": bidders with so-called equilibrium beliefs have rational expectations and anticipate errors of opponents, while bidders with Nash beliefs do not anticipate errors.

A model containing these belief structures as special cases obtains if we allow bidders to believe their opponents play a quantal response equilibrium (McKelvey and Palfrey, 1995, QRE)[22] and to logit respond with their own, presumably higher precision. That is, bidders κ-logit respond to a λ-QRE. I refer to this model as asymmetric QRE (AQRE). Note the minor but important difference to asymmetric logit equilibrium (Weizsäcker, 2003), where all precisions are common knowledge, while AQRE players believe their opponents simply play the QRE with precision λ and do not acknowledge that $\kappa \neq \lambda$.

Definition 2. Given a payoff structure $\tilde{\Pi}$, a strategy profile σ is a

- λ-QRE if all bidders $i \in N$ choose $\sigma_i = Logit_i(\sigma_{-i}|\tilde{\Pi}, \lambda)$
- (κ, λ)-AQRE if there exists a λ-QRE σ' such that $\sigma_i = Logit_i(\sigma'_{-i}|\tilde{\Pi}, \kappa)$

AQRE nests rational expectations for $\kappa = \lambda$, Level-1 for $\lambda = 0$, Nash beliefs for $\lambda = \infty$, and allows for a continuum in-between these extremes. Thus, AQRE is flexible enough to let the data speak for itself, and in addition, it is parsimonious, nesting the three models by adding just one parameter. There are two difficulties associated with using AQRE. One is that the underlying QRE needs to be computed explicitly. The insight of Bajari and Hortacsu (2005) allowing to avoid the fixed point computation underlying QRE—by exploiting rational expectations and using observed behavior as beliefs—is infeasible as observed behavior forms an AQRE and subjects do not have rational expectations. Hence, the QRE needs to be computed explicitly, but computing a QRE of an auction is not straightforward, as mixed bidding functions are rather complex. Thanks to the above result that subjects' strategies are one-dimensional, AQRE is computationally feasible using current technology, however.[23] That is, Result 1 allows us to compute QRE and AQRE, which in turn endogenizes the belief assumptions made in the literature.

The probably more prominent difficulty with using (A)QRE relates to the concern that a sufficiently generalized QRE can fit everything (see e.g. Haile et al., 2008). I address this concern in two ways. On the one hand, I will report on robustness checks using the best known alternative belief models, namely level-k (Stahl and Wilson, 1995; Nagel, 1995), cognitive hierarchy (Camerer et al., 2004) and noisy introspection (Goeree and Holt, 2004). Secondly, I will explicitly verify the fallacy to overfitting by examining predictive adequacy. To be safe, I evaluate descriptive, predictive and inferential adequacy of models. *Descriptive adequacy* quantifies goodness-of-fit in-sample and is measured by Bayes information criterion (Schwarz, 1978) using the number of subjects as number of observations. *Predictive adequacy* (Hey et al., 2010) measures the reliability of

[22]QRE is the standard model in behavioral game theory, and successfully captures behavior in e.g. the centipede game (Fey et al., 1996), the traveler's dilemma (Capra et al., 1999), public goods games (Goeree et al., 2002a), monotone contribution games (Choi et al., 2008), and beauty contests (Breitmoser, 2012).

[23]For illustration, consider a grid with 100 different normalized bids over which the bidders randomize. In an auction with 5 bidders and say 100 possible signals, evaluating the payoff function is possible by simulation using quasi-random numbers, which in turn can be implemented in a massively parallel manner on a GPU (which is reasonably standard, see e.g. Breitmoser, 2012). On top of it, finding the fixed point for the distribution over 100 normalized bids is possible, but finding the fixed point for 100×100 probabilities (allowing for logistic errors) is not yet possible using "regular" computers.

predictions across experiments, by fitting the parameters to one information condition, using the estimate to predict the remaining data, evaluating the likelihood, and rotating such that all data sets are used as training data.[24] Structural analysts of empirical auctions are concerned mainly about *inferential adequacy*, i.e. the accuracy of the object values inferred from bids (Bajari and Hortacsu, 2005). I use an out-of-sample version where, given parameters estimated using one data set, the out-of-sample predictions for the other treatments are computed and then, for each bid in these treatments, the posterior expectation of the underlying signal is evaluated. This conditional expectation is called inferred signal. The inferential adequacy is the mean absolute deviation (MAD) to the actual signals. The appendix contains formal definitions, and the supplementary material reports on robustness checks using the mean squared deviation (MSD).[25]

The remaining details of the specification are standard and relegated to Appendix A. Significance is reported exactly at the levels used above, 0.05 for "weak" significance and 0.005 to reflect the Bonferroni correction, based on bootstrapped likelihood ratio test statistics and using nested or non-nested Vuong tests, as appropriate. I control for subject heterogeneity by allowing that all parameters are distributed randomly across subjects ("mixed logit"), which adequately captures the unimodality of bids. The appendix specifies the (standard) likelihood function and the numerical approach to its maximization.

4.2 Which payoff structure is most adequate?

The analysis proceeds in four steps. First, I analyze which payoff structure captures behavior most adequately under the general belief model AQRE, i.e. nesting the three standard models: rational expectations, naive beliefs, and Nash beliefs.

Question 1. *Allowing for all of the standard belief structures, which payoff structure captures bidding: expected payoffs, risk aversion, cursedness, or projection?*

The results, presented in Table 4, are rather clear-cut: Type projection generally is most adequate, corroborating the compatibility with the stylized facts, and in most cases the differences to the other payoff structures are highly significant. The descriptive adequacy of projection shows that a fairly constant degree of projection fits behavior across conditions. its predictive adequacy shows that the fit is robust, i.e. the estimated degree of projection is robust, indicating that type projection may be a behavioral primitive. Predictive adequacy also clarifies the striking differences to the other concepts. Both risk aversion and cursedness significantly improve on expected payoffs descriptively (i.e. in-sample) but fail to consistently improve on it predictively (out-of-sample). Their behavioral content in relation to expected payoffs is not robust. In turn, type projection

[24] The tendency to distinguish descriptive and predictive adequacy is a rather recent development in analyses of decision-theoretic models (Wilcox, 2008; Hey et al., 2010), learning models (Erev and Roth, 1998; Camerer and Ho, 1999; Tang, 2003; Ho et al., 2008), and simple games (Blanco et al., 2011; Shapiro et al., 2014). I am not aware of existing analyses in Bayesian games in general or auctions in particular.

[25] Briefly, both descriptive and predictive adequacy are likelihood based, with the BIC representing a correction for the amount of overfitting induced by using free parameters (Schwarz, 1978). Predictive adequacy by definition avoids free parameters in the evaluation stage. These likelihood-based measures are attractive due to their well-known limiting properties (consistency and efficiency). The usage of MAD and MSD in inferential adequacy complements these measures and follows Bajari and Hortacsu (2005).

