Bayesian Implementation and Rent Extraction in a Multi-Dimensional Procurement Problem

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Bayesian Implementation and Rent Extraction in a Multi-Dimensional Procurement Problem[†]

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ABSTRACT: We consider a multi-dimensional procurement problem in which sellers have private information about their costs and about a possible design flaw. The information about the design flaw is necessarily correlated. We solve for the optimal Bayesian procurement mechanism that implements the efficient allocation under the constraint that sellers are protected by limited liability. We show that the rents obtained from reporting costs truthfully can be used to reduce the rents sellers must get for reporting the flaw. We compare the optimal Bayesian mechanism to the optimal ex post incentive compatible mechanism that is informationally less demanding.

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1 Introduction

In the procurement of a complex good, sellers are often better informed about the optimal design of the good than the buyer. For example, a seller may see that the design proposed by the buyer has a serious flaw or that there is a possibility for a design improvement. The buyer is aware that sellers may have superior information and wants to induce them to contribute their knowledge at the design stage. However, if the good is procured with a price-only auction, a seller who sees a possibility for a design improvement will not reveal this information. It is a weakly dominant strategy to keep the information to himself, bit more aggressively in the auction, and then - after he received the contract - renegotiate the design with the buyer once he is in a bilateral monopoly position where he can extract some share of the surplus (Herweg and Schmidt, 2017). But if the design is changed late via renegotiation, both parties have to incur inefficient adjustment costs. Therefore, the buyer wants to use a more sophisticated auction that induces sellers to reveal their ideas early.

In this paper we derive the optimal Bayesian incentive compatible mechanism that implements the efficient allocation at minimal cost to the buyer. This is a mechanism design problem with two dimensional private information. Sellers have private information about their costs and they have private information about the existence of a flaw (the possibility of a design improvement). As is standard in the literature we assume that the sellers' cost types are uncorrelated. However, the information about a design flaw must be correlated, because a seller can observe a flaw only if the flaw exists. Crémer and McLean (1985, 1988) have shown that if types are correlated, then it is possible to extract all rents from the seller by using penalties in certain states of the world to deter agents from lying. The problem is that these penalties may have to be very large and that it is often impossible to enforce them if agents are protected by limited liability. Our contribution to this literature is to show that the rents that the agents obtain in one dimension (private information about their cost types) can be used to relax the limited liability constraints. Thus, it is possible to exploit the correlation of sellers' types even if they are protected by limited liability. Furthermore, there exists a range of parameter values for which the buyer can fully extract the information rents in the dimension where types are correlated.

The problem that sellers conceal private information about flaws and design improvements in order to renegotiate later seems to be widespread. Bajari and Tadelis (2011) report that sellers read plans and specifications carefully to find flaws that they want to use strategically

in order to renegotiate later. Then they bid more aggressively to get the contract. In the construction industry this is called *opportunistic bidding* or *bid your claims* (Mohamed, Khoury, and Hafez, 2011). Bajari, McMillan, and Tadelis (2009); Bajari, Houghton, and Tadelis (2014) and Iimi (2013) provide additional empirical evidence on this behavior. Moreover, there is a lot of empirical evidence that contract renegotiation is costly and inefficient (Crocker and Reynolds, 1993; Chakravarty and MacLeod, 2009; Bajari, Houghton, and Tadelis, 2014).

We model this problem as follows. There is one buyer who wants to procure an indivisible good. With some probability the design that she came up with is plagued by a flaw. It is less expensive to fix the flaw early (before the contract is allocated) than late (via renegotiation). There are two sellers with two-dimensional private information. Each seller is privately informed about his production cost. Moreover, if the buyer's design is flawed, each seller privately observes the flaw with some probability. Thus, a seller's type consists of a cost type and a flaw type, where the latter is binary. While we assume that sellers' cost types are independently distributed, sellers' flaw types – by the nature of the model – are correlated: A seller can observe the flaw only if it exists.

Herweg and Schmidt (2018) point out three inefficiencies that arise if a buyer uses a price-only auction. First, there is the inefficiency that arises if a flaw is fixed with delay because a seller concealed his information at the auction stage and revealed it only later in renegotiation. Second, a seller who spotted a flaw and bids more aggressively may win the auction even though he is not the seller with the lowest production costs. Third, the contract may be awarded to a seller who is not aware of the flaw, while some of his competitors are. In this case the design improvements are never implemented – there is no renegotiation. How can these inefficiencies be avoided at minimal cost by using a more sophisticated auction?

In our model we consider a buyer who wants to implement the efficient allocation at the lowest possible transfers to sellers. Efficiency requires that the good is produced by the seller with the lowest cost and that each seller reports the flaw early, if he observed it. The optimal mechanism must be Bayesian incentive compatible to induce sellers to report their types truthfully and it must ensure that sellers do not make losses ex post because they are protected by limited liability.

For the one-dimensional screening problem it is well-known that the buyer can fully extract any information rents of the sellers, if types are correlated and agents have unlimited funds (Crémer and McLean, 1988). If agents are protected by limited liability, however, the

buyer does not benefit from the (imperfect) correlation of types. We show that in a multi-dimensional screening problem, the buyer can use the rents the agents obtain in one dimension to reduce the rents they obtain in the dimension where types are correlated, even if the limited liability constraints have to be satisfied.

The optimal mechanism has the following properties: In expectations, each seller obtains the standard rent from a Vickrey auction for revealing his cost type. Moreover, there is an additional fixed payment for revealing the flaw. This payment is higher the lower the correlation of sellers' flaw types. If the correlation is sufficiently high, the fixed payment is equal to zero (no information rent for sellers who observed the flaw). Note that if only one seller reports the flaw, then the other seller – who did not observe it or decided to conceal it – is punished. The limited liability constraint restricts this punishment to loosing the information rent obtained for reporting the cost type. In order to keep the information rent for a seller who did not observe the flaw in expectations equal to the rent from the Vickrey auction, the rent obtained if both sellers did not observe the flaw has to be higher than the rent from the Vickrey auction.

The optimal Bayesian incentive compatible mechanism does not allow for a separate (i.e. sequential) elicitation of cost and flaw types. Furthermore, the optimal fixed payment obtained for reporting the flaw crucially depends on all distribution functions: the likelihood of the flaw to exist, the probability with which it is detected by sellers, and the distribution of sellers' production costs. Due to these high informational demands, the practical applicability of the optimal Bayesian mechanism may be limited. This is why we derive the optimal ex post incentive compatible (EPIC) mechanism in our companion paper, Herweg and Schmidt (2018). The EPIC mechanism does not require any ex ante knowledge of the underlying distributions, nor does it require that the buyer knows the payoff consequences of a flaw ex ante, i.e., the EPIC mechanism is informationally robust. However, it may also be significantly more expensive, because the buyer has to pay a larger information rent to the sellers. In the current paper we derive conditions under which the loss in profits for the buyer is small when using the EPIC instead of the Bayesian mechanism.

Related Literature: Our paper contributes to three strands of the literature. First, there is a large literature on optimal procurement mechanisms. Important early contributions include McAfee and McMillan (1986) and Laffont and Tirole (1993). Our approach is more closely related to the literature on optimal scoring auctions. In a scoring auction sellers submit bids on price and design. The contract is assigned to the seller who comes up with the best

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proposal, i.e., the highest score. Scoring auctions outperform price-only auctions, if the buyer can commit not to renegotiate (Dasgupta and Spulber, 1989-1990; Che, 1993).[1] The properties and limits of scoring auctions are further analyzed by Asker and Cantillon (2008, 2010); Herweg and Schmidt (2017).[2] A crucial difference of our approach to these papers is that in our model the seller who points out the possibility of a design improvement does not have to be the seller who carries out production. This is also the case in Che, Iossa, and Rey (2016). Their model focuses on how to give incentives for innovation, while in our model sellers observe the design improvement for free but have to be induced to report it early. Furthermore, in contrast to Che, Iossa, and Rey (2016), we study a multidimensional screening problem. We show that the rents obtained in one dimension can be used to reduce rents in another dimension by relaxing the limited liability constraint.

