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Reciprocity in Dynamic Employment Relationships*

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This paper explores the optimal provision of dynamic incentives for employees with reciprocal preferences. Building on the presumption that a relational contract can establish a norm of reciprocity, I show that generous upfront wages that activate an employee's reciprocal preferences are more important when he is close to retirement. In earlier stages, "direct" performance-pay promising a bonus in exchange for effort is used more extensively. Then, a longer remaining time horizon increases the employer's commitment which is generally determined by her future profits. Moreover, since future profits are affected by the employee's reciprocal preferences, the norm of reciprocity already shapes the incentive system at the beginning of his career. I also show that more competition might magnify the use of reciprocity-based incentives, and that a formal commitment to paying non-discretionary wages in the future can boost the employer's credibibility.

Keywords: reciprocity, relational contracts, dynamic incentives

JEL Classification Numbers: C73, D21, D86, D90, D91

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1 Introduction

How to motivate their employees is of utmost importance for firms. Many use some form of incentive pay and, since individual contribution to firm value can rarely be measured objectively, often rely on informal, subjective assessments of performance (Kampkötter and Sliwka (2016), Frederiksen et al. (2017)). Alternatively, social norms might induce employees to increase their effort in response to generous wages (Akerlof, 1982). Indeed, a vast amount of evidence indicates that "standard" incentives as well as gift-exchange considerations motivative employees (see DellaVigna et al., 2019, for a recent contribution). However, there is less understanding of whether pay-for-performance and generous wages merely constitute two substitutable components of a firm's "incentive toolkit" (as studies within static settings indicate, see Englmaier and Leider, 2012a; DellaVigna et al., 2019), or whether more intricate interactions prevail – in particular taking into account the long-run nature of employment relationship and the inherent incompleteness of real-world labor contracts.

In this paper, I incorporate the notion that an informal relational contract might not only determine an employee's performance-based pay, but also establish a norm to reciprocate generous wages. I derive a tractable setup to explore how individual responsiveness to such a norm affects the optimal provision of informal incentives over the course of an employee's career. In its early stages, direct incentives where a bonus is promised in exchange for effort are more important because a longer remaining time horizon allows the employer to credibly promise a higher bonus. In later stages, generous wages assume a more significant role and gradually replace direct incentives. Moreover, the norm of reciprocity already shapes the incentive system at the beginning of a career. Then, it determines the future surplus of the employment relationship which constrains the power of direct incentives.

The idea that a relational contract can establish a norm to reciprocate goes back to the law scholar Ian Macneil. He developed a norms-based approach to contracting, in which a relational contract is a manifestation of the norms supposed to govern the behavior of the involved parties (Macneil, 1980; Macneil, 1983). This perception is supported by recent evidence from Kessler and Leider (2012) and Krupka et al. (2017) who demonstrate that contracts – in particular informal "handshake agreements" – can generate inherent enforcement mechanisms by establishing norms that parties feel obliged to honor. I build upon this idea and assume that the norm of reciprocity is enforced by an employee's social preferences. These are not only individual- or relationship-specific, but also shaped by the history of the employment relationship – in the sense that they deteriorate once the firm has broken a promise. This notion borrows from

recent experimental evidence showing that norms and thus social preferences seem to respond to a person's environment. For example, Kimbrough and Vostroknutov (2016) state that a small change in context can substantially alter the norms governing a situation, which consequently influences the extent of the prosociality of actions. Fehr and Schurtenberger (2018) demonstrate that norms pushing for prosocial behavior can quickly emerge. The persistence of these norms relies heavily on the ability to punish norm violations.

More precisely, I set up a repeated principal–agent model with a predetermined last period. The risk-neutral agent can exert costly effort which benefits the risk-neutral principal and is observable but not verifiable. Hence, formal court-enforceable contracts cannot be used to motivate the agent. Instead, both parties may form a self-enforcing relational contract, which determines bonus payments the principal is supposed to make to reward the agent's effort. In addition, the relational contract specifies a *norm of reciprocity*, implying that a generous wage payment by the principal is supposed to be reciprocated by the agent via higher effort. The agent responds to this norm because he has preferences for reciprocity. These preferences might be individual- or match-specific (for example because the agent has developed sentiments for the principal; see Akerlof, 1982) and are activated by non-discretionary wage components, by which I mean payments that are not paid as a reward for past effort. Thus, the principal has the following two means to provide incentives: (i) directly by promising a bonus to be made *after* the agent has exerted effort and (ii) indirectly via the norm of reciprocity and paying a non-discretionary wage *before* the agent is exerting effort. For the former, the principal uses "relational incentives", where the bonus is enforced by repeated game incentives. For the latter, the principal uses "reciprocity-based incentives". My specification allows to separate both incentive tools and designate each payment to exactly one of the two. Thus, besides having a larger intuitive appeal compared to a setting in which all payments are regarded as a gift (hence also direct performance pay the agent is supposed to receive anyway), my approach simplifies the analysis of optimal incentives within the relational contract.

The agent's responsiveness to the norm of reciprocity is affected by the history of the game. If the principal reneges on a promised bonus, not only does the relational contract break down (as is standard in the literature), but also the agent's preferences for reciprocity toward the principal disappear. In addition to the aforementioned experimental literature demonstrating the path dependency of social preferences, such a *norm function* is inspired by Cox et al. (2007) and Cox et al. (2008). They develop an approach to modeling reciprocity grounded in neoclassical preference theory. Preferences for reciprocity are stronger if actions upset the status quo. In my setting, the status quo

corresponds to the equilibrium prescribed by the relational contract, and (downward) deviations by the principal not only constitute a violation of the relational contract, but also affect the agent's reciprocal preferences. Moreover, in essence the specification of the agent's reciprocal preferences is similar to the expectation-based loss aversion model of Kőszegi and Rabin (2006). As in Kőszegi and Rabin (2006), the "behavioral" component of the agent's preferences in my setting evaluates realized outcomes by relating them to (expected) equilibrium outcomes.

Importantly, the norm function allows the use of relational incentives even though there exists a predetermined last period. Because the agent's preferences for reciprocity disappear once the principal reneges on a promised bonus and because the principal's profits in the last period of the game are higher with reciprocal preferences than without, her behavior in the penultimate period affects her profits in the last period. This interaction between relational and reciprocity-based incentives carries over to earlier periods and enables the principal to credibly promise an effort-based bonus. The maximum size of this bonus is determined by the so-called dynamic enforcement (DE) constraint, which states that a bonus must not exceed the difference between future discounted profits on and off the equilibrium path. Since *future* on-path profits increase in the extent of the agent's reciprocal preferences, the principal can also provide stronger relational incentives *today* if the agent is more reciprocal. This source of complementarity between relational incentives and the agent's reciprocal preferences is amended by an additional channel. The (DE) constraint in a given period is relaxed and more effort can be implemented if she pays a generous wage in this period, implying that reciprocity-based preferences are particularly valuable whenever the constraint binds. Therefore, relational and reciprocity-based incentives are complements and more effort is implemented with a combination of the two, a result that has received empirical support from Boosey and Goerg (2018). Both are dynamic substitutes, however, in the sense that relational incentives are gradually replaced by reciprocity-based incentives over time. The reason is that the (DE) constraint is tighter in later periods (having fewer remaining periods reduces the principal's future profits), which amplifies the benefits of reciprocity-based incentives as time passes.

This implies that a profit-maximizing incentive scheme has the highest effort in the early stages of the employment relationship, where it remains until the (DE) constraint starts to bind. Then, the principal's reduced credibility effectively constrains her ability to pay a sufficiently high bonus. This decreases effort, which in turn lets the principal respond with an increase in the non-discretionary fixed wage and consequently place a higher weight on reciprocity-based incentives. Overall, effort gradually decreases in the final periods of the employment relationship. In line with this result, there is

indeed evidence that the productivity of workers declines once they approach retirement (Haltiwanger et al., 1999; Skirbekk 2004; Lallemand and Rycx, 2009).

Effort is also higher if the agent has more pronounced preferences for reciprocity. This result has received empirical support from Dohmen et al. (2009), who use data from the German Socio-Economic Panel (SOEP) which also contains information on the reciprocal inclinations of individuals. Moreover, the positive effect of reciprocal preferences on effort is stronger if reciprocity-based incentives are more important (i.e., in later periods when the (DE) constraint binds). This outcome is in line with evidence provided by Fahn et al. (2017). Using the same data and approach as Dohmen et al. (2009), they show that the positive interaction between reciprocal inclinations and effort is substantially stronger for older workers close to retirement.

Next, I investigate a number of additional implications of having relational contracts with reciprocal agents. First, I explore how labor market competition affects the optimal dynamic incentive scheme and show that more intense competition for workers can actually intensify the use of reciprocity-based incentives. For this, I follow Schmidt (2011) and assume that more intense competition for workers decreases the principal's outside option and increases the wage the agent must at least be offered. I also assume that this "minimum wage" serves as a reference wage for the norm of reciprocity in the sense that the agent only perceives higher wages as generous. Better outside opportunities generally reduce the relationship surplus and consequently the potential strength of relational incentives. Now, more intense labor market competition has opposing effects on the principal's outside option and the agent's reference wage. If the effect on the agent's reference wage dominates, more intense labor market competition reduces the rent generated in the relationship. This restricts the power of relational incentives and consequently magnifies the importance of reciprocity-based incentives. Otherwise, more intense labor market competition allows for stronger relational incentives.

Second, I allow the principal to formally commit to pay a (non-discretionary) wage in the subsequent period. This resembles actual labor market contracts which often have longer time horizons and specify fixed wage payments for their whole term. Since employment protection laws or a minimum tenure of a labor contract constrain the use of firing threats and hence efficiency wages, a formal commitment to paying non-discretionary wages in the future seems to be detrimental to the provision of incentives. In my case, though, commiting to a non-discretionary wage in the future allows to credibly promise a higher discretionary bonus today. The reason is that the future wage has to be paid even if the principal has reneged on today's bonus, whereas the agent only reciprocates to this wage if the bonus has been paid.

In a number of extensions, I explore the robustness of my results, for simplicity all

within a two-period setting. In Section 5.1, I let the agent's preferences for reciprocity not merely be triggered by non-discretionary, but by all realized payments (i.e., including by wages paid in response to past effort). Then, only upfront wages and no bonuses are used to compensate the agent.

Some additional results are generated in Section 5.2, where I allow for asymmetric information on the agent's reciprocal preferences. There, I assume that the agent might either be reciprocal (as in the previous analysis) or selfish (i.e., without any reciprocal preferences). If the likelihood of facing a reciprocal agent is high, a "separating contract" is optimal for the principal. This incorporates high effort in the first period which however will only be exerted by the reciprocal type, whereas the selfish type shirks and is subsequently fired. If the likelihood of facing a selfish agent is high, it might be optimal for the principal to offer a "pooling contract". This incorporates low effort in the first period which is exerted by both types. In the second period, the selfish type collects the wage and subsequently shirks. The pooling contract resembles outcomes derived in the reputation literature (see Mailath and Samuelson (2006) for an overview), in which the presence of even a small proportion of "commitment types" can motivate selfish agents to cooperate in a finitely repeated game because it allows them to maintain a reputation for (potentially) being cooperative. Furthermore, it is a common perception that in lab experiments with repeated interaction, selfish types who imitate cooperative (or "fair") types are responsible for driving high cooperation in early periods (Fehr et al., 2009).

However, the existence of the pooling contract in my setting relies on a perfect Bayesian equilibrium being played at which *any* deviation from equilibrium effort lets the principal assign probability 1 to facing the selfish type. Nonetheless, even if preferred by the principal, such a pooling contract may not satisfy the intuitive criterion (Cho and Kreps, 1987). Deviation to a higher effort than that specified by the pooling contract would only be incentive compatible for the reciprocal type, but not for the selfish type. Such an upward deviation would thus reveal the agent to be reciprocal and allow for an adjustment of the second-period wage that makes the principal and reciprocal agent better off. Then, only a separating contract can be sustained, which can have implications for the interpretation of many experimental results. High effort in early periods and low effort in later periods might also be due to an early separation of types, followed by a relational contract between the remaining matches. Indeed, the experimental exercise conducted by Brown et al. (2004) generates this outcome. Their theoretical explanation (i.e., some players have fairness preferences, whereas those without imitate the fair players early on) can only account for the observed effort dynamics, but not for the high amount of separations in initial periods. Therefore, I provide a com-

plementary interpretation of the higher cooperation in lab experiments with repeated interaction.

In Section 5.3, I explore the implications of negative reciprocity in the sense that the agent wants to retaliate if the principal has reneged on a promised payment. If the agent's preferences for negative reciprocity are sufficiently strong, the results are as in the main part of the paper, even if the agent's preferences for positive reciprocity do not disappear after a deviation by the principal. I also incorporate recent evidence that individuals get accustomed to generous wages (Jayaraman et al., 2016, Sliwka and Werner, 2017). To do so, I assume that the wage paid in the first period becomes the second period's reference wage that has to be surpassed before the agent responds to the norm of reciprocity (Section 5.4). Then, the backloading of reciprocity-based incentives is even more pronounced because a positive first-period wage – albeit still inducing higher effort via the norm of reciprocity – increases the reference wage later on. Finally, in Section 5.5, I assume that the norm of reciprocity relates to the material rent the agent is bound to receive in a period, which equals the difference between on-path payments and effort costs. Then, the principal is less inclined to pay an upfront wage because also the bonus activates the agent's preferences for reciprocity.

Related Literature

One of the most robust, thoroughly researched outcomes in behavioral economics is that individuals not only maximize their own material payoffs, but also take others' well-being into account when making decisions (DellaVigna, 2009). Many individuals seem to possess social preferences, where an important component is captured by preferences for intrinsic reciprocity. A plethora of research since Fehr et al. (1993) and Fehr et al. (1998) has found experimental support for the existence of reciprocal preferences (see Camerer and Weber (2013) for an overview of experimental research, or DellaVigna and Pope (2018) for more recent evidence). Most of these exercises have been careful to rule out repeated interaction in order to isolate the effect of social preferences. However, to matter in the workplace, reciprocal preferences should not be marginalized by repeated game considerations. It is thus crucial to understand how repeated interaction affects the optimal provision of incentives for reciprocal individuals (Sobel, 2005). Some experimental studies have approached this question and disentangled the two motives for cooperation. Reuben and Suetens (2012) use an infinitely repeated prisoner's dilemma to assess the relative importance of strategic motives (i.e., driven by repeated interaction) and intrinsic reciprocity and find that cooperation is mostly driven by strategic concerns. Similarly, Dreber et al. (2014) find that strategic motives seem to

be more important than social preferences in an infinitely repeated prisoner's dilemma. Cabral et al. (2014) conduct an infinitely repeated veto game to distinguish between explanations of generous behavior. They find strategic motives to be the predominant motivation, but also present evidence for the importance of intrinsic reciprocity. Hence, experimental evidence suggests that repeated game incentives are an important mode to support cooperation even for individuals with reciprocal preferences. However, a sound understanding of how firms optimally design dynamic incentive schemes for reciprocal agents is still lacking. The present paper addresses this gap by providing a tractable theoretical framework that incorporates the norm of reciprocity into a relational contracting framework.

The theoretical literature on intrinsic reciprocity can be arranged along the lines of whether reciprocal behavior is merely triggered by outcomes or whether the counterpart's intentions matter as well. The classic gift exchange approach developed by Akerlof (1982) is an example of outcome-based reciprocity, where firms can strategically use wages above the market-clearing level to induce their employees to work harder. Applying this idea to a moral hazard framework with reciprocal agents, Englmaier and Leider (2012a) show that generous compensation can not only be a substitute for performance-based pay, but may also increase profits. On the contrary, Rabin (1993) claims that the perceived kindness of an action should be the driving force to induce reciprocal behavior and develops the techniques for incorporating intentions into game theory. Dufwenberg and Kirchsteiger (2004) apply this psychological game theory to extensive games. Segal and Sobel (2007) develop a more tractable approach, demonstrating how a player's preferences over strategies might be represented as a weighted average of the utility from outcomes of the individual and his opponents. Netzer and Schmutzler (2014), however, state that the extent to which intention-based reciprocity can explain gift exchange in the workplace is limited. They argue that, if only intentions matter, a self-interested firm cannot benefit from its employees' reciprocal preferences. Falk and Fischbacher (2006) develop a theory incorporating both aspects, outcomes and intentions. They assume that an action is perceived as kind if the opponent has the option to treat someone less kind. Their exercise incorporates evidence that while individuals respond to outcomes, those responses are considerably stronger if the choices are at the counterpart's discretion (see Falk et al., 2006; Fehr et al., 2009; Camerer and Weber, 2013). Cox et al. (2007) and Cox et al. (2008) develop a theoretical framework that can generate such results without having to resort to psychological game theory. Under their approach, which is based on neoclassical preference theory, individuals merely respond to observable events and opportunities instead of beliefs about others' intentions or types. I build upon these ideas and apply them to a dynamic setting.

Some papers have explicitly incorporated norms into (static) models as drivers of reciprocal behavior (López-Pérez, 2008; Kessler and Leider, 2012; Kimbrough and Vostroknutov, 2016), stating that deviating from the behavior specified by a norm is costly for individuals. These studies thus present alternative approaches for how a norm can enforce cooperative behavior.

I also contribute to the literature on relational contracts. Bull (1987) and MacLeod and Malcomson (1989) derive relational contracts with observable effort, whereas Levin (2003) shows that those also take a rather simple form in the presence of asymmetric information. Malcomson (2013) delivers an extensive overview of the literature on relational contracts. Within this broader area, a few papers have investigated the implications of incorporating "behavioral" components into a relational contracting framework. Dur and Tichem (2015) incorporate social preferences into a model of relational contracts and show that altruism undermines the credibility of termination threats. Contreras and Zanarone (2017) assume that employees suffer when their formal wage is below that of their colleagues. They show that these "social comparison costs" can be managed by having a homogeneous formal governance structure, while achieving the necessary customizations through relational contracts. Fahn and Hakenes (2019) show that teams can serve as a commitment device for present-biased individuals. To the best of my knowledge, the present paper is the first to incorporate intrinsic preferences for reciprocity into a relational contracting framework, and the first to show that the norm of reciprocity allows for the formation of relational contracts even with a predefined last period.

2 Model Setup

2.1 Environment and Technology

There is one risk-neutral principal ("she") and one risk-neutral agent ("he"). At the beginning of every period $t \in \{1,...,T\}$, with $1 < T < \infty$, the principal decides whether to make an employment offer to the agent or not ($d_t^P \in \{0,1\}$). In case an offer is made ($d_t^P = 1$), it specifies an upfront wage $w_t \in \mathbb{R}_+$.[1] The agent's acceptance/rejection decision is described by $d_t^A \in \{0,1\}$. Upon acceptance ($d_t^A = 1$), the agent receives w_t and chooses an effort level $e_t \in \mathbb{R}_+$, which is associated with effort costs $c(e) =$

[1] The non-negativity constraint simplifies the definition of reciprocity below in a sense that I do not have to differentiate between positive and negative payments. Moreover, I want to rule out negative up-front wages which could allow the principal to extract "reciprocity rents" that the agent enjoys later.

$e^3/3$.[2] Effort generates a deterministic output $e_t\theta$, with $\theta > 0$, which is subsequently consumed by the principal. Afterwards, the principal can pay a discretionary bonus $b_t \in \mathbb{R}_+$. If the principal refrains from making an offer ($d_t^P = 0$) or if the agent rejects an offer made by principal ($d_t^A = 0$), both consume their outside option utilities, which (for now) are set to zero. Moreover, the principal and agent share a discount factor $\delta \in (0,1]$.

2.2 Relational Contract, Preferences, and the Norm of Reciprocity

Neither effort nor output is verifiable; however, they can be observed by both parties. Therefore, only relational but no formal incentive contracts are feasible. The relational contract is a self-enforcing agreement determined by principal and agent and constitutes a subgame perfect equilibrium of the game. In addition to the standard components of a game – players, information, action spaces, preferences and equilibrium concept – I incorporate a norm function that activates the norm of reciprocity and maps the game's history into the agent's preferences. Before introducing this norm, I formally describe histories and feasible strategies. A discussion of several assumptions made with respect to the norm of reciprocity follows in Section 2.3.

Histories and feasible strategies The events in period t are denoted by $h_t = \left(d_t^P, w_t, d_t^A, e_t, b_t\right)$, with h_t being public information. A *history* of length $t-1$, h^{t-1} (for $t \geq 2$) collects the events up to, and including, time $t-1$, i.e. $h^{t-1} := (h_\tau)_{\tau=1}^{t-1}$. The set of histories of length $t-1$ is denoted by $\mathscr{H}^{t-1}$ (and $\mathscr{H}^0 = \{\emptyset\}$). I focus on pure strategies. For the agent, a pure strategy specifies what wage offers to accept in each period as a function of the previous history, and what level of effort to exert as a function of the previous history and current-period wages. Formally, it is a sequence of mappings $\left\{\sigma_t^A\right\}_{t=1}^T$, where, for each $t \leq T$, $\sigma_t^A = (d_t^A, e_t)$, and $d_t^A : \mathscr{H}^{t-1} \times \{0,1\} \times \mathbb{R}_+ \to \{0,1\}$, $(h^{t-1}, d_t^P, w_t) \mapsto d_t^A(h^{t-1}, d_t^P, w_t)$ and $e_t : \mathscr{H}^{t-1} \times \{0,1\} \times \mathbb{R}_+ \times \{0,1\} \to \mathbb{R}_+$, $(h^{t-1}, d_t^P, w_t, d_t) \mapsto e_t(h^{t-1}, d_t^P, w_t, d_t)$.

In each period, a pure strategy for the principal specifies her wage offer as a function of the previous history as well as the bonus payment as a function of the previous history, current-period wages and effort. Formally, it is a sequence of mappings $\left\{\sigma_t^P\right\}_{t=1}^T$, where, for each $t \leq T$, $\sigma_t^P = (d_t^P, w_t, b_t)$, and $d_t^P : \mathscr{H}^{t-1} \to \{0,1\}$, $(h^{t-1}) \mapsto d_t^P(h^{t-1})$,

[2] I assume this specific functional form for analytical tractability. Other (convex) cost functions deliver similar results as long as the third derivative is positive. A positive third derivative is needed for an interior solution in Section 3.1.

$w_t : \mathscr{H}^{t-1} \times \{0,1\} \to \mathbb{R}_+$, $(h^{t-1}, d_t^P) \mapsto w_t(h^{t-1})$, $b_t : \mathscr{H}^{t-1} \times \{0,1\} \times \mathbb{R}_+ \times \{0,1\} \times \mathbb{R}_+ \to \mathbb{R}_+$, $(h^{t-1}, d_t^P, w_t, d_t, e_t) \mapsto b_t(h^{t-1}, d_t^P, w_t, d_t, e_t)$.

