Selling Dreams: Endogenous Optimism in Lending Markets

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Abstract

We propose a simple model of borrower optimism in competitive lending markets with asymmetric information. Borrowers in our model engage in self-deception to arrive at a belief that optimally trades off the anticipatory utility benefits and material costs of optimism. Lenders' contract design shapes these benefits and costs. The model yields three key results. First, the borrower's motivated cognition increases her material welfare, regardless of whether or not she ends up being optimisitic in equilibrium. Our model thus helps explain why wishful thinking is not driven out of markets. Second, in line with empirical evidence, a low cost of lending and a booming economy lead to optimism and the widespread collateralization of loans. Third, equilibrium collateral requirements may be inefficiently high.

Keywords: optimal expectations, motivated cognition, wishful thinking, financial crisis, lending markets, screening.
JEL: D86, D82, G33.

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1 Introduction

Decades of research in psychology and behavioral economics document people's tendency to become optimistic by means of self-deception, biased updating, and selective recall.[1] The literature on motivated cognition argues that people value optimism because, among other things, it enables them to savor their prospective riches and to feel less anxious about an uncertain future. At the same time, optimism is kept in check by the costs associated with the bad decisions it gives rise to. But while the idea that beliefs are responsive to the material costs of bad decisions is present in almost all economic models of motivated cognition, it appears to be at odds with evidence of costly optimism among entrepreneurs, CEOs and prospective homeowners.[2] Should the market not provide strong incentives for these individuals to resist the lure of optimism?

In this paper, we show that competitive lending markets may actually reward the human tendency to engage in motivated cognition and therefore rationalize its prevalence among market actors. Moreover, we show that explicitly modeling borrowers' tendency to self-deceive can shed light on various observed features of lending markets.

Consider a borrower in need of a loan for a risky investment project. Suppose that she is privately informed about her risk of failure, which may be either high or low. In our leading example the borrower is an entrepreneur, but we may also think of her as a prospective home owner who faces uncertainty over her future income. Our model's key assumption is that the borrower may engage in self-deception about her risk. If a high-risk borrower believes that she has a low risk, then she obtains additional utility from anticipation. But biased beliefs may come at the cost of agreeing to contractual terms that are detrimental to her material payoffs in light of her actual risk. Following Brunnermeier and Parker (2005), the borrower chooses the belief that optimally trades off these anticipatory utility benefits and material costs of optimism: she forms optimal expectations.

Lenders are rational and compete over the borrower. They design contracts that specify a repayment if the project succeeds and borrower collateral that is seized if the project fails. By setting contractual terms, lenders exert significant influence over the costs and benefits that the borrower associates with being optimistic.

[1] See Kunda (1990) and Bénabou and Tirole (2016) for surveys of these literatures in psychology and economics, respectively.

[2] Leading economic models include Akerlof and Dickens (1982), Brunnermeier and Parker (2005), Bénabou and Tirole (2002). See Koellinger et al. (2007) and Dawson et al. (2014) for evidence on optimistic entrepreneurs, Malmendier and Tate (2008) for overconfident CEOs, and Arkes (2001) for overconfidence leading to faulty forecasts.

In this setting, collateral requirements can serve as a screening device because, in expectation, a low-risk borrower finds it cheaper to pledge costly collateral than a high-risk borrower (as in Bester 1985 and Besanko and Thakor 1987). However, when faced with a borrower who may self-deceive, a lender who wishes to separate risk types needs to assure not only incentive compatibility, but also realism on behalf of the high-risk type. We show that keeping borrowers realistic is a more stringent requirement than incentive compatibility and requires giving up an additional material rent to the high-risk borrower.

As an alternative to screening, lenders may offer pooling contracts, which allow the high-risk borrower to become optimistic. The pooling allocation that arises features positive collateral requirement, not as a screening device, but as a means to extract surplus from the self-deceived high-risk borrower to the actual low-risk borrower, who lenders compete over. Such pooling with positive collateral requirements does not occur in the benchmark without anticipatory utility concerns.

In equilibrium, borrowers are screened when the weight they place on anticipatory utility is low and pooled when anticipatory utility concerns are more important. When borrowers are separated, high-risk borrowers have to be paid for their realism. When borrowers are pooled, high-risk borrowers piggy back on the favorable contractual terms aimed at low risks. In both cases, the high-risk borrower's motivated cognition i.e. her anticipatory utility concerns and ability to self-deceive, allows her to obtain higher *material* payoffs.

By showing that motivated cognition may be profitable, our model helps explain its prevalence in market settings. The adaptiveness of optimal expectations does not obtain in individual decision-making problems, where they always leads to weakly lower material payoffs (Brunnermeier and Parker, 2005). Our result counters criticism from Von Hippel and Trivers (2011), who argue that utility from beliefs is unlikely to be a plausible motive for self-deception as it would not survive cultural or natural selection.

Our second main result is that collateralization and optimism of high-risk borrowers occurs when the cost of lending is low and when the economy is booming. A low risk-free rate and profitable projects increase the spoils accruing to a successful borrower and thus make the dream of being successful more attractive. This in turn makes it more expensive to keep high-risk borrowers from self-deceiving, so lenders give up on their attempt to screen and “sell dreams” instead.

In a 2009 speech, former chairman of the US Federal Reserve Bank, Ben Bernanke, put forward the notion that active screening of borrowers had been all but abandoned prior to the 2008 financial crisis, partly because of the "cheap credit" environment:[3]

> Financial institutions reacted to the surplus of available funds by competing aggressively for borrowers, and, in the years leading up to the crisis, credit to both households and businesses became relatively cheap and easy to obtain. One important consequence was a housing boom in the United States, a boom that was fuelled in large part by a rapid expansion of mortgage lending. Unfortunately, much of this lending was poorly done, involving, for example, little or no down payment by the borrower or insufficient consideration by the lender of the borrower's ability to make the monthly payments.

In line with this characterization, our model captures how a decrease in the risk-free rate, for example due to an influx of foreign savings, can lead lenders in an "aggressively" competitive market to relinquish the screening of borrowers and move toward a pooling allocation with lower repayments (cheap credit) and higher collateral requirements (more mortgage backed loans) on high-risk types. This move is associated with more optimistic beliefs on behalf of borrowers. A correlated bad realization of credit risk then entails widespread disillusionment and a high prevalence of foreclosures. The negative correlation between the risk-free rate and the likelihood of collateral use in loan contracts is also supported by evidence on Spanish business loans (Jiménez et al., 2006).[4]

To study welfare in our setting, we need to consider an emotionally enriched net present value of the investment project, which takes agents' anticipatory utility concerns into account. We find that although anticipatory utility concerns imply that some positive collateral is optimal, collateral in equilibrium may be inefficiently high and justify policy intervention.