Table 4: Analysis of the payoff structure (for all measures of adequacy: less is better)

(a) Inexperienced subjects (first five auctions), aggregated across conditions (IPV, APV, CV)

Adequacy	Exp Payoff		Risk Aversion		Projection		Cursedness
Descriptive	4108	$\gg$	3911	$\gg$	3742	$\ll$	3967
Predictive	4261	$\approx$	4203	$\gg$	4034	$\ll$	4449
Inferential	2226	$\ll$	2636	$\gg$	1759	$\ll$	2110
Average $(\lambda, \kappa, \alpha)$	$45, 0.12$		$0.39, 15, 0.34$		$11, 3.1, 0.44$		$20, 1.3, 0.73$

(b) Experienced subjects (last five auctions), aggregated across conditions (IPV, APV, CV)

Adequacy	Exp Payoff		Risk Aversion		Projection		Cursedness
Descriptive	4005	$\gg$	3573	$\gg$	3377	$\ll$	3805
Predictive	4069	$\gg$	3799	$>$	3686	$\ll$	4200
Inferential	4460	$>$	4004	$\approx$	3498	$\approx$	3650
Average $(\lambda, \kappa, \alpha)$	$47, 0.05$		$0.34, 14, 0.24$		$18, 3.3, 0.48$		$130, 0.21, 0.78$

Note: The row "Average $(\lambda, \kappa, \alpha)$" lists the average estimates of precision λ, belief parameter κ (of AQRE), and degree α of risk aversion/cursedness/projection (depending on model). Significance at 0.05 is indicated by $<, >$, and significance at 0.005 is indicated by $\ll, \gg$ (which implements the Bonferroni correction).

does not only yield higher predictive adequacy than expected payoffs for both inexperienced and experienced subjects—it fits better out-of-sample than expected payoffs does in-sample. Thus, type projection is of robust relevance in bidding. The results on inferential adequacy are similar, though not quite significant in all cases.

Result 6. *Allowing for the general belief structure, type projection is the dominant model of the payoff structure. It is most adequate by all measures, for both experienced and inexperienced subjects, and it uniquely improves on expected payoffs out-of-sample.*

4.3 How are beliefs formed?

Based on these results, let us use type projection as payoff structure and identify the belief structure. Besides equilibrium (QRE) and asymmetric QRE, I will consider noisy introspection (NI, Goeree and Holt, 2004), cognitive hierarchy (CHM, Camerer et al., 2004), and level-*k* (Nagel, 1995; Stahl and Wilson, 1995), see Appendix A.

Question 2. *Given the identified model of the payoff structure (type projection), which belief structure captures bidding?*

First, to provide context, let me briefly review existing results. In small normal-form games with dominated strategies, subjects exhibit low depth of reasoning: They do not choose dominated strategies but fail to take into account that opponents reason similarly (Costa-Gomes et al., 2001; Weizsäcker, 2003; Costa-Gomes and Weizsäcker, 2008). In games without dominated strategies, in particular in games with unique mixed equilibria, equilibrium beliefs are most adequate (Goeree et al., 2003; Brunner et al., 2011). In large normal form games, beliefs tend to be in-between these extremes: subjects may underestimate the precision of others, but not as extremely as level-1 (Goeree et al., 2002a; Costa-Gomes and Crawford, 2006; Breitmoser, 2012). This can be captured by

Table 5: Analysis of the belief structures (for all measures of adequacy: less is better)

(a) Inexperienced subjects (first five auctions), aggregated across conditions (IPV, APV, CV)

Adequacy	Level-k		CHM		QRE		AQRE		NI
Descriptive	3760	$\approx$	3762	$\approx$	3771	$>$	3742	$\approx$	3751
Predictive	4091	$>$	4014	$\ll$	4082	$>$	4034	$\approx$	4035
Inferential	1962	$\ll$	2210	$\gg$	2110	$\gg$	1759	$\ll$	2116
Average (λ, κ, ρ)	$43, 4.8, 0.44$		$47, 7.1, 0.44$		$45, 0.4$		$11, 3.1, 0.44$		$16, 0.54, 0.5$

(b) Experienced subjects (last five auctions), aggregated across conditions (IPV, APV, CV)

Adequacy	Level-k		CHM		QRE		AQRE		NI
Descriptive	3404	$\ll$	3435	$\approx$	3406	$>$	3377	$\ll$	3454
Predictive	3644	$\ll$	3697	$\gg$	3599	$\ll$	3686	$\gg$	3609
Inferential	3370	$\ll$	3565	$\approx$	3508	$\approx$	3498	$>$	3251
Average (λ, κ, ρ)	$29, 11, 0.42$		$29, 8, 0.42$		$52, 0.45$		$18, 3.3, 0.48$		$16, 0.62, 0.52$

Note: The row "Average (λ, κ, ρ)" lists the average estimates of precision λ, belief parameter κ (depending on model), and degree ρ of projection (depending on model). Significance at 0.05 is indicated by $<, >$, and significance at 0.005 is indicated by $\ll, \gg$.

e.g. AQRE with $\kappa > \lambda > 0$ and NI with $1 > \kappa > 0$. Auctions are similarly large games, and following Goeree et al. (2002b) equilibrium beliefs are adequate (i.e. QRE). Bajari and Hortacsu (2005) show that equilibrium beliefs are about as adequate as Nash beliefs.[26]

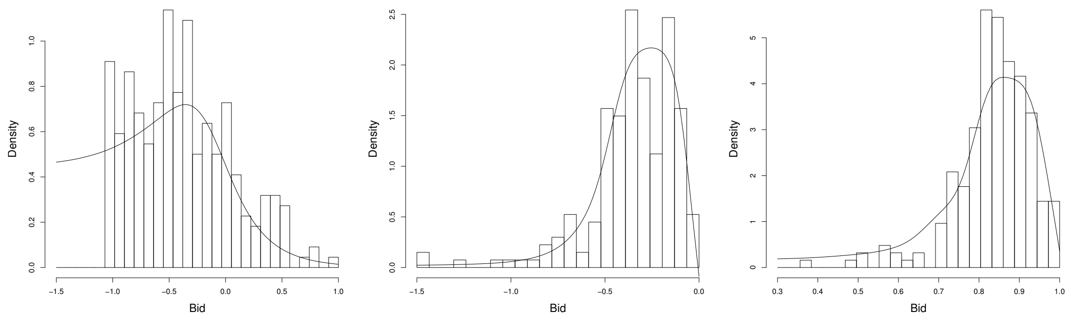
The results of the current analysis, provided in Table 5, largely corroborate these observations. To organize the results, let us take the unique one-parametric model (QRE) as benchmark and ask which of the two-parametric models improve on it consistently. As for inexperienced subjects, the only model that improves on QRE consistently (by all three measures) is AQRE, but in two of the three cases, the significance of the differences is not robust to the Bonferroni correction. Thus, I say that AQRE weakly improves on QRE for inexperienced subjects. As for experienced subjects, no model consistently improves on QRE, which has the highest predictive adequacy and thus fits most robustly.

Result 7. *Inexperienced subjects underestimate the precision of others, which is captured best by AQRE. Beliefs approach rational expectations (QRE) as subjects gain experience.*

Thus, in line with the literature, inexperienced bidders exhibit comparably noisy beliefs (relating to Crawford and Iriberri, 2007), though level-*k* is not the most adequate model, which confirms the observations that bid distributions are unimodal (Result 2). Experienced bidders are well described holding equilibrium beliefs (relating to Goeree et al., 2002b, and Bajari and Hortacsu, 2005). To illustrate the goodness-of-fit, Figure 3 plots the predicted densities of QRE with projection over the histograms of normalized bids. These plots refer to inexperienced subjects; the respective plots for experienced subjects are similar and provided as supplementary material.

[26] In turn, Crawford and Iriberri (2007) show that for a specific assumption of level-0 behavior and assuming expected payoffs, level-*k* models may fit better than equilibrium beliefs.

Figure 3: The predictions of QRE with projection (solid lines) in relation to the data



Note: The histograms plot the normalized bids for each information condition in standard auctions (see Table 1), always aggregated across treatments, focusing on inexperienced subjects. On top, the solid line depicts the predicted choice probabilities of QRE with projection, equally averaged across treatments.