Relational Contract and the Norm of Reciprocity The relational contract is agreed upon at the beginning of the game. It "activates" the norm of reciprocity and stipulates *reference functions* which specify history-dependent actions players are *supposed* to take. For the agent, the relational contract determines an acceptance function $\hat{d}_t^A(h^{t-1}, d_t^P, w_t)$ as well as an effort function $\hat{e}_t(h^{t-1}, d_t^P, w_t, d_t)$, with $\left(\hat{d}_t, \hat{e}_t\right) \in \sigma_t^A$. For the principal, the relational contract determines an offer function $\hat{d}_t^P(h^{t-1})$, a wage function $\hat{w}_t(h^{t-1}, d_t^P)$ and a bonus function $\hat{b}_t(h^{t-1}, w_t, d_t, e_t)$, with $\left(\hat{d}_t^P, \hat{w}_t, \hat{b}_t\right) \in \sigma_t^P$.

The norm of reciprocity states how the agent is supposed to reciprocate against "non-discretionary" upfront wages, that is, wages that are not paid as a reward for past effort. To incorporate this notion, the total wage $\hat{w}_t(h^{t-1}, d_t^P)$ is split into a discretionary part $\hat{w}_t^d(h^{t-1}, d_t^P)$ and a "non-discretionary" component. The latter is not a (direct) function of the agent's past choices and defined as $\hat{w}_t^{nd}\left(h^{t-1} \setminus \left\{e^{t-1}, d^{A,t-1}\right\}, d_t^P\right)$, where $e^{t-1} := (e_\tau)_{\tau=1}^{t-1}$ and $d^{A,t-1} := \left(d_\tau^A\right)_{\tau=1}^{t-1}$. Note that this specification does not rule out an indirect relationship between the agent's effort and w_t^{nd}, since the latter is a function of the principal's previous actions which itself are affected by the agent's behavior. Whereas the bonus and discretionary wage constitute the "direct" incentive system that grants payments as a reward for previously exerted effort, w_t^{nd} stipulates subsequent effort by the agent who adheres to the norm of reciprocity.

The agent's responsiveness to the norm of reciprocity is given by his utility function, which – in period t – equals

$$u_t = d_t \left(b_t + w_t - c(e_t) + \eta_t w_t^{nd} e_t \theta\right).$$

The term $\eta_t \in [0, \infty)$ captures the agent's inherent preferences for positive reciprocity (negative reciprocity is considered in Section 5.3) and lets the principal's output enter the agent's utility whenever $w_t^{nd} > 0$.[3] Its value in a given period depends on the history via a *norm function*, which takes the following form: When the relational contract is agreed upon at the beginning of the game, the reciprocity parameter is activated with $\eta_1 = \eta \geq 0$. The value η depends on the agent's individual characteristics, but also on the match-specific relationship between the principal and the agent.[4] η_t remains at η if the principal's actions so far have been consistent with the reference functions

[3] In a more general setting, the norm of reciprocity would be activated only if w_t^{nd} exceeded some reference wage (which might be determined by a minimum wage or the agent's outside option). Here, such a reference wage would equal zero; in Section 4.1, I allow for positive reference wages.

[4] For example because the agent develops sentiment for the principal when working for her (Akerlof, 1982).

specified by the relational contract. Otherwise, it drops to zero and remains there in all subsequent periods.[5] Therefore, in all periods $t \geq 2$,

$$\eta_t = \begin{cases} \eta & \textbf{if } d_\tau^P = \hat{d}_\tau^P(h^{\tau-1}),\, b_\tau \geq \hat{b}(h^{\tau-1}, d_\tau^P, w_\tau, d_\tau^A, e_\tau),\, w_\tau \geq \hat{w}(h^{\tau-1}, d_\tau^P), \textbf{all } \tau \leq t \\ 0 & \textbf{otherwise.} \end{cases}$$

Thus, η_t does *not* drop to zero after a deviation by the agent (and if no bonus is paid in response), capturing the idea that the agent's general "goodwill" towards the principal depends on the latter's behavior, not on his own. Hence, the agent's reciprocal inclinations towards the principal disappear once the latter refuses to make an offer she was supposed to make, or if she does not compensate the agent accordingly. This also includes deviations with respect to w_t^{nd} because otherwise, the principal could potentially reduce the non-discretionary wage after a deviation by the agent, which would contradict the definition of w_t^{nd} to be independent of the agent's previous effort.

The principal has no preferences for reciprocity and only maximizes her (monetary) payoffs:

$$\pi_t = d_t \left(e_t \theta - b_t - w_t \right).$$

Now, a subgame perfect equilibrium determines equilibrium functions $d_t^P(h^{t-1})$, $w_t(h^{t-1}, d_t^P)$, $d_t^A(h^{t-1}, d_t^P, w_t)$, $e_t(h^{t-1}, d_t^P, w_t, d_t^A)$, and $b_t(h^{t-1}, d_t^P, w_t, d_t^A, e_t)$. In addition, for every history, I impose the consistency requirements $\hat{d}_t^P = d_t^P$, $\hat{w}_t = w_t$, $\hat{d}_t^A = d_t^A$, $\hat{e}_t = e_t$, and $\hat{b}_t = b_t$. Incorporating these restrictions to a subgame perfect equilibrium, I denote the resulting equilibrium as a subgame norm-perfect equilibrium (SNPE).

In an SNPE in which $d_t^P = d_t^A = 1$ in all the periods of the game, the following recursive relationships hold in all periods $t \in \{1, ..., T\}$ for the principal's profits Π_t and the agent's utility U_t, where I set $\Pi_{T+1} = U_{T+1} = 0$:

$$\begin{aligned} \Pi_t &= e_t \theta - b_t - w_t + \delta \Pi_{t+1} \\ U_t &= b_t + w_t - c(e_t) + \eta_t w_t^{nd} e_t \theta + \delta U_{t+1}. \end{aligned}$$

In what follows, the objective is to characterize an SNPE that maximizes the principal's profits at the beginning of the game, Π_1.

Before characterizing such a profit-maximizing SNPE in Section 3, I discuss the assumptions made with respect to the agent's preferences for reciprocity and the finite horizon of the game.

[5] This definition can also be applied to settings in which the principal does not observe effort and output is not verifiable. Then, the bonus could be a function of output, and η_t would also drop to zero if the principal reneged on paying it.

2.3 Discussion of Assumptions

Reciprocity I build on the original gift exchange idea by Akerlof (1982) and assume that the principal can strategically "use" the norm of reciprocity. The agent understands the purpose of a gift received by the profit-maximizing principal but still reciprocates. This presumption is supported by experimental evidence presented by Malmendier and Schmidt (2017), who show that subjects reciprocate to gifts even though they understand that the giver is selfish and expects something in return.

The agent's responsiveness to the norm of reciprocity, the value η_t, depends on whether realized behavior deviates from the reference functions determined by the relational contract. Thus, the "behavioral" component of the agent's preferences is not only a function of the principal's past actions, but also of (expected) equilibrium behavior. This approach is inspired by the expectation-based loss aversion model of Kőszegi and Rabin (2006), where the extent of the agent's "behavioral" reaction to some realized event relates this event to expected equilibrium outcomes. It also builds upon Cox et al. (2008) who assume that an action by one player is perceived as more (less) generous – and consequently causes a stronger reciprocal reaction – if it allows the other player to obtain a higher (lower) monetary payoff (Cox et al. (2008), Definitions 1 and 2; Axiom R). Furthermore, (positive or negative) reciprocal reactions are stronger whenever an action upsets the status quo compared with this same action if it only upholds the status quo (Cox et al. (2008), Axiom S, Part 1). I capture the first aspect by assuming that realized payments trigger reciprocal behavior by the agent. Concerning the second aspect, I account for the "standard" role of a relational contract in the sense that it establishes a direct incentive system in which payments are promised in return for effort. Hence, wages and bonuses paid as a reward for past effort do *not* trigger reciprocal behavior, and η drops to zero after the principal refuses to compensate the agent for his performance. I also show that the latter is not needed if negative reciprocity is explicitly considered (Section 5.3) and that my main results are robust to letting the agent's reciprocal preferences respond to all payments (Sections 5.1 and 5.5). Although my approach might at first sight seem more complicated than a setting in which the agent reciprocates to all payments (i.e., also performance pay the agent is bound to receive anyway after high effort), it actually simplifies the analysis of the dynamic relational contract because both means to provide incentives can easily be separated.[6]

[6] One could also argue that, if performance pay was very generous in relation to the agent's effort cost, it should be regarded as a gift. However, such a payment could be split into a part that compensates the agent for his effort costs and one that grants him a rent. Paying this rent up front (and anticipating that the agent exerts effort accordingly) would then be equivalent to paying a non-discretionary wage in my setting.

Moreover, η drops to zero only after a deviation by the principal but not by the agent, hence the agent's general willingness to reciprocate gifts from the principal only depends on the latter's behavior. Otherwise, the agent would directly punish himself with a deviation, an implication I want to rule out.

Furthermore, I assume that reciprocity only enters the agent's stage game payoffs. This notion is consistent with evidence delivered by Bellemare and Shearer (2009), who show that a gift causes a positive effort response – but that this effect is only temporary. In Section 5.4, I also analyze a situation in which a positive wage today increases tomorrow's reference wage.

The reciprocity term in the agent's utility function contains θ, representing the extent to which the principal benefits from the agent's effort. This is in line with evidence that an important factor for reciprocity is the agent's assessment of the value generated for the principal (Hennig-Schmidt et al., 2010; Englmaier and Leider, 2012b).

Finally, I assume that the principal knows η. In Section 5.2, I explore the consequences of asymmetric information concerning the agent's preferences for reciprocity.

Finite Time Horizon I analyze a game of T periods, and most of the results on the dynamics of the employment relationship rely on the time horizon being finite. Whereas many real-life employment relationships either have a pre-defined ending date or an increasing probability of termination (which could be captured by a decreasing discount factor and generate the same dynamics, because those rely on the gradual reduction of future profits), most people work in multi-worker firms that continue to exist when workers retire. In my setting, this would imply that the principal also has the option to hire other agents for the job under consideration – after period T or potentially even before. Taking this into account, my results survive as long as multilateral punishments are not feasible (e.g., because deviations in one relationship cannot be observed by other (prospective) employees). With multilateral punishments, the principal's commitment in the employment relationship would not necessarily be smaller in the later periods of an employment relationship (which drives the dynamics in my setting). However, although deviations have to be private information of one match to render multilateral relational contracts (as in Levin, 2002) unfeasible, it would be fine for outsiders to observe whether the agent is employed or is fired. Then, only a premature termination could be punished by any "new" agent. This would make it costly for the principal to replace the agent early on, leaving my results valid.

If I completely ruled out punishments by prospective new agents in the case of a premature termination, the opportunity to employ other agents would manifest in a positive outside option for the principal, which I explore in Section 4.1. There, I assume

that this outside option is sufficiently small for the principal to never have an incentive to terminate an employment relationship on the equilibrium path. This could be due to replacement costs when hiring a new agent, like search costs or other labor market frictions, or direct replacement costs. Moreover, a sufficiently small outside option of the principal also rules out the use of efficiency wages, because any firing threat would not be credible (note that firing the agent would correspond to set $d^P = 0$).

3 Results

3.1 Reciprocity Spot Contract

I first derive a profit-maximizing spot contract and hence omit the time subscripts. Besides serving as a benchmark, such a contract will also be offered in the final period, T. In a spot contract, $b = 0$ because the principal has no incentive to make a payment to the agent after the latter has exerted effort. Therefore, the only means to incentivize the agent is a positive non-discretionary wage. Since $w = w^{nd}$, I omit the "*nd*" superscript in this section. Given w, and presuming he decides to work for the principal, the agent chooses effort to maximize his per-period utility $u = w - e^3/3 + \eta we\theta$.

The conditions for using the first order approach hold, and thus the agent's incentive compatibility (IC) constraint yields

$$e^* = \sqrt{\eta w \theta}. \tag{IC}$$

The principal sets w to maximize her expected per-period profits $\pi = e^*\theta - w$. Here, she has to take into account that accepting the contract must be optimal for the agent. This is captured by the agent's individual rationality (IR) constraint,

$$w - \frac{(e^*)^3}{3} + \eta w e^* \theta \geq 0. \tag{IR}$$

Concluding, the principal's problem is to

$$\max_w e^*\theta - w,$$

subject to (IR) and (IC) and the non-negativity constraint $w \geq 0$.

Lemma 1 *The profit-maximizing reciprocity spot contract has* $w = \eta\theta^3/4$ *and* $e^* = \eta\theta^2/2$. *Therefore,* $\pi = \eta\theta^3/4$ *and* $u = \eta\theta^3/4 + \eta^3\theta^6/12$.

The proof can be found in the Appendix.

Intuitively, a positive wage lets the agent partially internalize the principal's payoff, which is why he reciprocates and selects a positive effort level. Because this interaction is stronger for a more reciprocal agent, a higher η induces larger values of w, e^*, π, and u.

3.2 Relational Contract

Now, I analyze how a relational contract is used to incentivize the agent. Two aspects are of particular interest, namely the enforceability of the relational contract and how the norm of reciprocity affects outcomes. These aspects are explored in the next subsections, where I derive the properties of a profit maximizing relational contract.

3.2.1 Preliminaries and Optimization Problem

The relational contract determines payment functions $w_t^d(h^{t-1}, d_t^P)$, $w_t^{nd}\left(h^{t-1} \setminus \left\{e^{t-1}, d^{A,t-1}\right\}, d_t^P\right)$, and $b_t(h^{t-1}, d_t^P, w_t, d_t^A, e_t)$. The promise to make these payments must be credible, which is captured by dynamic enforcement (DE) constraints for each period t,

$$-b_t + \delta\Pi_{t+1} \geq \delta\tilde{\Pi}_{t+1}, \tag{DE}$$

as well as individual rationality (IR) constraints, $\Pi_t \geq \tilde{\Pi}_{t+1}$. Because $b_t \geq 0$, (IR) are implied by (DE) constraints and can hence be omitted. Π_{t+1} describes the principal's on-path and $\tilde{\Pi}_{t+1}$ her off-path continuation profits. The (DE) constraint indicates that discretionary payments are only feasible if $\Pi_{t+1} > \tilde{\Pi}_{t+1}$, i.e., if future equilibrium play can be made contingent on the principal's current behavior. Generally, relational contracts require a (potentially) infinite time horizon because of a standard unraveling argument that can be applied once a predetermined last period exists. If the equilibrium outcome in the last period is unique, the same holds for all preceding periods. In my case, however, the situation is different because the norm function lets η_t drop to zero once the principal refuses to make a specified payment. Moreover, the "standard" grim trigger punishment is imposed afterward and relational contracts are no longer feasible (adapting Abreu (1988) to my setting in the sense that any obseravable deviation from agreed upon behavior should be punished by a reversion to a player's minmax payoff). This implies that the principal's continuation profits are $\tilde{\Pi}_{t+1} = 0$ if she has deviated in any $\tau \leq t$, and her behavior in period $t < T$ indeed affects her future profits. Hence, not only does the relational contract determine whether a given payment "activates" the

agent's reciprocal preferences, but the latter are also a prerequisite for the relational contract to work.

In the next step, I explore the agent's incentives to exert equilibrium effort. Those are generally determined by a combination of reciprocity-based incentives (via a positive w_t^{nd}) and relational incentives (via b_t and w_t^d). Recall that my specification of the norm function implies that after a deviation by the agent, the reciprocity parameter remains at η. This indicates that the agent does not necessarily deviate to an effort level of zero. Moreover, since effort is public information, it is without loss to only specify a positive bonus $b_t \geq 0$ if the agent has exerted equilibrium effort and no bonus otherwise. Thus, the agent's (IC) constraint (which must hold in every period t) equals

$$-\frac{(e_t^*)^3}{3} + \eta w_t^{nd} e_t^* \theta + b_t + \delta U_{t+1} \geq -\frac{(\tilde{e}_t)^3}{3} + \eta w_t^{nd} \tilde{e}_t \theta + \delta \tilde{U}_{t+1}, \qquad \text{(IC)}$$

where $\tilde{U}_{t+1}$ is the agent's continuation utility after a deviation by himself. Moreover, if the agent deviates, he will choose an effort level $\tilde{e}_t = \text{argmax}\left(-e^3/3 + \eta w_t^{nd} e\theta\right)$, i.e., $\tilde{e}_t = \sqrt{\eta w_t^{nd} \theta}$. $\tilde{e}_t$ is the effort the agent would select if he only responded to the norm of reciprocity. Relational incentives using subsequent discretionary payments are needed to motivate the agent to exert additional effort $e_t^* - \tilde{e}_t$.

An (IR) constraint $U_t \geq 0$ must also hold in every period but is implied by (IC) because payments are assumed to be non-negative and because the right-hand side of (IC) cannot be smaller than zero (after a deviation, the agent can always secure a utility of zero by exerting no effort and then leaving the principal).

Concluding, the principal's problem is to maximize

$$\Pi_1 = \sum_{t=1}^{T} \delta^{t-1} \pi_t,$$

subject to a (DE) and (IC) constraints for every period t.

First, I derive a number of preliminary results, which substantially simplify the problem and are collected in Lemma 2.

Lemma 2 *There exists a profit-maximizing equilibrium which has the following properties in all periods t:*

- $w_t = w_t^{nd}$
- *(IC) holds as an equality*
- $U_t = \tilde{U}_t$

- *The equilibrium is sequentially optimal, hence the problem is equivalent to maximizing each π_t.*

The proof can be found in the Appendix.

First, it is without loss of generality to only use bonus payments for the provision of relational incentives. Thus, $w_t = w_t^{nd}$ for the remainder of the paper, and all upfront wages are non-discretionary. This simplifies the separation of "standard" direct incentives (which are denoted "relational incentives" and provided by b_t) from those relying on the norm of reciprocity (denoted "reciprocity-based incentives" and provided by w_t).

Second, the (IC) constraint binds in every period. If it did not bind, the bonus b_t could be slightly reduced, which would increase profits and relax the (DE) constraint without violating the (IC) constraint. Thus, the agent does not receive a rent for relational incentives. However, he enjoys a rent whenever $w_t (= w_t^{nd}) > 0$, i.e., when reciprocity-based incentives are provided. Importantly, though, these "warm-glow" rents cannot be used to provide relational incentives in earlier periods: If the agent was bound to lose them after a deviation (for example because of a firing threat as with efficiency wages), the upfront wage would not be non-discretionary anymore, and the agent would not reciprocate.[7]

Third, $U_t = \tilde{U}_t$ then follows because it is without loss of generality to provide relational incentives only with a current bonus. Thus, continuation play generally is not affected by the agent's actions, which finally implies that the profit-maximizing relational contract is sequentially optimal.

Collecting all results, binding (IC) constraints as well as $U_t = \tilde{U}_t$ yield $b_t = e_t^3/3 - \eta w_t e_t \theta + 2/3 \left(\sqrt{\eta w_t \theta}\right)^3$. Plugging this into the principal's profits and (DE) constraints, the optimization problem becomes to maximize

$\pi_t = e_t\theta - b_t - w_t = e_t\theta - \left(e_t^3/3 - \eta w_t e_t \theta + 2/3 \left(\sqrt{\eta w_t \theta}\right)^3\right) - w_t$ in every period t, subject to

$$\frac{e_t^3}{3} - \eta w_t \theta e_t \leq \delta \Pi_{t+1} - \frac{2}{3}\left(\sqrt{\eta w_t \theta}\right)^3. \tag{DE}$$

There, note that the enforceability of relational contracts is generally determined by a comparison of today's effort costs with the discounted future (net) payoffs generated in the relationship. Only if the latter are large enough are they sufficient to cover today's costs of exerting effort. Here, two additional terms enter if the (non-discretionary)

[7] This would be different if either bonus or discretionary wages also triggered direct reciprocal responses by the agent. Then, the respective payments would merely assume a larger relative weight in the optimal incentive scheme (see Sections 5.1 and 5.5).

wage is positive; first, the agent's preferences for reciprocity reduce the necessary bonus payment to implement a given effort level; second, if the agent deviates, he still selects positive effort.

3.2.2 Reciprocity-Based Incentives in a Relational Contract

In this section, I derive the conditions the use of reciprocity-based incentives in a relational contract. Different from relational incentives, those require a rent going to the agent, thus a positive wage is not paid in any case. First, I abstract from issues of enforceability and assume that the (DE) constraint does not bind (i.e., it is satisfied for the principal's preferred effort level). This situation is equivalent to one in which formal contracts based on effort would be feasible.

Lemma 3 *Assume the (DE) constraint does not bind in period $t < T$. Then, there exists a $\overline{\eta} > 0$ such that setting a strictly positive wage is optimal for $\eta > \overline{\eta}$, whereas the optimal wage is zero for $\eta \leq \overline{\eta}$.*

The proof can be found in the Appendix.

Lemma 3 implies that even if the principal's discounted future on-path profits are sufficiently large to not restrict her in setting the most preferred effort-based bonus b_t, she might still decide to grant the agent a rent. This is because the agent's responsiveness to the norm of reciprocity reduces his effective effort costs, but only in combination with a strictly positive upfront wage w_t. The principal thus faces a trade-off between the higher costs when paying a positive wage and the higher effort the agent is willing to exert in response. If the agent's preferences for reciprocity are sufficiently large, the latter effect dominates. In the following, I refer to the effort and wage levels for a non-binding (DE) constraint as the *first-best* levels.

In the next step, I assess how the agent's preferences for reciprocity affect the outcomes if her (DE) constraint binds.

Lemma 4 *Assume the (DE) constraint binds in period $t < T$. Then, equilibrium effort is smaller than with a non-binding (DE) constraint. Moreover, if paying a fixed wage is optimal in the situation with a non-binding (DE) constraint (i.e., if $\eta > \overline{\eta}$), the fixed wage is now larger. Otherwise (i.e., if $\eta \leq \overline{\eta}$), there exists a $\tilde{\eta}_t < \overline{\eta}$ such that setting a strictly positive wage is optimal for $\eta > \tilde{\eta}_t$, whereas the optimal wage equals zero for $\eta \leq \tilde{\eta}_t$. Finally, $\tilde{\eta}_t$ is increasing in δ.*

The proof can be found in the Appendix.

Besides reducing effective effort costs, an upfront wage also relaxes the principal's (DE) constraint by decreasing the bonus that must be paid for implementing a given effort level. Therefore, if the (DE) constraint binds, the wage is generally larger than when it does not bind.

All this implies that reciprocity-based incentives can improve the power of relational incentives for a given value of η, and vice versa (this complementarity between the two means to provide incentives is further fueled by a positive effect of η on future profits, see Proposition 2). Indeed, such a combination yields more efficient and profitable outcomes than the use of only one of them – a result that has received empirical support from Boosey and Goerg (2018). They conduct a lab experiment in which a manager and a worker interact for two periods. The worker can spend time completing a series of real effort tasks and is paid an upfront wage in every period. In addition, the principal may have the opportunity to pay a fixed bonus between the two periods, after the first period output has been observed. Boosey and Goerg (2018) find that average output is considerably larger with this option than in those treatments in which the principal either cannot pay a bonus (in which case, a positive effort is still observed, indicating that the participants have reciprocal preferences), or the bonus can be paid at the beginning or end of the game. This supports my result that a relational contract can boost productivity with agents who are known to be reciprocal and that a relational contract then can even be sustained with a finite time horizon.

3.2.3 Relational and Reciprocity-Based Incentives as Dynamic Substitutes

In this section, I characterize how, for a given value of η, the interaction between relational and reciprocity-based incentives evolves. The (DE) constraint might or might not bind in any period $t < T$ depending on discount factor δ, reciprocity parameter η, and productivity θ. Furthermore, the (DE) constraint becomes tighter in later periods.