Second, our paper is related to the literature on optimal screening with correlated types. In their seminal contribution, Crémer and McLean (1988) provide necessary and sufficient conditions for full surplus extraction (FSE) with Bayesian and dominant strategy screening mechanisms. They show that FSE is feasible, if (roughly speaking) agents' types are "correlated". McAfee and Reny (1992) extended this result to continuous type spaces.[3] All of this literature considers one-dimensional screening problems.

Finally, our paper contributes to the extensive and growing literature in economics and computer science on multi-dimensional screening. Armstrong (1996) shows that if the mechanism designer wants to maximize profits, the optimal mechanism (usually) excludes some types. Rochet and Choné (1998) show that it is often optimal to induce "bunching" of types.[4] To the best of our knowledge, our paper is the first that points out how the rents an agent obtains from one dimension can be used to reduce the rents from the other dimension by relaxing the limited liability constraint.

The rest of the paper is organized as follows. In the next section we introduce the model

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[1]If this commitment is not feasible, the optimal scoring auction may collapse to a price-only auction, see Herweg and Schwarz (2018).

[2]For a recent survey on scoring auctions in procurement see Camboni and Adani (2018).

[3]FSE may fail to hold if agents are risk averse (Robert, 1991), if type spaces are infinite (Heifetz and Neeman, 2006), and if types are endogenously determined (Obara, 2008). More recently, Farinha Luz (2013) characterizes the revenue maximizing mechanism with rich type spaces in the sense of Bergemann and Morris (2005). He generalizes the FSE result to situations where agents obtain information from sources other than their own payoff characteristics.

[4]See also Daskalakis, Deckelbaum, and Tzamos (2013). In multi-dimensional screening problems, the characterization of robust mechanisms is often simpler than of optimal Bayesian mechanisms. An important recent contribution in this literature is Carroll (2017) who introduces the concept of *generalized virtual values*. Similar approaches are also used by Sher and Vohra (2015) and Cai, Devanur, and Weinberg (2016).

and formally state the mechanism design problem. This problem is solved in Section 3, where we proceed in several steps. First, in Subsection 3.1, we show how the optimization problem can be simplified significantly. Then, in Subsection 3.2, we analyze the special cases where flaw types are either perfectly correlated or not correlated at all. Finally, in Subsection 3.3, we derive the optimal direct mechanism. Comparative statics of the optimal mechanism and an equivalent indirect mechanism are discussed in Section 4. The final Section 5 summarizes the main findings and discusses open questions for future research. Proofs that are not presented in the main text are deferred to Appendix A.

2 The Model

2.1 Players, Preferences, and Information Structure

A buyer (female) wants to procure an indivisible good. She proposes design D_0 which suffers from a flaw with probability $p \in (0, 1)$. If there is a flaw, the flaw can be fixed by switching from design D_0 to design D_f. The buyer can procure design D_f only if a seller pointed out the flaw to her in the initial design.

There are two sellers (male), indexed by $i = 1, 2$, who can produce the good. For simplicity we assume that a seller's production cost is the same for both designs. Seller i's cost to produce any design is c_i, where $c_i \in [\underline{c}, \bar{c}] \equiv \mathcal{C}$ is distributed according to c.d.f. $H(c_i)$ and density $h(c_i) > 0$ for all $c_i \in \mathcal{C}$. Sellers' costs are uncorrelated and drawn from the same distribution function. If the flaw in the design exists, seller i observes this with probability $q \in (0, 1)$ (which is independent of his cost c_i). Moreover, the probabilities with which the two sellers observe the design flaw (given that it exists) are assumed to be uncorrelated.

Thus, a seller i's private information is two dimensional and denoted by $\theta_i = (c_i, F_i) \in \mathcal{C} \times \mathcal{F}$, where $\mathcal{F} \equiv \{\emptyset, f\}$: $F_i = \emptyset$ means that seller i did not observe the flaw, while $F_i = f$ means seller i observed the flaw. Information regarding the flaw is partially verifiable in the following sense: A seller can claim to have spotted the flaw only if he indeed observed it. If one of the sellers reports the flaw to the buyer, the buyer learns that design D_f is optimal.

Importantly, we assume that a seller who observed the flaw has a strategic incentive to conceal this information. If seller i knows that design D_0 is flawed and is awarded with a contract to deliver design D_0, then he can use this information in order to renegotiate the contract with the buyer ex post. By renegotiating the contract and thus fixing the flaw, the

surplus can be increased. When the contract is renegotiated parties are in a bilateral monopoly position and the seller gets the amount $S > 0$ from the renegotiation surplus.[5] However, from an ex ante perspective it is more efficient to fix the flaw early (at the contracting stage) rather than late (via renegotiation). The reason is that renegotiating an existing contract causes delays, adjustment costs, and legal expenses if parties disagree on who is responsible fixing the flaw.[6]

The structure of the model (including the distribution functions) is common knowledge. The buyer uses a direct mechanism, i.e., she asks each seller i to report his type $\hat{\theta}_i = (\hat{c}_i, \hat{F}_i) \in \mathcal{C} \times \mathcal{F}$. Note that seller i can report any $\hat{F}_i \in \{\emptyset, f\}$ only if $F_i = f$, otherwise $\hat{F}_i = \emptyset$. The mechanism specifies a design $D(\cdot)$, probabilities ω_i with which seller i obtains the contract (has to produce and deliver the good), and transfers t_i received by seller i and paid by the buyer. The mechanism has to satisfy the Bayesian incentive compatibility constraints: It must be optimal for each seller i to report his type θ_i truthfully given his beliefs about the competitor's type and expecting that the competitor announces his type truthfully. Furthermore, the mechanism has to be such that sellers do not make losses ex post, i.e., sellers are protected by limited liability. This is a plausible assumption in many applications, because sellers often have limited funds and can declare bankruptcy if the ex post profit is negative. Thus the mechanism is an "at-will contract", because sellers can freely walk away ex post. Note that if the mechanism is such that sellers never make losses ex post, then they will voluntarily participate in the mechanism (the Bayesian participation constraint is automatically satisfied).

2.2 The Optimization Program

The buyer wants to implement the efficient allocation at the lowest expected transfer payments. Efficiency requires (i) that the contract is allocated to the seller with the lowest production cost and (ii) that a seller who detected the flaw reports it truthfully. Thus, the buyer's optimizing problem boils down to minimizing the expected transfers subject to the constraints of efficiency, Bayesian incentive compatibility, and limited liability. In order to formally set up the optimization program, let $t_{\hat{F}_i, \hat{F}_j}(\hat{c}_i, \hat{c}_j)$ be the transfer received by seller i if he reports to be of type $(\hat{c}_i, \hat{F}_i)$ and the competitor, seller j, reports to be of type $(\hat{c}_j, \hat{F}_j)$.

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[5]In the Herweg and Schmidt (2018) model the renegotiation surplus that accrues to the seller is denoted by $(1 - \alpha)S^R$.

[6]For a more detailed justification of the assumption that renegotiation is inefficient, see the discussions in Herweg and Schmidt (2017, 2018). A psychological foundation for inefficient renegotiation based on loss aversion is provided by Herweg and Schmidt (2015).