4.4 Are subjects additionally risk averse?

Subjects may be projecting or cursed in addition to being risk averse. Potentially, the focus on determining the "single best" explanation, which shapes the existing literature and to some extent also the present analysis up to this point (we do allow for generalized belief structures), is inadequate and misrepresents behavior. This possibility is examined in this third analytical step.

Question 3. *Is a generalized payoff structure allowing for risk aversion besides type projection or cursedness more adequate than plain type projection?*

On a qualitative basis, type projection explains overbidding in private value auctions as well as risk aversion, and in addition it explains randomization and skewness. This captures behavior more comprehensively both in-sample and out-of-sample, suggesting that risk aversion may be behaviorally insignificant once we consider type projection behaviorally relevant. However, since risk aversion explains overbidding in private value auctions and cursedness explains overbidding in common value auctions, the notion that subjects are both, risk averse and cursed, may adequately capture behavior.

The analytical approach is as before. Based on the above results, I focus on equilibrium beliefs; robustness checks are provided as supplementary material. The results, reported in Table 6, are clear and can be summarized succinctly. The in-sample differences are small and insignificant, i.e. type projection describes behavior comprehensively and does not miss out on any aspect captured by the other models despite its relative parsimony. This corroborates its compatibility with the stylized facts compiled above, see Table 3. The predictive adequacy significantly improves with type projection on its own, indicating that its parsimony indeed improves robustness. Augmenting type projection by risk aversion improves the inferential adequacy when subjects are experienced, which may be of relevance in empirical work.[27]

[27] One caveat is that large and diverse data sets are required to reliably estimate both degree of projection and degree of risk aversion (risk aversion on its own lacks inferential adequacy). This seems to be the

Table 6: Evaluation of models with multiple motives

(a) Inexperienced subjects (first five auctions), aggregated across conditions (IPV, APV, CV)

Adequacy	Proj + RA		Projection		Curse + RA
Descriptive	3772	$\approx$	3771	$\approx$	3790
Predictive	4160	$\gg$	4082	$\ll$	4235
Inferential	2127	$\approx$	2110	$\ll$	2312
Average pars	$46, 0.37, 0.88$		$45, 0.4$		$20, 0.73, 0.64$

(b) Experienced subjects (last five auctions), aggregated across conditions (IPV, APV, CV)

Adequacy	Proj + RA		Projection		Curse + RA
Descriptive	3378	$\approx$	3406	$\approx$	3424
Predictive	3732	$\gg$	3599	$\ll$	3762
Inferential	2916	$\ll$	3508	$\approx$	3209
Average pars	$67, 0.31, 0.71$		$52, 0.45$		$68, 0.72, 0.46$

Note: The tables report results for QRE with projection. The order of average parameters is (λ, ρ, α) for "Proj + RA", (λ, ρ) for "Projection", and (λ, χ, α) for "Curse + RA", where λ is the QRE-precision, and α, ρ, χ are the degrees of risk aversion, projection, and cursedness, respectively. Significance at 0.05 is indicated by $<, >$, and significance at 0.005 is indicated by $\ll, \gg$.

Table 7: Average precision and degrees of projection as a function of experience

	CV 1st price		CV 2nd price		APV		IPV		Pooled	
	λ	ρ	λ	ρ	λ	ρ	λ	ρ	λ	ρ
Inexperienced	1.7	1	30	0.58	18	0.3	26	0.42	45	0.4
Experienced	8.2	0.28			45	0.65	41	0.46	52	0.45

Note: Given the estimates for QRE with projection, the tables lists mean precision λ and mean degree of projection ρ. The underlying distributions are log-normal and truncated normal, respectively.

Result 8. *Complementing type projection by risk aversion does not improve model adequacy in-sample (descriptively) or out-of-sample (predictively), but it improves inferential adequacy for experienced subjects.*

4.5 Is the projection bias robust to experience?

Finally, let us look at the differences between inexperienced and experienced subjects. The main purpose is to evaluate whether the projection bias possibly disappears as subjects gain experience, which would limit its relevance for applied work.

Question 4. *Are projection bias, precision, and heterogeneity robust to experience?*

Table 7 presents the means of precision and degree of projection for each information condition, separately for inexperienced and experienced subjects. In all information

case in the present analysis but is unlikely to be satisfied in field work. Another caveat is that the models considered here are estimated by maximum likelihood, and thus inferential adequacy is a side effect. If inferential adequacy is the main objective, a different estimator may be appropriate.

Table 8: Analysis of significance of subject heterogeneity

(a) Inexperienced subjects (first five auctions)

Adequacy	Homog.		Heterog.
Descriptive	3893	$\gg$	3771
Predictive	4075	$\approx$	4082
Inferential	2108	$\approx$	2110
Average pars	$17, 0.43$		$45, 0.4$

(b) Experienced subjects (last five auctions)

Adequacy	Homog.		Heterog.
Descriptive	3570	$\gg$	3406
Predictive	4030	$\gg$	3599
Inferential	4613	$\gg$	3508
Average pars	$11, 0.61$		$52, 0.45$

Note: The tables report results for QRE with projection, assuming either homogeneous or heterogeneous subjects (the latter as in the model used so far and as described in Appendix A). Significance at 0.05 is indicated by $<, >$, and significance at 0.005 is indicated by $\ll, \gg$.

conditions, the mean precision increases as subjects gain experience, to the extent that behavior almost converged to projection equilibrium without errors in private value auctions. The degree of projection is on average constant, slightly increasing for private value auctions and substantially decreasing for common value auctions. In the latter case, the degree of projection is initially very high ($\rho = 1$) but declines to one of the lowest values across conditions as subjects gain experience. The high initial value indicates that inexperienced subjects struggle comprehending common values, and the subjects struggling the most actually go bankrupt in CV auctions. Bankrupt subjects are removed from the experiment and therefore not present in the pool of experienced subjects, which slightly biases the average degree of projection in CV auctions in relation to the other auctions. Aside from that, the mean degree of projection is near 0.5, which indicates that type projection is indeed a constant factor in bidding.

Next, I investigate how the extent of subject heterogeneity varies with experience. Contrary to the heterogeneous model considered so far, I now consider the homogeneous model where subjects are collectively described by a representative agent with "average" precision λ and "average" degree of projection ρ. The procedure is otherwise equal to above. The results are presented in Table 8. As for inexperienced subjects, allowing for heterogeneity improves the goodness-of-fit descriptively (in-sample), but neither predictively nor inferentially. In this case, allowing for heterogeneity induces overfitting. As for experienced subjects, allowing for heterogeneity highly significantly improves on the representative-agent model according to all three measures. This complements the earlier finding that experienced subjects exhibit higher precision and rational expectations, suggesting that experienced subjects understand the auctions and their opponents, enabling them to bid according to their preferences and individual differences become visible.

Result 9. *As subjects gain experience, their average precision increases, the average degree of projection remains largely constant, and subjects exhibit heterogeneity.*

5 Conclusion

This paper introduces type projection equilibrium as a model of bidding in auctions. Type projection was an ex-ante plausible candidate to be behaviorally relevant, as it is robustly observed in psychological research and intuitively applies to all (symmetric) Bayesian games, such as auctions. Yet, despite the large amounts of studies dedicated to either, auctions in economics and projection in psychology, the only published paper suggesting a potential link between bidding and projection appears to be Engelmann and Strobel (2012). After deriving theoretical predictions on overbidding, mixedness and skewness, I show that these mostly novel predictions are borne out in the data and that type projection substantially improves on existing models such as CRRA, cursedness, and level-*k*, both in-sample and out-of-sample. The previous drawback of structural auction analyses, that results depend on the identifying assumption about strategic beliefs, is resolved using a novel model of strategic beliefs (asymmetric QRE) nesting all models typically used.