Lemma 5 *For every $\delta > 0$, the (DE) constraint in period $T-1$ holds for first-best effort and wage levels if η is sufficiently large. For any values η and θ, the (DE) constraint in period $T-1$ does not hold for first-best effort and wage levels if the discount factor is sufficiently small.*

Furthermore, $\Pi_{t-1} > \Pi_t$ for all $t \leq T$.

The proof can be found in the Appendix.

The principal's commitment in a relational contract is determined by what she has to lose given she deviates. If the discount factor is small, she cares less about a potential

reduction in future profits and is therefore less willing to pay a bonus to compensate the agent for his effort. Conversely, a larger reciprocity parameter η increases future profits and reduces today's effective effort costs. The second part of Lemma 5 states that on-path profits decline over time. As time passes, the remaining time horizon and consequently the number of periods in which profits can be generated falls. This also triggers a reinforcing effect. Since $\Pi_T > 0$, the (DE) constraint allows a larger effort level in period $T-1$ than in period T. Then, per-period profits in period $T-1$ are higher than those in period T, and implementable effort in period $T-2$ is even larger than in period $T-1$, and so on.

All this implies that if the (DE) constraint binds in a given period $\tilde{t}$, it will also bind in all *subsequent* periods $t > \tilde{t}$. If it is slack in a given period $\hat{t}$, it will also be slack in all *previous* periods $t < \hat{t}$. This yields the following effort and compensation dynamics.

Proposition 1 *Equilibrium effort is weakly decreasing over time, i.e., $e_t^* \leq e_{t-1}^*$. Moreover, $e_t^* < e_{t-1}^*$ implies $e_{t+1}^* < e_t^*$, whereas $e_{t+1}^* = e_t^*$ implies $e_t^* = e_{t-1}^*$.*

The equilibrium wage is weakly increasing over time and the bonus weakly decreasing, i.e., $w_t \geq w_{t-1}$ and $b_t \leq b_{t-1}$. Moreover, $w_t > w_{t-1}$ and $b_t < b_{t-1}$ imply $w_{t+1} > w_t$ and $b_{t+1} < b_t$, whereas $w_{t+1} = w_t$ and $b_{t+1} = b_t$ imply $w_{t+1} = w_t$ and $b_t = b_{t-1}$.

The agent's total compensation, $w_t + b_t$, might increase or decrease over time.

The proof can be found in the Appendix.

Proposition 1 indicates that effort and compensation are time-invariant in the early stages of the employment relationship, as long as the future is sufficiently valuable for the (DE) constraint not to bind. Once the end of the employment relationship is close and the (DE) constraint binds, effort and bonus profiles become downward-sloping and the wage profile upward-sloping. This is because the principal can no longer credibly promise her preferred bonus. On the one hand, this reduces equilibrium effort. On the other hand, the principal might respond with a wage increase that raises equilibrium effort due to the agent's preferences for reciprocity. The effort increase caused by a higher wage does not fully compensate for the effort reduction caused by the binding (DE) constraint, though, because the costs of implementing one additional unit of effort are now higher with reciprocity-based incentives than with relational incentives. Over time, the (DE) constraint is further tightened (Lemma 5).

Hence, toward the end of an employment relationship, relational incentives are gradually replaced by reciprocity-based incentives (bonus $\downarrow$, wage $\uparrow$), with the substitution however being incomplete (effort $\downarrow$). The dynamics of the agent's total compensation,

$w_t + b_t$, are not necessarily monotone and depend on the relative importance of relational and reciprocity-based incentives.

Consistent with the described effort dynamics, there is evidence that a worker's productivity decreases once he approaches retirement. Using US data, Haltiwanger et al. (1999) find that a firm's productivity is higher if it has a lower proportion of workers older than 55. Skirbekk (2004) reports that older workers generally have lower productivity and are overpaid relative to their productivity. Using Belgian data, Lallemand and Rycx (2009) show that having a high share of workers above 49 is harmful for a firm's productivity. Reduced effort in the last periods of an employment relationship has also been observed in many lab experiments (e.g., Brown et al., 2004; Fehr et al., 2009). Those results have mainly been attributed to selfish individuals imitating those with social preferences in early periods to collect rents later on. I further explore this aspect in Section 5.2 and show that my model can deliver an alternative explanation of many of the results presented by Brown et al. (2004).

Before going on, note that wage and bonus dynamics rely on my specification of reciprocity and make use of the result that it is without loss (but not uniquely optimal) to exclusively use b_t to provide relational incentives in period t, whereas upfront wages only provide reciprocity-based incentives (Lemma 2). Thus, I am cautious relating these results to empirical evidence (such as the wide-spread occurrence of upward sloping wage curves over employees' careers; see Waldman, 2012 for a summary). The gradual replacement of relational incentives (the agent responds to promised payments to be made *after* effort has been exerted) with reciprocity-based incentives (the agent responds to a "gift" *before* effort is exerted), however, is robust to different specifications of reciprocity (see sections 5.1 and 5.5)

Payoffs

Now, I present results on the dynamics of players' payoffs. Whereas the principal's per-period profits decrease over time (once (DE) binds), the opposite is true for the agent's per-period utilities. This result is also driven by the gradual replacement of relational with reciprocity-based incentives; because of the binding (IC) constraint, the agent only collects a rent with the latter.

Lemma 6 *The principal's per-period profits π_t are weakly decreasing over time, i.e., $\pi_t \leq \pi_{t-1}$. Moreover, $\pi_t < \pi_{t-1}$ implies $\pi_{t+1} < \pi_t$, whereas $\pi_{t+1} = \pi_t$ implies $\pi_t = \pi_{t-1}$.*

The agent's per-period utility u_t is weakly increasing over time, i.e., $u_t \geq u_{t-1}$. Moreover, $u_t > u_{t-1}$ implies $u_{t+1} > u_t$, whereas $u_{t+1} = u_t$ implies $u_t = u_{t-1}$.

The proof can be found in the Appendix.

3.3 Reciprocity

In the previous sections, I derived the properties of a profit-maximizing relational contract for a given value of η. Now, I explore how the agent's responsiveness to the norm of reciprocity affects effort during his career.

Proposition 2 *Equilibrium profits Π_t and effort e_t increase in η. This positive effect is stronger if the (DE) constraint binds (i.e., in later periods).*

The proof can be found in the Appendix.

First, a higher η directly raises e_t (and consequently profits) for a given $w_t > 0$ due to the reduction in effective effort costs. Second, there is an indirect effect. Because future profits also increase in η, the (DE) constraint in period t is relaxed, which further leads to higher effort and profits. This interaction provides an additional source for the complementarity between relational and reciprocity-based incentives. Moreover, Proposition 2 indicates that the positive effect of η on effort is stronger if the principal's (DE) constraint binds (i.e., in the later stages of the agent's career). Then, the incentive system places more weight on reciprocal incentives, and the role of η intensifies.

Evidence on the generally positive relationship between reciprocity and effort has been provided by Dohmen et al. (2009) based on data from the SOEP, an annual panel survey representative of the German population that contains a wide range of questions on the personal and socioeconomic situation as well as labor market status and income of respondents. In a number of years (2005, 2010, and 2015), it contained questions designed to capture individual reciprocal inclinations. As a measure of (non-verifiable) effort, Dohmen et al. (2009) use overtime work, finding that individuals with stronger reciprocal inclinations are more likely to work overtime. Moreover, Fahn et al. (2017) find evidence that is consistent with a stronger effect of reciprocity on effort towards the end of an individual's career. They confirm the results of Dohmen et al. (2009), and also show that the positive link between reciprocity and overtime is significantly more pronounced for older workers close to retirement.

4 Further Implications

In the next two subsections, I further explore how the agent's reciprocal preferences affect the optimal design of relational contracts in various environments. First, I analyze how different levels of the relationship surplus shape the interplay between relational and reciprocity-based incentives. The size of the relationship surplus is to a large extent

determined by players' outside opportunities, hence I argue that the results from this section can generate insights on the role of labor market competition. Second, I show that a formal commitment to pay future (non-discretionary) wages – a feature shared by many real-world employment relationships – can increase the principal's commitment in the relational contract.

4.1 Relationship Surplus and Competition

An important question in (behavioral) economics relates to the effect of labor market competition on the relevance and persistence of social preferences. A number of theoretical and empirical contributions indicate that social preferences are driven out by competition if contracts are complete (Fehr and Schmidt, 1999; Dufwenberg et al., 2011). With incomplete contracts (such as in the present setting), however, the situation is different (Fehr and Fischbacher, 2002; Schmidt, 2011). Schmidt (2011) analyzes how labor market competition might affect the utilization of fairness preferences by firms. Applying a static model, he shows that induced effort levels are the same for all degrees of competition and that only rents are shifted between firms and workers.

In this section, I discuss how competition shapes the optimal relational utilization of reciprocity-based preferences. I show that in a more competitive labor market, the principal actually makes *more* use of reciprocity-based incentives if more intense labor market competition reduces the rent generated by the relational contract. My approach to model labor market competition follows Schmidt (2011), where the degree of competition determines the outside options of principal and agent. This reduced-form approach substantially simplifies the analysis and still allows me to generate a number of insights. The principal's outside option equals $\overline{\Pi} \geq 0$ and is the same in every period t. As discussed in Section 2.3, this view can be supported by the presumption that T reflects only the agent's time horizon, whereas the firm's is potentially infinite. Then, $\overline{\Pi}$ would include the profits from hiring new agents once the current employment relationship is terminated (with multilateral punishments not being feasible). Moreover, $\overline{\Pi}$ is assumed to be smaller than profits in period T (because of sufficiently high costs of replacing the agent with a new one). Hence a premature on-path termination and the use of efficiency wages are not optimal. A larger degree of competition for workers is likely to decrease $\overline{\Pi}$ because more intense competition increases the costs and time incurred in finding a new agent.

The agent's outside opportunities are captured by the wage $\overline{w} \geq 0$ he could secure when working for a different employer. Note that this specification is less restrictive than it might seem for a setting in which most incentives are informal. Because the agent

does not receive a rent from relational incentives, his value from being employed (by his current or alternative employers) is solely determined by upfront wages. Thus, one could let the agent's outside option be constituted by a more general increasing function of his outside wage. This, however, would not affect my results qualitatively but only complicate the exposition. Moreover, $\overline{w}$ is constant over time (although one might expect older employees to have worse outside opportunities), which is not material for my results as long as the marginal effect of competition on $\overline{w}$ is the same in all periods. More intense competition for workers increases the outside wage $\overline{w}$. Importantly, $\overline{w}$ now also constitutes the new reference wage that determines whether wages are regarded as generous and "activate" the norm of reciprocity. Therefore, the agent's per-period payoff in period t amounts to

$$u_t = w_t + b_t + \eta_t \left(w_t - \overline{w}\right) \theta e_t - \frac{e_t^3}{3},$$

which already incorporates that a wage below $\overline{w}$ would not be accepted by the agent.

First, I characterize effort and wage in a spot reciprocity contract.

Lemma 7 *Effort in the profit-maximizing spot reciprocity contract is independent of* $\overline{w}$ *and* $\overline{\Pi}$*. Moreover,* $\partial w/\partial \overline{w} = 1$*, and* $\partial w/\partial \overline{\Pi} = \partial e/\partial \overline{w} = \partial e/\partial \overline{\Pi} = 0$*.*

The proof can be found in the Appendix.

The principal responds to a higher $\overline{w}$ with an increase in w_t to keep incentives constant. Therefore, labor market competition does not affect the importance of the norm of reciprocity for the optimal provision of incentives in a *static setting*. $\overline{w}$ only causes a redistribution of rents, whereas the outcomes are entirely independent of $\overline{\Pi}$ (as long as $\pi = \eta\theta^3/4 - \overline{w} \geq \overline{\Pi}$, which I implicitly assume and which implies that exercising the outside option is not optimal for the principal). This replicates the results of Schmidt (2011), who presents equivalent findings for the case of fairness preferences.

To derive a profit-maximizing relational contract with positive outside options, I first characterize the agent's (IC) constraint for a general $\overline{w} \geq 0$:

$$b_t - \frac{(e_t)^3}{3} + \eta \left(w_t - \overline{w}\right) e_t \theta \geq \frac{2}{3} \left(\sqrt{\eta \left(w_t - \overline{w}\right) \theta}\right)^3.$$

The outside wage $\overline{w}$ enters the agent's (IC) constraint only via the associated increase in the reference wage. This is different from a "standard" efficiency wage effect, where a better outside option of an employee directly reduces his incentives to work hard.

Here, any firing threat (which is an important component of the efficiency wage mechanism) would not be credible because keeping the agent provides the principal with a payoff above $\overline{\Pi}$.

The principal's (DE) constraint amounts to

$$-b_t + \delta\Pi_{t+1} \geq \delta\overline{\Pi}.$$

(DE) is tightened by a larger $\overline{\Pi}$, and thus the principal can ceteris paribus commit to a *larger* bonus if facing tighter competition for labor.

The general structure of a profit-maximizing relational contract is as in my main model, with constant wage and effort levels as long as (DE) is slack as well as upward-sloping wage and downward-sloping effort profiles once (DE) becomes binding. Still, $\overline{\Pi}$ and $\overline{w}$ crucially affect the importance of reciprocity-based incentives, as described in Proposition 3.

Proposition 3 *Larger values of $\overline{w}$ and/or $\overline{\Pi}$ tighten the (DE) constraint.*

If (DE) does not bind in period t, $\partial w_t/\partial\overline{w} = 1$. Moreover, $\partial w_t/\partial\overline{\Pi} = \partial e_t/\partial\overline{w} = \partial e_t/\partial\overline{\Pi} = 0$.

If (DE) binds in period t, $\partial w_t/\partial\overline{w} > 1$ and $\partial w_t/\partial\overline{\Pi} > 0$. Moreover, $\partial e_t/\partial\overline{w} < 0$ and $\partial e_t/\partial\overline{\Pi} < 0$.

Finally, for given values of $\overline{w}$ and $\overline{\Pi}$, the effort and compensation dynamics are as in Proposition 1

The proof can be found in the Appendix.

As larger values of $\overline{w}$ and $\overline{\Pi}$ have no direct effect on the optimal provision of incentives, the principal implements the same effort for all values of $\overline{w}$ and $\overline{\Pi}$ if (DE) does not bind (i.e., in earlier periods of the employment relationship). Then, as in a reciprocity spot contract, a higher $\overline{w}$ causes a mere redistribution of rents from the principal to the agent (and $\partial w_t/\partial\overline{w} = 1$).

However, higher values of $\overline{w}$ and $\overline{\Pi}$ reduce the principal's future profits. This tightens the (DE) constraint and, once the constraint binds, restricts the principal's possibility of using relational incentives. As in the main analysis (see Lemma 1 and Proposition 1), she mitigates the necessary effort reduction by expanding reciprocity-based incentives and raising w_t beyond the increase induced by a larger $\overline{w}$. Hence, $\partial w_t/\partial\overline{w} > 1$ and $\partial w_t/\partial\overline{\Pi} > 0$ if (DE) binds.

Now, more competition in the labor market increases $\overline{w}$ but reduces $\overline{\Pi}$, both of which have opposite effects. If the effect of a lower $\overline{\Pi}$ dominates, a more competitive labor market allows the effort reduction induced by a binding (DE) to materialize at a

later point. Moreover, effort is generally higher and fixed wages are lower, and thus reciprocity-based incentives assume a *less* important role. The opposite happens if the effect of a higher $\overline{w}$ dominates and more intense labor market competition reduces the relationship surplus. Then, the (DE) constraint binds earlier if the labor market is more competitive, effort is lower and wages are higher. All this is driven by the reduced commitment in the relational contract, letting reciprocity-based incentives become more important in a profit-maximizing dynamic incentive scheme.

4.2 Long-Term Commitment

So far, I have assumed that although future wages are part of the (informally specified) relational contract, there is no legal obligation for the principal to pay any upfront wage at the beginning of any period. In most real-world employment relationships, though, a formal employment contract specifies fixed wage payments for months and years in advance.[8] Such an unconditional commitment to wage payments seems counterproductive for the provision of (formal or relational) incentives, in particular if legal institutions – such as employment protection laws – or contractual arrangements limit the use of firing threats and thus the effectiveness of efficiency wages. In the following, I show that the ability to formally commit to future non-discretionary wage payments can increase profits, by increasing the principal's credibility when providing relational incentives. For the following analyis, I set $T = 2$.

I assume that, at the beginning of the first period, the principal not only can pay w_1, but in addition formally commit to an amount $\hat{w}_2$ paid at the beginning of the second period. The agent reacts reciprocally towards $\hat{w}_2$ in period 2, when this payment is actually made. Committing to future payments also comes with a cost, and the agent reacts less strongly to a wage that has been set in the past, in the sense that only $\gamma\hat{w}_2$ enters the "reciprocity term" in the agent's utility function, with $\gamma \leq 1$. When the second period comes, the principal can further increase the wage by $w_2 - \hat{w}_2$ (hence w_2 amounts to the total second-period wage), to which the agent fully reciprocates. Finally, the principal is not able to fire the agent at the end of the first/beginning of the second period.

Note that, without costs of commitment, setting a positive $\hat{w}_2$ would always be (weakly) optimal because equilibrium outcomes are deterministic. In reality, there also might be forces outside the model that make it costly for the principal to reduce her future flexibility, for example demand fluctuations that let the size of a firm's optimal

[8] These contracts then might or might not include additional performance-based pay. Indeed, there is evidence that even though the compensation of most employees is somehow tied to their performance, a substantial share of their salary is not (Lemieux et al. (2009), p. 22).

workforce vary.

Now, the agent's second-period (on-path) utility amounts to

$$u_2 = w_2 - \frac{e_2^3}{3} + \eta\left(\gamma\hat{w}_2 + w_2 - \hat{w}_2\right) e_2\theta,$$

and $e_2 = \sqrt{\left[w_2 - (1-\gamma)\,\hat{w}_2\right]\eta\theta}$.

At the beginning of period $t = 2$, the principal takes $\hat{w}_2$ as given and sets w_2 to maximize

$$\begin{aligned}\pi_2 =& e_2\theta - w_2\\ =& \sqrt{\left[w_2 - (1-\gamma)\,\hat{w}_2\right]\eta\theta}\,\theta - w_2,\end{aligned}$$

which yields $w_2 = \eta\theta^3/4 + (1-\gamma)\,\hat{w}_2$, $e_2 = \eta\theta^2/2$, and $\pi_2 = \eta\theta^3/4 - (1-\gamma)\,\hat{w}_2$.

Thus, the implemented second-period effort level is independent of $\hat{w}_2$. Profits π_2 are decreasing in $\hat{w}_2$ for $\gamma < 1$, thus it would be optimal to set $\hat{w}_2 = 0$ from the perspective of period 2. However, a positive $\hat{w}_2$ can be optimal from the perspective of period 1 because it relaxes the principal's dynamic enforcement constraint, $-b_1 + \delta\pi_2 \geq -\delta\hat{w}_2$:

Proposition 4 *Assume the principal can commit to $\hat{w}_2$, as specified above.*

- *If $\eta^2\theta^3$ is sufficiently large, (DE) does not bind. Then, $\hat{w}_2 = 0$.*
- *Otherwise, (DE) binds. In this case, $\hat{w}_2 > 0$ if γ is sufficiently large.*

The proof can be found in the Appendix.

A positive $\hat{w}_2$ entails direct costs because it triggers less reciprocal behavior than a payment that is initiated today. Therefore, $\hat{w}_2 = 0$ if the (DE) constraint is slack. If (DE) binds, though, a positive $\hat{w}_2$ is optimal if γ is not too small. The reason is that the agent receives $\hat{w}_2$ in any case, irrespective of whether the principal has paid the first-period bonus or not. However, the agent only reacts reciprocally to $\hat{w}_2$ in case b_1 has been paid. Thus, the costs of a deviation go up for the principal, and her increased vulnerability allows to credibly promise a higher bonus.[9]

[9] This mechanism is reminiscent of outcomes in Ramey and Watson (1997) or Englmaier and Fahn (2019). There, physical investments increase the future rent generated by the relational contract by more than reservation values. Thus, investments increase players' commitment, and "overinvestments" can be optimal.

5 Extensions and Robustness

In the following, I change some of the previously made assumptions and explore how this affects my results, still using a simplified setting with $T = 2$.

5.1 Reciprocity Triggered by all Current Payments

First, I let the agent's preferences for reciprocity be triggered by *all* realized current payments. Then, wages paid as a reward for previously exerted effort (and not only w_t^{nd}) also induce the agent to reciprocate. This does not hold for the bonus, however, which is paid after effort has been exerted (in Section 5.5, I let the norm of reciprocity extend to expected bonus payments). Therefore, only upfront wages are used to provide incentives because they can assume the role of the bonus and additionally induce reciprocal behavior.

To formally underpin this claim, the agent's second-period effort still maximizes $u_2 = w_2 - c(e_2) + \eta w_2 \theta e_2$; hence,

$$e_2^* = \sqrt{\eta w_2 \theta}.$$

Different from before, the principal does not maximize π_2 when selecting w_2, hence the profit-maximizing equilibrium is not sequentially optimal. Instead, w_2 is also a function of e_1 and set to maximize the principal's total discounted profit stream, Π_1. The agent's first-period effort must satisfy his (IC) constraint. Here, I assume that once the agent deviates, $b_1 = 0$, and w_2 is set such that π_2 is myopically maximized (in which case $w_2^* = \eta\theta^3/4$, $e_2^* = \eta\theta^2/2$, and $u_2 = \eta\theta^3/4 + \eta^3\theta^6/12$).[10] Therefore, if the agent deviates, he chooses $\tilde{e}_1$ to maximize $\tilde{u}_1 = w_1 - e_1^3/3 + \eta w_1 \theta e_1$, and thus $\tilde{e}_1 = \sqrt{\eta w_1 \theta}$.

All this implies that the agent's (IC) constraint equals

$$\begin{aligned} & b_1 - \frac{e_1^3}{3} + \eta w_1 \theta e_1^* + \delta \left[w_2 + \frac{2\left(\sqrt{\eta w_2 \theta}\right)^3}{3} \right] \\ \geq & \frac{2\left(\sqrt{\eta w_1 \theta}\right)^3}{3} + \delta \left(\frac{\eta\theta^3}{4} + \frac{\eta^3\theta^6}{12} \right). \end{aligned} \tag{IC}$$

The principal is only willing to make equilibrium payments if her (DE) constraint holds,

$$-b_1 + \delta\left(e_2\theta - w_2\right) \geq 0. \tag{DE}$$

Furthermore, η drops to zero if w_2 differs from the amount promised at the beginning

[10] As before, a firing threat, which would maximize the power of incentives, is not credible.

of period 1.

Then, the principal sets w_1, w_2, and b_1 to maximize $\Pi_1 = e_1^*\theta - w_1 - b_1 + \delta\,(e_2^*\theta - w_2)$, subject to (IC) and (DE), and taking into account that $e_2^* = \sqrt{\eta w_2 \theta}$.