The buyer solves the following program:

$$\min \quad \mathbb{E}_{c_1,c_2,F_1,F_2}\left[t_{F_1,F_2}(c_1,c_2) + t_{F_2,F_1}(c_2,c_1)\right] \tag{1}$$

subject to: $\forall\, (c_i, F_i) \in \mathcal{C} \times \mathcal{F}$

$$D(F_1, F_2) = \begin{cases} D_0 & \text{if } F_1 = F_2 = \emptyset, \\ D_f & \text{otherwise.} \end{cases} \tag{ED}$$

$$\omega_i = \omega(c_i, c_j) = \begin{cases} 1 & \text{if } c_i < c_j \\ 0.5 & \text{if } c_i = c_j \\ 0 & \text{if } c_i > c_j. \end{cases} \tag{EP}$$

$$t_{F_i,F_j}(c_i,c_j) - \omega(c_i,c_j)c_i \geq 0 \tag{LL}$$

$$\mathbb{E}_{c_j,F_j}\left[t_{\emptyset,F_j}(c_i,c_j) - \omega(c_i,c_j)c_i \,\Big|\, F_i = \emptyset\right] \geq \tag{$\text{IC}_{\emptyset\emptyset}$}$$
$$\mathbb{E}_{c_j,F_j}\left[t_{\emptyset,F_j}(\hat{c}_i,c_j) - \omega(\hat{c}_i,c_j)c_i \,\Big|\, F_i = \emptyset\right] \quad \forall\, \hat{c}_i \in \mathcal{C}$$

$$\mathbb{E}_{c_j,F_j}\left[t_{f,F_j}(c_i,c_j) - \omega(c_i,c_j)c_i \,\Big|\, F_i = f\right] \geq \tag{IC_{ff}}$$
$$\mathbb{E}_{c_j,F_j}\left[t_{f,F_j}(\hat{c}_i,c_j) - \omega(\hat{c}_i,c_j)c_i \,\Big|\, F_i = f\right] \quad \forall\, \hat{c}_i \in \mathcal{C}$$

$$\mathbb{E}_{c_j,F_j}\left[t_{f,F_j}(c_i,c_j) - \omega(c_i,c_j)c_i \,\Big|\, F_i = f\right] \geq \tag{$\text{IC}_{f\emptyset}$}$$
$$\mathbb{E}_{c_j,F_j}\left[t_{\emptyset,F_j}(\hat{c}_i,c_j) - \omega(\hat{c}_i,c_j)c_i \,\Big|\, F_i = f\right] + (1-q)\mathbb{E}_{c_j}[\omega(\hat{c}_i,c_j)]S \quad \forall\, \hat{c}_i \in \mathcal{C}$$

Constraint (ED) ensures that the buyer procures the efficient design, given the available information. If at least one seller observes the flaw, the design is adjusted to D_f. Constraint (EP) requires efficient production, i.e. the seller with the lower cost has to produce the good. We assume that sellers can freely walk away ex post, which implies that ex post profits have to be non-negative in any state. This is required by the limited liability constraint (LL), which is sometimes called an ex-post participation constraint. Finally, three incentive compatibility constraints need to be satisfied. First, a seller who did not spot the flaw must find it optimal to report his cost truthfully ($\text{IC}_{\emptyset\emptyset}$). Second, a seller who spotted the flaw and reports it truthfully must also report his cost truthfully (IC_{ff}). Third, a joint deviation – misreporting on costs and concealing the flaw – of a seller who spotted the flaw is also not profitable ($\text{IC}_{f\emptyset}$). Importantly, a seller who spotted the flaw and concealed it but received the contract nevertheless benefits from contract renegotiation if and only if his competitor did not observe (and report) the flaw.

Note that a seller who spotted the flaw assigns a different probability to the event that the competitor spotted the flaw than a seller who did not observe the flaw. This makes the

types of the sellers correlated. Seller i with type $F_i = \emptyset$ expects that seller $j \neq i$ is of type $F_j = f$ with probability

$$prob(F_j = f | F_i = \emptyset) = \frac{pq(1-q)}{1-pq} =: \hat{q}. \tag{2}$$

If seller i is of type $F_i = f$, he expects that the competitor is of type $F_j = f$ with probability q. Note that $0 < \hat{q} < q < 1$. If the flaw exists with certainty, $p \to 1$, we have $\hat{q} \to q$ and sellers' types are uncorrelated. If the flaw is observed with certainty (given that it exists), $q \to 1$, so we have $\hat{q} \to 0$ and types are perfectly correlated; i.e., seller i observes the flaw if and only if seller j observes it as well.

As it is well-known for the one-dimensional screening problem with unlimited liability if agents' types are correlated, the principal can elicit agents' private information without leaving an information rent to them (Crémer and McLean, 1988). This full extraction of the surplus requires severe punishment of the agents in some states of the world. Such severe punishments are not feasible if agents are protected by limited liability. We will show that in the multi-dimensional screening problem the rents obtained in one dimension (truthful revelation of the cost type) can be used to relax the limited liability constraint and therefore to cut back on the information rent in the other dimension (flaw type) by exploiting the fact that agents' types are correlated.

2.3   Discussion of Modeling Assumptions

Flaws vs. design improvements: Throughout the paper we talk about design flaws, but the model is equally applicable to the situation of design improvements. With probability p a design improvement is feasible. If this is the case, a seller is aware of the design improvement with probability q. Moreover, if a seller is aware of the design improvement and obtained the contract for the (non-improved) design D_0, he can make a profit of $S > 0$ by renegotiating the contract.

Objective function: We assume that the buyer wants to implement the efficient allocation. This is the case if the buyer is a government agency that wants to maximize social welfare. The focus on the efficient allocation also allows us not to specify the buyer's loss in utility if the flawed design is implemented. If the buyer wants to maximize expected profits, she trades off paying an information rent for obtaining information about design improvements with the expected loss in utility from procuring a suboptimal design.

Renegotiation: We do not model a specific renegotiation game explicitly because we want to keep this part of the model as general as possible and because we want to focus on the mechanism design problem.[7] Note, however, that by assuming that the seller gets a fixed amount S from renegotiation we rule out the possibility that the buyer "designs" the renegotiation game.[8]

Partial verifiability: The type of mechanism design problem that we analyze is known as a mechanism design problem with "partially verifiable information" (Green and Laffont, 1986). The message $\hat{F}_i = f$ can be sent by seller i only if his true type is $F_i = f$. The message $\hat{F}_i = \emptyset$ can always be sent. In such a setup with multiple agents, the *revelation principle* can be applied only if the set of feasible reports (the evidentiary structure) is strongly normal (Bull and Watson, 2007). This assumption is obviously satisfied in our model with binary flaw types.[9]

3   Analysis

We start out by reformulating and simplifying the buyer's minimization program which requires some additional notation. Then we solve the minimization problem for two special cases, perfect correlation and no correlation of sellers' flaw types. This provides some intuition for our main result. Finally, we solve the general problem.

3.1   The Relaxed Minimization Problem

It is useful to define a seller's expected profit from taking part in the mechanism. Let

$$u_{F_i F_j}(c) = \mathbb{E}_{c_j}\left[t_{F_i,F_j}(c,c_j) - \omega(c,c_j)c\right] \tag{3}$$

be the expected utility of a type (c, F_i) who reports his type truthfully and whose competitor is of flaw type F_j. The expectation is taken only with respect to the competitor's cost type c_j, not with respect to his flaw type F_j. Note that the first subscript refers to the seller's

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[7]In Herweg and Schmidt (2018) we model the renegotiation game explicitly.

[8]The design of renegotiation games and how this can be beneficial to the buyer is analyzed by Aghion, Dewatripont, and Rey (1990, 1994). For a critical discussion of the literature on renegotiation design, see Hart (1995, p. 77-78). From a technical perspective, we analyze a mechanism design problem where a seller who is informed about the flaw and obtains the contract can generate an additional surplus of S if none of the sellers reported information f to the buyer. How he can generate this additional surplus is not important for the mechanism design problem. Contract renegotiation is one (and our preferred) interpretation.

[9]For a more detailed discussion see Herweg and Schmidt (2018).

own flaw type and the second to the competitor's flaw tape. Similarly, we can define the expected transfer of seller i who reports (c, F_i) truthfully and whose competitor is of flaw type F_j, formally $t_{F_i F_j}(c) \equiv \mathbb{E}_{c_j}[t_{F_i, F_j}(c, c_j)]$. Using (3) we can rewrite the expected transfer as

$$t_{F_i F_j}(c) = u_{F_i F_j}(c) + [1 - H(c)]c. \tag{4}$$

Recall that by efficient production seller i has to produce the good if his cost type is lower than the competitor's cost type. Thus, seller i's expected production costs are $[1 - H(c)]c$.