The degree of projection, being around 0.5, is largely constant across information conditions and robust to experience—while subjects' beliefs approach rational expectations and their precision in maximizing utility increases with experience. Finally, type projection provides a comprehensive explanation in the sense that complementing it by say risk aversion or level-*k* does not improve model adequacy. This is compatible with the observations that bidders exhibit loser regret rather than risk aversion (Filiz-Ozbay and Ozbay, 2007) and that the gender differences in bidding (Casari et al., 2007; Ham and Kagel, 2006; Chen et al., 2013; Pearson and Schipper, 2013) do not relate to differences in risk aversion (Schipper, 2015).[28] Overall, the results are consistent and clear, strongly suggesting that type projection is a factor of behavior in auctions, and by extension in type-symmetric Bayesian games, which suggests ample opportunity for further research.

In this regard, three points may be worth noting. First, projection likely affects behavior not only in auctions, but similarly in other Bayesian games with symmetric type sets, including games where social preferences matter. In general, though, experimental work in economics tends to attribute deviations from Nash equilibrium either to preferences, such as risk aversion or inequity aversion, or to belief asymmetry, such as level-*k*. Intuitively, each of these influences affects behavior in general, but projection should not be neglected simply because the literature focused on other concepts so far: Judging by the psychological evidence, the relevance of projection appears to be rather universal.

Second, analysts of empirical auctions may consider projection at least alongside risk aversion in econometric analyses of bidding. This has both a downside and an upside. On the downside, projection equilibria are mixed and their computation may require information that analysts do not immediately have, e.g. the upper bound of values in private value auctions. Less information is required, and some tractability is gained (see Bajari and Hortacsu, 2005), if one is willing to neglect projection and assume "Nash beliefs" (bidders' beliefs are equilibrium strategies without errors). These assumptions are highly debatable, though. My results challenge the neglect of projection, and most analyses, including Crawford and Iriberri (2007) and above, show that subjects tend to underestimate the precision of others, i.e. the opposite of Nash beliefs. Further on the upside, projection equilibria fit much more robustly than received models across private and common values, which suggests that they are less prone to misspecification of the information conditions. This is promising as many empirical auctions take place in hybrid conditions (Haile, 2001; Goeree and Offerman, 2002). These advantages may well outweigh the additional computational burden, but more work clearly is required.

[28] Indeed, as an anonymous referee pointed out, the gender differences relate to progesterone which increases "social closeness" as its only other documented behavioral implication (Brown et al., 2009; Schipper, 2015), and social closeness seems to increase the affinity to projection (Clement and Krueger, 2002).

Finally, Engelmann and Strobel (2012) have shown that subjects are less likely to project if they are provided with the objective information in the best possible way. This suggests that the fallacy to projection may be subject to policy intervention, and future work may determine the best way of providing objective information. Further, to the degree that overbidding is due to risk aversion, educating subjects does not help efficiency. To the degree that overbidding is due to projection, educating subjects increases the efficiency in at least two ways: Subjects stop randomizing in equilibrium, which ensures that the bidder with the highest value wins, and in cases where not just the winners pay their bids (e.g. contests), a reduction of overbidding increases efficiency. Thus, the above findings also have novel policy implications.

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A Appendix

A.1 Alternative behavioral concepts

Cursed equilibrium The relation to cursed equilibrium (Eyster and Rabin, 2005) has been discussed in the Introduction: Both concepts assume that players have a mistaken understanding of the type distribution. Given the degree of cursedness $\chi \in [0,1]$, cursed players assign weight $1-\chi$ to the Bayesian case and χ to the event that their opponents' types are random and uninformative given the own signal. In the latter case, the opponents play the average strategy $\overline{\sigma}_i(a_{-i}|t_i) = \sum_{t_{-i}\in T_{-i}} \Pr(t_{-i}|t_i) \prod_{j\neq i} \sigma_j(a_j|t_j)$, and overall, cursed players expect payoffs

$$\pi_i^{Curse}\big(a_i|t_i,\sigma_{-i}\big) = (1-\chi) \sum_{t_{-i}\in T_{-i}} \sum_{a_{-i}\in A_{-i}} \Pr(t_{-i}|t_i)\; p_i\big[(a_i,a_{-i}),(t_i,t_{-i})\big] \prod_{j\neq i} \sigma_j(a_j|t_j)$$
$$+\chi \sum_{t_{-i}\in T_{-i}} \sum_{a_{-i}\in A_{-i}} \Pr(t_{-i}|t_i)\; p_i\big[(a_i,a_{-i}),(t_i,t_{-i})\big]\, \overline{\sigma}_i(a_{-i}|t_i). \tag{9}$$

A strategy profile $\sigma = (\sigma_1,\ldots,\sigma_n)$ is a χ-cursed equilibrium if $\sigma_i(\cdot|t_i) \in BR_{t_i}(\sigma_{-i}|\pi_i^{Curse})$ for all i and t_i. I am not aware of independent evidence supporting "random projection" as in cursed equilibrium (as opposed to projection of the own type), but cursed equilibrium appears well-suited to capture beliefs if type sets are asymmetric. Market interactions with one-sided incomplete information as analyzed in Eyster and Rabin (2005) are a prototypical example. In such asymmetric games, type projection appears less intuitive.

Risk aversion Cox et al. (1985, 1988) argue that a potential factor in bidding is constant relative risk aversion (CRRA), $u(p) = p^\alpha/\alpha$ with $\alpha \neq 0$, with expected utilities

$$\pi_i^{CRRA}\big(a_i|t_i,\sigma_{-i}\big) = \sum_{t_{-i}\in T_{-i}} \sum_{a_{-i}\in A_{-i}} \Pr(t_{-i}|t_i)\; u\big(p_i\big[(a_i,a_{-i}),(t_i,t_{-i})\big]\big) \prod_{j\neq i} \sigma_j(a_j|t_j). \tag{10}$$

CRRA utilities $u(\cdot)$ can equally be used to complement projection and cursedness. As it stands, risk aversion is the leading explanation of overbidding in private value auctions, but the more recent observations on loser regret, e.g. Filiz-Ozbay and Ozbay (2007) and Engelbrecht-Wiggans and Katok (2007), challenge this perspective (as discussed above).

Limited depth of reasoning The concepts discussed so far have in common that they are defined in terms of the payoff structure $\tilde{\pi}_i \in \{\pi_i, \pi_i^{CRRA}, \pi_i^{Proj}, \pi_i^{Curse}\}$. The players' beliefs about their opponents' strategies are taken as given. The complementary approach is to vary the belief structure, allowing that players deviate from BNE by violating rational expectations.[29] The seminal model in this strand literature, level-k, follows Stahl and Wilson (1995) and Nagel (1995); other belief structures are discussed below. Assuming level-0 randomizes uniformly, $\sigma^0(\cdot|t_i) = 1/|A_i|$ for i, t_i, and given a payoff structure

[29] Note that both cursedness and projection can equally be defined as concepts relaxing the belief structure. Above, they have been defined in terms of the payoff structure, as both Eyster and Rabin (2005) and the above definitions emphasize that an equilibrium assumption is maintained even under cursedness and projection (a BNE of an augmented game), while standard models of alternative belief structures (such as level-k) emphasize the non-equilibrium character of the predicted strategy profiles.