The structure of the optimal arrangement is similar to that in the main part, with two exceptions. First, it is optimal to set $b_1 = 0$. On the contrary, assume a profit-maximizing equilibrium has $b_1 > 0$. Then, a reduction in b_1 by a small $\varepsilon > 0$ together with an increase in w_2 by ε/δ does not affect (DE) and Π_1, but does relax (IC). Therefore, w_2 is *above* the level maximizing π_2 and bounded by the condition that second-period profits must be non-negative. This implies that the back-loading of upfront wages is more pronounced than before. Second, the principal's profits will be larger than those in the main model because the payments used to provide relational incentives also trigger reciprocal behavior, an aspect missing before.[11]

However, it is not possible anymore to easily separate the two means to provide incentives because w_2 is not only used to provide reciprocity-based incentives in the second, but also for relational incentives in the first period. Put differently, the second-period "warm-glow" is also at the discretion of the agent's first-period effort and thus used to provide relational incentives. Still, because relational incentives are only relevant in the first period, this section indicates that relational incentives continue to be more important in early stages of the relationship.

5.2 Asymmetric Information

So far, I have assumed that the principal is aware of the agent's η, for example because of personality tests used in the hiring process. In this section, I explore the potential implications of asymmetric information on the agent's reciprocal inclinations. I assume that the agent can either be a "reciprocal" type with $\eta > 0$ (with probability $p \in (0,1)$) or a "selfish" type with no reciprocal preferences (with probability $1-p$). Moreover, the agent's type is his private information. Assuming that the principal can design the incentive scheme and does so in a profit-maximizing way, she chooses one of the following two options. First, the principal asks for a first-period effort level that only the reciprocal, but not the selfish agent is willing to exert. Then, the selfish agent collects the first-period wage, but is subsequently detected and fired (because he would exert no effort in the second period). I call this a "separation contract". Second, the effort request is sufficiently low that it satisfies the selfish type's (IC) constraint. In this case, the agent's effort choice cannot be used to screen agents and both types are also employed

[11] With the equilibrium values from the main part and with w_2 adjusted to include the first-period bonus, the (IC) constraint would now be slack.

in the second period. Only then does the selfish agent – after collecting w_2 – shirk by exerting zero effort. I call this arrangement a "pooling contract".

I retain the setting of Section 5.1 where the norm of reciprocity is triggered by all realized payments. Although this does not allow for a clear distinction between relational and reciprocity-based incentives anymore, it simplifies the analysis of asymmetric information because, in a separation contract, the reciprocal agent takes into account that he will only remain employed if he exerts equilibrium effort in the first period. Therefore, his incentives to exert first-period effort are not only determined by the corresponding direct incentives, but also by the benefits of not being regarded as the selfish type. Taking this into account, only future wages are used to motivate the agent (Section 5.1).

Now, I derive a Perfect Bayesian Equilibrium where *any* deviation by the agent lets the principal assign probability 1 to facing the selfish type. Then, a separation and a pooling contract are both feasible. The (IC) constraints, one for the selfish type (ICS), and one for the reciprocal type (ICR), already taking into account that $e_2 = \sqrt{\eta w_2 \theta}$, amount to

$$-\frac{e_1^3}{3} + \delta w_2 \geq 0 \tag{ICS}$$

$$-\frac{e_1^3}{3} + \eta w_1 \theta e_1 + \delta \left[w_2 + \frac{2\left(\sqrt{\eta w_2 \theta}\right)^3}{3} \right] \geq -\frac{\tilde{e}_1^3}{3} + \eta w_1 \theta \tilde{e}_1, \tag{ICR}$$

with $\tilde{e}_1 = \sqrt{\eta w_1 \theta}$. Different from Section 5.1, a deviation from the equilibrium effort now results in a termination and henceforth zero off-path continuation utilities. For any effort level $e_1 \geq \tilde{e}_1$ (ICS) is tighter than (ICR) (this is shown in the proof to Proposition 5). Therefore, if the principal offered the profit-maximizing contract designed for a reciprocal type (which involves a binding (ICR) constraint), this would automatically result in a separation of types. Moreover, effort in a pooling contract will be determined by a binding (ICS) constraint.

Proposition 5 *In a profit-maximizing perfect Bayesian equilibrium at which any deviation from equilibrium effort induces the principal to assign probability 1 to facing a selfish type, a pooling contract is optimal if p is sufficiently small. If p is sufficiently large, a separating contract is optimal.*

The proof can be found in the Appendix.

Generally, the principal faces the following trade-off. First, with a pooling contract, the first-period effort is rather low (determined by a binding (ICS) constraint); however, it is exerted by both types. Then, only the reciprocal type exerts effort in the second period, whereas both are paid w_2. In this case, the principal's expected profits are $\Pi_1^P = e_1\theta - w_1 + \delta\left[p\left(\sqrt{w_2\eta\theta}\theta - w_2\right) - (1-p)w_2\right]$, and outcomes resemble those in the classical reputation literature (see Mailath and Samuelson, 2006). Second, with a separating contract, the first-period effort is higher for a given w_2 (and determined by a binding (ICR) constraint), however only exerted by the reciprocal type. Then, both types are paid w_1, whereas the selfish type is fired and only the reciprocal type remains employed in the second period. In this case, the principal's expected profits are $\Pi_1^S = -w_1 + p\left[e_1\theta + \delta\left(\sqrt{w_2\eta\theta}\theta - w_2\right)\right]$. If p is sufficiently small, the principal prefers a pooling contract.

This pooling contract, however, relies on the assumption that the reciprocal type cannot reveal himself by choosing a higher effort level. But this restriction generally does not survive the Intuitive Criterion as a refinement of a perfect Bayesian equilibrium (Cho and Kreps, 1987). Assume that, in a pooling contract, an agent chooses an effort level slightly higher than equilibrium effort. Since the selfish type's (IC) constraint binds, whereas the reciprocal type's is slack, a deviation to a higher effort level should indicate that the principal in fact faces the reciprocal type. If the principal responds to this revelation by offering the profit-maximizing second-period wage for the reciprocal type and if this gives the latter a higher utility than equilibrium play, an upward deviation by the reciprocal type indeed increases his utility. To support the relevance of this argument, in the proof to Proposition 5, I show that for low p and consequently a pooling contract,[12] $e_1^* = \sqrt[3]{3\delta p^2\eta\theta^3}$ and $w_2 = e_1^3/3\delta = p^2\eta\theta^3$. If the reciprocal type deviates and chooses an effort level $e_1^* + \varepsilon$, the principal will take this as a signal that she faces the reciprocal type and might instead offer $w_2 = \eta\theta^3/4$ (the second-period wage that maximizes her profits with a reciprocal type; see the proof to Lemma 1). This wage also increases the reciprocal type's utility for $p < 1/2$.

Although a more general characterization of an optimal arrangement under asymmetric information is beyond the scope of this paper, note that a large amount of experimental evidence indicates that cooperation is larger in repeated than in one-shot interactions, even with a predefined last period. This is usually attributed to selfish types imitating those with social preferences to collect future rents (Fehr et al., 2009). I aim to provide support for a complementary story that takes into account that indi-

[12] More precisely, for $p^2 \leq \left(\sqrt{\frac{2}{\theta}}\right)^3/3\delta\eta$.

viduals with social preferences also behave strategically.[13] If the uninformed party can determine the incentive scheme, and in particular ask for a certain effort level, pooling equilibria at which a selfish type imitates a reciprocal type are much harder to maintain. Then, an early separation of types can be achieved by requiring an effort level that just satisfies the reciprocal type's (IC) constraint, with the remaining matches thereafter having a relational contract that produces outcomes resembling my main results (high effort in early periods, declining effort once the last period approaches). Such results have indeed been observed in the lab experiments conducted by Brown et al. (2004). They compare different settings, in particular one in which players (among whom one side assumes the role of firms and the other side represents workers) have the option to form long-term relationships or are randomly matched in each of the 15 rounds. Firms pay wages in every period and ask for effort from "their" workers, who subsequently select their effort levels. Brown et al. (2004) find that effort is significantly larger in the treatment with long-term relationships, where effort only falls in the last two periods. They present a theoretical explanation where some players have fairness preferences and where those without imitate the fair players early on, which mirrors the pooling contract in my setting. However, they observe many separations early on (70 percent in period 1, 65 percent in period 2) but few separations in later periods, which indicates that their outcomes rather resemble separating contracts.

The setting in Brown et al. (2004) admittedly differs from my theoretical model in several ways. For example, students who assume the role of firms might also have social preferences, whereas only the agent is reciprocal in my setting. Nevertheless, the presented theoretical analysis, together with a careful analysis of experimental results such as those presented by Brown et al. (2004), justify the notion that not only the "selfish types mimic fair types" story of a pooling contract might contribute to explaining many experimental results. In particular, if players do not face an inflexible environment such as a standard prisoner's dilemma, the possibility of separating types early on and subsequently having a relational contract might also contribute to the high cooperation observed in repeated, but finite, gift-exchange experiments.

5.3 Negative Reciprocity

So far, I have focused on the positive effects of the norm of reciprocity. I have abstracted from any potential "dark side" of reciprocal preferences in the sense that if an agent is granted a lower payment than expected, he wants to actively harm the principal.

[13] This is not assumed in most of the reputation literature (Mailath and Samuelson, 2006), where "commitment" types automatically choose cooperative actions.

The potential consequences of negative reciprocity have been explored by, for example, Dufwenberg and Kirchsteiger (2004), Dohmen et al. (2009), and Netzer and Schmutzler (2014). In this section, I introduce negative reciprocity and show that it leads to the same results as in the main part of this paper, even if η does not drop to zero after a deviation by the principal. This section therefore also serves as a robustness exercise to show that my results continue to hold if the agent's preferences are unaffected by the principal's behavior.

I use the approach introduced by Hart and Moore (2008), who assume that the terms of a contract provide reference points and determine a party's ex post performance. If someone receives less than he feels entitled to, he shades on performance, thereby causing a deadweight loss that has to be borne by the other party. Such an assumption has received empirical support from Malmendier and Schmidt (2017), who show that individuals exert negative reciprocity upon a potential gift giver if they expected a gift but did not receive one. I adapt the setting of Hart and Moore (2008) to my environment and assume that the relational contract also determines the agent's reference point.

Therefore, the agent feels entitled to the equilibrium bonus b_1^*. If he receives a lower bonus, his period-1 utility decreases by $\underline{\eta}\,(b_1^* - b_1)$, where $\underline{\eta} \geq 0$ and b_1 is the bonus actually paid by the principal. Moreover, the agent can reduce this utility loss via shading (e.g., by sabotaging the principal), by an amount σ at the agent's discretion. I assume that the agent still has to be employed by the principal to shade and the principal can fire the agent before making the choice whether to pay the bonus. Hence, she can escape the shading costs σ but would then also sacrifice potential future profits.[14]

All this implies that the utility stream of the agent, conditional on not being fired, amounts to

$$\begin{aligned} U_1 =& b_1 + w_1 - c(e_1^*) + \eta w_1 \theta e_1^* - \max\left\{\left[\underline{\eta}\,(b_1^* - b_1) - \sigma\right], 0\right\} \\ &+ \delta\left(w_2 - c(e_2^*) + \eta w_2 \theta e_2^*\right). \end{aligned}$$

The principal's payoff stream if she does not fire the agent before paying the bonus amounts to

$$\begin{aligned} \Pi_1 =& e_1\theta - w_1 - b_1 - \sigma \\ &+ \delta\left(e_2\theta - w_2\right). \end{aligned}$$

Since shading is not costly for the agent, it is optimal to set $\sigma = \underline{\eta}\,(b_1^* - b_1)$ (for $b_1 \leq b_1^*$). Furthermore, the second-period effort and wage equal $w_2 = \eta\theta^3/4$ and $e_2^* =$

[14] Thus, a bonus is still not feasible in the last period of the game.

$\eta\theta^2/2$, respectively; hence, the second-period profits are $\pi_2 = \eta\theta^3/4$ (see Lemma 1).

The principal faces two decisions. First, which bonus $b_1 \in [0, b_1^*]$ to pay, and second whether to fire the agent. Concerning the first decision, if the principal decides to pay a bonus $b_1 \leq b_1^*$ (and not fire the agent), her profits amount to

$$\begin{aligned}\Pi_1 =& e_1\theta - w_1 + \left(\underline{\eta} - 1\right) b_1 - \underline{\eta} b_1^* \\ &+ \delta\frac{\eta\theta^3}{4}.\end{aligned}$$

This immediately reveals that $b_1 = 0$ is optimal for $\underline{\eta} < 1$, whereas $b_1 = b_1^*$ for $\underline{\eta} \geq 1$. $b_1 = b_1^*$ on the equilibrium path. Thus, $\underline{\eta} < 1$ also implies $b_1^* = b_1 = 0$, and only reciprocity spot contracts are feasible in this case.

Now assume $\underline{\eta} \geq 1$. Then, the principal sets $b_1 = b_1^*$ if she does not fire the agent. She will terminate the relationship, however, if the bonus is larger than the period-2 profits, i.e., if $b_1^* > \delta\pi_2$.

The principal's optimization problem becomes maximizing $\pi_1 = e_1^*\theta - b_1^* - w_1$, subject to the agent's binding (IC) constraint, which yields $b_1^* = (e_1^*)^3/3 - \eta w_1 e_1^*\theta + 2/3\left(\sqrt{\eta w_1\theta}\right)^3$, as well as subject to $b_1^* \leq \delta\pi_2$. The last condition is equivalent to the (DE) constraint, and thus the problem in this section is the same as the optimization problem in the main part of the paper.

These results are collected in Lemma 8.

Lemma 8 *The profit-maximizing equilibrium with negative reciprocity, and a constant norm function $\eta_t(h^{t-1}) = \eta \ \forall h^{t-1}$, has the following characteristics:*

- *If $\underline{\eta} < 1$, $b_1^* = 0$. Moreover, $e_1^* = e_2^* = \eta\theta^2/2$ and $w_1 = w_2 = \eta\theta^3/4$.*
- *If $\underline{\eta} \geq 1$, $b_1^* > 0$, and outcomes are as characterized in Section 3.2, with $w_1 < w_2 = \eta\theta^3/4$ and $e_1^* > e_2^* = \eta\theta^2/2$, as well as $de_2^*/d\eta > de_1^*/d\eta$.*

5.4 Adjustment of the Reference Wage

Some evidence points toward a declining effect of gifts in long-term interactions. Gneezy and List (2006) conduct a field experiment in which they permanently increase the wages of recruited workers. Although workers respond with an immediate effort increase, this is only temporary, and effort falls to an amount only slightly above the initial level. Jayaraman et al. (2016) explore the effects of a mandated 30% wage increase for tea pluckers in India. They find that productivity substantially increases immediately after the wage raise. However, it starts falling again in the second month after

the change and returns to its initial level after four months. Sliwka and Werner (2017) examine how reciprocal effort is affected by the timing of wage increases. They find that a permanent wage raise only temporarily increases effort and that the only way to permanently benefit from an individual's reciprocal behavior is to constantly raise wages.

This evidence suggests that individuals adapt to wage increases and update their reference wages. In the following, I incorporate this evidence and assume that the reference wage above which the agent is willing to reciprocate depends on his past wages. More precisely, the agent starts with a reference wage of zero. In the second period, the first-period wage w_1 becomes the new reference wage.[15] Hence, the agent's utilities are

$$\begin{aligned} u_1 &= b_1 + w_1 - c(e_1^*) + \eta w_1 e_1^* \theta \\ u_2 &= b_2 + w_2 - c(e_2^*) + \max\left\{0,\, \eta\left(w_2 - w_1\right) e_2^* \theta\right\}. \end{aligned}$$

First, I compute the profit-maximizing spot reciprocity contract in the last period. Then, no bonus is paid, and – taking into account that setting $w_2 \geq w_1$ is optimal – effort maximizes $-e_2^3/3 + \eta\left(w_2 - w_1\right) e_2 \theta$. As shown in Lemma 7, effort is unaffected by the higher reference wage; hence, $e_2^* = \eta\theta^2/2$ and $w_2^* = \eta\theta^3/4 + w_1$.

The outcomes for an optimal relational contract are given in Lemma 5.4.

Lemma 9 *Assume the second-period reference wage is equal to w_1. Then, $w_1 < w_2$. Moreover, the (DE) constraint might or might not bind.*

- *If it does not bind, $de_1^*/d\eta < de_2^*/d\eta$. Furthermore, there exists a $\overline{\eta} > 0$ such that the optimal wage is zero for $\eta \leq \overline{\eta}$. In this case, $e_1^* > e_2^*$. For $\eta > \overline{\eta}$, setting a strictly positive wage is optimal, and e_1^* can be smaller or larger than e_2^*.*
- *If it binds, there exists a $\tilde{\eta} > 0$ such that the optimal wage equals zero for $\eta \leq \tilde{\eta}$, whereas it is strictly positive for $\eta \geq \tilde{\eta}$. In both cases, e_1^* can be smaller or larger than e_2^*.*

$\overline{\eta}$ can be smaller or larger than $\tilde{\eta}$, and both are larger than if the second-period reference wage equals zero independent of w_1.

The proof can be found in the Appendix.

[15] This resembles the setting of DellaVigna et al. (2017), who apply a similar assumption (with the exception that the reference point path is exogenous, whereas it is endogenous in my setting) to a model of reference dependent loss aversion.

The principal is reluctant to trigger the agent's reciprocal preferences in the first period. In particular, if δ is large, she wants to maintain this opportunity until later when relational contracts are no longer feasible. Therefore, the threshold for η above which a positive first-period wage is paid is larger than that in the main part of the paper – implying that the backloading of reciprocity-based incentives is more pronounced than with a constant reference wage. A higher w_1 also does not necessarily relax the (DE) constraint anymore (which implies that $\tilde{\eta}$ does not have to be smaller than $\overline{\eta}$). This is because a positive first-period wage has two effects on the tightness of the (DE) constraint. On the one hand, as in the main part, the necessary bonus to implement a certain effort level is reduced, which relaxes the constraint. On the other hand, future profits are reduced via the adjustment of the reference wage, which tightens the constraint. Moreover, e_1^* is not necessarily larger than e_2^* because the reluctance to pay a positive w_1 also reduces the agent's willingness to exert effort in the first period.

5.5 Reciprocity Triggered by Rents

Finally, I explore the implications of reciprocity being triggered by the agent's material rent, in contrast to only by monetary payments. Thus, I assume that the agent's per-period utilities are

$$\begin{aligned} u_1 &= (b_1 + w_1 - c(e_1))(1 + \eta e_1 \theta) \\ u_2 &= (w_2 - c(e_2))(1 + \eta e_2 \theta). \end{aligned}$$

Importantly, when choosing his effort level, the agent also reciprocates on the equilibrium bonus of this period *before* it is paid. Hence, the principal is less inclined to pay a positive fixed wage in the first period. Only if a sufficiently tight (DE) constraint considerably restrains the bonus is w_1 positive.

Formally, effort in the second period is given by the agent's first order condition,

$$-e_2^2 - \frac{4}{3} e_2^3 \eta \theta + w_2 \eta \theta = 0.$$

This is taken into account by the principal who sets w_2 to maximize $\pi_2 = e_2\theta - w_2$.

In the first period, the principal's (DE) constraint still equals $-b_1 + \delta \pi_2 \geq 0$, whereas the agent's (IC) constraint becomes

$$\left(b_1 + w_1 - \frac{(e_1^*)^3}{3}\right)(1 + \eta e_1^* \theta) \geq \left(w_1 - \frac{(\tilde{e}_1)^3}{3}\right)(1 + \eta \tilde{e}_1 \theta). \tag{IC}$$

Here, $\tilde{e}_1$ is characterized by $-\tilde{e}_1^2 - \frac{4}{3}\tilde{e}_1^3\eta\theta + w_1\eta\theta = 0$, and $e_1^* > \tilde{e}_1$ if $b_1 > 0$.

Lemma 10 *Assume that the agent's preferences for reciprocity are triggered by his material rent. Then, the (DE) constraint binds given $T = 2$ and $\delta \leq 1$. Moreover, there exists a $\tilde{\eta} > 0$ such that the optimal wage equals zero for $\eta \leq \tilde{\eta}$, whereas it is strictly positive for $\eta \geq \tilde{\eta}$.*

In any case, $e_1^* > e_2^*$ and $w_1 < w_2$.

The proof can be found in the Appendix.

With $T = 2$, second-period profits cannot be sufficiently large for a non-binding (DE) constraint given $\delta \leq 1$. However, in a more general setting with more than two periods, (DE) might indeed be slack. In this case, the proof to Lemma 10 reveals that paying a positive wage would not be optimal because the purpose of a positive wage – triggering the agent's reciprocal inclinations – can equivalently be achieved by a bonus, which additionally allows for higher effort via the use of relational incentives. With a binding (DE) constraint, the principal might pay a fixed wage in the first period, but only if η is large enough.

6 Conclusion

I showed that the norm of reciprocity can have important implications for the optimal provision of dynamic incentives. Relational and reciprocity-based incentives reinforce each other and should optimally be used in combination. At the beginning of an employee's career, relational incentives assume a larger role because a longer remaining time horizon increases a firm's commitment. Once the end of the career approaches, reciprocity-based incentives gradually become more important. More intense competition for empoyees increases the importance of the norm of reciprocity for the provision of incentives if a lower relationship surplus reduces the effectiveness of relational incentives. Finally, formally committing to paying non-discretionary wages in the future can raise a firm's profits because this increases its commitment in the relational contract.

Appendix

Proof of Lemma 1 I maximize profits $\pi = e\theta - w$, taking into account that effort equals $e = \sqrt{\eta w\theta}$, and that the agent's (IR) constraint, $u = w - e^3/3 + \eta we\theta = w + (2/3)\sqrt{\eta w\theta}^3 \geq 0$, must be satisfied. Naturally, the latter holds for any $w \geq 0$.

The principal's first order condition equals

$$\frac{d\pi}{dw} = \frac{de}{dw}\theta - 1 = 0,$$

which yields

$$w = \frac{\eta\theta^3}{4}.$$

Hence,

$$e^* = \frac{\eta\theta^2}{2},$$

and $\pi = \eta\theta^3/4$, $u = \eta\theta^3/4 + \eta^3\theta^6/12 > 0$. ■

Proof of Lemma 2 The principal maximizes Π_1, subject to (IC) and (DE) constraints for every period,

$$b_t - \frac{(e_t^*)^3}{3} + \eta w_t^{nd} e_t^*\theta + \delta U_{t+1} \geq -\frac{(\tilde{e}_t)^3}{3} + \eta w_t^{nd}\tilde{e}_t\theta + \delta\tilde{U}_{t+1} \qquad \text{(IC)}$$

$$-b_t + \delta\Pi_{t+1} \geq 0. \qquad \text{(DE)}$$

$\tilde{U}_{t+1}$ describes the agent's off-path continuation utility after a deviation by himself. After the principal deviates (downwards), η drops to zero, and continuation payoffs of principal and agent are zero. For this proof, I hence mostly focus on deviations by the agent, hence assume (DE) constraints hold on and off the equilibrium path and η remains constant. I only have to take care of the possibility of upward deviations by the principal (i.e., increasing future payments of w_t^{nd} after a deviation by the agent), which do not reduce η. If these upward deviations are optimal off the equilibrium path, they affect the size of $\tilde{U}_t$ and hence the agent's incentives to provide equilibrium effort.