This paper contributes to a growing literature on consumers with behavioral biases that face rational firms (see Spiegler (2011) and Kőszegi (2014) for surveys). Crucially, for an analysis that treats beliefs as an endogenous outcome, whether or not a borrower ultimately self-deceives in our model is determined in the equilibrium. Our approach

[3]Speech delivered at the Morehouse College, Atlanta, Georgia. Available online at `http://www.federalreserve.gov/newsevents/speech/bernanke20090414a.htm`

[4]Moreover, pooling of risk types with positive collateral requirements is in line with empirical evidence that collateral use is prevalent across risk classes and not necessarily correlated with ex-ante measures of risk (Berger and Udell, 1990), even though the use of collateral is responsive to changes in ex-ante information asymmetry (Berger et al., 2011).

is therefore distinct from previous work that studies optimal contract design with optimists and assumes that the level of optimism is fixed and exogenous (Sandroni and Squintani, 2007; Landier and Thesmar, 2009; de la Rosa, 2011; Spinnewijn, 2013). In Sandroni and Squintani (2007), for example, exogenous optimism leads to an insurance provider having no choice but to offer the same contract to overly optimistic high-risk agents and low-risk agents. Similar pooling of risk-types can be a feature of our equilibrium, but there also exist parameter values such that borrowers remain realistic.

Several theoretical papers investigate the role of anticipatory utility in belief formation (Akerlof and Dickens, 1982; Caplin and Leahy, 2001; Caplin and Eliaz, 2003; Brunnermeier and Parker, 2005; Brunnermeier et al., 2007; Bénabou, 2013), but do not address contracting between rational actors and wishful thinkers. Exceptions are Immordino et al. (2011) and Immordino et al. (2015), who explore the interaction between a rational principal and an agent with motivated beliefs in a moral hazard setting. Their focus on managerial incentives and moral hazard leads them to emphasize the selection of managers on the basis of cognitive traits and does not readily translate into insights relevant to lending markets. In Schwardmann (2019), sellers of preventative health care optimally respond to consumers that may self-deceive about their health risk but are not privately informed.

The idea that people manipulate their beliefs in the service of belief-based utility has empirical support. Experimental subjects self-deceive in order to savor the anticipation of higher payoffs (Mayraz, 2011; Alladi, 2018; Coutts, 2019) and to avoid feeling anxious about the threat of an aversive stimulus (Engelmann et al., 2019). They also process information in a way that helps them maintain a positive self-image (Eil and Rao, 2011; Mobius et al., 2011), but is sensitive to material incentives (Zimmermann, 2020). Oster et al. (2013) find that people at risk of Huntington disease avoid diagnostic tests in order to remain optimistic about their health risk, but that their propensity to do so is responsive to the costs of optimism. Kunda (1990) reviews a sizable psychology literature that provides the seminal evidence for the assertion that belief formation is often driven by the affective benefits of holding biased beliefs.

2 Setup

A risk-neutral borrower seeks a fixed-sized investment G for a project that may either succeed and yield a return $y > 0$ or fail and yield no return. The borrower's risk of failure θ can either be high (θ_H) or low (θ_L), with $0 < \theta_L < \theta_H < 1$. Lenders cannot observe the borrower's risk but know the proportion of high-risk types $\nu = \mathbb{P}(\theta = \theta_H) \in (0,1)$. The expected surplus generated by both high- and low-risk projects is positive, i.e. $G \leq (1-\theta_H)y < (1-\theta_L)y$, so it is efficient for both borrower types to obtain funding.

A contract specifies a repayment R if the project is successful and a non-negative amount of collateral C that is transferred from the borrower to the lender if the project fails. Lenders are risk-neutral and the expected profit from contracting with a borrower of type θ is given by

$$\Pi(\theta, R, C) = (1-\theta) R + \theta\delta(C)C - G$$

where $\delta(C)$ is the lender's valuation of collateral. Assets are heterogeneous in their transferability and the borrower pledges the most transferable assets, with a relatively low wedge between private and market value, first. We assume that the valuation of collateral takes the following functional form

$$\delta(C) = \max(1-\chi C, 0)$$

Perfect transferability of the first unit, i.e. $\delta(0) = 1$, assures an interior solution.[5]

The expected *material* payoff of a borrower of type θ, if she accepts contract (R, C), is given by

$$U(\theta, R, C) = (1-\theta)(y-R) - \theta C$$

The borrower also obtains anticipatory utility from expecting the payoff associated with contract (R, C) evaluated at her subjective belief $\tilde{\theta}$

$$U\left(\tilde{\theta}, R, C\right) = \left(1-\tilde{\theta}\right)(y-R) - \tilde{\theta}C$$

The key assumption of our model is that the high-risk borrower may self-deceive into believing that she has low risk, i.e. $\tilde{\theta}_H \in \{\theta_L, \theta_H\}$.[6] The borrower correctly anticipates

[5] This assumption is similar to insurance models, in which the risk premium associated with a small deviation around full insurance is also of second order and the screening technology (partial insurance) therefore comes at no cost at the margin around the efficient (full insurance) allocation.

[6] This belief choice is equivalent to the "censorship of adverse evidence" process with a naive future self in Bénabou and Tirole (2002). Note that the low-risk borrower never wants to be pessimistic. We further assume that the low-risk borrower cannot become optimistic, so $\tilde{\theta}_L = \theta_L$. This assumption is innocent in the sense that a lender would want to keep low-risk borrowers realistic and could do so for

that after adopting subjective belief $\tilde{\theta}$ she will pick the contract that yields maximum expected utility as evaluated using the subjective belief $\tilde{\theta}$. We denote this belief-contract choice mapping by $(R, C)(\tilde{\theta})$. As in Brunnermeier and Parker (2005) the borrower chooses her belief to maximize a weighted sum of material payoffs and anticipatory utility. For a borrower with actual risk θ and belief $\tilde{\theta}$ this is given by

$$U\left(\theta, (R, C)(\tilde{\theta})\right) + sU\left(\tilde{\theta}, (R, C)(\tilde{\theta})\right)$$

where the parameter $s \geq 0$ captures the weight the borrower places on anticipatory feelings relative to material payoffs. The magnitude of s captures the borrower's innate anxiety as well as the relative salience of future outcomes and will be higher for projects characterized by long delays until the resolution of uncertainty.

Timing.

- **t=0:** Lenders offer finite menus of contracts. The resulting aggregate menu is $\mathcal{C}$.
- **t=1:** The borrower observes both her type $\theta \in \{\theta_L, \theta_H\}$ and the menus of contracts available. She chooses her belief $\tilde{\theta} \in \{\theta_L, \theta_H\}$ so as to maximize $U(\theta, (R, C)(\tilde{\theta})) + sU(\tilde{\theta}, (R, C)(\tilde{\theta}))$.
- **t=2:** The borrower chooses her favored contract $(R, C) \in \mathcal{C}$ given her belief $\tilde{\theta}$, denoted $(R, C)(\tilde{\theta})$. Anticipatory payoffs $U(\tilde{\theta}, (R, C)(\tilde{\theta}))$ are realized.
- **t=3:** Material payoffs $U(\theta, (R, C)(\tilde{\theta}))$ and profits $\Pi(\theta, (R, C)(\tilde{\theta}))$ are realized.