We also have to define a seller's expected profit from taking part in the mechanism, where expectations are taken with respect to the competitor's flaw and cost type. Let $\bar{u}_{F_i F_i} \equiv \mathbb{E}_{F_j}[u_{F_i, F_j}(c)]$ be the overall expected profit of seller i who is of flaw type $F_i \in \{\emptyset, f\} \equiv \mathcal{F}$ and cost type $c \in \mathcal{C}$. For the overall expected profit – denoted by a bar – the first subscript refers to the true flaw type and the second one to the announced flaw type. With this notation the incentive compatibility constraints for pure misreporting of the costs, ($\text{IC}_{\emptyset\emptyset}$) and ($\text{IC}_{ff}$), can be written as follows. For all $(c, \hat{c}) \in \mathcal{C}^2$ let

$$\bar{u}_{\emptyset\emptyset}(c) \equiv (1 - \hat{q})u_{\emptyset\emptyset}(c) + \hat{q}u_{\emptyset f}(c) \geq (1 - \hat{q})u_{\emptyset\emptyset}(\hat{c}) + \hat{q}u_{\emptyset f}(\hat{c}) \tag{5}$$

and

$$\bar{u}_{ff}(c) \equiv (1 - q)u_{f\emptyset}(c) + qu_{ff}(c) \geq (1 - q)u_{f\emptyset}(\hat{c}) + qu_{ff}(\hat{c}), \tag{6}$$

respectively.

So far, we considered the incentives to misreport the production cost. Now, we consider the incentives to reveal the observed flaw truthfully. A necessary condition for ($\text{IC}_{f\emptyset}$) to hold is that all cost types c who report the cost truthfully also have an incentive to report the observed flaw truthfully. In other words, a pure misreporting of the flaw type is unprofitable. In order to formally define the incentive compatibility constraint regarding pure misreporting of the flaw, let

$$\bar{u}_{f\emptyset}(c) \equiv (1 - q)u_{\emptyset\emptyset}(c) + qu_{\emptyset f}(c). \tag{7}$$

If sellers truthfully report the cost type, they also truthfully report the flaw type if for all $c \in \mathcal{C}$:

$$\bar{u}_{ff}(c) \geq \bar{u}_{f\emptyset}(c) + (1 - q)[1 - H(c)]S. \tag{$\text{IC}_{f\emptyset}^{Relaxed}$}$$

By concealing the flaw the seller can make a profit of $S > 0$ if he obtains the contract and the competitor has not reported the flaw.

In the following we will focus on the buyer's *relaxed optimization program*. As the relaxed program we define the minimization problem where the constraint $(IC_{f\emptyset})$ is replaced by the constraint $(IC_{f\emptyset}^{Relaxed})$. The relaxed program guarantees that a seller has no incentive to solely misreport his cost or to solely misreport his flaw type. Joint deviations – misreporting the cost and the flaw type – are ignored. If joint deviations are unprofitable, the solution to the relaxed problem is also a solution to the buyer's original problem. After having solved the relaxed problem, we will show that the obtained solution also solves the original optimization problem.

Lemma 1. *The relaxed minimization program is given by*

$$\min \int_{\underline{c}}^{\bar{c}} \left[pq \ \bar{u}_{ff}(c) + (1 - pq) \ \bar{u}_{\emptyset\emptyset}(c) \right] h(c)dc \tag{8}$$

subject to: $\forall \ c \in \mathcal{C}$

$$\bar{u}_{\emptyset\emptyset}(c) = \bar{u}_{\emptyset\emptyset}(\bar{c}) + \int_c^{\bar{c}} [1 - H(z)]dz \tag{9}$$

$$\bar{u}_{ff}(c) = \bar{u}_{ff}(\bar{c}) + \int_c^{\bar{c}} [1 - H(z)]dz \tag{10}$$

$$\bar{u}_{ff}(c) \geq \bar{u}_{f\emptyset}(c) + (1 - q)[1 - H(c)]S \tag{11}$$

$$\bar{u}_{\emptyset\emptyset}(c) = (1 - \hat{q})u_{\emptyset\emptyset}(c) + \hat{q}u_{\emptyset f}(c) \tag{12}$$

$$\bar{u}_{ff}(c) = (1 - q)u_{f\emptyset}(c) + qu_{ff}(c) \tag{13}$$

$$\bar{u}_{f\emptyset}(c) = (1 - q)u_{\emptyset\emptyset}(c) + qu_{\emptyset f}(c) \tag{14}$$

$$u_{F_i F_j}(c) \geq 0 \quad \forall \ (F_i, F_j) \in \mathcal{F}^2 \tag{15}$$

The buyer minimizes the expected transfer payments. With probability p the flaw exists and is then observed with probability q by a seller. With probability $1 - pq$ a seller does not report the flaw, either because it does not exist or because he did not observe it even though it exists.

It is well-known from the one-dimensional screening literature that the expected utilities are pinned down by incentive compatibility up to a constant. The equality constraints (9) and (10) are equivalent to the incentive compatibility constraints regarding pure misreporting of the cost. Inequality constraint (11) is the relaxed incentive compatibility constraint $(IC_{f\emptyset}^{Relaxed})$ – pure misreporting of the flaw type is not profitable.

Equality constraints (12)–(14) are the definitions of the overall expected utility terms. Finally, constraint (15) ensures that the ex post utilities are non-negative, i.e., the limited liability constraint holds.

3.2 Two Special Cases

In this subsection we analyze two special cases. This gives us an upper and a lower bound on the information rent obtained by a seller who is informed about the flaw.

Uncorrelated flaw types: Suppose that the flaw in design D_0 exists with certainty but the buyer does not know how to fix it. Each seller independently knows with probability q how to fix the design flaw. Formally, suppose that $p = 1$ and thus $\hat{q} = q$, which implies that sellers' types are uncorrelated.

With $\hat{q} = q$ we have $\bar{u}_{f\emptyset}(c) \equiv \bar{u}_{\emptyset\emptyset}(c)$. Thus, the constraints simplify to: $\forall\, c \in \mathcal{C}$

$$\bar{u}_{\emptyset\emptyset}(c) = \bar{u}_{\emptyset\emptyset}(\bar{c}) + \int_c^{\bar{c}} [1 - H(z)]dz \tag{16}$$

$$\bar{u}_{ff}(c) = \bar{u}_{ff}(\bar{c}) + \int_c^{\bar{c}} [1 - H(z)]dz \tag{17}$$

$$\bar{u}_{ff}(c) \geq \bar{u}_{\emptyset\emptyset}(c) + (1-q)[1 - H(c)]S \tag{18}$$

$$\bar{u}_{\emptyset\emptyset}(\bar{c}) \geq 0, \quad \bar{u}_{ff}(\bar{c}) \geq 0 \tag{19}$$

The first two constraints ensure that each seller reports his cost truthfully irrespective of his flaw type. The third constraint ensures that solely misreporting the flaw is not profitable. Finally, the expected utilities of the least efficient type, $c_i = \bar{c}$, have to be non-negative. This, together with incentive compatibility, ensures that all ex post profits are weakly positive.

The buyer wants to choose $\bar{u}_{ff}(c)$ and $\bar{u}_{\emptyset\emptyset}(c)$ as low as possible. Hence, specifying $\bar{u}_{\emptyset\emptyset}(\bar{c}) = 0$ is optimal. The least efficient type who did not observe the flaw does not obtain a rent. All other cost types who did not observe the flaw obtain the standard information rent for revealing the cost, $R_c = \int_c^{\bar{c}} [1 - H(z)]dz$.

The buyer also wants to choose the lowest feasible $\bar{u}_{ff}(\bar{c})$. Conditions (17) and (18) hold, if for all $c \in \mathcal{C}$:

$$\bar{u}_{ff}(\bar{c}) + \int_c^{\bar{c}} [1 - H(z)]dz \geq \int_c^{\bar{c}} [1 - H(z)]dz + (1-q)[1 - H(c)]S$$

$$\Longleftrightarrow \bar{u}_{ff}(\bar{c}) \geq (1-q)[1 - H(c)]S \tag{20}$$

The above condition has to hold for all c and is hardest to satisfy for $c = \underline{c}$. Intuitively, type $c = \underline{c}$ has the strongest incentives to conceal that he observed the flaw because he obtains the contract with certainty. Thus, it has to hold that $\bar{u}_{ff}(\bar{c}) \geq (1-q)S$. From this we obtain that it is optimal to specify

$$\bar{u}_{ff}(c) = (1-q)S + \int_c^{\bar{c}} [1 - H(z)]dz. \tag{21}$$

This mechanism corresponds to the ex-post incentive compatible mechanism derived in Herweg and Schmidt (2018). A seller who spotted the flaw and reveals it obtains a bonus of the size of the profits he could obtain from contract renegotiation but only if he is the only one who reported the flaw. The (expected) information rent for revealing the flaw is $R_f = (1-q)S$. Additionally, each seller obtains the rent from the standard Vickrey auction. The mechanism obviously satisfies the joint incentive compatibility constraint $(\text{IC}_{f\emptyset})$.