First, it is without loss of generality to set $w_t^d = 0$ after all histories: Assume $w_t^d > 0$. Reducing it to zero and increasing b_{t-1} by δw_t^d leaves all payoffs and constraints unaffected. In the following, I hence assume that $w_t^d = 0$ in all periods t and for all histories.

Second, w_t^{nd} is – by definition – independent of the agent's past effort choices. Thus, I need to prove consistency, in the sense that it is indeed optimal for the principal to

not change w_t^{nd} after a deviation by the agent. In this respect, I assume that all w_t^{nd} in a profit-maximizing relational contract are smaller than the wage in a reciprocity spot contract (Lemma 1) and later verify that this is indeed optimal (see Proposition 1). Then, firing the agent after he did not perform accordingly is *not* subgame perfect and can thus be ruled out as a potential response of the principal. The reason is that, with non-discretionary wages not exceeding the profit-maximizing wage in a reciprocity spot contract, the principal makes positive profits even if no relational incentives are provided on top (if some w_t^{nd} were larger, I would have to check whether spot contracts with "too high" wages still generate positive profits). This also implies that the principal does not lower non-discretionary wages after a deviation (because this would let η drop to zero).

Third, I need to verify that increasing w^{nd} after a deviation by the agent is not optimal (otherwise, a deviation might increase the agent's continuation utility and consequently his incentives to deviate). Note that, according to the definition of w_t^{nd}, the agent only reciprocates to wage components that are independent of the agent's past effort, hence increasing the wage after a deviation of the agent would not induce a stronger reciprocal reaction. However, this (arguably unintuitive) assumption is less restrictive than it might seem. The reason is that the agent does not enjoy a rent from relational incentives (see below). Therefore, even if higher values of w_t^{nd} increased the agent's reciprocal reaction accordingly, raising w_t^{nd} would not be subgame perfect. Instead, the principal would continue to offer the on-path contract, thereby maximizing her (off-path) profits and minimizing $\tilde{U}_{t+1}$ (which would take the same value as with the current setup). All this implies that $\tilde{U}_t$ contains the same non-discretionary wage stream $\left\{w_\tau^{nd}\right\}_{\tau=t}^{T}$ as U_t. Moreover, I can set $\tilde{U}_t = \sum_{\tau=t}^{T}\left(w_\tau^{nd} - \frac{\tilde{e}_\tau^3}{3} + \eta w_\tau^{nd}\tilde{e}_\tau\theta\right)$, which corresponds to the agent's minmax payoff given $\left\{w_\tau^{nd}\right\}_{\tau=t}^{T}$ (and provided $w_t \geq 0$, which rules out negative upfront payments to extract the agent's "reciprocity rent). The reason is that, using standard arguments, a series of spot contracts always constitutes on equilibrium of such a finitely repeated game.

Fourth, I show that for given values w_t^{nd} and e_t, it is (weakly) optimal for the principal to set $b_t - \frac{e_t^3}{3} + \eta w_t^{nd} e_t\theta = -\frac{\tilde{e}_t^3}{3} + \eta w_t^{nd}\tilde{e}_t\theta$ in all periods. To do so, I proceed sequentially and start with period $t = 1$:

- Assume $b_1 - \frac{e_1^3}{3} + \eta w_1^{nd} e_1\theta > -\frac{\tilde{e}_1^3}{3} + \eta w_1^{nd}\tilde{e}_1\theta$. Reduce b_1 by a small $\varepsilon > 0$, which increases Π_1 and relaxes the first-period (DE) constraint.

- Assume $b_1 - \frac{e_1^3}{3} + \eta w_1^{nd} e_1\theta < -\frac{\tilde{e}_1^3}{3} + \eta w_1^{nd}\tilde{e}_1\theta$. Because w^d has been set to zero in all periods, this implies that there exists at least one period $\tau > 1$ in which $b_\tau - \frac{e_\tau^3}{3} + \eta w_\tau^{nd} e_\tau\theta > -\frac{\tilde{e}_\tau^3}{3} + \eta w_\tau^{nd}\tilde{e}_\tau\theta$. Assume τ_1 is the first of these periods.

Reduce b_{τ_1} by $\varepsilon > 0$ and increase b_1 by $\delta^{\tau_1 - 1}\varepsilon$. Proceed until either $b_1 - \frac{e_1^3}{3} + \eta w_1^{nd} e_1 \theta = -\frac{\tilde{e}_1^3}{3} + \eta w_1^{nd} \tilde{e}_1 \theta$ or $b_{\tau_1} - \frac{e_{\tau_1}^3}{3} + \eta w_{\tau_1}^{nd} e_{\tau_1} \theta = -\frac{\tilde{e}_{\tau_1}^3}{3} + \eta w_{\tau_1}^{nd} \tilde{e}_{\tau_1} \theta$. In the latter case, move to the second period $\tau_2 > \tau_1$ in which $b_{\tau_2} - \frac{e_{\tau_2}^3}{3} + \eta w_{\tau_2}^{nd} e_{\tau_2} \theta > -\frac{\tilde{e}_{\tau_2}^3}{3} + \eta w_{\tau_2}^{nd} \tilde{e}_{\tau_2} \theta$ (such a period must exist as long as $b_1 - \frac{e_1^3}{3} + \eta w_1^{nd} e_1 \theta < -\frac{\tilde{e}_1^3}{3} + \eta w_1^{nd} \tilde{e}_1 \theta$), reduce b_{τ_2} by $\varepsilon > 0$ and increase b_1 by $\delta^{\tau_2 - 1}\varepsilon$, and so on. Continue until $b_1 - \frac{e_1^3}{3} + \eta w_1^{nd} e_1 \theta = -\frac{\tilde{e}_1^3}{3} + \eta w_1^{nd} \tilde{e}_1 \theta$.

In period $t = 2$, proceed accordingly if $b_2 - \frac{e_2^3}{3} + \eta w_2^{nd} e_2 \theta \neq 0$, as well as in all following periods.

It follows that $u_t = w_t^{nd} + b_t - \frac{e_t^3}{3} + \eta w_t^{nd} e_1 \theta = w_t^{nd} - \frac{\tilde{e}_t^3}{3} + \eta w_t^{nd} \tilde{e}_t \theta$, and consequently that $U_t = \tilde{U}_t$ in all periods t, and that all (IC) constraints hold as equalities.

Finally, these results imply that there exists a sequentially optimal profit-maximizing equilibrium, in the sense that maximizing Π_1 is equivalent to maximizing each per-period profit π_t, subject to (DE) and binding (IC) constraints. This is because the agent's incentives to exert effort in any period t are solely determined by payments made in period t, w_t^{nd} and b_t. There, b_t is bounded by the principal's future payoff stream, thus maximizing each π_t ceteris paribus maximizes Π_1, but also yields the largest maximum feasible values of b_t without adverse effect on (IC) constraints.

■

Proof of Lemma 3 If the (DE) constraint does not bind in a period t, the principal maximizes profits $\pi_t = e_t \theta - \left((e_t)^3/3 - \eta w_t e_t \theta + 2/3 \left(\sqrt{\eta w_t \theta} \right)^3 \right) - w_t$, subject to $w_t \geq 0$.

The Lagrange function equals $L_t = e_t \theta - (e_t)^3/3 + \eta w_t e_t \theta - 2/3 \left(\sqrt{\eta w_t \theta} \right)^3 - w_t + \lambda_{w_t} w_t$, where $\lambda_{w_t} \geq 0$ represents the Lagrange parameter for the agent's limited liability constraint, giving first order conditions

$$\frac{\partial L_t}{\partial e_t^*} = \theta - (e_t)^2 + \eta w_t \theta = 0$$
$$\frac{\partial L_t}{\partial w_t} = \eta \theta \left(e_t - \sqrt{\eta w_t \theta} \right) - 1 + \lambda_{w_t} = 0.$$

First, assume that $\lambda_{w_t} = 0$. Then, the first order conditions yield $w_t = \left(\eta^2 \theta^3 - 1 \right)^2 / (4 \eta^3 \theta^3)$ and $e_t^* = \left(1 + \eta^2 \theta^3 \right) / (2 \eta \theta)$. Second, assume that $\lambda_{w_t} > 0$ and hence $w_t = 0$. Then, $e_t^* = \sqrt{\theta}$ and $\pi_t = \frac{2}{3} \left(\sqrt{\theta} \right)^3$. To establish the existence of $\overline{\eta}$, note that $d\pi_t / dw_t \mid_{w_t = 0} = \sqrt{\eta^2 \theta^3} - 1$. This is positive for $\eta > \sqrt{1/\theta^3}$, hence a strictly positive wage is optimal in this case and not otherwise. ■

Proof of Lemma 4 Including the respective (DE) constraints, the Lagrange function of the principal's maximization problem in a period t becomes

$$L_t = e_t\theta - e_t^3/3 + \eta w_t e_t\theta - 2/3\left(\sqrt{\eta w_t\theta}\right)^3 - w_t + \lambda_{DE_t}\left[\delta\Pi_{t+1} - \frac{2}{3}\left(\sqrt{\eta w_t\theta}\right)^3 - e_t^3/3 + \eta w_t\theta e_t\right] + w_t\lambda_{w_t},$$

where $\lambda_{w_t} \geq 0$ represents the Lagrange parameter for the agent's limited liability constraint and $\lambda_{DE_t} \geq 0$ represents the Lagrange parameter for the principal's dynamic enforcement constraint.

First order conditions are

$$\frac{\partial L}{\partial e_t^*} = \theta - e_t^2 + \eta w_t\theta + \lambda_{DE_t}\left[-e_t^2 + \eta w_t\theta\right] = 0$$
$$\frac{\partial L}{\partial w_t} = \eta\theta e_t - \eta\theta\sqrt{\eta w_t\theta} - 1 + \lambda_{DE_t}\left[-\eta\theta\sqrt{\eta w_t\theta} + \eta\theta e_t\right] + \lambda_{w_t} = 0.$$

First, assume that $\lambda_{w_t} = 0$. Then, the first order conditions yield $w_t = \frac{\left(\eta^2\theta^3\left(1+\lambda_{DE_t}\right)-1\right)^2}{4\eta^3\theta^3\left(1+\lambda_{DE_t}\right)^2}$ and $e_t^* = \frac{1+\eta^2\theta^3\left(1+\lambda_{DE_t}\right)}{2\eta\theta\left(1+\lambda_{DE_t}\right)}$. It follows that, given $\lambda_{DE_t} > 0$ and $\eta > \overline{\eta}$ (i.e., $\eta^2\theta^3 - 1$, implying that $w_t > 0$ if (DE) does not bind), $\frac{\left(\eta^2\theta^3\left(1+\lambda_{DE_t}\right)-1\right)^2}{4\eta^3\theta^3\left(1+\lambda_{DE_t}\right)^2} > \frac{\left(\eta^2\theta^3-1\right)^2}{4\eta^3\theta^3}$ and $\frac{1+\eta^2\theta^3\left(1+\lambda_{DE_t}\right)}{2\eta\theta\left(1+\lambda_{DE_t}\right)} < \frac{1+\eta^2\theta^3}{2\eta\theta}$.

Second, assume that $\lambda_{w_t} > 0$ and hence $w_t = 0$. Then, $e_t^* = \sqrt{\theta/\left(1+\lambda_{DE_t}\right)}$. To establish the existence of $\tilde{\eta}$, note that $\partial L/\partial w_t\,|_{w_t=0} = \sqrt{\eta^2\theta^3\left(1+\lambda_{DE_t}\right)} - 1 = 0$. This is positive for $\eta > \sqrt{1/\theta^3\left(1+\lambda_{DE_t}\right)}$, and thus a strictly positive wage is optimal in this case and not otherwise. Finally, for $\lambda_{DE_t} > 0$, $\tilde{\eta} = \sqrt{1/\theta^3\left(1+\lambda_{DE_t}\right)} < \overline{\eta} = \sqrt{1/\theta^3}$. Moreover, $\tilde{\eta}$ increases in δ because λ_{DE_t} decreases in δ (see the proof to Lemma 5). ■

Proof of Lemma 5 The (DE) constraint in period $T-1$ (where on-path continuation profits are $\Pi_T = \eta\theta^3/4$) equals $(e_t^*)^3/3 - \eta w_t\theta e_t^* \leq \delta\eta\theta^3/4 - 2/3\left(\sqrt{\eta w_t\theta}\right)^3$. First, note that for $\eta \leq \overline{\eta}$, in which case the first-best wage is zero and first-best effort equals $\sqrt{\theta}$, the (DE) constraint equals $(\sqrt{\theta})^3/3 \leq \delta\eta\theta^3/4$. This cannot hold if $\eta \leq \overline{\eta} = \sqrt{1/\theta^3}$, even for $\delta \to 1$.

Therefore, assume $\eta > \overline{\eta}$ for the remainder of this proof. Then, first-best effort and wage levels are $e = \left(1+\eta^2\theta^3\right)/(2\eta\theta)$ and $w = \left(\eta^2\theta^3 - 1\right)^2/(4\eta^3\theta^3)$, and the (DE)

constraint in period $T-1$ becomes

$$\frac{3\eta^2\theta^3-1}{6\eta^3\theta^3} \leq \delta\frac{\eta\theta^3}{4}. \tag{1}$$

Because $\eta > \overline{\eta}$, the left-hand-side is strictly positive. Therefore, the constraint is violated for first-best effort and wage levels if $\delta \to 0$.

To show that first-best effort can be implemented in period $T-1$ if η is sufficiently large, I compute the derivative of the left-hand-side of 1 and obtain $\left(1-\eta^2\theta^3\right)/2\eta^4\theta^3$, which is negative for $\eta > \bar{\eta}$. Moreover, $\lim_{\eta\to\infty}\frac{3\eta^2\theta^3-1}{6\eta^3\theta^3} = 0$, whereas the right-hand side of 1 is strictly positive and increasing in η. Therefore, 1 is satisfied if η is sufficienty large.

Concerning the second part of the Lemma, recall that the equilibrium is sequentially efficient, hence the principal's maximization problem is equivalent to maximizing $\pi_t = e_t^*\theta - b_t^* - w_t$ in every period t, subject to the (DE) constraint $(e_t^*)^3/3 - \eta w_t\theta e_t^* \leq \delta\Pi_{t+1} - \frac{2}{3}\left(\sqrt{\eta w_t\theta}\right)^3$. It follows that, for a given w_t, the maximum implementable effort in period t is ceteris paribus strictly increasing in Π_{t+1}, whereas per-period profits π_t are consequently weakly increasing in Π_{t+1}. Furthermore, per-period profits in periods $t < T$ can be expressed as functions of Π_{t+1}, i.e. $\pi_t(\Pi_{t+1})$, with $\pi_t' \geq 0$.

The profit-maximizing spot reciprocity contract is the principal's optimal choice in the last period T, hence $\pi_T = \Pi_T = \eta\theta^3/4$. In all previous periods, the principal still has the option to implement the spot reciprocity contract (by setting $b_t^* = 0$ and $w_t = \eta\theta^3/4$), therefore $\pi_t \geq \pi_T\,\forall t$.

Now, I apply proof by induction to verify that $\Pi_{t-1} > \Pi_t$. First, $\Pi_{T-1} > \Pi_T$ because

$$\Pi_{T-1} = \pi_{T-1} + \delta\Pi_T \geq \pi_T + \delta\Pi_T = \Pi_T(1+\delta) > \Pi_T.$$

For the inductin step, assume that $\Pi_t > \Pi_{t+1}$. Since $\pi_t'(\Pi_{t+1}) \geq 0$, $\pi_{t-1} \geq \pi_t$. Therefore, $\Pi_{t-1} = \pi_{t-1} + \delta\Pi_t \geq \pi_t + \delta\Pi_t > \pi_t + \delta\Pi_{t+1} = \Pi_t$, which completes the proof. ■

Proof of Proposition 1 First, assume $\eta > \overline{\eta} = \sqrt{1/\theta^3}$, hence $w_t > 0\,\forall t$. Furthermore, in Lemmas 3 and 4, I have established that $w_t = \frac{\left(\eta^2\theta^3\left(1+\lambda_{DE_t}\right)-1\right)^2}{4\eta^3\theta^3\left(1+\lambda_{DE_t}\right)^2}$ and $e_t^* = \frac{1+\eta^2\theta^3\left(1+\lambda_{DE_t}\right)}{2\eta\theta\left(1+\lambda_{DE_t}\right)}$, where λ_{DE_t} is the Lagrange parameter associated with the (DE) constraint in period t. Hence, $w_t = w_{t-1}$ and $e_t^* = e_{t-1}$ if $\lambda_{DE_t} = \lambda_{DE_{t-1}} = 0$. By Lemma 4, if $\lambda_{DE_{t-1}} = 0$ but $\lambda_{DE_t} > 0$, then $w_t > w_{t-1}$ and $e_t^* < e_{t-1}$. Finally, assume that $\lambda_{DE_{t-1}} > 0$. First, I show that in this case also $\lambda_{DE_t} > 0$: Plugging $w_{t-1} = \frac{\left(\eta^2\theta^3\left(1+\lambda_{DE_{t-1}}\right)-1\right)^2}{4\eta^3\theta^3\left(1+\lambda_{DE_{t-1}}\right)^2}$ and

$e^*_{t-1} = \frac{1+\eta^2\theta^3\left(1+\lambda_{DE_{t-1}}\right)}{2\eta\theta\left(1+\lambda_{DE_{t-1}}\right)}$ into the binding (DE) constraint for period $t-1$ yields

$$\frac{3\eta^2\theta^3\left(1+\lambda_{DE_{t-1}}\right)-1}{6\eta^3\theta^3\left(1+\lambda_{DE_{t-1}}\right)^3} = \delta\Pi_t.$$

I can also treat $\lambda_{DE_{t-1}}$ as a function of Π_t. By the implicit function theorem, $\frac{\partial\lambda_{DE_{t-1}}}{\partial\Pi_t} = \frac{2\delta\eta^3\theta^3\left(1+\lambda_{DE_{t-1}}\right)^4}{1-2\eta^2\theta^3\left(1+\lambda_{DE_{t-1}}\right)} < 0$ (since $\eta > \overline{\eta}$ implies $\eta^2\theta^3 > 1$). Hence, Lemma 5 yields $\lambda_{DE_{t-1}} < \lambda_{DE_t}$, which implies $\lambda_{DE_{t-1}} > 0 \Rightarrow \lambda_{DE_t} > 0$. Furthermore, if $\lambda_{DE_t} = 0$ in a period t, this also holds for all previous periods.

The wage schedule is increasing in periods $t < T$ since $\frac{\partial w_t}{\partial\lambda_{DE_t}} = \frac{\left(\eta^2\theta^3\left(1+\lambda_{DE_t}\right)-1\right)}{2\eta^3\theta^3\left(1+\lambda_{DE_t}\right)^3} > 0$, whereas the effort path is decreasing because of $\frac{\partial e^*_t}{\partial\lambda_{DE_t}} = \frac{-1}{2\eta\theta\left(1+\lambda_{DE_t}\right)^2} < 0$. Finally, wage and effort in period T are $e^*_T = \frac{\eta\theta^2}{2}$ and $w_T = \frac{\eta\theta^3}{4}$, respectively. $e^*_T < e^*_t$ for all $t < T$ follows from $\frac{\eta\theta^2}{2} < \frac{1+\eta^2\theta^3\left(1+\lambda_{DE_t}\right)}{2\eta\theta\left(1+\lambda_{DE_t}\right)}$ $\left(\Leftrightarrow \eta^2\theta^3\left(1+\lambda_{DE_t}\right) < 1+\eta^2\theta^3\left(1+\lambda_{DE_t}\right)\right)$. $w_T > w_t$ for all $t < T$ follows from $\frac{\eta\theta^3}{4} > \frac{\left(\eta^2\theta^3\left(1+\lambda_{DE_t}\right)-1\right)^2}{4\eta^3\theta^3\left(1+\lambda_{DE_t}\right)^2}$ $\left(\Leftrightarrow 2\eta^2\theta^3\left(1+\lambda_{DE_t}\right) > 1\right)$.

For the remainder of the proof, assume $\eta \leq \overline{\eta}$, hence $w_t = 0$ and $e^*_t = \sqrt{\theta}$ if $\lambda_{DE_t} = 0$. As before, $\lambda_{DE_t} = 0$ implies $\lambda_{DE_{t-1}} = 0$, and $\lambda_{DE_t} > 0$ implies $\lambda_{DE_{t+1}} > \lambda_{DE_t}$.

The following cases still have to be explored:

- $\lambda_{DE_t} > 0$ and $w_t > 0$. Then, $w_t = \frac{\left(\eta^2\theta^3\left(1+\lambda_{DE_t}\right)-1\right)^2}{4\eta^3\theta^3\left(1+\lambda_{DE_t}\right)^2}$ and $e^*_t = \frac{1+\eta^2\theta^3\left(1+\lambda_{DE_t}\right)}{2\eta\theta\left(1+\lambda_{DE_t}\right)}$, and the previous analysis regarding wages w_τ and effort levels e_τ, for $\tau > t$, can be applied. The previous analysis can also be applied if $\lambda_{DE_{t-1}} > 0$ and $w_{t-1} > 0$.

  Now, assume $\lambda_{DE_{t-1}} > 0$ and $w_{t-1} = 0$. Then, $e_{t-1} = \sqrt{\theta/\left(1+\lambda_{DE_{t-1}}\right)}$ (see the proof to Lemma 4), and I have to show that

  $$\sqrt{\frac{\theta}{\left(1+\lambda_{DE_{t-1}}\right)}} > \frac{1+\eta^2\theta^3\left(1+\lambda_{DE_t}\right)}{2\eta\theta\left(1+\lambda_{DE_t}\right)}.$$

  In the proof to Lemma 4, I haven proven that $w_{t-1} = 0$ implies $\eta \leq \sqrt{1/\theta^3\left(1+\lambda_{DE_{t-1}}\right)}$, which can be re-written to $\sqrt{\theta/\left(1+\lambda_{DE_{t-1}}\right)} \geq \eta\theta^2$. Therefore, it is sufficient to show that $\eta\theta^2 > \left[1+\eta^2\theta^3\left(1+\lambda_{DE_t}\right)\right]/\left[2\eta\theta\left(1+\lambda_{DE_t}\right)\right]$, which becomes $\eta > \sqrt{1/\theta^3\left(1+\lambda_{DE_t}\right)}$. This, however, is implied by $w_t > 0$ (see the proof to Lemma 4).

- $\lambda_{DE_t} = 0$ and $\lambda_{DE_{t+1}} > 0$, with $w_{t+1} = 0$. Now, $e^*_{t+1} < e^*_t$ follows from $e^*_t =$

$\sqrt{\theta/(1+\lambda_{DE_t})}$, $e^*_{t+1} = \sqrt{\theta/\left(1+\lambda_{DE_{t+1}}\right)}$ and $\lambda_{DE_{t+1}} > \lambda_{DE_t}$.