Equilibrium concept. In competitive screening models, pure-strategy subgame-perfect equilibria may fail to exist (Rothschild and Stiglitz, 1976). Offers that are profitable type by type may invite a deviation because they may be less profitable on aggregate than cross-subsidizing offers. But cross-subsidization cannot occur in equilibrium because it is vulnerable to cream skimming. Wilson (1977), Miyazaki (1977) and Spence (1978) (hereafter WMS) argue that cream-skimming deviations that attract only low-risk types rely on a rather implausible form of cooperation on behalf of competitors: when facing an incumbent offer that features cross-subsidization from low-risk to high-risk borrowers, poaching low-risk borrowers is profitable for a competing firm only if the original firm carries on servicing high-risk borrowers at a loss.

free by means of latent threat contract.

The WMS equilibrium concept we will use assures existence by ruling out profitable cream-skimming deviations.[7] Existence in turn facilitates the interpretation of our comparative statics.

Definition 1 *An allocation is a WMS equilibrium if no lender can offer a menu of contracts that earns positive profits and continues to be profitable after competitors have dropped all contracts rendered unprofitable by the addition of the new contracts.*

In any equilibrium allocation, we denote by (R_H, C_H) the contract chosen at $t = 2$ by a borrower with belief $\tilde{\theta} = \theta_H$ and by (R_L, C_L) the contract chosen by a borrower with belief $\tilde{\theta} = \theta_L$, regardless of whether a borrower's self-assessed type coincides with her actual type. If high-risk borrowers are realistic, then each borrower type picks the contract intended for them. If, on the other hand, high-risk borrowers are optimistic in equilibrium and $\tilde{\theta_H} = \theta_L$, then contract (R_H, C_H) is the contract that would have been selected by a realistic high-risk borrower off the equilibrium path.

3 Results

3.1 Equilibrium allocation

In this section, we characterize the WMS equilibrium. We begin by introducing two candidate allocations that, depending on parameter values, make up the equilibrium. Competition between lenders implies that both allocations maximize the self-assessed low-risk borrower's utility under some constraints.

The *best separating allocation* screens borrowers and, therefore, necessarily keeps high-risk borrowers from deceiving themselves. It solves

$$\begin{aligned} &\underset{\{C_H, C_L, R_H, R_L\}}{Max} \quad U(\theta_L, R_L, C_L) \\ &s.t. \quad \begin{cases} \nu\Pi(\theta_H, R_H, C_H) + (1-\nu)\,\Pi(\theta_L, R_L, C_L) \geq 0 & \langle P \rangle \\ \Pi(\theta_H, R_H, C_H) \leq 0 & \langle P_H \rangle \\ (1+s)\,U(\theta_H, R_H, C_H) - U(\theta_H, R_L, C_L) - sU(\theta_L, R_L, C_L) \geq 0 & \langle OE_H \rangle \end{cases} \end{aligned} \tag{1}$$

[7] Netzer and Scheuer (2014) and Mimra and Wambach (2019) provide explicit game-theoretic foundations for WMS allocations.The construction of Netzer and Scheuer (2014) features a vanishing cost of contract withdrawal and posits that firms may only withdraw their entire menu of contract offers but not individual contracts, while that of Mimra and Wambach (2019) introduces several withdrawal periods and the possibility of firm entry at each withdrawal stage.

Constraint P is a zero-profit constraint and states that the lender must break even on average across borrowers. Constraint P_H rules out cross-subsidies from high-risk to low-risk borrowers. OE_H is the optimal expectations constraint. It imposes that, at $t = 1$, the high-risk borrower prefers remaining realistic and picking (R_H, C_H) over self-deceiving and receiving material payoff $U(\theta_H, R_L, C_L)$, evaluated using her objective risk, and anticipatory payoffs $sU(\theta_L, R_L, C_L)$, evaluated using her subjective risk.

The screening of borrowers also requires that contracts are incentive compatible at $t = 2$. A realistic high-risk borrower has to prefer contract (R_H, C_H) over contract (R_L, C_L). Incentive compatibility is absent from program 1 because it is implied by the OE_H constraint. Intuitively, a high-risk borrower merely mimicking a low risk and a high-risk borrower actually believing that she has a low risk make the same choice and obtain the same material payoffs. However, the latter is better off in anticipatory utility terms.

The exact shape of the best separating allocation depends on the proportion of high-risk borrowers. But in all cases, low risks separate by pledging collateral, which, in expectation, is cheaper for them than for high risks.

Definition 2 *The best separating allocation* $\mathcal{M}_{\langle 1\rangle} := \left\{\left(R_{L\langle 1\rangle}, C_{L\langle 1\rangle}\right), \left(R_{H\langle 1\rangle}, C_{H\langle 1\rangle}\right)\right\}$ *is the 4-uple that solves* (1)*. If constraint* P_H *is slack (low proportion of high-risk borrowers* ν*), then* $\mathcal{M}_{\langle 1\rangle}$ *is uniquely characterised by*

$$
\begin{aligned}
&(1+s)U(\theta_H, R_H, C_H) = U(\theta_H, R_L, C_L) + sU(\theta_L, R_L, C_L)\\
&\nu\Pi(\theta_H, R_H, C_H) + (1-\nu)\Pi(\theta_L, R_L, C_L) = 0\\
&C_H = 0\\
&C_L = \frac{1}{2(1+s)}\frac{(\theta_H - \theta_L)\,\nu}{\theta_L\chi(1-\theta_L)\,(1-\nu)}
\end{aligned}
$$

If constraint P_H *is binding (high proportion of high-risk borrowers* ν*), then* $\mathcal{M}_{\langle 1\rangle}$ *is uniquely characterised by*

$$
\begin{aligned}
&(1+s)U(\theta_H, R_H, C_H) = U(\theta_H, R_L, C_L) + sU(\theta_L, R_L, C_L)\\
&\Pi(\theta_H, R_H, C_H) = 0\\
&\Pi(\theta_L, R_L, C_L) = 0\\
&C_H = 0.
\end{aligned}
$$

Next we turn to the *selling dreams allocation*. It features a single contract (R_L, C_L) with $U(\theta_L, R_L, C_L) \geq 0$ and allows the high-risk borrower to become optimistic. The single contract implies that self-deception is costless because high risks cannot choose a "wrong" contract in material terms. The *selling dreams allocation* solves the following program.

$$\begin{aligned} \underset{\{C_L \geq 0, R_L \geq 0\}}{Max} \quad & U(\theta_L, R_L, C_L) \\ s.t. \quad & \nu \Pi(\theta_H, R, C) + (1-\nu)\Pi(\theta_L, R, C) \geq 0 \quad \langle P \rangle \end{aligned} \tag{2}$$

Constraint P is a zero-profit constraint and states that the lender must break even on average across borrowers. When only a single contract is offered, competition presents firms with the sole challenge of attracting self-assessed low-risk borrowers, while offering contracts that are profitable given borrowers' average objective risk.