$\tilde{\pi}_i \in \{\pi_i, \pi_i^{CRRA}, \pi_i^{Proj}, \pi_i^{Curse}\}$, player i has level-k depth of reasoning, $k \geq 1$, if he plays $\sigma^k(\cdot|t_i) \in BR_{t_i}(\sigma_{-i}^{k-1}|\tilde{\pi}_i)$ for all t_i. In a similar manner, level-k has been applied to auctions by Crawford and Iriberri (2007).

A.2 Belief models

As described above, I endow all models with logistic errors. Noisy introspection (Goeree and Holt, 2004) is a model inspired by relaxing rationalizability through allowing for logistic errors. Each type plays a λ-logit response to the belief that his opponents play a $\lambda \cdot \kappa$-logit response to the belief their opponents play a $\lambda \cdot \kappa^2$-logit response to their belief, and so on, using $\kappa \in [0,1]$. The model contains quantal response equilibrium and level-1 as special cases, for $\kappa = 1$ and $\kappa = 0$, respectively.

Definition 3 (Noisy introspection, NI)**.** Given $\tilde{\pi}_i \in \{\pi_i, \pi_i^{CRRA}, \pi_i^{Proj}, \pi_i^{Curse}\}$, a strategy profile $\sigma = (\sigma_1, \ldots, \sigma_n)$ is consistent with (λ, κ)-noisy introspection if all types $t_i \in T_i$ of all players $i \in N$ choose $\sigma_i(\cdot|t_i) = Logit_{t_i}(\sigma_{-i}^1|\tilde{\pi}_i, \lambda)$ with

$$\sigma_i^k(\cdot|t_i) = Logit_{t_i}(\sigma_{-i}^{k+1}|\tilde{\pi}_i, \lambda \cdot \kappa^k) \tag{11}$$

The cognitive hierarchy model (Camerer et al., 2004) adapts the level-k model by assuming that level-k players do not play a logit response to the belief that all opponents are level $k-1$, but a logit response to the belief that the opponents are at any level $k' < k$ (including level-0). Players are assumed to have rational expectations about the relative frequencies of these levels, and overall levels are assumed to have Poisson distribution in the population. Given the Poisson distribution, let $f(k) = \Pr(level = k)$ denote the relative frequency of level k overall (given distribution parameter κ), and define the conditional probability $g(k'|k) = \Pr(level = k' \,|\, level < k)$. The level-0 strategy is uniform randomization, $\sigma^0(\cdot|t_i) = 1/|A_i|$.

Definition 4 (Cognitive hierarchy model, CHM)**.** Given $\tilde{\pi}_i \in \{\pi_i, \pi_i^{CRRA}, \pi_i^{Proj}, \pi_i^{Curse}\}$, a strategy profile $\sigma = (\sigma_1, \ldots, \sigma_n)$ is consistent with (λ, κ)-cognitive hierarchy if all types $t_i \in T_i$ of all players $i \in N$ choose $\sigma(\cdot|t_i) = \sum_{k \geq 0} f(k) \cdot \sigma^k(\cdot|t_i)$ with

$$\sigma^k(\cdot|t_i) = Logit_{t_i}(\tau_{-i}^{k-1}|\tilde{\pi}_i, \lambda) \qquad \text{and} \qquad \tau^{k-1}(\cdot|t_i) = \sum_{k'=0}^{k-1} g(k'|k) \cdot \sigma^{k'}(\cdot|t_i). \tag{12}$$

I use the parsimonious approach of Camerer et al. (2004) to capture the distribution of levels via Poisson also to complete the level-k model. Again, $f(k) = \Pr(level = k)$ denotes the relative frequency of level k in the population (given distribution parameter κ), and the level-0 strategy is uniform randomization, $\sigma^0(\cdot|t_i) = 1/|A_i|$.

Definition 5 (Level-k)**.** Given a payoff structure $\tilde{\pi}_i \in \{\pi_i, \pi_i^{CRRA}, \pi_i^{Proj}, \pi_i^{Curse}\}$, a strategy profile $\sigma = (\sigma_1, \ldots, \sigma_n)$ is consistent with (λ, κ)-level-k if all types $t_i \in T_i$ of all players $i \in N$ choose $\sigma(\cdot|t_i) = \sum_{k \geq 0} f(k) \cdot \sigma^k(\cdot|t_i)$ with

$$\sigma^k(\cdot|t_i) = Logit_{t_i}(\sigma_{-i}^{k-1}|\tilde{\pi}_i, \lambda). \tag{13}$$

A.3 Descriptive, predictive, and inferential adequacy

To be clear, let me introduce some notation. D_e denotes the data set associated with experiment e, $D = \cup_e D_e$ denotes the pooled data, and define $D_{-e} = D \setminus D_e$ (the data sets used here are listed in Table 1). Given a model, $\mathbf{p}$ denotes a generic parameter vector and $\mathbf{p}^\star(D')$ denotes the maximum likelihood estimate given data set D'. Finally, $|\mathbf{p}|$ denotes the the dimensionality of $\mathbf{p}$, $|D'|$ denotes the number of subjects in data set D', and $ll(\mathbf{p}|D')$ denotes the log-likelihood of the model with parameters $\mathbf{p}$ given data D'.

First, I measure descriptive adequacy by Bayes information criterion (Schwarz, 1978), using the number of subjects as number of observations.

Definition 6 (Descriptive adequacy)**.** $BIC = -ll\big(\mathbf{p}^\star(D)\,|\,D\big) + |p^\star(D)|/2 \cdot \log|D|$

Second, I measure predictive adequacy by fitting the parameters to one information condition, using the estimate to predict the remaining data, and rotating such that all data sets are used as training data. The predictive adequacy contains no penalty term as in BIC, as by definition no parameter is adjusted to the data set used in the validation stage, i.e. no degree of freedom is used. To be aligned with the other measures, I report the absolute values of the log-likelihoods, which implies that "less is better" for all measures.

Definition 7 (Predictive adequacy)**.** $LL_{pred} = -\sum_e ll\big(\mathbf{p}^\star(D_e)\,|\,D_{-e}\big)/(m-1)$ using m as number of experiments analyzed

Finally, the inferential adequacy also is evaluated out-of-sample, but now, we infer signals from bids rather than predicting bids from signals (following Bajari and Hortacsu, 2005). Given an observation and a set of parameters (estimated using training data), the theoretical bidding function for the respective out-of-sample treatment is determined and the expectation of the signal conditional on the observed bid is computed. This conditional expectation is called inferred signal. The inferential adequacy is the mean absolute deviation (MAD) to the actual signals. The supplementary material additionally lists the results for the mean squared deviation (MSD), which are very similar. Formally, let $m(\mathbf{p}|D')$ denote the mean absolute deviation of inferred signals to actual signals if inference is made using parameter vector $\mathbf{p}$ on data set D'.