- $\lambda_{DE_t} = 0$ and $\lambda_{DE_{t+1}} > 0$, with $w_{t+1} > 0$. Now, $e^*_t = \sqrt{\theta}$ and $e^*_{t+1} = \frac{1+\eta^2\theta^3\left(1+\lambda_{DE_{t+1}}\right)}{2\eta\theta\left(1+\lambda_{DE_{t+1}}\right)}$, and I have to show that

$$\begin{aligned}\sqrt{\theta} &> \frac{1+\eta^2\theta^3\left(1+\lambda_{DE_{t+1}}\right)}{2\eta\theta\left(1+\lambda_{DE_{t+1}}\right)} \\ \Leftrightarrow \left(1+\lambda_{DE_{t+1}}\right)&\left(2\sqrt{\eta^2\theta^3}-\eta^2\theta^3\right) > 1.\end{aligned}$$

  Again, $w_{t+1} > 0$ implies $\left(1+\lambda_{DE_{t+1}}\right) > 1/\eta^2\theta^3$ (see the proof to Lemma 4), hence it is sufficient to prove that (taking into acount that $\eta \leq \overline{\eta}$ implies $2\sqrt{\eta^2\theta^3} - \eta^2\theta^3 > 0$)

$$\begin{aligned}\frac{\left(2\sqrt{\eta^2\theta^3}-\eta^2\theta^3\right)}{\eta^2\theta^3} &\geq 1 \\ \Leftrightarrow 2\sqrt{\eta^2\theta^3}&\left(1-\sqrt{\eta^2\theta^3}\right) \geq 0,\end{aligned}$$

  which holds because of $\eta \leq \overline{\eta}$.

Concerning the bonus, note that the binding (IC) constraint delivers $b_t = (e^*_t)^3/3 - \eta w_t e^*_t\theta + 2/3\left(\sqrt{\eta w_t\theta}\right)^3$. It follows that, if $w_t = \frac{\left(\eta^2\theta^3\left(1+\lambda_{DE_t}\right)-1\right)^2}{4\eta^3\theta^3\left(1+\lambda_{DE_t}\right)^2} > 0$,

$$b_t = \frac{3\eta^2\theta^3\left(1+\lambda_{DE_t}\right)-1}{6\eta^3\theta^3\left(1+\lambda_{DE_t}\right)^3},$$

with $\frac{db_t}{\left(1+\lambda_{DE_t}\right)} = \frac{-2\eta^2\theta^3\left(1+\lambda_{DE_t}\right)+1}{2\eta^3\theta^3\left(1+\lambda_{DE_t}\right)^4} < 0$.

Moreover, if $w_t = 0$, then

$$b_t = \frac{\left(\sqrt{\frac{\theta}{\left(1+\lambda_{DE_t}\right)}}\right)^3}{3},$$

with $db_t/d\left(1+\lambda_{DE_t}\right) < 0$. Bonus dynamics then are computed equivalently to wage and effort dynamics.

Concerning total compensation, I focus on the case $\eta^2\theta^3 > 1$, hence $w_t = \frac{\left(\eta^2\theta^3\left(1+\lambda_{DE_t}\right)-1\right)^2}{4\eta^3\theta^3\left(1+\lambda_{DE_t}\right)^2} > 0$. Then,

$$\frac{d\left(w_t+b_t\right)}{d\left(1+\lambda_{DE_t}\right)} = \frac{-2\eta^2\theta^3\left(1+\lambda_{DE_t}\right)+1+\eta^2\theta^3\left(1+\lambda_{DE_t}\right)^2-\left(1+\lambda_{DE_t}\right)}{4\eta^3\theta^3\left(1+\lambda_{DE_t}\right)^4},$$

which is negative for $\lambda_{DE_t} \to 0$. To show that this expression can also be positive, note that a binding (DE) constraint delivers

$$\frac{3\eta^2\theta^3\left(1+\lambda_{DE_t}\right)-1}{6\eta^3\theta^3\left(1+\lambda_{DE_t}\right)^3} = \delta\Pi_{t+1},$$

hence $3\eta^2\theta^3\left(1+\lambda_{DE_t}\right)-1 \geq 0$. At $3\eta^2\theta^3\left(1+\lambda_{DE_t}\right)-1=0$, the numerator of $d\left(w_t+b_t\right)/d\left(1+\lambda_{DE_t}\right)$ becomes $\left(3\eta^2\theta^3-2\right)/9\eta^2\theta^3 > 0$.

■

Proof of Lemma 6. First, I consider the case $w_t > 0$, hence $\eta^2\theta^3\left(1+\lambda_{DE_t}\right)-1>0$. Then,

$$\begin{aligned}
u_t &= w_t + b_t - \frac{e_t^3}{3} + \eta w_t e_t^* \theta \\
&= w_t + 2/3\left(\sqrt{\eta w_t \theta}\right)^3 \\
&= \frac{\left(\eta^2\theta^3\left(1+\lambda_{DE_t}\right)-1\right)^2}{4\eta^3\theta^3\left(1+\lambda_{DE_t}\right)^2}\left[1+\frac{\left(\eta^2\theta^3\left(1+\lambda_{DE_t}\right)-1\right)}{3\left(1+\lambda_{DE_t}\right)}\right] \text{ and} \\
\frac{\partial u_t}{\partial\left(1+\lambda_{DE_t}\right)} &= \frac{\left(\eta^2\theta^3\left(1+\lambda_{DE_t}\right)-1\right)}{2\eta^3\theta^3\left(1+\lambda_{DE_t}\right)^3}\left[1+\frac{2\left(\eta^2\theta^3\left(1+\lambda_{DE_t}\right)-1\right)}{6\left(1+\lambda_{DE_t}\right)}\right] \\
&\quad + \frac{\left(\eta^2\theta^3\left(1+\lambda_{DE_t}\right)-1\right)^2}{12\eta^3\theta^3\left(1+\lambda_{DE_t}\right)^3} > 0
\end{aligned}$$

Moreover,

$$\begin{aligned}
\pi_t &= e\theta - w - b \text{ and} \\
\frac{\partial \pi_t}{\partial\left(1+\lambda_{DE_t}\right)} &= \frac{\partial e}{\partial\left(1+\lambda_{DE_t}\right)}\theta - \frac{\partial w}{\partial\left(1+\lambda_{DE_t}\right)} - \frac{\partial b}{\partial\left(1+\lambda_{DE_t}\right)} \\
&= -\frac{\eta^2\theta^3\left(1+\lambda_{DE_t}\right)\lambda_{DE_t} + \left(\eta^2\theta^3\left(1+\lambda_{DE_t}\right)-1\right)}{2\eta^3\theta^3\left(1+\lambda_{DE_t}\right)^4} < 0
\end{aligned}$$

Finally,

$$\lim_{\lambda_{DE_t}\to\infty} u_t = \frac{\left(\eta^4\theta^6 - \frac{2\eta^2\theta^3}{\left(1+\lambda_{DE_t}\right)} + \frac{1}{\left(1+\lambda_{DE_t}\right)^2}\right)}{4\eta^3\theta^3}\left[1 + \frac{\left(\eta^2\theta^3 - \frac{1}{\left(1+\lambda_{DE_t}\right)}\right)}{3}\right]$$
$$= \frac{\eta\theta^3}{4}\left[1 + \frac{\eta^2\theta^3}{3}\right] = u_T$$

and

$$\lim_{\lambda_{DE_t}\to\infty} \pi_t = \frac{\frac{1}{\left(1+\lambda_{DE_t}\right)} + \eta^2\theta^3}{2\eta\theta}\theta - \frac{\left(\eta^4\theta^6 - \frac{2\eta^2\theta^3}{\left(1+\lambda_{DE_t}\right)} + \frac{1}{\left(1+\lambda_{DE_t}\right)^2}\right)}{4\eta^3\theta^3}$$
$$- \frac{3\eta^2\theta^3 - \frac{1}{\left(1+\lambda_{DE_t}\right)}}{6\eta^3\theta^3\left(1+\lambda_{DE_t}\right)^2} = \frac{\eta\theta^3}{4} = \pi_T$$

Second, I consider the case $w_t = 0$.
Then

$$u_t = 0,$$
$$\pi_t = \sqrt{\frac{\theta}{\left(1+\lambda_{DE_t}\right)}}\left(\theta - \frac{\theta}{3\left(1+\lambda_{DE_t}\right)}\right) \text{ and}$$
$$\frac{\partial\pi_t}{\partial\left(1+\lambda_{DE_t}\right)} = -\sqrt{\frac{\theta^3}{\left(1+\lambda_{DE_t}\right)^3}}\frac{\lambda_{DE_t}}{2\left(1+\lambda_{DE_t}\right)} < 0$$

Finally, note that $w_t > 0$ for λ_{DE_t} sufficiently large, hence the first case also applies to $\lambda_{DE_t} \to \infty$. ■

Proof of Proposition 2. First, note that $e_T^* = \eta\theta^2/2$, which is obviously increasing in η. Second, assume that a positive wage is optimal in any period $t < T$ (i.e., if $\eta > \overline{\eta}$ with a non-binding (DE) constraint and $\eta > \tilde{\eta}$ with a binding (DE) constraint). Then, $e_t^* = \frac{1+\eta^2\theta^3\left(1+\lambda_{DE_t}\right)}{2\eta\theta\left(1+\lambda_{DE_t}\right)}$, with $\lambda_{DE_t} \geq 0$, and

$$\frac{\partial e_t^*}{\partial\eta} = \frac{\eta^2\theta^3\left(1+\lambda_{DEt}\right)-1}{2\eta^2\theta\left(1+\lambda_{DEt}\right)} - \frac{1}{2\eta\theta\left(1+\lambda_{DE_t}\right)^2}\frac{\partial\lambda_{DE_t}}{\partial\eta} > 0.$$

There, $\partial\lambda_{DE_t}/\partial\eta \leq 0$ because λ_{DE_t} is decreasing in Π_{t+1} (see the proof to Proposition 1), and because profits in all periods increase in η: This is obviously true for $\pi_T = \eta\theta^3/4$. Therefore, (DE) constraints in all periods $\tau < T$ are relaxed. Moreover, the agent's (IC) constraints in all periods $\tau < T$ are relaxed by a higher η if $w_\tau > 0$ and stay unaffected if $w_\tau = 0$.

Now, assume that $w_t = 0$ is optimal in any period $t < T$. Then, $e_t^* = \sqrt{\theta/(1+\lambda_{DE_t})}$, with $\lambda_{DE_t} \geq 0$, and

$$\frac{\partial e_t^*}{\partial \eta} = -\frac{1}{2}\sqrt{\frac{\theta}{(1+\lambda_{DE_t})^3}}\frac{\partial \lambda_{DE_t}}{\partial \eta} \geq 0.$$

The second part ($\partial e_t^*/\partial\eta$ is larger if $\lambda_{DE_t} > 0$) immediately follows. ■

Proof of Lemma 7. For a given $w \geq \overline{w}$, the agent chooses an effort level that maximizes $u = w + \eta(w-\overline{w})\theta e - e^3/3$, hence $e^* = \sqrt{\eta(w-\overline{w})\theta}$. Taking this into account, the principal maximizes profits $\pi = e^*\theta - w = \sqrt{\eta(w-\overline{w})\theta}\theta - w$, subject to $w \geq \overline{w}$. First ignoring the latter constraint, the principal's first-order condition equals

$$\frac{\eta\theta^2}{2\sqrt{\eta(w-\overline{w})\theta}} - 1 = 0.$$

This yields

$$w = \frac{\eta\theta^3}{4} + \overline{w},$$

which is larger than $\overline{w}$.

Hence,

$$e^* = \frac{\eta\theta^2}{2},$$

and $\pi = \frac{\eta\theta^3}{4} - \overline{w}$, $u = \frac{\eta\theta^3}{4} + \frac{\eta^3\theta^6}{12} + \overline{w}$. ■

Proof of Proposition 3. In any period t, the principal maximizes
$\pi_t = e_t\theta - \left((e_t)^3/3 - \eta(w_t-\overline{w})e_t\theta + 2/3\left(\sqrt{\eta(w_t-\overline{w})\theta}\right)^3\right) - w_t$, subject to (DE) and $w_t \geq \overline{w}$. First, I assume that (DE) does not bind (which is possible if η and/or δ are sufficiently large – see the proof to Lemma 3). Then, the Lagrange function equals

$$\begin{aligned} L_t =& e_t\theta - (e_t)^3/3 + \eta(w_t-\overline{w})e_t\theta \\ & - \frac{2}{3}\left(\sqrt{\eta(w_t-\overline{w})\theta}\right)^3 - w_t + \lambda_{w_t}(w_t-\overline{w}), \end{aligned}$$

with first order conditions

$$\frac{\partial L_t}{\partial e_t^*} = \theta - (e_t)^2 + \eta\,(w_t - \overline{w})\,\theta = 0$$
$$\frac{\partial L_t}{\partial w_t} = \eta\,\theta\left(e_t - \sqrt{\eta\,(w_t - \overline{w})\,\theta}\right) - 1 + \lambda_{w_t} = 0$$

I start with $\lambda_{w_t} = 0$. Then, the first order conditions yield $w_t = \left(\eta^2\theta^3 - 1\right)^2/(4\eta^3\theta^3) + \overline{w}$ and $e_t^* = \left(1 + \eta^2\theta^3\right)/(2\eta\theta)$. Now, assume that $\lambda_{w_t} > 0$ and hence $w_t = \overline{w}$. Then, $e_t^* = \sqrt{\theta}$. Moreover, note that $d\pi_t/dw_t\,|_{w_t=\overline{w}} = \sqrt{\eta^2\theta^3} - 1$. This is positive for $\eta > \sqrt{1/\theta^3}$, hence a strictly positive wage is optimal in this case and not otherwise. Therefore, effort levels in both cases ($w_t > 0$ and $w_t = 0$) are not affected by $\overline{w}$, as well as the threshold $\overline{\eta}$ above which $w_t > 0$ is optimal. Therefore, equilibrium effort is independent of $\overline{w}$. It follows that e_t^* and w_t are independent of $\overline{\Pi}$.

Now, I include the respective (DE) constraints, which yields the Lagrange function of the principal's maximization problem in a period t

$$\begin{aligned} L_t = {} & e_t\theta - e_t^3/3 + \eta\,(w_t - \overline{w})\,e_t\theta - 2/3\left(\sqrt{\eta\,(w_t - \overline{w})\,\theta}\right)^3 - w_t \\ & + \lambda_{DE_t}\left[\delta\left(\Pi_{t+1} - \overline{\Pi}\right) - \frac{2}{3}\left(\sqrt{\eta\,(w_t - \overline{w})\,\theta}\right)^3 - e_t^3/3 + \eta\,(w_t - \overline{w})\,\theta e_t\right] \\ & + \lambda_{w_t}\,(w_t - \overline{w}), \end{aligned}$$

where $\lambda_{w_t} \geq 0$ represents the Lagrange parameter for the agent's limited liability constraint and $\lambda_{DE_t} \geq 0$ represents the Lagrange parameter for the principal's dynamic enforcement constraint.

First-order conditions are

$$\begin{aligned} \frac{\partial L}{\partial e_t^*} = {} & \theta - e_t^2 + \eta\,(w_t - \overline{w})\,\theta + \lambda_{DE_t}\left[-e_t^2 + \eta\,(w_t - \overline{w})\,\theta\right] = 0 \\ \frac{\partial L}{\partial w_t} = {} & \eta\,\theta e_t - \eta\,\theta\sqrt{\eta\,(w_t - \overline{w})\,\theta} - 1 \\ & + \lambda_{DE_t}\left[-\eta\,\theta\sqrt{\eta\,(w_t - \overline{w})\,\theta} + \eta\,\theta e_t\right] + \lambda_{w_t} = 0. \end{aligned}$$

I start with $\lambda_{w_t} = 0$. Then, the first order conditions yield $w_t = \frac{\left(\left(1+\lambda_{DE_t}\right)\eta^2\theta^3 - 1\right)^2}{\left(1+\lambda_{DE_t}\right)^2 4\eta^3\theta^3} + \overline{w}$ and $e_t = \frac{1+\left(1+\lambda_{DE_t}\right)\eta^2\theta^3}{\left(1+\lambda_{DE_t}\right)2\eta\theta}$. It follows that, given $\lambda_{DE_t} > 0$ and $\eta^2\theta^3 - 1 \geq 0$, $\frac{\left(\eta^2\theta^3\left(1+\lambda_{DE_t}\right)-1\right)^2}{4\eta^3\theta^3\left(1+\lambda_{DE_t}\right)^2} > \frac{\left(\eta^2\theta^3-1\right)^2}{4\eta^3\theta^3}$ and $\frac{1+\eta^2\theta^3\left(1+\lambda_{DE_t}\right)}{2\eta\theta\left(1+\lambda_{DE_t}\right)} < \frac{1+\eta^2\theta^3}{2\eta\theta}$.

Now, assume that $\lambda_{w_t} > 0$ and hence $w_t = \overline{w}$. Then, $e_t^* = \sqrt{\theta/\left(1+\lambda_{DE_t}\right)}$. To

show that both cases, $w_t = \overline{w}$ and $w_t > \overline{w}$, are feasible, note that $\partial L/\partial w_t \mid_{w_t=\overline{w}} = \sqrt{\eta^2\theta^3(1+\lambda_{DE_t})} - 1 = 0$. This is positive for $\eta > \sqrt{1/\theta^3(1+\lambda_{DE_t})}$, hence a strictly positive wage is optimal in this case and not otherwise.

Now, I show that Π_t is decreasing in $\overline{w}$. This implies that (DE) is more likely to bind for a higher $\overline{w}$, and thus – once (DE) binds – λ_{DE_t} increases with $\overline{w}$ (see the proof to Lemma 5). First, I have already shown (in the proof to Lemma 7) that $\Pi_T = \pi_T$ is decreasing in $\overline{w}$. Therefore, (DE) in period $T-1$ is tightened, and consequently profits π_{T-1} and Π_{T-1} are reduced for larger values of $\overline{w}$. This tightens the (DE) constraint in period $T-2$ and reduces profits π_{T-2} and Π_{T-2}, and so on. Therefore, Π_t is decreasing in $\overline{w}$ for all t, and λ_{DE_t}, if positive, is increasing.

Therefore,

$$\frac{\partial e_t^*}{\partial \overline{w}} = -\frac{1}{2\eta\theta(1+\lambda_{DE_t})^2}\frac{\partial \lambda_{DE_t}}{\partial \overline{w}} < 0$$
$$\frac{\partial w_t}{\partial \overline{w}} = \frac{\left((1+\lambda_{DE_t})\eta^2\theta^3 - 1\right)}{2\eta^3\theta^3(1+\lambda_{DE_t})^3}\frac{\partial \lambda_{DE_t}}{\partial \overline{w}} + 1 > 1.$$

if $w_t > \overline{w}$. If $w_t = \overline{w}$,

$$\frac{\partial e_t^*}{\partial \overline{w}} = -\frac{1}{2}\sqrt{\frac{\theta}{(1+\lambda_{DE_t})^3}}\frac{\partial \lambda_{DE_t}}{\partial \overline{w}} < 0$$
$$\frac{\partial w_t}{\partial \overline{w}} = 1.$$

Finally, λ_{DE_t} increases in $\overline{\Pi}$ because a larger $\overline{\Pi}$ tigthens (DE) (see the proof to Lemma 5).

Therefore,

$$\frac{\partial e_t^*}{\partial \overline{\Pi}} = -\frac{1}{2\eta\theta(1+\lambda_{DE_t})^2}\frac{\partial \lambda_{DE_t}}{\partial \overline{\Pi}} < 0$$
$$\frac{\partial w_t}{\partial \overline{\Pi}} = \frac{\left((1+\lambda_{DE_t})\eta^2\theta^3 - 1\right)}{2\eta^3\theta^3(1+\lambda_{DE_t})^3}\frac{\partial \lambda_{DE_t}}{\partial \overline{\Pi}} + 1 > 1.$$

■

Proof of Proposition 4. First, I give a more precise description of the outcomes if the principal can commit to $\hat{w}_2$, to which the agent reacts with $\gamma\hat{w}_2$:

If $\eta^2\theta^3 > \frac{3+\sqrt{9-6\delta}}{3\delta}$, (DE) does not bind. Then, $\hat{w}_2 = 0$ and $w_1 > 0$.

For $\eta^2\theta^3 \leq \frac{3+\sqrt{9-6\delta}}{3\delta}$, (DE) binds. In this case,

- $w_1 = \hat{w}_2 = 0$ if $\eta^2\theta^3 \leq \frac{3}{4}\delta$ and $\gamma \leq \left(\frac{3}{4}\delta\sqrt{\eta^2\theta^3}\right)^{\frac{2}{3}}$
- $w_1 = 0$ and $\hat{w}_2 > 0$ if $\eta^2\theta^3 \leq \frac{3}{4}\delta$ and $\gamma > \left(\frac{3}{4}\delta\sqrt{\eta^2\theta^3}\right)^{\frac{2}{3}}$, or if $\eta^2\theta^3 > \frac{3}{4}\delta$ and $\gamma > \eta^2\theta^3$.
- $w_1 > 0$ if $\gamma \leq \eta^2\theta^3$ and $\eta^2\theta^3 > \frac{3}{4}\delta$. Then, $\hat{w}_2 > 0$ if $\eta^2\theta^3 < \frac{\gamma^2\left(1+2\sqrt{\frac{1}{4}-\frac{\delta}{6\gamma}}\right)}{\delta}$ and $\gamma \geq \frac{3}{4}\delta$, and $\hat{w}_2 = 0$ otherwise.

To prove this result, note that the principal sets e_1,b_1, w_1 and $\hat{w}_2$ to maximize

$$\Pi_1 = e_1\theta - b_1 - w_1 + \delta\left(\frac{\eta\theta^3}{4} - (1-\gamma)\hat{w}_2\right),$$

subject to

$$b_1 - \frac{(e_1^*)^3}{3} + \eta w_1 e_1^*\theta \geq 2/3\left(\sqrt{\eta w_1\theta}\right)^3 \quad \text{(IC)}$$

$$-b_1 + \delta\left(\frac{\eta\theta^3}{4} + \gamma\hat{w}_2\right) \geq 0 \quad \text{(DE)}$$

$$w_1 \geq 0$$

$$\hat{w}_2 \geq 0.$$

As before, (IC) binds in a profit-maximizing equilibrium. Solving (IC) for b_1 and substituting it into Π_1 and (DE) yields the Lagrange function

$$\begin{aligned} L =& e_1\theta - \frac{(e_1^*)^3}{3} + \eta w_1 e_1^*\theta - 2/3\left(\sqrt{\eta w_1\theta}\right)^3 - w_1 + \delta\left[\frac{\eta\theta^3}{4} - (1-\gamma)\hat{w}_2\right] \\ &+ \lambda_{DE}\left[-\frac{(e_1^*)^3}{3} + \eta w_1 e_1^*\theta - 2/3\left(\sqrt{\eta w_1\theta}\right)^3 + \delta\left(\frac{\eta\theta^3}{4} + \gamma\hat{w}_2\right)\right] + \lambda_{\hat{w}_2}\hat{w}_2 + \lambda_{w_1}w_1. \end{aligned}$$

First-order conditions are

$$\begin{aligned}
\frac{\partial L}{\partial e_1} =& \theta - \left(e_1^2 - \eta w_1 \theta\right)(1+\lambda_{DE}) = 0 \\
&\Rightarrow (1+\lambda_{DE}) = \frac{\theta}{\left[e_1^2 - \eta w_1 \theta\right]} \\
\frac{\partial L}{\partial w_1} =& -1 + \left(\eta e_1 \theta - \sqrt{\eta w_1 \theta}\eta\theta\right)(1+\lambda_{DE}) + \lambda_{w_1} = 0 \\
&\Rightarrow \lambda_{w_1} = 1 - \frac{\eta\theta^2}{\left(e_1 + \sqrt{w_1 \eta \theta}\right)} \\
\frac{\partial L}{\partial \hat{w}_2} =& -\delta(1-\gamma) + \lambda_{DE}\delta\gamma + \lambda_{\hat{w}_2} = 0. \\
&\Rightarrow \lambda_{\hat{w}_2} = \delta(1-\gamma) - \lambda_{DE}\delta\gamma
\end{aligned}$$

The last condition already reveals that $\hat{w}_2 = 0$ if $\lambda_{DE} = 0$.