Definition 3 *The selling dreams allocation is the unique pooling contract* $\mathcal{M}_{\langle 2 \rangle} := \left\{ R_{L\langle 2 \rangle}, C_{L\langle 2 \rangle} \right\}$ *such that:*

$$\begin{aligned} C_L &= \frac{\nu\left(\theta_H - \theta_L\right)}{\left(1-\theta_L\right) \chi \left(\theta_L + \nu\left(\theta_H - \theta_L\right)\right)} \\ \nu \Pi\left(\theta_H, R_L, C_L\right) &+ (1-\nu)\Pi\left(\theta_L, R_L, C_L\right) = 0 \end{aligned}$$

Even though collateral is not used as a screening device in this pooling contract, the optimal amount of collateral $C_{L\langle 2 \rangle}$ is positive. This reflects the difference in beliefs: lenders know the expected risk $\nu\theta_H + (1-\nu)\theta_L$, but face borrowers who are convinced of being low risk. Borrowers therefore favor reducing repayment and increasing collateral at a higher rate than their average risk warrants. The optimal amount of collateral then reflects the lenders' trade-off between exploiting the difference in beliefs and limiting the inefficiency of liquidating collateral.

Using the allocations defined above, we can now characterize the unique WMS allocation as a function of parameter s, the weight of anticipatory utility concerns.

Proposition 1 *There exists* $s^* > 0$ *such that*

- *for* $s < s^*$*, the unique equilibrium is that high-risk borrowers are realistic* ($\tilde{\theta}_H = \theta_H$) *and the equilibrium contracts are given by the best separating allocation* $\mathcal{M}_{\langle 1 \rangle}$,
- *for* $s > s^*$*, the unique equilibrium allocation is that high-risk borrowers self-deceive* ($\tilde{\theta}_H = \theta_L$) *and the equilibrium contracts are given by the selling dreams allocation* $\mathcal{M}_{\langle 2 \rangle}$.
- *Both allocations are equilibria for* $s = s^*$.

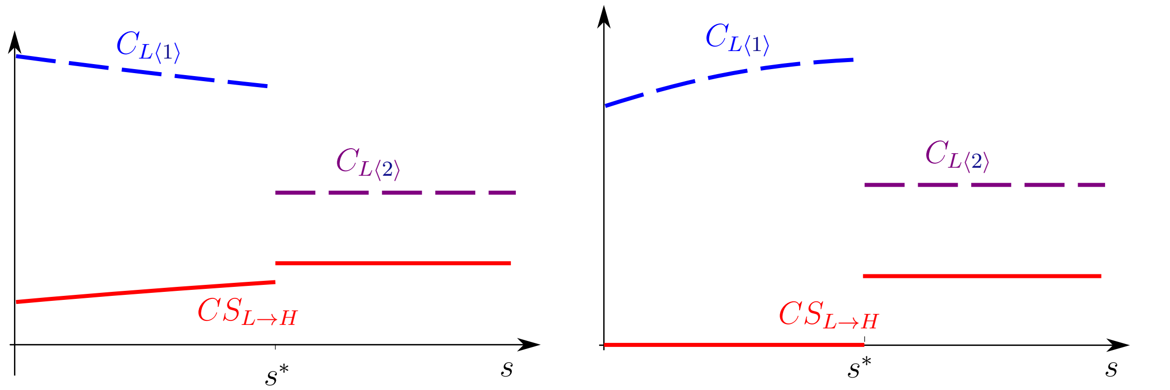
The proof of Proposition 1 is in Appendix A. The WMS equilibrium features the best separating allocation for low s and the selling dreams allocation for high s. Loosely speaking, the equilibrium allocation maximizes the utility of the low-risk borrower subject to whether or not it is preferable for her to have the high-risk borrower self-deceive. Allocations which fail to maximize the utility of the low-risk borrower are vulnerable to entry because they allow an entrant to poach low-risk borrowers away from an incumbent lender and make positive profits on them.

The equilibrium nests the rational benchmark as the case of $s = 0$. When $s = 0$ borrowers are always screened and the OE_H constraint in program (1) reduces to a more standard incentive compatibility constraint on high-risk borrowers. To achieve incentive compatibility, low risks are required to pledge positive collateral, whereas the allocation for high risks features the materially efficient level of collateral $C_H = 0$.

When $s \in (0, s^*)$, the optimal expectation constraint plays a similar role to incentive compatibility, but is a more stringent requirement. If the cross-subsidy constraint P_H is slack in program (1), then the tightening of the optimal expectation constraint associated with an increase in s translates into a combination of lowered repayment for high-risk borrowers and lower collateral requirements on low-risk borrowers. The material rent that has to be given up to the high-risk borrower to keep her realistic increases in s and the utility of the low-risk borrower decreases. If the cross-subsidy constraint P_H is binding in program (1), then the tightening of the optimal expectation constraint translates into a higher collateral requirement on low risks, thereby reducing material efficiency and, since firms make zero-profit type-by-type, the utility of low-risk borrowers.

When s increases beyond threshold s^* screening is abandoned in favor of pooling risk types. The selling dreams allocation and the optimism it induces then yield more utility to the low-risk borrower and become the equilibrium.

Figure 1 depicts how equilibrium collateral of the low-risk type (C_L) and cross-subsidy from low- to high-risk type ($CS_{L\to H}$) change with s, both for the case of a low (panel A) and the case of a high proportion of high risks (panel B). The fact that the cross-subsidy is weakly increasing in s gives rise to the adaptiveness result in the next section. Panel B shows the case in which constraint (P_H) of program (1) is binding and the WMS equilibrium of our game coincides with the standard Rothschild-Stiglitz equilibrium that does not rely on the withdrawal of unprofitable contracts after entry. In the case of Panel A, the Rothschild-Stiglitz equilibrium of our game does not exist.



A - constraint P_H slack in (1) (low ν)   B - constraint P_H binding in (1) (high ν)

Figure 1: Equilibrium collateral of self-perceived low risks (C_L) and cross-subsidy from low to high risks ($CS_{L\to H}$).

3.2 Adaptiveness

Next, we turn to the question of whether we should expect market forces to eliminate motivated cognition. To this end, we study the effect of motivated cognition on the high-risk borrower's material payoffs. If this effect is positive, then the ability to self-deceive would be favored or selected for even before a borrower knows her risk type.[8] The following proposition follows directly from the cross-subsidy to high-risk borrowers implied by the equilibrium allocation.