Definition 8 (Inferential adequacy)**.** $MAD = \sum_e m\big(\mathbf{p}^\star(D_e)\,|\,D_{-e}\big)$

A.4 Subject heterogeneity, likelihood function and maximization

The precision parameters λ and κ are bounded at zero and have independent gamma distributions, whereas the degrees of risk aversion, projection and cursedness are bounded at both 0 and 1 and have independent beta distributions. Thus, each subject is described by a parameter vector $\mathbf{p} \in \mathbf{P}$ with joint density $f()$. Using $o_s = (o_{s,t})$ to describe the observations of subject $s \in S$ at time $t \in T$, and $\sigma(o_{s,t}|f)$ as the probability of observation $o_{s,t}$ under density f, the individual likelihood given the observations o_s of subject s is

$$l_s(f|o_s) = \int_{\mathbf{P}} \prod_{t \in T} \sigma(o_{s,t}|\mathbf{p}) \cdot f(\mathbf{p}) d\mathbf{p}. \tag{14}$$

The predictions $\sigma(o_{s,t}|f)$ implicitly depend also on the underlying belief model, e.g. QRE or AQRE. The integral is evaluated by simulation, using quasi random numbers, see Train (2003) and e.g. the supplement to Bellemare et al. (2008). Aggregating across subjects, the log-likelihood of the respective model with parameter density f is

$$ll(f) = \sum_{s \in S} \log l_s(f|o_s). \tag{15}$$

QREs are computed using a homotopy method leaning on Turocy (2005). Parameters are estimated by maximizing the log-likelihood, sequentially applying two maximization algorithms. Initially, I use the robust, gradient-free NEWUOA algorithm (Powell, 2006) and I verify convergence using a Newton-Raphson algorithm. The estimates are tested by extensive cross-analysis to ensure that global maxima are found. All parameter estimates are provided as supplementary material.

A.5 Proof of Proposition 1

Step 1 (Best responses) If $r - r_\star \in (\underline{d}, \overline{d})$, then bidding r wins the auction with non-degenerate probability and

$$\begin{aligned}
\frac{d}{dr}\tilde{\Pi}_0(r|r_\star) &= \frac{d}{dr}\int_{\underline{d}}^{r-r_\star} \big(\tilde{v}(d) - r\big) f_D(d)\,dd \\
&= \big(\tilde{v}(r-r_\star) - r\big) \cdot f_D(r-r_\star) - F_D(r-r_\star) \\
&\propto \big(\tilde{v}(r-r_\star) - r\big) - \frac{F_D(r-r_\star)}{f_D(r-r_\star)}.
\end{aligned}$$

Since we know

$$\begin{aligned}
\tilde{v}(d) &= (1-\delta) \cdot \frac{d}{2} \\
f_D(d) &= \frac{2\varepsilon - |d|}{4\varepsilon^2} \\
F_D(d) &= \begin{cases} (2\varepsilon + d) \cdot f_D(d)/2, & \text{if } d < 0 \\ 1 - (2\varepsilon - d) \cdot f_D(d)/2, & \text{if } d \geq 0, \end{cases}
\end{aligned}$$

it follows that $\tilde{V} = \int_{\underline{d}}^{\overline{d}} \tilde{v}(d) f_D(d)\,dd = 0$ and the BNE (for $\rho = 0$), the zero of $\tilde{\Pi}_0'(r|r_\star)$ in case $r = r_\star$, is

$$\tilde{v}(0) - r^{BNE} = \frac{F_D(0)}{f_D(0)} \qquad \Leftrightarrow \qquad 0 - r^{BNE} = \frac{1/2}{1/2\varepsilon} \qquad \Leftrightarrow \qquad r^{BNE} = -\varepsilon.$$

Thus, the best response to $r_\star$, which solves $\tilde{\Pi}_0'(r|r_\star) = 0$, can be characterized as

$$BR(r_\star) = \begin{cases} \frac{\delta r_\star - 2\varepsilon}{2+\delta}, & \text{if } r \geq r^{BNE} \\ \frac{\delta + 4\varepsilon + 1 - \sqrt{\delta^2 + (-4r_\star + 8\varepsilon + 2)\delta + 4{r_\star}^2 - 4r_\star + 32\varepsilon^2 + 8\varepsilon + 1}}{2}, & \text{if } r < r^{BNE}. \end{cases}$$

This implies that $BR(r_\star)$ is weakly increasing in $r_\star$ (strategic complementarity) and that (i) $BR(r_\star) > r_\star$ if $r_\star < r^{BNE}$ and (ii) $BR(r_\star) < r_\star$ if $r_\star > r^{BNE}$.

Step 2 (Concavity) The second derivative with respect to $r_\star$ satisfies

$$\begin{aligned}
\frac{d^2}{dr^2}\tilde{\Pi}_0(r|r_\star) &= (\tilde{v}'(r-r_\star)-1)\cdot f_D(r-r_\star) + \big(\tilde{v}(r-r_\star)-r\big)\cdot f_D'(r-r_\star) - f_D(r-r_\star) < 0 \\
&= \left(\frac{1-\delta}{2}-1\right)\cdot\frac{2\varepsilon-|d|}{4\varepsilon^2} - \left(\frac{(1-\delta)\,d}{2}-r\right)\cdot\frac{1}{4\varepsilon^2} - \frac{2\varepsilon-|d|}{4\varepsilon^2} \\
&= \frac{1-\delta}{2}\cdot\frac{2\varepsilon-|d|}{4\varepsilon^2} - \left(\frac{(1-\delta)\,d}{2}-r\right)\cdot\frac{1}{4\varepsilon^2} - \frac{2\varepsilon-|d|}{2\varepsilon^2} \\
&\leq \frac{1-\delta}{2}\cdot\frac{2\varepsilon}{4\varepsilon^2} + \frac{r}{4\varepsilon^2} - \frac{2\varepsilon-|d|}{2\varepsilon^2} \\
&\propto (1-\delta)\cdot\varepsilon + r - 2\cdot(2\varepsilon-|d|) \leq r + 2\cdot|d| - 3\varepsilon
\end{aligned}$$

which is negative if $r+2\cdot|d| \leq 3\varepsilon$.

Thus, if $r-r_\star \in (\underline{d},\overline{d})$ and $r+2\cdot|d| \leq 3\varepsilon$, then $d\tilde{\Pi}_0(r|r_\star)/dr$ is decreasing in r. Critically, concavity therefore obtains if $r, r_\star \leq \tilde{V} = 0$. Note that this statement also holds true for $r - r_\star = \underline{d}$, i.e. for the maximal r winning the auction with zero probability, if we consider the directional derivative $dr > 0$.

Step 3 (Upper bound $\bar{r} \leq \tilde{V}$) By $r-r_\star \in (\underline{d},\overline{d})$ and $\tilde{v}$ being non-decreasing, we know that if $r > \tilde{V}$,

$$\tilde{\Pi}_0(r\,|\,r_\star) = \int_{\underline{d}}^{r-r_\star}\big(\tilde{v}(d)-r\big)f_D(d)\,dd \leq \int_{\underline{d}}^{\overline{d}}\big(\tilde{v}(d)-r\big)f_D(d)\,dd = \tilde{V}-r < 0,$$

where the (first) weak inequality follows from the assumption that $\tilde{v}$ is non-decreasing.

Thus, if $r > \tilde{V}$, then $\tilde{\Pi}_0(r|r_\star) < 0$ in response to any $r_\star$ with $r-r_\star \in (\underline{d},\overline{d})$. By corollary, the statement holds true equally in response to any mixed strategy σ. As a result, in response to any σ, if $r > \tilde{V}$, then

$$\tilde{\Pi}_\rho(r|\sigma) = (1-\rho)\,\tilde{\Pi}_0(r|\sigma) + \rho\,(\tilde{V}-r)F_\sigma(r) < 0.$$

Negativity directly follows from $\tilde{V}-r<0$ and $\tilde{\Pi}_0(r|\sigma)<0$ (Step 3). In turn, zero payoffs are generally feasible by making a bid that loses with certainty, i.e. any $r \leq \inf\{r'|\sigma(r')>0\} + \underline{d} - \overline{d}$, and bidding $r > \tilde{V}$ is therefore not optimal in response to any σ. That is, $\bar{r} := \sup BR(\sigma)$ for all σ.