Perfect correlation: Suppose that if the flaw exists, then it is observed by both sellers. Formally, we have $q = 1$ and thus $\hat{q} = 0$. Sellers' flaw types are perfectly correlated: Either both sellers are of type $F_i = \emptyset$ or of type $F_i = f$.

In this case, a seller not only knows his flaw type but also his competitor's flaw type. This implies that $\bar{u}_{ff}(c) = u_{ff}(c)$ and $\bar{u}_{\emptyset\emptyset}(c) = u_{\emptyset\emptyset}(c)$. Thus, the set of constraints the buyer needs to satisfy simplifies to: $\forall\ c \in \mathcal{C}$

$$u_{\emptyset\emptyset}(c) = u_{\emptyset\emptyset}(\bar{c}) + \int_c^{\bar{c}} [1 - H(z)]dz \tag{22}$$

$$u_{ff}(c) = u_{ff}(\bar{c}) + \int_c^{\bar{c}} [1 - H(z)]dz \tag{23}$$

$$u_{ff}(c) \geq u_{\emptyset f}(c) \tag{24}$$

$$u_{F_i F_j}(c) \geq 0 \quad \forall\ (F_i, F_j) \in \mathcal{F}^2 \tag{25}$$

The first two constraints reflect incentive compatibility with regard to costs. The final constraint is the limited liability constraint. Note that constraint (24), the incentive compatibility constraint regarding the flaw type, has become much simpler.

The buyer wants to minimize $u_{\emptyset\emptyset}(c)$ and $u_{ff}(c)$. Here it is optimal to set the utility of the out-of-equilibrium event that only the competitor reported the flaw as low as possible, i.e., $u_{\emptyset f}(c) \equiv 0$ (for all c). Put verbally, a seller who concealed the flaw is "punished" by not obtaining a rent. This also allows the buyer to set the utility of the least efficient type, $c = \bar{c}$, so that for this type the limited liability constraint is binding – irrespective of whether he observed the flaw or not. Formally, it is optimal to set $u_{\emptyset\emptyset}(\bar{c}) = u_{ff}(\bar{c}) = 0$. Importantly, this implies that the buyer can elicit information regarding the flaw without paying an information rent, i.e., $R_f = 0$. The sellers obtain an information rent solely for revealing their private information regarding the production cost, which is given by $R_c = \int_c^{\bar{c}} [1 - H(z)]dz$.

The buyer has one degree of freedom in specifying the optimal mechanism, namely $u_{f\emptyset}(c)$. By choosing $u_{f\emptyset}(c) > 0$ the buyer creates a strict incentive for the informed sellers to report the detected flaw truthfully. Finally, it is straightforward to see that a seller has no incentive

to misreport in both dimension because $u_{\emptyset f}(c) \equiv 0$.

3.3 Solving the General Problem

In order to solve the general relaxed minimization program, we further rewrite the constraint $(\text{IC}_{f\emptyset}^{Relaxed})$, so that it depends only on the functions $\bar{u}_{ff}(\cdot)$ and $\bar{u}_{\emptyset\emptyset}(\cdot)$ but not on $\bar{u}_{f\emptyset}(\cdot)$.

Using (14) allows us to rewrite (11) (constraint $\text{IC}_{f\emptyset}^{Relaxed}$) as follows. For all $c \in \mathcal{C}$:

$$\bar{u}_{ff}(c) \geq (1-q)u_{\emptyset\emptyset}(c) + qu_{\emptyset f}(c) + (1-q)[1-H(c)]S. \tag{26}$$

From the condition (9) and the definition (12) it follows that

$$u_{\emptyset f}(c) = \frac{1}{\hat{q}}\bar{u}_{\emptyset\emptyset}(\bar{c}) + \frac{1}{\hat{q}}\int_c^{\bar{c}}[1-H(z)]dz - \frac{1-\hat{q}}{\hat{q}}u_{\emptyset\emptyset}(c). \tag{27}$$

Substituting (27) for $u_{\emptyset f}(c)$ into inequality (26) and rearranging yields

$$\bar{u}_{ff}(\bar{c}) \geq \frac{q}{\hat{q}}\bar{u}_{\emptyset\emptyset}(\bar{c}) - \frac{q-\hat{q}}{\hat{q}}u_{\emptyset\emptyset}(c) + \frac{q-\hat{q}}{\hat{q}}\int_c^{\bar{c}}[1-H(z)]dz + (1-q)[1-H(c)]S. \tag{28}$$

The buyer wants to minimize $\bar{u}_{ff}(\bar{c})$, so it is optimal to choose the highest feasible $u_{\emptyset\emptyset}(c)$. From (27) it follows that maximizing $u_{\emptyset\emptyset}(c)$ is equivalent to minimizing $u_{\emptyset f}(c)$. Thus, it is optimal to specify $u_{\emptyset f}(c) \equiv 0$ for all $c \in \mathcal{C}$. This result is intuitive: With correlated types, if a seller does not report the flaw but the competitor does, then the seller who did not report the flaw is punished by not obtaining a rent. From this it follows immediately that

$$u_{\emptyset\emptyset}(c) = \frac{1}{1-\hat{q}}\bar{u}_{\emptyset\emptyset}(\bar{c}) + \frac{1}{1-\hat{q}}\int_c^{\bar{c}}[1-H(z)]dz. \tag{29}$$

Types are correlated only imperfectly, and thus – in order to maintain incentive compatibility with respect to cost – the rent a seller obtains if both are uninformed has to be increased. Condition (29) allows us to state the incentive compatibility constraint with respect to the flaw as

$$\bar{u}_{ff}(\bar{c}) \geq \frac{1-q}{1-\hat{q}}\bar{u}_{\emptyset\emptyset}(\bar{c}) - \frac{q-\hat{q}}{1-\hat{q}}\int_c^{\bar{c}}[1-H(z)]dz + (1-q)[1-H(c)]S, \tag{30}$$

which has to hold for all c.

The above considerations allow us to restate the buyer's relaxed optimization problem in a simplified form:

$$\min \int_{\underline{c}}^{\bar{c}}\left\{pq\bar{u}_{ff}(c) + (1-pq)\bar{u}_{\emptyset\emptyset}(c)\right\}h(c)dc \tag{31}$$

subject to $\forall\, c \in \mathcal{C}$:

$$\bar{u}_{\emptyset\emptyset}(c) = \bar{u}_{\emptyset\emptyset}(\bar{c}) + \int_c^{\bar{c}} [1 - H(z)]dz \tag{32}$$

$$\bar{u}_{ff}(c) = \bar{u}_{ff}(\bar{c}) + \int_c^{\bar{c}} [1 - H(z)]dz \tag{33}$$

$$\bar{u}_{ff}(\bar{c}) \geq \frac{1-q}{1-\hat{q}} \bar{u}_{\emptyset\emptyset}(\bar{c}) - \frac{q - \hat{q}}{1 - \hat{q}} \int_c^{\bar{c}} [1 - H(z)]dz + (1-q)[1 - H(c)]S \tag{34}$$

$$\bar{u}_{ff}(\bar{c}) \geq 0, \quad \bar{u}_{\emptyset\emptyset}(\bar{c}) \geq 0 \tag{35}$$

The buyer has to insure that (i) a seller who did not observe the flaw reports his cost truthfully, (ii) a seller who observed and reports the flaw reports his cost truthfully, (iii) a seller who detected the flaw reports it – irrespective of his cost type, and (iv) no seller type ever makes a loss.