Proposition 2 *The high-risk borrower's material payoffs are weakly higher when she has anticipatory utility concerns (i.e. $s > 0$) than when she has no anticipatory utility concerns (i.e. $s = 0$). Her material payoffs weakly increase with the weight on anticipatory concerns as long as $s \leq s^*$.*

When $s > s^*$ and the high-risk borrower picks contract $(R_{L\langle 2\rangle}, C_{L\langle 2\rangle})$, the increase in material payoffs comes from the optimistic high-risk type being offered a contract that is designed according to borrowers' average risk and makes zero-profit on average. The high-risk type therefore benefits from a cross-subsidy.

When $0 < s < s^*$ and and the high-risk borrower picks contract $\left(R_{H\langle 1\rangle}, C_{H\langle 1\rangle}\right)$ in equilibrium, material payoffs are independent of s when the cross-subsidy constraint

[8] Ex post, a low-risk borrower is hurt by motivated cognition on behalf of high risks. But neither selection nor a borrower's selfish "ex ante preference" over whether to be able to self-deceive would take this negative ex-post externality into account.

is binding, but increasing in s when the cross-subsidy constraint is slack. In this latter case, the threat of self-deception is most efficiently averted by giving up a higher rent to the high risks. The reason that giving up additional rents on the equilibrium path is more efficient than the off-equilibrium-path threat of increasing the low-risk type's collateral requirements, is that, off the equilibrium path, the high-risk borrower is optimistic and and, hence, less impressed by this threat. When s is higher, the borrower at $t = 1$ places more weight on the unimpressed potential optimist at $t = 2$.

Brunnermeier and Parker (2005) address concerns that agents with optimal expectations might be driven to extinction by agents with rational beliefs by pointing out that optimal expectations respond to the costs of mistakes and that they are therefore harder to exploit than a fixed bias. Moreover, they highlight that some environments favor agents who take on more risk and that there is a biological link between happiness and better health.

We argue that, in strategic environments, optimal expectations can make an agent better off in material terms. Far from being a reason for agents to be driven out of markets, the behavioral trait may therefore help agents thrive in competitive environments. Our results imply that it may make sense for parents to encourage their children to envision a rosy future, dream big and believe in themselves. They also vindicate a sizable popular self-help literature that trades in similar advice.

In an influential paper, Von Hippel and Trivers (2011) argue that our ability to self-deceive into higher confidence has evolved in strategic environments, where deceiving ourselves makes us more effective at deceiving others. They make the point that affective benefits like anticipatory utility are not a plausible driver of self-deception because happiness is not an evolutionary end in itself – natural selection only cares about material payoffs. Our model contradicts this point by demonstrating that anticipatory utility motives can yield material benefits in interactive environments characterized by asymmetric information.

3.3 Pre-crisis lending

We explore the comparative statics of the equilibrium allocation by asking how shifts in parameters impact on the threshold s^*. A shift in parameters that decreases s^* makes it more likely that we observe the selling dreams allocation with optimistic high-risk borrowers, while the likelihood of observing the best separating allocation increases with parameter shifts that increase the s^* threshold. For simplicity, we focus on the case where parameters are such that P_H is slack in program (1), so the best separating allocation features a positive cross-subsidy.

Proposition 3 *The threshold s^* and, therefore, the likelihood that we observe realism and the separation of borrower types is i) increasing in the opportunity cost of funds G and ii) decreasing in the return of the project y.*

The proof of Proposition 3 is found in Appendix B. The incidence of optimism and the collateralization of high-risk loans is inversely related to the cost of funds G. We would therefore expect more optimism in an economy in which lenders or banks are able to borrow at a low risk-free rate. The intuition for this result is the following. Competitive markets channel the rents created by a decrease in the cost of funds to the borrowers. The lower is G, the higher are the returns that accrue to the borrower when the project is a success because less needs to be repaid to the lender for the lender to break even. This provides a cognitive incentive for the high-risk borrower to self-deceive into thinking that the state in which these larger returns are realized occurs relatively more often. As a result, the optimal expectations constraint tightens, making screening borrowers more expensive. In our equilibrium, high-risk types only pledge collateral when $s > s^*$. So more loans feature collateral when interest rates are low, which is what Jiménez et al. (2006) find in a large sample of Spanish business loans.

Other things equal, an increase in y has a similar effect on the borrower's returns in the good state of the world as a decrease in G and therefore also increases the incentive to believe that a project's likelihood of success is high. The model predicts that optimism arises during economic booms and when interest rates are low, and gives way to widespread disillusionment and excessive transfers of collateral after a crash. Therefore, the comparative statics of our model provide a possible mechanism behind Ben Bernanke's description of consumer lending before the 2008 financial crisis featured in the introduction.

3.4 Policy relevance

In the presence of anticipatory utility concerns, it is not obvious what a social planner's objective should be. We may envision a social planner who only cares about material outcomes. She dislikes inefficiency and therefore finds the use of any collateral requirements unappealing.

Alternatively, a social planner might take the borrowers' emotions into account. This is the stance we will take. Under the resulting welfare function, anticipatory utility and the potential of self-deception imply a new role for collateral requirements. A high-risk borrower's overall utility is maximized when she deceives herself and chooses a contract with moderate collateral requirements. In dealing with a borrower who values her dream of future income streams and who believes the good state of the world to be more likely to occur than it actually is, it is optimal to increase the spoils of a good realization at the expense of a harsher experience in case of a bad realization.[9] This is achieved by means of positive collateral requirements.

The low-risk borrower's realistic beliefs imply that she should never pledge any collateral in the welfare-maximizing allocation. Neither selling dreams nor best separating allocation feature optimism and positive collateral on the high-risk and no collateral on the low-risk type. Moreover, for a policy maker to implement this allocation, she would require an informational advantage over lenders that is unlikely to exist.

Instead, consider the less informationally burdensome optimal pooling allocation that maximizes the expected utility of borrowers prior to finding out their type and subject to lender zero-profit. It solves

$$\begin{aligned} \underset{C_P, R_P}{Max} \quad & \nu U(\theta_H, R_P, C_P) + (1-\nu) U(\theta_L, R_P, C_P) + sU(\theta_L, R_P, C_P) \\ s.t. \quad & (1+s)\nu \Pi(\theta_H, R_P, C_P) + (1+s)(1-\nu)\Pi(\theta_L, R_P, C_P) \geq 0 \end{aligned} \tag{4}$$

The optimal pooling allocation features the following collateral requirement

$$C_{P\langle 4\rangle} = \frac{s\nu(\theta_H - \theta_L)}{2\chi \mathbb{E}[\theta]((1-\theta_L)(1+s) - \nu(\theta_H - \theta_L))}$$

which is strictly smaller than collateral in the selling dreams allocation $C_{L\langle 2\rangle}$. Therefore, we can state the following proposition.