In the following, I analyze all potential cases and derive conditions for each of those to hold.

1.) Slack (DE)

This case mirrors the results derived in Lemma 3, derived for the specific case of two periods: $\lambda_{DE} = 0$ yields

$$e_1^2 = \theta(1+\eta w_1), \tag{2}$$

and we have to disginguish between the two cases $w_1 = 0$ and $w_1 > 0$.

A) $w_1 = 0$ First-order conditions yield $\lambda_{w_1} > 0 \Leftrightarrow e_1 > \eta\theta^2$, whereas $e_1 = \sqrt{\theta}$ follows from (2), hence $\eta^2\theta^3 < 1$ is a necessary condition for this case to hold.

Finally, a slack (DE) constraint at these values requires

$$\eta^2\theta^3 \geq \left(\frac{4}{3\delta}\right)^2,$$

which is at odds with $\eta^2\theta^3 < 1$. Hence, if only one period is left, a non-binding (DE) constraint requires $w_1 > 0$ (for $\delta \leq 1$), also see the proof to Lemma 5.

B) $w_1 > 0$ First-order conditions yield $\lambda_{w_1} = 0 \Leftrightarrow w_1 = \frac{\left(\eta\theta^2 - e_1\right)^2}{\eta\theta}$, whereas $e_1 = \frac{1+\eta^2\theta^3}{2\eta\theta}$ – and consequently $w_1 = \left(\frac{\eta^2\theta^3 - 1}{2\eta\theta}\right)^2 / \eta\theta$ – follows from (2).

Also note that $\lambda_{w_1} = 1 - \frac{\eta\theta^2}{\left(e_1 + \sqrt{w_1\eta\theta}\right)} = 0 \Leftrightarrow \sqrt{w_1\eta\theta} = \eta\theta^2 - e_1$ and $e_1 \leq \eta\theta^2$. This implies $\eta^2\theta^3 \geq 1$, and a non-binding (DE) constraint at these values requires

$$-\frac{\left(\frac{1+\eta^2\theta^3}{2\eta\theta}\right)^3}{3}+\left(\frac{\eta^2\theta^3-1}{2\eta\theta}\right)^2\frac{1+\eta^2\theta^3}{2\eta\theta}-2/3\left(\sqrt{\left(\frac{\eta^2\theta^3-1}{2\eta\theta}\right)^2}\right)^3+\delta\frac{\eta\theta^3}{4}\geq 0.$$

Since $\eta^2\theta^3 \geq 1$, this condition becomes

$$2-6\eta^2\theta^3+3\delta\eta^4\theta^6 \geq 0,$$

and either holds for $\eta^2\theta^3 > \left(3+\sqrt{9-6\delta}\right)/3\delta$ or $\eta^2\theta^3 < \left(3-\sqrt{9-6\delta}\right)/3\delta$. Again, since $\eta^2\theta^3 \geq 1$, (DE) does not bind for

$$\eta^2\theta^3 > \frac{3+\sqrt{9-6\delta}}{3\delta}.$$

2.) Binding (DE)

Assume (DE) binds, hence $\eta^2\theta^3 \leq \left(3+\sqrt{9-6\delta}\right)/3\delta$. Moreover, first-order conditions reveal that $e_1^2 < \theta\left(1+\eta w_1\right)$. In the following, I explore all four potential cases separately and assess the conditions for each of them to hold.

A) $w_1 = 0$**,** $\hat{w}_2 = 0$ First-order conditions yield $\lambda_{w_1} > 0 \Leftrightarrow 1-\frac{\eta\theta^2}{\left(e_1+\sqrt{w_1\eta\theta}\right)} > 0 \Leftrightarrow e_1 > \eta\theta^2$. Recall that $e_1^2 < \theta$ must hold as well, hence $w_1 = 0$ if the necessary condition

$$\eta^2\theta^3 < 1$$

is satisfied.

Moreover, a binding (DE) constraint, taking into account $w_1 = \hat{w}_2 = 0$, yields $e_1 = \sqrt[3]{\delta\frac{3\eta}{4}}\theta$.

$\lambda_{\hat{w}_2} > 0$ also requires $\delta\left(1-\gamma\right)-\lambda_{DE}\delta\gamma > 0$ and consequently $1-\frac{\theta}{\left[e_1^2-\eta w_1\theta\right]}\gamma > 0$, which is equivalent to

$$\gamma < \left(\delta\frac{3\eta}{4}\right)^{\frac{2}{3}}\theta.$$

Concluding, the requirements for this case are

- $e_1 = \sqrt[3]{\delta\frac{3\eta}{4}}\theta < \sqrt{\theta} \Leftrightarrow \eta^2\theta^3 < \frac{16}{9\delta^2}$, which always holds for $\eta^2\theta^3 < 1$
- $e_1 = \sqrt[3]{\delta\frac{3\eta}{4}}\theta > \eta\theta^2 \Leftrightarrow \eta^2\theta^3 < \frac{3}{4}\delta$, which is tougher than $\eta^2\theta^3 < 1$

- $\gamma \leq \left(\delta \frac{3\sqrt{\eta^2\theta^3}}{4}\right)^{\frac{2}{3}}$

B) $w_1 = 0$**,** $\hat{w}_2 > 0$ As in case A), first-order conditions yield $\lambda_{w_1} > 0 \Leftrightarrow 1 - \frac{\eta\theta^2}{\left(e_1+\sqrt{w_1\eta\theta}\right)} > 0 \Leftrightarrow e_1 > \eta\theta^2$, as well as $e_1^2 < \theta$, which give the necessary condition

$$\eta^2\theta^3 < 1.$$

Moreover, $\lambda_{\hat{w}_2} = \delta - \frac{\theta}{e_1^2}\delta\gamma = 0 \Leftrightarrow e_1 = \sqrt{\theta\gamma}$, and a binding (DE) constraint yields

$$\hat{w}_2 = \frac{\frac{(\sqrt{\theta\gamma})^3}{3} - \delta\frac{\eta\theta^3}{4}}{\delta\gamma},$$

Concluding, the requirements for this case are

- $e_1 > \eta\theta^2 \Leftrightarrow \gamma > \eta^2\theta^3$
- $\hat{w}_2 > 0 \Leftrightarrow \gamma > \left(\delta \frac{3\sqrt{\eta^2\theta^3}}{4}\right)^{\frac{2}{3}}$; the latter is tighter than $\gamma > \eta^2\theta^3$ if $\eta^2\theta^3 < \frac{3}{4}\delta$

Therefore, the requirements become

- $\gamma > \eta^2\theta^3$ if $\eta^2\theta^3 \geq \frac{3}{4}\delta$
- $\gamma > \left(\delta \frac{3\sqrt{\eta^2\theta^3}}{4}\right)^{\frac{2}{3}}$ if $\eta^2\theta^3 < \frac{3}{4}\delta$

C) $w_1 > 0$**,** $\hat{w}_2 > 0$ First-order conditions yield $\lambda_{w_1} = 1 - \frac{\eta\theta^2}{\left(e_1+\sqrt{w_1\eta\theta}\right)} = 0 \Leftrightarrow \sqrt{w_1\eta\theta} = \eta\theta^2 - e_1 \Rightarrow \eta\theta^2 \geq e_1$.

Moreover, $\lambda_{w_1} = 0 \Leftrightarrow w_1 = \frac{\left(\eta\theta^2 - e_1\right)^2}{\eta\theta}$, as well as $\lambda_{\hat{w}_2} = 0 \Leftrightarrow 1 - \frac{\theta}{\left[e_1^2 - \eta w_1\theta\right]}\gamma = 0$. Combining these two conditions delivers

$$e_1 = \frac{\eta^2\theta^3 + \gamma}{2\eta\theta}.$$

$$\frac{\eta^2\theta^3 + \gamma}{2\eta\theta} \leq \eta\theta^2 \Leftrightarrow \gamma \leq \eta^2\theta^3$$

The condition for a binding (DE) constraint, $e_1^2 < \theta\left(1 + \eta w_1\right)$, becomes $0 < 1 - \gamma$, hence is always satisfied at these values.

Finally, a binding (DE) constraint yields

$$\hat{w}_2 = \frac{\left(\frac{\eta^2\theta^3+\gamma}{2\eta\theta}\right)^3/3 - \left(\frac{\eta^2\theta^3-\gamma}{2\eta\theta}\right)^2 \frac{\eta^2\theta^3+\gamma}{2\eta\theta} + \frac{2}{3}\left(\sqrt{\left(\frac{\eta^2\theta^3-\gamma}{2\eta\theta}\right)^2}\right)^3 - \delta\frac{\eta\theta^3}{4}}{\delta\gamma}$$

Since $\gamma \leq \eta^2\theta^3$,

$$\hat{w}_2 = \frac{\frac{3\eta^2\theta^3\gamma^2-\gamma^3}{6} - \delta\frac{\left(\eta^2\theta^3\right)^2}{4}}{\eta^3\theta^3\delta\gamma}.$$

This implies that $\hat{w}_2$ is positive for $\eta^2\theta^3 \in \left(\frac{\gamma^2\left(1-2\sqrt{\frac{1}{4}-\frac{\delta}{6\gamma}}\right)}{\delta}, \frac{\gamma^2\left(1+2\sqrt{\frac{1}{4}-\frac{\delta}{6\gamma}}\right)}{\delta}\right)$, which requires $\gamma \geq \frac{2}{3}\delta$. The upper bound is consistent with $\gamma \leq \eta^2\theta^3$ if $\gamma \geq \frac{3}{4}\delta$. The lower bound is consistent with $\gamma \leq \eta^2\theta^3$ if

$$2\gamma\sqrt{\frac{1}{4}-\frac{\delta}{6\gamma}} \leq \gamma - \delta.$$

There, the right hand side is positive for $\gamma \geq \delta$. Then, this condition becomes $\gamma \leq \frac{3}{4}\delta$, which contradicts $\gamma \geq \delta$. Thus, $\eta^2\theta^3 \in \left(\gamma, \frac{\gamma^2\left(1+2\sqrt{\frac{1}{4}-\frac{\delta}{6\gamma}}\right)}{\delta}\right)$ if $\gamma > \frac{3}{4}\delta$.

Concluding, the requirements for this case are

- $\gamma < \eta^2\theta^3$, $\gamma \geq \frac{3}{4}\delta$ (thus also $\eta^2\theta^3 > \frac{3}{4}\delta$)
- $\eta^2\theta^3 \in \left(\gamma, \frac{\gamma^2\left(1+2\sqrt{\frac{1}{4}-\frac{\delta}{6\gamma}}\right)}{\delta}\right)$.

D) $w_1 > 0$, $\hat{w}_2 = 0$ First-order conditions yield $\lambda_{w_1} = 0 \Leftrightarrow w_1 = \frac{\left(\eta\theta^2-e_1\right)^2}{\eta\theta}$, whereas a binding (DE) constraint implies $e_1^2 < \theta\left(1+\eta w_1\right) \Leftrightarrow e_1 < \frac{1+\eta^2\theta^3}{2\eta\theta}$.

Also note that $\lambda_{w_1} = 1 - \frac{\eta\theta^2}{\left(e_1+\sqrt{w_1\eta\theta}\right)} = 0 \Leftrightarrow \sqrt{w_1\eta\theta} = \eta\theta^2 - e_1 \Rightarrow \eta\theta^2 \geq e_1$.

Moreover, a binding (DE) constraint, taking into account $w_1 = \frac{\left(\eta\theta^2-e_1\right)^2}{\eta\theta}$ and $\hat{w}_2 = 0$, yields

$$-\frac{e_1^3}{3} + \left(\eta\theta^2-e_1\right)^2 e_1 - 2/3\left(\sqrt{\left(\eta\theta^2-e_1\right)^2}\right)^3 + \delta\left(\frac{\eta\theta^3}{4}\right) = 0. \tag{3}$$

Since $e_1 \leq \eta\theta^2$, this condition becomes

$$\frac{4}{3}e_1^3 - 4e_1^2\eta\theta^2 + 3e_1\eta^2\theta^4 - \frac{2}{3}\eta^3\theta^6 + \delta\frac{\eta\theta^3}{4} = 0 \tag{4}$$

There, note that the partial derivative of the left-hand side of the (IC) constraint with respect to e_1, $-e_1^2 + \eta w_1\theta = -2\eta\theta^2 e_1 + \eta^2\theta^4$, must be negative because otherwise, a higher e_1 would relax (IC), contradicting that it binds. Hence, $e_1 > \eta\theta^2/2$. At $e_1 = \eta\theta^2/2$, the left hand side of (4) equals $\delta\frac{\eta\theta^3}{4} > 0$. Moreover, the partial derivative of the left hand side of (4) with respect to e_1 equals $4e_1^2 - 8e_1\eta\theta^2 + 3\eta^2\theta^4$, which is negative for the relevant range $e_1 \in \left[\frac{\eta\theta^2}{2}, \eta\theta^2\right]$. Thus, within this range the left-hand side of (4) is minimized for $e_1 = \eta\theta^2$ and then equals $-\eta^2\theta^3/3 + \delta/4$. All this implies that, if $-\eta^2\theta^3/3 + \delta/4 \leq 0 \Leftrightarrow \eta^2\theta^3 \geq 3\delta/4$, there is a $e_1 \in \left[\frac{\eta\theta^2}{2}, \eta\theta^2\right]$ solving (4).

Finally, first-order conditions reveal that $\lambda_{\hat{w}_2} > 0 \Leftrightarrow 1 - \frac{\theta}{\left[e_1^2 - \eta w_1\theta\right]}\gamma > 0$, hence

$$1 - \frac{\theta}{\left[2\eta\theta^2 e_1 - \eta^2\theta^4\right]}\gamma > 0.$$

Since $e_1 \geq \frac{\eta\theta^2}{2}$, this becomes

$$\gamma < 2\eta\theta e_1 - \eta^2\theta^3 \leq \eta^2\theta^3.$$

Hence, necessary conditions for this case to be feasible are $\gamma < \eta^2\theta^3$ and $\eta^2\theta^3 \geq 3\delta/4$. Then, this case holds if the conditions of C) are not satisfied. ■

Proof of Proposition 5. First, I show that, for $p \to 1$, a separating contract yields higher profits than a pooling contract. There, note that, in any profit-maximizing equilibrium, (ICS), the selfish type's (IC) constraint, is tighter than (ICR), the reciprocal type's (IC) constraint:

$$-\frac{e_1^3}{3} + \delta w_2 \geq 0 \tag{ICS}$$

$$-\frac{e_1^3}{3} + \eta w_1\theta e_1 + \delta\left[w_2 + \frac{2\left(\sqrt{\eta w_2\theta}\right)^3}{3}\right] \geq \frac{2}{3}\left(\sqrt{\eta w_1\theta}\right)^3. \tag{ICR}$$

With $w_1 = 0$, (ICS) is tighter than (ICR) for any second-period wage w_2 because second-period utilities are larger for the reciprocal type. A strictly positive w_1 can only possibly be optimal for the principal if it further relaxes (ICR) ((ICS) is unaffected by w_1), which confirms that (ICS) is tighter than (ICR) in any profit-maximizing

equilibrium. This implies that a strictly higher effort level can be implemented with a separating contract (then however only exerted by the reciprocal type) than with a pooling contract (then exerted by both). For $p \to 1$, profits under both regimes approach $e_1\theta - w_1 + \delta\left(\sqrt{w_2\eta\theta}\theta - w_2\right)$, which is larger with a separating contract because of the higher effort implemented in this case.

To show that a pooling contract yields higher profits than a separating contract for $p \to 0$, I first assume that the principal offers a pooling contract and explore its properties. Then, I do the same with a separating contract, and finally compare both alternatives.

Pooling contract In any profit-maximizing equilibrium, (ICS) is tighter than (ICR). Therefore, (ICS) determines feasible effort in a pooling contract. This also implies that $w_1 = 0$, because a positive w_1 might only relax (ICR).

Now, the principal maximizes Π_1, subject to her own (DE) constraint, $pe_2\theta - w_2 \geq 0$, as well as the selfish agent's (IC) constraint, $-\frac{e_1^3}{3} + \delta w_2 \geq 0$. This will bind because, otherwise, the principal could ask for a higher first-period effort level without violating any constraint. Moreover, the reciprocal type exerts an effort level $e_2 = \sqrt{w_2\eta\theta}$ in the second period, whereas the selfish type's second period effort amounts to zero, hence $\Pi_1 = e_1\theta + \delta\left(p\sqrt{w_2\eta\theta}\theta - w_2\right)$.

Taking all this into account, the Lagrange function becomes

$$L = e_1\theta + \delta\left[p\sqrt{\frac{e_1^3}{3\delta}\eta\theta}\theta - \frac{e_1^3}{3\delta}\right] + \lambda_{DE}\left[p\sqrt{\frac{e_1^3}{3\delta}\eta\theta}\theta - \frac{e_1^3}{3\delta}\right],$$

and the first order condition

$$\frac{\partial L}{\partial e_1} = \theta + \left[\frac{p\eta\theta^2}{2\sqrt{\frac{e_1^3}{3\delta}\eta\theta}} - 1\right]\frac{e_1^2}{\delta}(\delta + \lambda_{DE}) = 0.$$

First, assume $\lambda_{DE} = 0$. Then, e_1^* is characterized by

$$2\sqrt{\frac{\eta\theta}{3\delta}}\left(\theta - (e_1^*)^2\right) + p\eta\theta^2\sqrt{e_1^*} = 0. \tag{5}$$

Second, assume $\lambda_{DE} > 0$. Then, e_1^* is determined by the binding (DE) constraint,

$$e_1^* = \sqrt[3]{3\delta p^2\eta\theta^3}.$$

To compute the condition for when (DE) actually binds, I plug $e_1^* = \sqrt[3]{3\delta p^2\eta\theta^3}$ into

the first order condition,

$$\begin{aligned}\theta + \left[\frac{p\eta\theta^2}{2\sqrt{\frac{e_1^3}{3\delta}\eta\theta}} - 1\right]\frac{e_1^2}{\delta}(\delta + \lambda_{DE}) \\ = \theta - \frac{1}{2}\frac{e_1^2}{\delta}(\delta + \lambda_{DE}) \\ = \theta - \frac{1}{2}\frac{\left(\sqrt[3]{3\delta p^2\eta\theta^3}\right)^2}{\delta}(\delta + \lambda_{DE}) = 0.\end{aligned}$$

Therefore, (DE) binds if $\theta - \frac{1}{2}\theta^2\left(\sqrt[3]{3\delta p^2\eta}\right)^2 \geq 0$, or

$$p^2 \leq \frac{\left(\sqrt{\frac{2}{\theta}}\right)^3}{3\delta\eta}.$$

In this case, which is the relevant case for $p \to 0$, the principal's profits with a pooling equilibrium are

$$\Pi_1^P = e_1^*\theta = \sqrt[3]{3\delta p^2\eta}\theta^2.$$

Otherwise, $\Pi_1^P = e_1\theta + \delta\left[p\sqrt{\frac{\left(e_1^*\right)^3}{3\delta}\eta\theta}\theta - \frac{\left(e_1^*\right)^3}{3\delta}\right] = e_1^*\left[\theta + \frac{\left(e_1^*\right)^2 - 2\theta}{3}\right]$, where e_1^* is characterized by (5).

Separating contract In case she offers a separating contract, the principal maximizes $\Pi_1 = p\left[e_1\theta + \delta\left(e_2\theta - w_2\right)\right] - w_1$, where $e_2 = \sqrt{w_2\eta\theta}$, subject to her own (DE) constraint, $e_2\theta - w_2 \geq 0$ (which is relevant in case the agent turns out to be reciprocal), the limited liability constraint, $w_1 \geq 0$, as well as the reciprocal agent's binding (IC) constraint,

$$\begin{aligned}-\frac{\left(e_1^*\right)^3}{3} + \eta w_1\theta e_1^* + \delta\left[w_2 + \frac{2\left(\sqrt{\eta w_2\theta}\right)^3}{3}\right] \\ = \frac{2\left(\sqrt{\eta w_1\theta}\right)^3}{3}. \qquad \text{(IC)}\end{aligned}$$

There, note that

$$\frac{de_1^*}{dw_1} = \eta\theta \frac{e_1^* - \sqrt{\eta w_1 \theta}}{\left(e_1^*\right)^2 - \eta w_1 \theta} = \frac{\eta\theta}{e_1^* + \sqrt{\eta w_1 \theta}}$$
$$\frac{de_1^*}{dw_2} = \frac{\delta\left[1 + \sqrt{\eta w_2 \theta}\eta\theta\right]}{\left(e_1^*\right)^2 - \eta w_1 \theta}.$$

Therefore, the Lagrange function becomes $L = p\left[e_1\theta + \delta\left(\sqrt{w_2\eta\theta}\theta - w_2\right) + \lambda_{DE}\left(\sqrt{w_2\eta\theta}\theta - w_2\right)\right] - w_1 + \lambda_{w_1} w_1$, with first order conditions

$$\begin{aligned}
\frac{\partial L}{\partial w_1} =& p\frac{\eta\theta}{e_1 + \sqrt{\eta w_1 \theta}}\theta - 1 + \lambda_{w_1} = 0 \\
\frac{\partial L}{\partial w_2} =& p\left[\frac{\delta\left[1 + \sqrt{\eta w_2 \theta}\eta\theta\right]}{e_1^2 - \eta w_1 \theta}\theta + \delta\left(\frac{\eta\theta^2}{2\sqrt{w_2\eta\theta}} - 1\right) + \lambda_{DE}\left(\frac{\eta\theta^2}{2\sqrt{w_2\eta\theta}} - 1\right)\right] = 0
\end{aligned}$$

For later use, note that the first condition implies that $w_1 = 0$ for $p \to 0$ (because e_1^* is bounded away from zero for any strictly positive δ).

First, assume $\lambda_{DE} = 0$, hence

$$\frac{1 + \sqrt{\eta w_2 \theta}\eta\theta}{e_1^2 - \eta w_1 \theta}\theta + \frac{\eta\theta^2}{2\sqrt{w_2\eta\theta}} - 1 = 0.$$

This, together with the reciprocal agent's (IC) constraint, determines outcomes if $w_1 = 0$. If $w_1 > 0$, outcomes are additionally given by

$$p\frac{\eta\theta^2}{e_1^* + \sqrt{\eta w_1 \theta}} - 1 = 0,$$

and an explicit characterization of the results is not feasible.