Step 4 (Any symmetric ρ-TPE is mixed) For the purpose of contradiction, fix any $\rho \in (0,1)$ and assume that a pure strategy equilibrium $r_\star$ exists. That is, given the expected payoffs of bidding r in response to $r_\star$,

$$\tilde{\Pi}_\rho(r\,|\,r_\star) = (1-\rho)\cdot\int_{\underline{d}}^{r-r_\star}\big(\tilde{v}(d)-r\big)f_D(d)\,dd + \rho\cdot s(b,b_\star,x)\;(\tilde{V}-r)$$

we assume $r_\star \in \arg\max_r \tilde{\Pi}_\rho(r \,|\, r_\star)$. Now, I distinguish two cases.

Case 1: $r_\star < \tilde{V}$. In this case, the bidder profits from the unilateral defection to $r_\star + \varepsilon$ for some $\varepsilon > 0$. For,

$$\begin{aligned}\lim_{\varepsilon\to 0} \tilde{\Pi}_\rho(r_\star+\varepsilon \,|\, r_\star) - \tilde{\Pi}_\rho(r_\star \,|\, r_\star) &= \rho \cdot \left[s(b_\star+\varepsilon, b_\star, x) - s(b_\star, b_\star, x)\right] \, (\tilde{V} - r_\star) \\ &= \rho \cdot [1 - 1/n] \, (\tilde{V} - r_\star) > 0,\end{aligned}$$

which is positive since $\rho > 0$, $n = 2$, and $r_\star < \tilde{V}$, the contradiction.

Case 2: $r_\star \geq \tilde{V}$. By Step 3, we can rule out $r_\star > \tilde{V}$, implying that $r_\star = \tilde{V}$ obtains and thus $\tilde{\Pi}_0(r_\star|r_\star) = 0$ as well as $\tilde{\Pi}_\rho(r_\star|r_\star) = 0$ for any ρ. Now consider the "infimal" winning bid $r_{\inf}$ which is the infimum of all bids that win the auction with positive probability in response to $r_\star$, namely $r_{\inf} = r_\star + \underline{d} - \overline{d}$. The expected payoff of bidding $r_{\inf}$ is zero, by $\underline{d} < \overline{d}$ we obtain $r_{\inf} < r_\star$, and by concavity of $\tilde{\Pi}_0$ for all $r, r_\star \leq \tilde{V}$ it follows that the bidder can profitably deviate to any $r : r_{\inf} < r < r_\star$, i.e. bidding $r_\star \geq \tilde{V}$ is not optimal in response to any $r_\star \geq \tilde{V}$ (the contradiction).

Step 5 (Lower bound $\underline{r} \geq r^{BNE}$) At the lower bound, the directional derivative with respect to $dr < 0$ is

$$\tilde{\Pi}'_\rho(r|\sigma)\big|_{r=\underline{r}, dr<0} = (1-\rho)\, \tilde{\Pi}'_0(r|\sigma),$$

which must be non-negative in equilibrium. Otherwise, one gains from deviating unilaterally toward putting probability mass on bids $r < \underline{r}$. Thus, $\tilde{\Pi}'_0(\underline{r}|\sigma) \geq 0$ if $\rho \in (0,1)$. Second, the directional derivative with respect to $dr > 0$ is, noting $F_\sigma(\underline{r}) = 0$,

$$\tilde{\Pi}'_\rho(r|\sigma)\big|_{r=\underline{r}, dr>0} = \rho\, (\tilde{V} - r)\, \sigma(r) + (1-\rho)\, \tilde{\Pi}'_0(r|\sigma).$$

This directional derivative must be zero, since σ is mixed. By Step 3, $\tilde{V} - \overline{r} \geq 0$, by Step 4 (mixedness) we know $\underline{r} < \overline{r}$, and thus $\tilde{V} - \underline{r} > 0$. Given $\sigma(\underline{r}) \geq 0$ and $\rho \in (0,1)$, this implies $\tilde{\Pi}'_0(\underline{r}|\sigma) = 0$ (besides $\sigma(\underline{r}) = 0$). Next, I show that $\tilde{\Pi}'_0(\underline{r}|\sigma) = 0$ implies $\underline{r} \geq r^{BNE}$. For contradiction, assume $\underline{r} < r^{BNE}$. By Step 1, we know $BR(r) > \underline{r}$ for all $r \geq \underline{r}$ in this case, implying $d\tilde{\Pi}_0(\underline{r}|r)/dr > 0$ for all $r > \underline{r}$. Hence, $\tilde{\Pi}'_0(\underline{r}|\sigma) > 0$, the contradiction.

Step 6 (Left-skewness) Finally, I show that the density of any symmetric ρ-TPE strategy is increasing on its support. Taking the derivative of $\tilde{\Pi}_\rho$ with respect r in response to σ, we obtain for any r in the interior of σ's support

$$\tilde{\Pi}'_\rho(r|\sigma)\big|_{r\in(\underline{r},\overline{r})} = (1-\rho)\, \tilde{\Pi}'_0(r|\sigma) + \rho\, (\tilde{V} - r)\, \sigma(r) - \rho F_\sigma(r).$$

Along the support of the mixed equilibrium, $\tilde{\Pi}'_\rho(r|\sigma) = 0$ is satisfied, implying for all interior $r \in (\underline{r}, \overline{r})$,

$$\sigma(r) = \frac{1}{\tilde{V} - r} \cdot \left(F_\sigma(r) - \frac{1-\rho}{\rho} \cdot \tilde{\Pi}'_0(r|\sigma) \right).$$

Here, $F_\sigma(r)$ is (weakly) increasing in r by virtue of being a cumulative distribution function, $\tilde{V} - r$ is decreasing in r since $\tilde{V}$ is constant, and $\tilde{\Pi}_0'(r|\sigma)$ is decreasing in r by concavity of Π_0 (given $r, \bar{r} \leq \tilde{V}$). Hence, $\sigma(r)$ is increasing in r on σ's support. $\square$

Figure 4: First-price auctions, affiliated private values (KHL87). Inexperienced subjects (a–b) vs. experienced subjects (c–d). Plots are histograms of $r = (Bid - Signal)/\varepsilon$

(a) Inexp: $N = 6, \varepsilon = 6$ (b) Inexp: $N = 6, \varepsilon = 12$ (c) Exp: $N = 6, \varepsilon = 12$ (d) Exp: $N = 6, \varepsilon = 24$

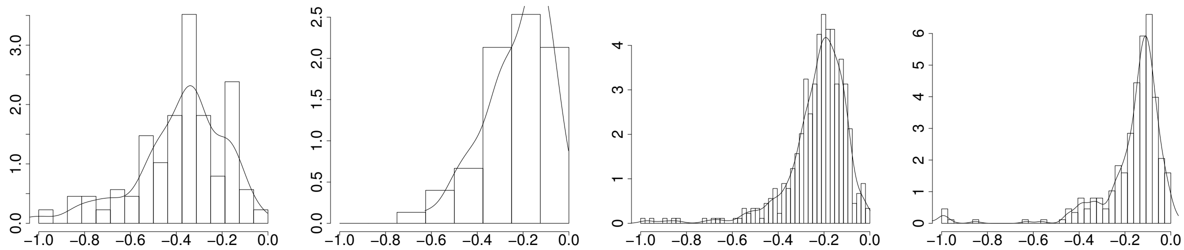
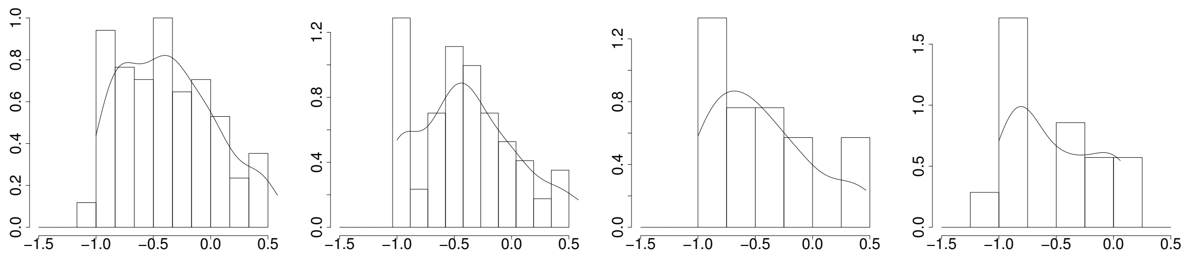