Proposition 4 *When $s > s^*$, equilibrium collateral requirements are too high in the sense that a cap on collateral at $C_{P\langle 4\rangle}$ would improve overall welfare.*

The collateral requirement in the selling dreams allocation is designed to be desirable to a self-perceived low-risk borrower at $t = 2$. On the contrary, the optimal pooling collateral takes into account the high-risk borrower's material payoffs, evaluated at her actual risk θ_H, and therefore features lower collateral. Clearly, the optimal pooling

[9] This "high reward - high cost of failure" contract that is favored in the presence of over-optimistic expectations is rather reminiscent of the financing of entrepreneurs. On the the other end of the spectrum, a realist favors the low stakes payoff structure associated with zero collateral that may be viewed as an analogy for steady employment.

allocation is also favorable to the best separating allocation, which requires collateral from the low-risk type and not the high-risk type, instead of the other way around, as would be socially efficient.

4 Conclusion

This paper shows that competitive lending markets with asymmetric information reward a predisposition to engage in motivated cognition. We also argue that acknowledging and modeling motivated cognition by market actors may improve our understanding of lending markets. Finally, we find that equilibrium allocations in our model generally do not maximize welfare.

Previous empirical work on unrealistic optimism in financial markets has generally focused on the effect of optimism on behavior. For example, Landier and Thesmar (2009) show that optimists take on more short-term debt than realists. In our model optimism is an outcome, which points to an interesting empirical endeavor that treats unrealistic optimism as a dependent variable. For example, we may explore whether the presence of unrealistic optimism, as measured by the wedge between ex-ante subjective expectations and ex-post realizations of risk, is impacted upon by exogenous variation in the risk-free rate or entrepreneurial profits. Our model also makes predictions that may be tested even if a suitable measure of optimism cannot be constructed. For example, a decrease in the risk free rate or an increase in entrepreneurial profits is predicted to decrease the correlation between a borrower's risk and her likelihood of pledging collateral.

While our results only speak to lending markets, it seems likely that self-deception also shapes interactions between firms and their consumers in other settings. For example, people frequently engage in wishful thinking about their health. This suggests that insurance providers may, like the lenders in our model, be concerned with shaping their insuree's cognitive incentives.

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Appendix

A Proof of Proposition 1

A.1 Notation and preliminary results

In order to use duality arguments, we define a number of parametrized optimization programs and their corresponding value functions. These programs are all related to programs (1) and (2) by either nesting them, being relaxed versions, or being dual programs. With some abuse of notation, we use the same notation to denote the program itself and its maximized value function.

We nest program (1) by allowing the lender's profit to be a free parameter π.

$$\mathcal{V}(\pi) = \underset{\{C_H, C_L, R_H, R_L\}}{Max} \quad U(\theta_L, R_L, C_L)$$

$$s.t. \quad \begin{cases} \nu\Pi(\theta_H, R_H, C_H) + (1-\nu)\Pi(\theta_L, R_L, C_L) \geq \pi \\ \Pi(\theta_H, R_H, C_H) \leq 0 \\ (1+s)\,U(\theta_H, R_H, C_H) - U(\theta_H, R_L, C_L) - sU_B(\theta_L, R_L, C_L) \geq 0 \end{cases}$$

Program (1) is simply given by $\mathcal{V}(0)$. The relaxed program $\hat{\mathcal{V}}(\pi)$ then obtains from program $\mathcal{V}(\pi)$ by omitting the cross-subsidy constraint $\Pi(\theta_H, R_H, C_H) \leq 0$.

The dual program $\mathcal{Q}(v_L)$ represents profit maximization, subject to a minimum utility target of v_L for low-risk borrowers.

$$\mathcal{Q}(v_L) = \underset{\{C_H, C_L, R_H, R_L\}}{Max} \quad \nu\Pi(\theta_H, R_H, C_H) + (1-\nu)\Pi(\theta_L, R_L, C_L)$$

$$s.t. \quad \begin{cases} U(\theta_L, R_L, C_L) \geq v_L \\ \Pi(\theta_H, R_H, C_H) \leq 0 \\ (1+s)\,U(\theta_H, R_H, C_H) - U(\theta_H, R_L, C_L) - sU_B(\theta_L, R_L, C_L) \geq 0 \end{cases}$$

The program obtained by relaxing the cross-subsidy constraint in $\mathcal{Q}(v_L)$ is denoted $\hat{\mathcal{Q}}(v_L)$.

We proceed similarly with program (2), defining the nesting program $\mathcal{W}(\pi)$, and dual program $\mathcal{T}(v_L)$

$$\mathcal{W}(\pi) = \underset{\{C_L \geq 0, R_L \geq 0\}}{Max} \quad U(\theta_L, R_L, C_L)$$

$$s.t. \quad \nu\Pi(\theta_H, R_L, C_L) + (1-\nu)\Pi(\theta_L, R_L, C_L) \geq \pi$$

$$\begin{aligned}\mathcal{T}(v_L) = & \underset{\{C_L \geq 0, R_L \geq 0\}}{Max} \quad \nu\Pi(\theta_H, R_L, C_L) + (1-\nu)\Pi(\theta_L, R_L, C_L) \\ & s.t. \quad U(\theta_L, R_L, C_L) \geq v_L\end{aligned}$$

Lemma 1 *Programs $\mathcal{T}(v_L)$ and $\mathcal{W}(\pi)$ are dual, so for values of π and v_L such that the constraint sets are nonempty, $\mathcal{T}(v_L) \geq \pi$ iff $\mathcal{W}(\pi) \leq v_L$. Similarly, programs $\mathcal{Q}(v_L)$ and $\mathcal{V}(\pi)$ are dual, so $\mathcal{Q}(v_L) \geq \pi$ iff $\mathcal{V}(\pi) \leq v_L$. Similarly, $\hat{\mathcal{Q}}(v_L) \geq \pi$ iff $\hat{\mathcal{V}}(\pi) \leq v_L$.*

Lemma 2 *Define $\zeta(\nu, s, \pi) := -\Pi(\theta_H, R_H, C_H)$, the cross-subsidy in the unique solution to program $\hat{\mathcal{V}}(\pi)$. The unique solution is given by*

$$\begin{aligned}&(1+s)U(\theta_H, R_H, C_H) = U(\theta_H, R_L, C_L) + sU(\theta_L, R_L, C_L) \\ &\nu\Pi(\theta_H, R_H, C_H) + (1-\nu)\Pi(\theta_L, R_L, C_L) = \pi \\ &C_H = 0 \\ &C_L = \frac{1}{2(1+s)} \frac{(\theta_H - \theta_L)\,\nu}{\theta_L \chi (1-\theta_L)\,(1-\nu)}\end{aligned}$$

By the Le Châtelier principle, we have $\hat{\mathcal{V}}(\pi) > \mathcal{V}(\pi)$ if $\zeta(\nu, s, \pi) < 0$ and $\hat{\mathcal{V}}(\pi) = \mathcal{V}(\pi)$ iff $\zeta(\nu, s, \pi) \geq 0$.