Setting $\bar{u}_{\emptyset\emptyset}(\bar{c}) = 0$ is optimal because it reduces the buyer's expected cost and relaxes the constraint (34). This implies that

$$\bar{u}_{\emptyset\emptyset}(c) = \int_c^{\bar{c}} [1 - H(z)]dz. \tag{36}$$

A seller who did not observe the flaw (and thus does not report it) obtains an expected information rent that equals the standard information rent for revealing the cost type truthfully, $R_c = \int_c^{\bar{c}} [1 - H(z)]dz$. The actual information rent, however, depends on whether or not his competitor reported the flaw.

Now, we investigate a seller's incentive to conceal the flaw. In order to do so, we define

$$\psi(c) \equiv (1-q)[1 - H(c)]S - \frac{q - \hat{q}}{1 - \hat{q}} \int_c^{\bar{c}} [1 - H(z)]dz. \tag{37}$$

The incentive compatibility constraint (34) is equivalent to $\bar{u}_{ff}(\bar{c}) \geq \psi(c)$. The function $\psi(c)$ is the net revenue a seller of cost type c can obtain by concealing the flaw. If the seller of cost type c conceals the flaw, he may benefit from contract renegotiation. This requires that the seller obtains the contract, which happens with probability $1 - H(c)$, and that the competitor did not observe the flaw, which happens with probability $1 - q$. Concealing the flaw allows the seller also to obtain a higher information rent for revealing the cost type, namely $[1/(1-\hat{q})]R_c$, if the competitor did not observe the flaw. If, on the other hand, the competitor observed the flaw, the seller looses his information rent for revealing the cost type truthfully by misreporting the flaw type. The latter effect outweighs the former. Thus, misreporting the flaw type comes at the cost of loosing part of the information rent for revealing the cost type. When contemplating whether to conceal the flaw a seller trades off the positive effect

on potential surplus from renegotiation against the negative effect from a reduced information rent.

Let $c^* \in \arg\max_{c \in \mathcal{C}} \psi(c)$ be the cost type who benefits most from concealing the flaw. Note that the lower the cost type is, the higher is the (expected) benefit from contract renegotiation but the higher is also the information rent R_c that the seller may loose by misreporting his flaw type. We define $\psi^* := \max\{0, \psi(c^*)\}$ so that the optimal (lowest feasible) $\bar{u}_{ff}(\bar{c}) = \psi^*$. Thus,

$$\bar{u}_{ff}(c) = \psi^* + \int_c^{\bar{c}} [1 - H(z)]dz. \tag{38}$$

A seller who spotted the flaw obtains information rent $R_f = \psi^*$ for reporting it and information rent $R_c = \int_c^{\bar{c}} [1 - H(z)]dz$ for truthfully revealing his cost.

Now, we can state the main finding.

Proposition 1. *The buyer's preferred mechanism that implements the efficient allocation as a Bayes Nash equilibrium has the following properties:*

$$\bar{u}_{\emptyset\emptyset}(c) = \int_c^{\bar{c}} [1 - H(z)]dz \tag{i}$$

$$\bar{u}_{ff}(c) = \psi^* + \int_c^{\bar{c}} [1 - H(z)]dz \tag{ii}$$

$$u_{\emptyset f}(c) = 0. \tag{iii}$$

Note that by (i) and (iii) we have pinned down

$$u_{\emptyset\emptyset}(c) = \frac{1}{1 - \hat{q}} \int_c^{\bar{c}} [1 - H(z)]dz.$$

On the other hand, the buyer can freely choose $u_{ff}(c)$ and $u_{f\emptyset}(c)$, as long as (ii) is satisfied and all $u_{ff}(c)$ and $u_{f\emptyset}(c)$ are non-negative. For instance, the optimal mechanism can specify

$$u_{ff}(c) = u_{f\emptyset}(c) = \psi^* + \int_c^{\bar{c}} [1 - H(z)]dz.$$

According to Proposition 1, any seller obtains the standard information rent from the Vickrey auction for revealing his cost, $R_c = \int_c^{\bar{c}} [1 - H(z)]dz$. This information rent is exactly the expected marginal contribution of one seller to the joint surplus (the expected cost advantage).

The expected marginal contribution of a seller who reported the flaw to the joint surplus is at least $(1 - q)S$.[10] The information rent that a seller obtains for revealing the flaw is only

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[10]The marginal contribution to the joint surplus is exactly $(1 - q)S$ if the seller obtains all the surplus from renegotiation and renegotiation is efficient.

$R_f = \psi^* \in [0, (1-q)S]$. This information rent depends crucially on the correlation of sellers' types. Notice that

$$\frac{q - \hat{q}}{1 - \hat{q}} = \frac{q(1-p)}{1 - 2pq + pq^2}.$$

Thus, for $q \to 1$ (perfect correlation) we have $\psi^* = 0$; i.e., the buyer does not have to pay a rent for eliciting information regarding the design flaw. On the other hand, for $p \to 1$ (uncorrelated types) we have $c^* = \underline{c}$ and thus $\psi^* = (1-q)S$. In this case the mechanism coincides with the EPIC-mechanism derived in Herweg and Schmidt (2018).

The buyer can cut back on the information rent for reporting the flaw by exploiting that sellers' flaw types are correlated. This is reminiscent to the full surplus extraction mechanism derived by Crémer and McLean (1988). The Crémer-McLean mechanism, however, relies on unlimited liability on the side of the agents (the sellers). In our multi-dimensional screening problem with limited liability, the buyer can cut back on the information rent in the flaw dimension by putting the information rent from the cost dimension at stake. Technically, by reducing the information rent of seller who misreports the flaw, the buyer relaxes the limited liability constraint.

4   Comparative Statics and Practical Implementation

4.1   Comparative Statics

How does the information rent depend on the underlying distributions? We will focus on the rent that a seller obtains for revealing the flaw, $R_f = \psi^*$. The information rent obtained for revealing the cost type truthfully is the standard rent of a Vickrey auction.

Proposition 2. *In the optimal mechanism, the information rent obtained by a seller who observed the flaw for reporting it, $R_f = \psi^*$, is*

(i) *(weakly) increasing in the probability p with which the flaw exists.*

(ii) *(weakly) decreasing in the probability q with which a seller observes the flaw if it exists.*

If it is more likely that a flaw exists, then it is also more likely that a seller is informed about it and thus obtains an information rent. Proposition 2(i) shows that this is not the end of the story. The information rent is also higher the more likely it is that the flaw exists.

Thus, if p increases, the buyer not only has to pay the information rent more often but she also has to pay a higher rent.

If it is more likely that the flaw is detected by a seller, then it is more likely that sellers obtain an information rent for revealing the flaw. However, Proposition 2(ii) shows that the information rent that a seller obtains is reduced.

We have already seen that the information rent is maximized if sellers' types are uncorrelated ($q = 0$) and minimized if sellers' types are perfectly correlated ($p = 1$). From Proposition 2 it follows that the information rent R_f is decreasing in the correlation of sellers' flaw types. In order to see this, note that the Pearson correlation coefficient is given by

$$\rho = \frac{cov(F_1, F_2)}{\sigma(F_1)\sigma(F_2)} = \frac{q(1-p)}{1-pq} \in [0, 1], \tag{39}$$

where $\sigma(\cdot)$ is the standard deviation and $cov(\cdot)$ the covariance. It can readily be established that the correlation coefficient ρ is increasing in q and decreasing in p. Hence, the more sellers' types are correlated, the lower is the necessary information rent for revealing the flaw. The reason is that a seller who conceals the flaw expects with a high probability that the competitor reports the flaw. In this case the seller is punished by not obtaining any rent, $u_{\emptyset f} = 0$. Now, the higher the degree of correlation, the more likely it is that a seller is punished for misreporting the flaw type, which reduces his incentives to do so.

It is important to note that sellers are protected by limited liability and thus the possible punishment is bounded. The effect works solely via the increased probability of the punishment. If unbounded punishment is feasible, sellers do not obtain a rent for revealing the flaw as long as types are not uncorrelated. Then the Crémer and McLean (1988) mechanisms can be used.