Figure 5: First-price auctions with common values (KL86), inexperienced subjects (a–d) vs. experienced subjects(e–h). Plots are histograms of $r = (Bid - Signal)/\varepsilon$

(a) $N = 4, \varepsilon = 6$ (b) $N = 7, \varepsilon = 6$ (c) $N = 4, \varepsilon = 12$ (d) $N = 7, \varepsilon = 12$



(e) $N = 3-4, \varepsilon = 12, 18$ (f) $N = 5-7, \varepsilon = 12, 18$ (g) $N = 3-4, \varepsilon = 24, 30$ (h) $N = 5-7, \varepsilon = 24, 30$

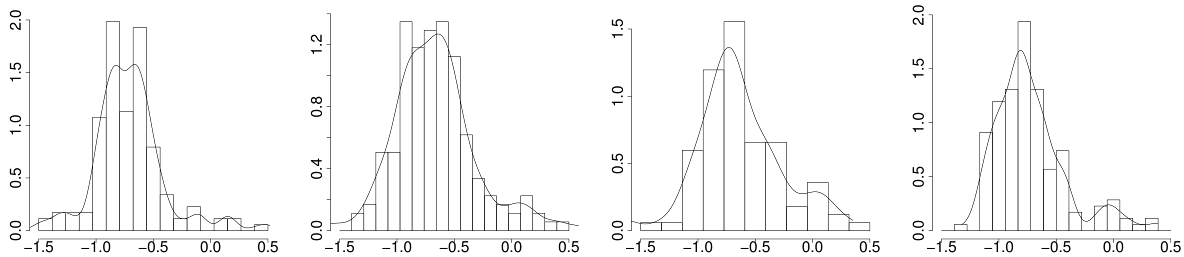


Figure 6: First-price auctions with independent private values (DKL89). Inexperienced subjects (a–c) and experienced subjects (d–f). Histograms of $Bid/Signal$

(a) Inexp: DKL89, $N = 3$ (b) Inexp: DKL89, $N = 6$ (c) Exp: DKL89, $N = 3$ (d) Exp: DKL89, $N = 6$

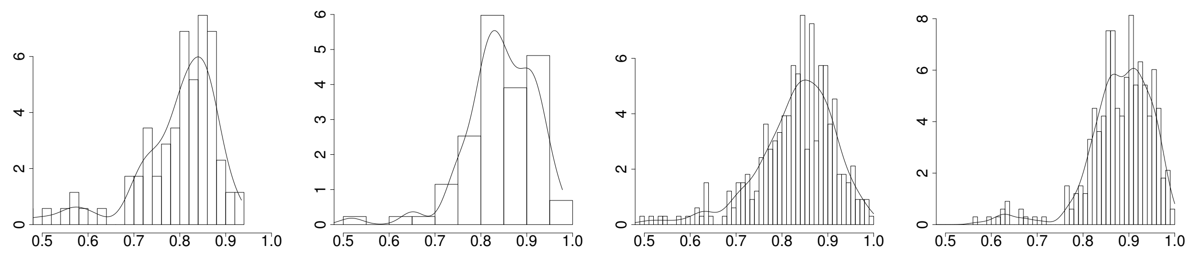


Table 9: Statistical tests of differences in the degree of overbidding and within-subject variance between auctions with affiliated private values and common values

	Degree of Overbidding			Within-Subject Variance			Between-Subj
Data	APV		CV	APV		CV	Variance
Inexperienced, $\varepsilon = 6$	-0.128	$\ll$	0.641	0.359	$\approx$	0.341	0.246
Inexperienced, $\varepsilon = 12$	0.104	$\ll$	0.523	0.052	$\ll$	0.4	0.147
Inexperienced, all w	-0.058	$\ll$	0.621	0.326	$\approx$	0.361	0.21
Experienced, $w \leq 18$	0.062	$\ll$	0.403	0.159	$<$	0.336	0.248
Experienced, $w \geq 24$	0.179	$<$	0.329	0.113	$\ll$	0.32	0.13
Experienced, all w	0.142	$\ll$	0.357	0.15	$\ll$	0.343	0.162

Description: The sole difference to Table 10 is that the comparison is between APV and CV auctions, instead of inexperienced and experienced subjects.

Table 10: Statistical tests of the degree of overbidding and within-subject variance (with respect to the degree of overbidding) as a function of experience

	Degree of Overbidding			Within-Subject Variance			Between-Subj
Data	Inexperienced		Experienced	Inexperienced		Experienced	Variance
Independent private values auctions (DKL89, KL93)							
$N = 3$	0.104	$\approx$	0.148	0.16	$\approx$	0.123	0.033
$N = 5$	0.08	$>$	-0.144	0.142	$<$	0.36	0.077
$N = 6$	-0.021	$<$	0.036	0.164	$\approx$	0.11	0.039
all N	0.05	$\approx$	0.04	0.156	$\approx$	0.178	0.119
all, contr. for N	0.05	$\approx$	0.041	0.155	$\approx$	0.179	0.103
Affiliated private values auctions (KHL87)							
$\varepsilon = 12$	0.104	$\approx$	0.062	0.051	$\approx$	0.172	0.17
All data	-0.058	$\ll$	0.142	0.331	$>$	0.15	0.134
All, contr. for w	0.058	$\approx$	0.04	0.192	$\approx$	0.209	0.117
Common value auctions (KL86)							
$N \leq 4$, $w \in \{12, 18\}$	0.538	$>$	0.228	0.415	$\approx$	0.343	0.208
$N \leq 4$, all w	0.63	$\gg$	0.316	0.37	$\approx$	0.363	0.233
$N \geq 5$, all w	0.613	$\gg$	0.389	0.344	$\approx$	0.309	0.254
all N, $w \in \{12, 18\}$	0.517	$\approx$	0.404	0.397	$\approx$	0.332	0.263
all N, all w	0.621	$\gg$	0.359	0.357	$\approx$	0.332	0.242
all N, all w, contr. for w	0.573	$\approx$	0.411	0.349	$\approx$	0.337	0.247

Description: The table reports the results of one set of statistical tests per row. Given the subset of data specified in column 1, two null hypotheses are simultaneously tested: (i) H_0 : the degree of overbidding does not differ between inexperienced and experienced subjects, and (ii) H_0 : the residual (i.e. within-subject) variances do not differ between them. These nulls are tested in regression models with the degree of overbidding as independent variable and the level of experience as independent variable (without intercept). $\gg, \ll$ indicate rejection of H_0 at the .005 level and $>, <$ indicate rejection at .05, where the p-values are bootstrapped as described above. Considering the Bonferroni correction for the multiple testing problem inherent in this analysis, results should be significant roughly at the .005 level. Terms such as the degree of overbidding are used as defined above (e.g. Table 2).