A.2 Characterising threshold s^*

The threshold s^* is characterized uniquely by equating the values of programs $\hat{\mathcal{V}}(0)$ and $\mathcal{W}(0)$, i.e. as the unique positive root of function $\Psi(s) = \hat{\mathcal{V}}(0) - \mathcal{W}(0)$. We first establish that $\Psi(0) > 0$. We have that

$$\Psi(0) = \frac{1}{4} \frac{\theta_H(\theta_H - \theta_L)^2 \nu^3}{\chi\,(1-\theta_L)\,\mathbb{E}[\theta]\,(1-\nu)\,(1-\mathbb{E}[\theta])\,\theta_L} > 0.$$

Next, we establish that Ψ is decreasing in s. Note first that the value of $\mathcal{W}(0)$ is independent of s, while $\frac{\partial \hat{\mathcal{V}}(0)}{\partial s}$ is proportional to the Lagrange multiplier of the optimal expectation constraint in $\hat{\mathcal{V}}$, denoted κ_H. Then

$$\frac{\partial \Psi}{\partial s} = -\frac{\kappa_H\,(\theta_H - \theta_L)\,(y - R_L + C_L)}{1+s} < 0.$$

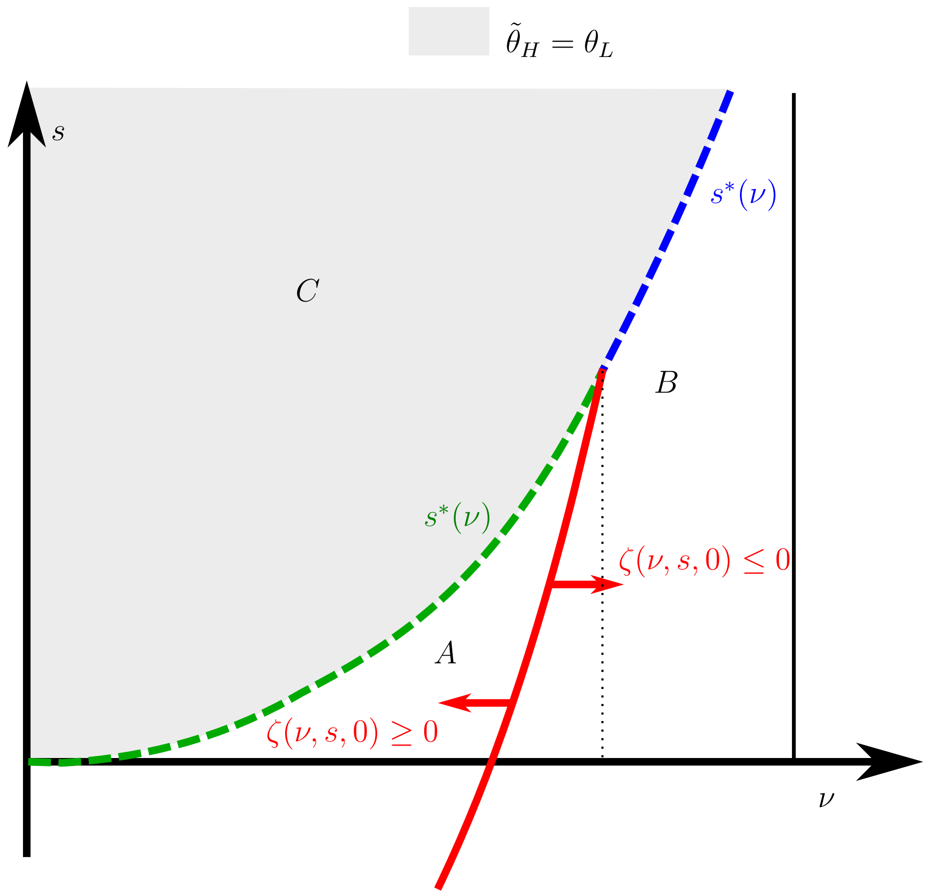

Figure A.1: Equilibrium allocations in (ν, s) space.

Finally, Ψ has a negative limit as s grows large, bounded above by

$$-4^{-1}\frac{\nu\theta_H - \nu\theta_L}{(1-\theta_L)(1-\theta_L)(1-\nu)\,\theta_L\,(1-\mathbb{E}[\theta])\,\chi\mathbb{E}[\theta]}.$$

Ψ is continuous by Berge's maximum theorem. This, along with monotonicity and boundary conditions, establishes the existence and uniqueness of threshold $s^* > 0$.

A.3 Equilibrium existence ($s \leq s^*$): separating offers with positive cross-subsidy

Suppose that parameters (ν, s) lie in the quadrant A of Figure A.1, excluding the locus $\zeta(\nu, s, 0) = 0$. We show that there can be no strictly profitable entry, taking into account withdrawal of unprofitable contracts.

a. No entrant can profitably attract high-risk borrowers only.
In the solution to $\hat{\mathcal{V}}(0)$, the cross-subsidy $\zeta(\nu, s, 0)$ is negative and $C_H = 0$, which characterizes the efficient allocation between lender and high-risk borrower, so there is no other allocation yielding positive profit and inducing self-selection.

b. No entrant can profitably attract both borrower types with a separating offer.
A hypothetical entrant would solve problem $\hat{\mathcal{Q}}(v_L)$, with $v_L = \mathcal{V}(0)$. Since the cross-subsidy is positive, $v_L = \hat{\mathcal{V}}(0)$ and therefore $\hat{\mathcal{Q}}(v_L) \leq 0$.

c. No entrant can profitably attract low-risk borrowers only.
Since the cross-subsidy is positive, entry would trigger withdrawal of high-risk borrower contracts. A hypothetical entrant would then have to generate positive profit by attracting both borrower types, which is ruled out by point b.

d. No entrant can profitably attract both borrower types with a pooling offer.
A hypothetical entrant would solve problem $\mathcal{T}(v_L)$, with $v_L = \hat{\mathcal{V}}(0)$. In quadrant A, $\hat{\mathcal{V}}(0) \geq \mathcal{W}(0)$ and therefore $\mathcal{T}(v_L) \leq 0$.