4.2   Implementation via an Indirect Mechanism

How can the optimal mechanism of Proposition 1 be implemented in practice? The first important observation is that the mechanism requires a joint elicitation of information on costs and flaws. The reason is that the rent an uniformed seller obtains for revealing his cost depends on whether the competitor observed the flaw. Thus, sequential procedures that first ask for design improvements and thereafter award the contract for the potentially improved design cannot be used.

In the following we describe an indirect mechanism that is equivalent to the direct mech-

anism of Proposition 1 and that could be used in practice. Each seller i has to place a bid $b_i = (p_i, F_i)$, where p_i is a price bid and $F_i = \{\emptyset, f\}$ denotes whether or not the seller reports the flaw. Let $\tau_{F_i, F_j}(p_i, p_j)$ be the transfer that seller i obtains when placing bid $b_i = (p_i, F_i)$ and his competitor places the bid $b_j = (p_j, F_j)$. The good has to be produced by the seller who places the lower price bid.

For $F_i = f$ the payment can be made independent of $F_j \in \{\emptyset, f\}$:

$$
\tau_{f,F_j} = \begin{cases} \psi^* & \text{if } p_i > p_j \\ \psi^* + p_j & \text{if } p_i \leq p_j \end{cases}
$$

The informed seller obtains a fixed payment ψ^* for reporting the flaw. If he has to produce the good, the price he obtains is determined by the price bid of his competitor. Here, the mechanism can be interpreted as augmented second-price auction.

The payment of a seller who did not observe the flaw depend on whether the competitor observed it. If the competitor observed the flaw the payment is

$$
\tau_{\emptyset,f} = \begin{cases} 0 & \text{if } p_i > p_j \\ p_i & \text{if } p_i \leq p_j \end{cases}
$$

If the competitor did not observe the flaw either, the payment is

$$
\tau_{\emptyset,\emptyset} = \begin{cases} 0 & \text{if } p_i > p_j \\ p_i + \frac{1}{1-\hat{q}}(p_j - p_i) & \text{if } p_i \leq p_j \end{cases}
$$

These payments are more complicated than the payments for a seller who observed the flaw. Here, the payment structure is not reminiscent to a standard second-price auction.

The indirect mechanism described above implements the same allocation as the direct mechanism described in Proposition 1. The indirect mechanism asks for price bids and whether or not a seller observed the flaw, which can be implemented in practice. The payment rules, however, are relatively complicated. In order to specify the payment $\tau_{\emptyset,\emptyset}$ the buyer has to know $\hat{q}$ perfectly, i.e., the buyer has to know the probability that the flaw exists (p) and the probability that it is detected by a seller (p) perfectly. Moreover, in order to specify τ_{f,F_j} the buyer has to be able to calculate ψ^*. This requires next to the knowledge of p and q also perfect knowledge of the distribution of production costs $H(c)$. Furthermore, all of this has to be common knowledge.

Thus, the optimal mechanism that implements the efficient allocation is informationally very demanding. Note that if $p = 0$ (a flaw never exists) the efficient allocation can be

implemented with a second-price auction, which does not require any prior knowledge of the distribution functions. In practice, the buyer often lacks this information. Instead of using this complicated mechanism, the buyer could use the following simple sequential procedure. First, each seller is asked whether he observed the flaw. If only one seller observed the flaw, he obtains a fixed payment $\tau_f = S$. Second, the buyer runs a standard second-price sealed-bid auction for the best design she is aware of. This mechanism is equivalent to the ex post incentive compatible mechanism derived in Herweg and Schmidt (2018). It implements the efficient allocation (satisfies (ex post) incentive compatibility (EPIC) and limited liability). The expected costs to the buyer are higher than under the Bayesian incentive compatible mechanism. The advantage of the ex post incentive compatible mechanism is that it does not require any prior knowledge of the distribution functions.

How large is the difference in payments for the buyer from using the simple EPIC mechanism instead of the Bayesian incentive compatible mechanism? The difference in the (expected) information rent obtained by an informed seller is $L = (1 - q)S - \psi^*$, which – for $\psi^* > 0$ – is given by

$$L = (1 - q)H(c^*)S + \frac{q - \hat{q}}{1 - \hat{q}} \int_{c^*}^{\bar{c}} [1 - H(z)]dz. \tag{40}$$

Suppose $c^* = \underline{c}$, which, for instance, is the case if S is relatively large. In this scenario, we have

$$L = \frac{q - \hat{q}}{1 - \hat{q}} \int_{\underline{c}}^{\bar{c}} [1 - H(z)]dz < \frac{q - \hat{q}}{1 - \hat{q}}(\bar{c} - \underline{c}). \tag{41}$$

If cost differences are rather low, $\bar{c} \to \underline{c}$, the "loss" to the buyer from using the simple mechanism is low ($L \approx 0$). Hence, in many situations where differences in production costs are not very large, the correlation of sellers' types is not too strong, but sellers' benefits from contract renegotiation are noticeable, the harm to the buyer from using the simple mechanism is low. In these cases the EPIC mechanism should be used. It is approximately optimal and does not require any prior knowledge of the likelihood of a flaw, sellers' abilities to detect one, and sellers' production costs.

5   Conclusion

Our paper derives the efficient Bayesian incentive compatible procurement mechanism when sellers have two dimensional private information. Each seller privately observes his cost type and whether or not he is able to benefit from potential contract renegotiation (may observe

a design flaw). In such a situation a standard price-only auction can be highly inefficient: A seller who spotted the flaw will keep this information to himself, bid more aggressively to win the auction, and then renegotiate when being in a bilateral monopoly position. If renegotiation involves large adjustment costs, the price-only auction is highly inefficient.

The optimal mechanism always allocates the contract to the seller with the lower cost and induces all informed sellers to report the design flaw to the buyer ex ante. By the very nature of the model, sellers' flaw types are correlated (a seller can observe the flaw only if it exists). We assume that sellers are protected by limited liability so that the buyer cannot exploit the correlation of types by using a Crémer and McLean (1988) mechanism. Nevertheless, the correlation of types allows the seller to cut back on the information rent. This is achieved by reducing the rent for revealing the cost truthfully if a seller misreports his flaw type. The optimal mechanism awards a fixed (expected) bonus for revealing the flaw and punishes a firm that does not report the flaw in case the competitor did report it.

The practicable applicability of the optimal mechanism may be limited due to its high informational demands. In particular, all underlying distributions have to be common knowledge. Moreover, the optimal mechanism does not allow for a sequential elicitation of information regarding the design and regarding the costs (award of contract).

Throughout the paper we focused on the implementation of the efficient allocation. If the buyer is a private enterprise, she may be interested in profit maximization instead of efficiency. In this case the traditional rent extraction versus efficiency tradeoff arises. The buyer may prefer not to procure the good at all if both sellers have rather high cost. This allows to cut back on the information rent sellers obtain for revealing the cost type. Reducing the information rent for revealing the cost type truthfully, however, has a potentially negative impact on the information rent for reporting the flaw. The buyer uses the former rent to punish a seller who misreports his flaw type, i.e., to reduce the latter rent. Thus, the revenue maximizing mechanism may be similar to the efficient mechanism. The analysis of the revenue maximizing Bayesian incentive compatible mechanism is an interesting topic for future research.

A Proofs

Proof of Lemma 1. Using the standard techniques (Mas-Colell, Whinston, and Green, 1995, p. 888) shows that incentive compatibility with respect to costs is equivalent to

$$\bar{u}'_{F_i F_i}(c) = -[1 - H(c)] \quad \text{for } F_i \in \{\emptyset, f\}. \tag{A.1}$$

Hence, Bayesian incentive compatibility is satisfied if and only if for all $\mathcal{F} \in \{\emptyset, f\}$

$$\bar{u}_{F_i F_i}(c) = \bar{u}_{F_i F_i}(\bar{c}) + \int_c^{\bar{c}} [1 - H(z)] dz. \tag{A.2}$$

This gives us constraints (9) and (10).