A.4 Equilibrium existence ($s \leq s^*$): separating offers with zero cross-subsidy

Suppose that parameters (ν, s) lie in quadrant B of Figure A.1. We show that there can be no strictly profitable entry, taking into account withdrawal of contracts with negative profits following entry.

a. No entrant can profitably attract high-risk borrowers only.
In the solution to $\mathcal{V}(0)$, the cross-subsidy $\zeta(\nu, s, 0)$ is zero and $C_H = 0$, which characterizes the efficient allocation between lender and high-risk borrower. So there can be no other allocation that yields positive profit and induces self-selection.

b. No entrant can profitably attract low-risk borrowers only.
Suppose a contract $(R_{L,d}, C_{L,d})$ improves on the candidate equilibrium contract (R_L, C_L) for both lender and low-risk borrower. Write Δ_1 and Δ_2, both nonnegative, with

$\Delta_1 + \Delta_2 > 0$, for the improvements in lender and borrower payoffs. Such a contract would violate the optimal expectation constraint, as shown by (5d)

$$U(\theta_H, R_H, C_H) + sU(\theta_H, R_H, C_H) - U(\theta_H, R_L, C_L) - sU(\theta_L, R_L, C_L) = 0 \tag{5a}$$

$$\Pi(\theta_L, R_{L,d}, C_{L,d}) = \Pi(\theta_L, R_L, C_L) + \Delta_1 \tag{5b}$$

$$U(\theta_L, R_{L,d}, C_{L,d}) = U(\theta_L, R_L, C_L) + \Delta_2 \tag{5c}$$

$$\begin{aligned}&(1+s)\,U(\theta_H, R_H, C_H) - U(\theta_H, R_{L,d}, C_{L,d}) - sU(\theta_L, R_{L,d}, C_{L,d}) \\ &= -(1-\theta_H)\,\Delta_2 - s(1-\theta_L)\,\Delta_2 \\ &\quad - \chi^{-1}\theta_L^{-1}(\theta_H - \theta_L)\left(-C_{L,d}\theta_L\chi + \sqrt{\chi\theta_L\left(\chi {C_{L,d}}^2\theta_L + \Delta_1 + \Delta_2\right)}\right) < 0.\end{aligned} \tag{5d}$$

Therefore, a hypothetical entrant must attract both borrower types.

c. No entrant can profitably attract both borrower types with a separating offer.

Since the cross-subsidy is zero, and $C_H = 0$, a hypothetical entrant cannot make a profit on high-risk borrowers and induce self-selection. Therefore, a hypothetical entrant's offer must satisfy the cross-subsidy constraint and solve problem $\mathcal{Q}(v_L)$, with $v_L = \mathcal{V}(0)$, instead of solving $\hat{\mathcal{Q}}(v_L)$. We have that $v_L = \mathcal{V}(0)$. Therefore, $\mathcal{Q}(v_L) \leq 0$.

d. No entrant can profitably attract both borrower types with a pooling offer.

A hypothetical entrant would solve problem $\mathcal{T}(v_L)$, with $v_L = \mathcal{V}(0)$. In quadrant B, $\mathcal{V}(0) \geq \mathcal{W}(0)$ and therefore $\mathcal{T}(v_L) \geq 0$.

A.5 Equilibrium existence ($s \geq s*$): pooling offers

We characterise the equilibrium in quadrant C of Figure A.1. Incumbent firms offer both the selling dreams allocation $\mathcal{M}_{\langle 2\rangle} := \left\{R_{L,\langle 2\rangle}, C_{L,\langle 2\rangle}\right\}$ and a latent, inactive contract $(R = G/(1-\theta_H), C = 0)$, not picked by either borrower type on the equilibrium path.

a. No entrant can profitably attract both borrower types with a pooling offer.

An entrant would at most get profit $\mathcal{T}(v_L)$, with $v_L = \mathcal{T}(0)$. Therefore, $\mathcal{T}(v_L) \leq 0$.

b. No entrant can profitably attract only low-risk borrowers.
Since borrowers make decisions based on their belief $\tilde{\theta}$, there is no possible separation without attracting high-risk borrowers.

c. No entrant can profitably attract only high-risk borrowers.
The latent offer makes zero-profit on high-risk borrowers and therefore precludes positive profits.

d. No entrant can profitably attract both borrower types with a separating offer.
The existence of the latent offer implies a nonpositive profit on high-risk borrowers, so the entrant must solve program $\mathcal{Q}(v_L)$, with $v_L = \mathcal{W}(0)$. In quadrant C, by construction, $\mathcal{W}(0) \geq \mathcal{V}(0)$, and therefore $\mathcal{Q}(v_L) \leq 0$.

A.6 Equilibrium uniqueness

Competition between lenders implies that candidate equilibrium offers must leave no aggregate profit to the incumbent lender. For $s < s^*$, consider any other zero-profit realism-inducing offer $M_{ac} := \{(R_{H,ac}, C_{H,ac}), (R_{L,ac}, C_{L,ac})\}$ as a candidate equilibrium. Lenders cannot make a strictly positive profit on high-risk borrowers. Indeed, since the incentive constraint (IC_L) is slack, a single contract $(R_{H,ac}-\epsilon, C_{H,ac})$ would otherwise be profitable for $\epsilon > 0$ small enough. It follows that any candidate equilibrium allocation must solve program (1) and therefore, must achieve a lower value than its solution. Thus, from the true equilibrium allocation $\mathcal{M}_{\langle 1\rangle}$, one can construct an improvement offer simply by using offer $\big\{(R_{L,\langle 1\rangle} + \delta, C_{L,\langle 1\rangle}), (R_{H,\langle 1\rangle}, C_{H,\langle 1\rangle})\big\}$, for δ small enough, which still induces separation and achieves strictly positive profit.

Likewise for $s > s^*$, the borrower-preferred pooling allocation generates the highest possible surplus among pooling allocations. Any other profitable offer generates less surplus, and therefore must leave room for entry.

B Proof of Proposition 3

Suppose that the solution to (1) features cross-subsidization. Threshold s^* is then implicitly defined by $\mathcal{W}(0) - \hat{\mathcal{V}}(0) = 0$. We have that $\frac{\partial \mathcal{W}(0)}{\partial s} = 0$ and $\frac{\partial \hat{\mathcal{V}}(0)}{\partial s} < 0$. So for any parameter α, $\frac{\partial s^*}{\alpha}$ is proportional to $\frac{\partial \left[\mathcal{W}(0) - \hat{\mathcal{V}}(0)\right]}{\partial \alpha}$. Note that

$$\frac{\partial\left[\mathcal{W}(0)-\hat{\mathcal{V}}(0)\right]}{y}=\frac{(1-\theta_L)\ \nu\, s\,(\theta_H-\theta_L)}{s\,(1-\theta_L)+(1-\mathbb{E}\,[\theta])}>0.$$

$$\frac{\partial\left[\mathcal{W}(0)-\hat{\mathcal{V}}(0)\right]}{G}=-\frac{(1-\theta_L)\ \nu\, s\,(\theta_H-\theta_L)}{(1-\mathbb{E}\,[\theta])\,(s\,(1-\theta_L)+(1-\mathbb{E}\,[\theta]))}<0.$$

An increase in y or decrease in G therefore decrease threshold s^* and make the pooling equilibrium and optimism more likely.