Now, we rewrite the buyer's target function, which is given by

$$T = 2 \int_{\underline{c}}^{\bar{c}} \bigg\{ prob(F_1 = F_2 = f) t_{ff}(c) + prob(F_1 = f, F_2 = \emptyset) t_{f\emptyset}(c)$$
$$+ prob(F_1 = \emptyset, F_2 = f) t_{\emptyset f}(c) + prob(F_1 = F_2 = \emptyset) t_{\emptyset\emptyset}(c) \bigg\} h(c) dc. \tag{A.3}$$

Notice that

$$prob(F_1 = F_2 = f) = p\, q^2 \tag{A.4}$$
$$prob(F_1 = f, F_2 = \emptyset) = prob(F_1 = \emptyset, F_2 = f) = p\, q(1 - q) \tag{A.5}$$
$$prob(F_1 = F_2 = \emptyset) = 1 - p\, q(2 - q). \tag{A.6}$$

Using the definition of the expected transfers (4), allows us to rewrite the target function of the buyer as follows:

$$T = 2 \int_{\underline{c}}^{\bar{c}} \bigg\{ pq[q u_{ff}(c) + (1 - q) u_{f\emptyset}(c)]$$
$$+ (1 - pq) \left[\frac{pq(1 - q)}{1 - pq} u_{\emptyset f}(c) + \frac{1 - pq(2 - q)}{1 - pq} u_{\emptyset\emptyset}(c) \right] \bigg\} h(c) dc$$
$$+ 2 \int_{\underline{c}}^{\bar{c}} [1 - H(c)] c\, h(c) dc. \tag{A.7}$$

In order to obtain the target function provided in the lemma, we use the definitions of $\bar{u}_{F_i F_i}(c)$ and ignore factors that are constant to the buyer. $\qquad\square$

Proof of Proposition 1. It remains to be shown that a seller who spotted the flaw has no incentive to conceal it *and* to misreport his cost.

We consider a seller of type (c, f). If this seller reports his type truthfully, his expected utility is $\bar{u}_{ff}(c) = \psi^* + \int_c^{\bar{c}}[1 - H(z)]dz$. Suppose, this seller reports type $(\hat{c}, \emptyset)$. The expected utility is

$$U(\hat{c}, \emptyset | c, f) = (1 - q)[t_{\emptyset\emptyset}(\hat{c}) - (1 - H(\hat{c}))c]$$
$$+ q[t_{\emptyset f}(\hat{c}) - (1 - H(\hat{c}))c] + (1 - q)[1 - H(\hat{c})]S, \quad \text{(A.8)}$$

which can be written as

$$U(\hat{c}, \emptyset | c, f) = (1 - q)[\underbrace{t_{\emptyset\emptyset}(\hat{c}) - (1 - H(\hat{c}))\hat{c}}_{=u_{\emptyset\emptyset}(c)} - (1 - H(\hat{c}))(\hat{c} - c)]$$
$$+ q[\underbrace{t_{\emptyset f}(\hat{c}) - (1 - H(\hat{c}))\hat{c}}_{=u_{\emptyset f}(c)=0} - (1 - H(\hat{c}))(\hat{c} - c)] + (1 - q)[1 - H(\hat{c})]S. \quad \text{(A.9)}$$

Thus, the expected profit from a joint deviation is

$$U(\hat{c}, \emptyset | c, f) = (1 - q)u_{\emptyset\emptyset}(\hat{c}) + [1 - H(\hat{c})](\hat{c} - c) + [1 - H(\hat{c})]S$$
$$= \frac{1 - q}{1 - \hat{q}} \int_{\hat{c}}^{\bar{c}}[1 - H(z)]dz + [1 - H(\hat{c})](\hat{c} - c) + [1 - H(\hat{c})]S. \quad \text{(A.10)}$$

Hence, there is no incentive to deviate if

$$\psi^* + \int_c^{\bar{c}}[1 - H(z)]dz \geq \frac{1 - q}{1 - \hat{q}} \int_{\hat{c}}^{\bar{c}}[1 - H(z)]dz + [1 - H(\hat{c})](\hat{c} - c) + [1 - H(\hat{c})]S. \quad \text{(A.11)}$$

We distinguish two case, understating and exaggerating the own cost.

Case (a) $\hat{c} < c$: There is no incentive to misreport if

$$\psi^* \geq \underbrace{(1 - q)[1 - H(\hat{c})]S - \frac{q - \hat{q}}{1 - \hat{q}} \int_{\hat{c}}^{\bar{c}}[1 - H(z)]dz}_{=\psi(\hat{c}) \leq \psi^*}$$
$$+ \underbrace{\int_{\hat{c}}^{c}[1 - H(z)]dz - (c - \hat{c})[1 - H(\hat{c})]}_{<0}. \quad \text{(A.12)}$$

The first line of the right-hand side is smaller that ψ^* by the definition of $\psi(\cdot)$ and ψ^*. The second line is negative because $1 - H(\cdot)$ is a strictly decreasing function and $c > \hat{c}$.

Case (b) $\hat{c} > c$: There is no incentive to misreport if

$$\psi^* \geq \underbrace{(1 - q)[1 - H(\hat{c})]S - \frac{q - \hat{q}}{1 - \hat{q}} \int_{\hat{c}}^{\bar{c}}[1 - H(z)]dz}_{=\psi(\hat{c}) \leq \psi^*}$$
$$\underbrace{- \int_c^{\hat{c}}[1 - H(z)]dz + (\hat{c} - c)[1 - H(\hat{c})]}_{<0}. \quad \text{(A.13)}$$

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The first line of the right-hand side is smaller that ψ^* by the definition of $\psi(\cdot)$ and ψ^*. The second line is negative because $1 - H(\cdot)$ is a strictly decreasing function and $c < \hat{c}$. $\square$

Proof of Proposition 2. Let

$$A(p, q) = \frac{q(1 - p)}{1 - 2pq + pq^2} \tag{A.14}$$

and note that $\partial A / \partial q > 0$ for $p < 1$ and $\partial A / \partial p < 0$ for $q < 1$. Thus,

$$\psi(c) = (1 - q)[1 - H(c)]S - A(p, q) \int_c^{\bar{c}} [1 - H(z)]dz. \tag{A.15}$$

(i) Consider two distributions $i = 1, 2$ that differ only in the probability p_i with which a flaw exists. Let $p_1 < p_2$. Let ψ_i be the function ψ for $p = p_i$ and let c_i^* be the corresponding cost type that maximizes ψ_i. Then, the following holds

$$\psi_1(c_1^*) = (1 - q)[1 - H(c_1^*)]S - A(p_1, q) \int_{c_1^*}^{\bar{c}} [1 - H(z)]dz \tag{A.16}$$

$$< (1 - q)[1 - H(c_1^*)]S - A(p_2, q) \int_{c_1^*}^{\bar{c}} [1 - H(z)]dz \tag{A.17}$$

$$\leq (1 - q)[1 - H(c_2^*)]S - A(p_2, q) \int_{c_2^*}^{\bar{c}} [1 - H(z)]dz = \psi_2(c_2^*). \tag{A.18}$$

The first inequality follows from the fact that $A(\cdot)$ is decreasing in p. The second inequality follows from the definition of c_2^*. Noting that $\psi^* = \max\{0, \psi(c^*)\}$ completes the first part of the proof.

(ii) Consider two distributions $i = 1, 2$ that differ only in the probability q_i with which a seller observes the flaw. Let $q_1 < q_2$. Let ψ_i be the function ψ for $q = q_i$ and let c_i^* be the corresponding cost type that maximizes ψ_i. Then, the following holds

$$\psi_2(c_2^*) = (1 - q_2)[1 - H(c_2^*)]S - A(p, q_2) \int_{c_2^*}^{\bar{c}} [1 - H(z)]dz \tag{A.19}$$

$$< (1 - q_1)[1 - H(c_2^*)]S - A(p, q_1) \int_{c_2^*}^{\bar{c}} [1 - H(z)]dz \tag{A.20}$$

$$\leq (1 - q_1)[1 - H(c_1^*)]S - A(p, q_1) \int_{c_1^*}^{\bar{c}} [1 - H(z)]dz = \psi_1(c_1^*). \tag{A.21}$$

The first inequality follows from the fact that $q_1 < q_2$ and that $A(\cdot)$ is increasing in q. The second inequality follows from the definition of c_1^*. $\square$

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