Now, assume $\lambda_{DE} > 0$. Then, a binding (DE) constraint implies $w_2 = \eta\theta^3$.

If $w_1 = 0$, (IC) yields

$$e_1^* = \sqrt[3]{3\delta\left[\eta\theta^3 + \frac{2\eta^3\theta^6}{3}\right]}$$

To compute the condition for when $w_1 = 0$ (if (DE) binds), I plug these values into

the first first order condition, $p\frac{\eta\theta^2}{\sqrt[3]{3\delta\left[\eta\theta^3+\frac{2\eta^3\theta^6}{3}\right]}}-1+\lambda_{w_1}=0$. Therefore, $w_1=0$ if

$$p\frac{\eta\theta^2}{\sqrt[3]{3\delta\left[\eta\theta^3+\frac{2\eta^3\theta^6}{3}\right]}}-1\leq 0,$$

$$p^3\leq\frac{3\delta\left[\eta\theta^3+\frac{2\eta^3\theta^6}{3}\right]}{\eta^3\theta^6}.$$

To compute the condition for when (DE) binds (if $w_1=0$), I plug these values into the second first order condition. Therefore, (DE) binds for

$$\frac{\delta\left[1+\eta\theta^2\eta\theta\right]}{\left(\sqrt[3]{3\delta\left[\eta\theta^3+\frac{2\eta^3\theta^6}{3}\right]}\right)^2}\theta-\frac{1}{2}\delta\geq 0,$$

or

$$\delta^2\leq\frac{8\theta^3\left[1+\eta^2\theta^3\right]^3}{9\left[\eta\theta^3+\frac{2\eta^3\theta^6}{3}\right]^2}.$$

The right hand side of this condition is larger than 1, and (DE) *always* binds if $w_1=0$. Therefore, (DE) always binds if $p\to 0$ because then, $w_1=0$ (see above). On a general note, though, wIwant to emphasize that this might change in a more general setup with a longer time horizon.

All this implies that, for $p\to 0$, profits with a separating contract are

$$\Pi_1^S=pe_1^*\theta=p\theta\sqrt[3]{3\delta\left[\eta\theta^3+\frac{2\eta^3\theta^6}{3}\right]}.$$

Comparison For $p\to 0$, profits with a pooling contract are $\Pi_1^P=\sqrt[3]{3\delta p^2\eta}\theta^2$, and $\Pi_1^S=p\theta\sqrt[3]{3\delta\left[\eta\theta^3+\frac{2\eta^3\theta^6}{3}\right]}$ for a separating contract. Therefore,

$$\begin{aligned}&\Pi_1^P>\Pi_1^S\\ \Leftrightarrow&\sqrt[3]{3\delta p^2\eta\theta^3}\theta\geq p\theta\sqrt[3]{3\delta\left[\eta\theta^3+\frac{2\eta^3\theta^6}{3}\right]}\\ \Leftrightarrow&1\geq p\left(1+\frac{2\eta^2\theta^3}{3}\right),\end{aligned}$$

which holds for $p\to 0$.

■

Proof of Lemma 9. The principal maximizes

$$\Pi_1 = e_1\theta - (e_1^*)^3/3 + \eta w_1 e_1^*\theta - 2/3\left(\sqrt{\eta w_1\theta}\right)^3 - w_1 + \delta\left(\frac{\eta\theta^3}{4} - w_1\right),$$

subject to $w_1 \geq 0$ and

$$\frac{(e_1^*)^3}{3} - \eta w_1\theta e_1^* + \frac{2}{3}\left(\sqrt{\eta w_1\theta}\right)^3 \leq \delta\left(\frac{\eta\theta^3}{4} - w_1\right). \tag{DE}$$

This yields the Lagrange function

$$\begin{aligned} L = {} & e_1\theta - (e_1)^3/3 + \eta w_1 e_1\theta - 2/3\left(\sqrt{\eta w_1\theta}\right)^3 - w_1 + \delta\left(\frac{\eta\theta^3}{4} - w_1\right) \\ & + \lambda_{w_1} w_1 + \lambda_{DE}\left[\eta w_1\theta e_1 + \delta\left(\frac{\eta\theta^3}{4} - w_1\right) - \frac{2}{3}\left(\sqrt{\eta w_1\theta}\right)^3 - \frac{(e_1)^3}{3}\right], \end{aligned}$$

where $\lambda_{w_1} \geq 0$ represents the Lagrange parameter for the agent's limited liability constraint, and $\lambda_{DE} \geq 0$ the Lagrange parameter for the principal's dynamic enforcement constraint.

First order conditions are

$$\begin{aligned} \frac{\partial L}{\partial e_1} = {} & \theta - e_1^2 + \eta w_1\theta + \lambda_{DE}\left[\eta w_1\theta - e_1^2\right] = 0 \\ \frac{\partial L}{\partial w_1} = {} & \eta e_1\theta - \eta\theta\sqrt{\eta w_1\theta} - 1 - \delta + \lambda_{w_1} \\ & + \lambda_{DE}\left[\eta\theta e_1^* - \delta - \eta\theta\sqrt{\eta w_1\theta}\right] = 0. \end{aligned}$$

First, assume $\lambda_{DE} = 0$. Then, I have to consider the two cases $w_1 = 0$ and $w_1 > 0$.

If $w_1 = 0$, $e_1^* = \sqrt{\theta}$ and $\Pi_1 = 2/3\left(\sqrt{\theta}\right)^3 + \delta\eta\theta^3/4$. Moreover, $d\Pi_1/dw_1\,|_{w_1=0} = \sqrt{\eta^2\theta^3} - 1 - \delta$, therefore $w_1 = 0$ for $\eta^2\theta^3 \leq (1+\delta)^2$, whereas $w_1 > 0$ for $\eta^2\theta^3 > (1+\delta)^2$. Recall that the condition for a positive wage in case (DE) is not binding in the main part (i.e., without an adjustment of the reference wage) equals $\eta^2\theta^3 > 1$.

Furthermore, $e_1^* > e_2^* \Leftrightarrow \eta^2\theta^3 < 4$, which holds because $\eta^2\theta^3 < (1+\delta)^2$. Moreover, $0 = w_1 < w_2 = \frac{\eta\theta^3}{4}$, and $\frac{de_1^*}{d\eta} = 0 < \frac{de_2^*}{d\eta}$.

To check the feasibility of the case $\lambda_{DE} = 0$ and $w_1 = 0$, I plug the respective values

into the (DE) constraint, and obtain

$$\frac{16}{9\delta^2} \leq \eta^2\theta^3.$$

This is consistent with $\eta^2\theta^3 \leq (1+\delta)^2$ if $3\delta(1+\delta) \geq 4$. Now, assume $\eta^2\theta^3 > (1+\delta)^2$ and $\lambda_{DE} = 0$. Hence, $\lambda_{w_1} = 0$, and the first order conditions yield $e_1 = \frac{(1+\delta)^2+\eta^2\theta^3}{2\eta\theta(1+\delta)}$ and $w_1 = \frac{\left[\eta^2\theta^3-(1+\delta)^2\right]^2}{4(1+\delta)^2\eta^3\theta^3}$. Moreover, $e_1^* > e_2^* \Leftrightarrow \delta\theta^3\eta^2 < (1+\delta)^2$, which only is consistent with $\eta^2\theta^3 > (1+\delta)^2$ if δ is sufficiently small. In any case, $w_1 < w_2$ and $de_1^*/d\eta < de_2^*/d\eta$, where the latter condition is equivalent to $\delta\eta^2\theta^3 > -(1+\delta)^2$.

To check the feasibility of the case $\lambda_{DE} = 0$ and $w_1 > 0$, I plug the respective values into the (DE) constraint, and obtain

$$2 \leq \delta\left(\frac{(1+\delta)^2\eta^2\theta^3 - 1}{(1+\delta)^2}\right) + (1+\delta)^2\frac{1}{3}\frac{(2-\delta)}{\eta^2\theta^3}.$$

The right hand side is increasing in η^2 if δ is large enough. Since $\eta^2\theta^3 > (1+\delta)^2$, this condition holds if it is satisfied for $\eta^2\theta^3 = (1+\delta)^2$. For this case, it becomes

$$\frac{4}{3} \leq \delta^2(2+\delta) + \frac{2}{3}\delta - \frac{\delta}{(1+\delta)^2}.$$

There, the right hand side is increasing in δ and, for $\delta \to 1$, approaches $3 + \frac{5}{12} > \frac{4}{3}$. Hence, this case is feasible if η and/or δ are large enough.

Now, assume that the (DE) constraint binds, hence $\lambda_{DE} > 0$.

First, I assume that $\lambda_{w_t} > 0$, hence $w_t = 0$ and $e_t^* = \sqrt{\theta/(1+\lambda_{DE_t})}$. To establish the existence of $\tilde{\eta}$, note that $\partial L/\partial w_t \mid_{w_t=0} = \left(\eta\theta\sqrt{\theta/(1+\lambda_{DE_t})} - \delta\right)(1+\lambda_{DE}) - 1$, which is positive for $\eta^2\theta^3 > (1+\delta(1+\lambda_{DE}))^2/(1+\lambda_{DE})$. This threshold is larger than with a non-binding (DE) if $\lambda_{DE} > \left(1-\delta^2\right)/\delta^2$, which might or might not hold. Moreover, provided $\eta^2\theta^3 \leq (1+\delta(1+\lambda_{DE}))^2/(1+\lambda_{DE})$, $e_1^* > e_2^* \Leftrightarrow \eta^2\theta^3(1+\lambda_{DE_t}) < 4$, which might or might not hold.

Second, I assume $\eta^2\theta^3 > (1+\delta(1+\lambda_{DE}))^2/(1+\lambda_{DE})$, hence $\lambda_{w_1} = 0$. Then, the first order conditions yield

$$e_1 = \frac{\eta^2\theta^3(1+\lambda_{DE}) + (1+\delta(1+\lambda_{DE}))^2}{2\eta\theta(1+\lambda_{DE})(1+\delta(1+\lambda_{DE}))}$$

$$w_1 = \frac{\left[\eta^2\theta^3(1+\lambda_{DE}) - (1+\delta(1+\lambda_{DE}))^2\right]^2}{4\eta^3\theta^3(1+\lambda_{DE})^2(1+\delta(1+\lambda_{DE}))^2}.$$

Now, $e_1^* > e_2^* \Leftrightarrow \frac{(1+\delta(1+\lambda_{DE}))^2}{\delta(1+\lambda_{DE})^2} > \eta^2\theta^3$, which might or might not be consistent with $\eta^2\theta^3 > (1+\delta(1+\lambda_{DE}))^2/(1+\lambda_{DE})$. ■

Proof of Lemma 10. In the second period, the principal maximizes $\pi_2 = e_2\theta - w_2$, where e_2 is given by

$$-e_2^2 - \frac{4}{3}e_2^3\eta\theta + w_2\eta\theta = 0.$$

This yields

$$\begin{aligned}
e_2^* =& \frac{\sqrt{1+4\eta^2\theta^3}-1}{4\eta\theta} \\
w_2 =& \frac{e_2^2 + \frac{4}{3}e_2^3\eta\theta}{\eta\theta} \\
\pi_2 =& e_2^*\left(\frac{8\eta^2\theta^3+1-\sqrt{1+4\eta^2\theta^3}}{12\eta^2\theta^2}\right) \\
u_2 =& \frac{(e_2^*)^2(1+\eta e_2^*\theta)^2}{\eta\theta}.
\end{aligned}$$

Recall that last-period profits in the main setup are $\eta\theta^3/4$, which is larger than the amount obtained here.

In the first period, at e_1^*, u_1 is decreasing in e_1. If it were increasing, the agent would further raise his effort level. This implies that (IC) is binding in a profit-maximizing equilibrium. If it were not binding, the principal could ask for a higher effort level without paying more.

Plugging the binding (IC) constraint,

$$b_1 = \frac{(e_t^*)^3}{3} - w_1 + \left(w_1 - \frac{(\tilde{e}_t)^3}{3}\right)\frac{(1+\eta\tilde{e}_1\theta)}{(1+\eta e_1^*\theta)},$$

into profits and the (DE) constraint yields the Lagrange function

$$\begin{aligned}
L =& e_1\theta - \frac{(e_t^*)^3}{3} - \left(w_1 - \frac{(\tilde{e}_t)^3}{3}\right)\frac{(1+\eta\tilde{e}_1\theta)}{(1+\eta e_1^*\theta)} \\
&+ \lambda_{w_1}w_1 + \lambda_{DE}\left[-\frac{(e_1^*)^3}{3} + w_1 - \left(w_1 - \frac{(\tilde{e}_1)^3}{3}\right)\frac{(1+\eta\tilde{e}_1\theta)}{(1+\eta e_1^*\theta)} + \delta\pi_2\right],
\end{aligned}$$

where $\lambda_{w_1} \geq 0$ represents the Lagrange parameter for the agent's limited liability constraint, and $\lambda_{DE} \geq 0$ the Lagrange parameter for the principal's dynamic enforcement

constraint.

First order conditions are

$$\begin{aligned}
\frac{\partial L}{\partial e_1} =& \theta - (e_1^*)^2 + \left(w_1 - \frac{(\tilde{e}_t)^3}{3}\right)\frac{\eta\theta\,(1+\eta\tilde{e}_1\theta)}{\left(1+\eta e_1^*\theta\right)^2} \\
&+ \lambda_{DE}\left[-(e_t^*)^2 + \left(w_1 - \frac{(\tilde{e}_t)^3}{3}\right)\frac{\eta\theta\,(1+\eta\tilde{e}_1\theta)}{\left(1+\eta e_1^*\theta\right)^2}\right] = 0 \\
\frac{\partial L}{\partial w_1} =& -\left(1-(\tilde{e})^2\frac{d\tilde{e}}{dw_1}\right)\frac{(1+\eta\tilde{e}\theta)}{\left(1+\eta e_1^*\theta\right)} - \left(w_1 - \frac{(\tilde{e})^3}{3}\right)\frac{\eta\theta}{\left(1+\eta e_1^*\theta\right)}\frac{d\tilde{e}}{dw_1} \\
&+ \lambda_{DE}\left[1-\left(1-(\tilde{e})^2\frac{d\tilde{e}}{dw_1}\right)\frac{(1+\eta\tilde{e}\theta)}{\left(1+\eta e_1^*\theta\right)} - \left(w_1 - \frac{(\tilde{e}_t)^3}{3}\right)\frac{\eta\theta}{\left(1+\eta e_1^*\theta\right)}\frac{d\tilde{e}}{dw_1}\right] \\
&+ \lambda_{w_1} = 0
\end{aligned}$$

Using $-\tilde{e}_1^2 - 4/3\tilde{e}_1^3\eta\theta + w_1\eta\theta = 0$, which implies $w_1 = \tilde{e}_1^2/\eta\theta + 4/3\tilde{e}_1^3$, those conditions become

$$\begin{aligned}
\frac{\partial L}{\partial e_1} &: \theta - \left((e_1^*)^2 - \tilde{e}_1^2\frac{(1+\eta\tilde{e}_1\theta)^2}{\left(1+\eta e_1^*\theta\right)^2}\right)(1+\lambda_{DE}) = 0 \\
\frac{\partial L}{\partial w_1} &: -\frac{(1+\eta\tilde{e}\theta)}{\left(1+\eta e_1^*\theta\right)}(1+\lambda_{DE}) + \lambda_{w_1} + \lambda_{DE} = 0
\end{aligned}$$

First, assume $\lambda_{DE} = 0$. Then, I have to consider the two cases $w_1 = 0$ and $w_1 > 0$. However, $w_1 > 0$ and consequently $\lambda_{w_1} = 0$ cannot be optimal, since in this case, the second condition would become $-(1+\eta\tilde{e}\theta)/(1+\eta e_1^*\theta) = 0$.

Therefore, $\lambda_{DE} = 0$ implies $w_1 = 0$; hence $\tilde{e} = 0$ and

$$e_1^* = \sqrt{\theta}.$$

Moreover $e_1^* = \sqrt{\theta} > \left(\sqrt{1+4\eta^2\theta^3} - 1\right)/(4\eta\theta) = e_2^*$ and $w_1 = 0 < \left(e_2^2 + \frac{4}{3}e_2^3\eta\theta\right)/(\eta\theta) = w_2$.

However, note that for two periods and $\delta \leq 1$, $\lambda_{DE} = 0$ is not feasible: For $w_1 = 0$, $b_1 = (\sqrt{\theta})^3/3$ and $e_1^* = \sqrt{\theta}$, the (DE) constraint becomes

$$-\frac{(\sqrt{\theta})^3}{3} + \delta\frac{\left(1+4\eta^2\theta^3\right)\sqrt{1+4\eta^2\theta^3} - 6\eta^2\theta^3 - 1}{24\eta^3\theta^3} \geq 0.$$

There, the second term increases in η and approaches $\delta 2\sqrt{\theta^3}/9$ for $\eta \to \infty$. Therefore, the constraint does not hold for any θ and η if $\delta \leq 1$.

Now, assume that (DE) binds. Again, I start with $w_1 = 0$. Then, $e_1^* = \sqrt{\theta/(1+\lambda_{DE})}$, and

$$\begin{aligned}\lim_{w_1 \to 0} \frac{\partial L}{\partial w_1} =& -\frac{(1+\lambda_{DE})}{\left(1+\eta e_1^* \theta\right)} + \lambda_{DE} \\ =& -\frac{(1+\lambda_{DE})}{\left(1+\sqrt{\frac{\eta^2\theta^3}{(1+\lambda_{DE})}}\right)} + \lambda_{DE},\end{aligned}$$

which is positive for

$$\eta^2\theta^3 > \frac{1+\lambda_{DE}}{\lambda_{DE}^2}.$$

Put differently,

$$e_1^* = \sqrt{\frac{\theta}{(1+\lambda_{DE})}},$$

if $\eta^2\theta^3\lambda_{DE}^2 - \lambda_{DE} - 1 \leq 0$, hence if $\lambda_{DE} \leq \left(1+\sqrt{1+4\eta^2\theta^3}\right)/(2\eta^2\theta^3)$. In this case,

$$\begin{aligned}e_1^* \geq& \sqrt{\frac{\theta}{\left(1+\frac{1+\sqrt{1+4\eta^2\theta^3}}{2\eta^2\theta^3}\right)}} \\ =& \sqrt{\frac{2\eta^2\theta^4}{2\eta^2\theta^3+1+\sqrt{1+4\eta^2\theta^3}}}\end{aligned}$$

This is larger than $e_2^* = \left(\sqrt{1+4\eta^2\theta^3}-1\right)/(4\eta\theta)$, if

$$12\eta^4\theta^6 > 0.$$

Therefore, $e_1^* > e_2^*$ and $w_1 < w_2$.

Now, assume that $\lambda_{DE} > \left(1+\sqrt{1+4\eta^2\theta^3}\right)/(2\eta^2\theta^3)$, hence $w_1 > 0$. Solving the first first order condition for λ_{DE} and plugging it into the second yields

$$\eta\theta^2 - \frac{\left[1+\eta\theta\left(e_1^*+\tilde{e}_1\right)\right]\left[e_1^*\left(1+e_1^*\eta\theta\right)+\tilde{e}_1\left(1+\eta\tilde{e}_1\theta\right)\right]}{\left(1+\eta e_1^*\theta\right)} = 0,$$

which, together with the binding (DE) constraint, determines e_1^* as well as $\tilde{e}_1$ (and consequently w_1). Making use of $-\tilde{e}_1^2 - \frac{4}{3}\tilde{e}_1^3\eta\theta + w_1\eta\theta = 0 \Rightarrow w_1 = \frac{\tilde{e}_1^2}{\eta\theta} + \frac{4}{3}\tilde{e}_1^3$, the

latter becomes

$$-\frac{\left(\left(e_1^*\right)^3-\tilde{e}_1^3\right)}{3}+\tilde{e}_1^2\left(e_1^*-\tilde{e}_1\right)\frac{\left(1+\tilde{e}_1\eta\theta\right)}{\left(1+\eta e_1^*\theta\right)}+\delta\pi_2=0,$$

In order to prove $e_1^*>e_2^*$ and $w_1<w_2$, I first show that e_1^* is increasing and $\tilde{e}_1$ is decreasing in $\delta\pi_2$:

$$\frac{de_1^*}{d\left(\delta\pi_2\right)}=\frac{\begin{vmatrix} 0 & -\frac{\eta\theta\left\{e_1^*e_1^*\eta\theta+\eta\tilde{e}_1\tilde{e}_1\theta+2\left(e_1^*+\tilde{e}_1\right)\left(1+\eta\tilde{e}_1\theta\right)\right\}+\left(1+2\eta\tilde{e}_1\theta\right)}{\left(1+\eta e_1^*\theta\right)} \\ -1 & -\frac{-\left(\left(e_1^*\right)^2+e_1^*\tilde{e}_1+\tilde{e}_1^2\right)+\left(e_1^*-\tilde{e}_1\right)\left(e_1^*+2\tilde{e}_1\right)}{3}+\frac{2\tilde{e}_1e_1^*+3e_1^*\tilde{e}_1^2\eta\theta-3\tilde{e}_1^2-4\tilde{e}_1^3\eta\theta}{\left(1+\eta e_1^*\theta\right)} \end{vmatrix}}{\begin{vmatrix} \frac{\tilde{e}_1\eta\theta\left(1+\eta\tilde{e}_1\theta\right)^2-\left[\left(1+\eta\theta\tilde{e}_1\right)\left(1+e_1^*\eta\theta\right)+\eta\theta e_1^*\left(1+2e_1^*\eta\theta\right)\right]\left(1+\eta e_1^*\theta\right)}{\left(1+\eta e_1^*\theta\right)^2} & -\frac{\eta\theta\left\{e_1^*e_1^*\eta\theta+\tilde{e}_1\tilde{e}_1\eta\theta+2\left(e_1^*+\tilde{e}_1\right)\left(1+\eta\tilde{e}_1\theta\right)\right\}+\left(1+2\eta\tilde{e}_1\theta\right)}{\left(1+\eta e_1^*\theta\right)} \\ -\left(e_1^*\right)^2+\tilde{e}_1^2\frac{\left(1+\tilde{e}_1\eta\theta\right)^2}{\left(1+\eta e_1^*\theta\right)^2} & \tilde{e}_1^2+\frac{2\tilde{e}_1e_1^*+3e_1^*\tilde{e}_1^2\eta\theta-3\tilde{e}_1^2-4\tilde{e}_1^3\eta\theta}{\left(1+\eta e_1^*\theta\right)} \end{vmatrix}}$$

$$=\frac{\begin{vmatrix} 0 & -\frac{\eta\theta\left\{e_1^*e_1^*\eta\theta+\eta\tilde{e}_1\tilde{e}_1\theta+2\left(e_1^*+\tilde{e}_1\right)\left(1+\eta\tilde{e}_1\theta\right)\right\}+\left(1+2\eta\tilde{e}_1\theta\right)}{\left(1+\eta e_1^*\theta\right)} \\ 1 & -\frac{-\left(\left(e_1^*\right)^2+e_1^*\tilde{e}_1+\tilde{e}_1^2\right)+\left(e_1^*-\tilde{e}_1\right)\left(e_1^*+2\tilde{e}_1\right)}{3}+\frac{2\tilde{e}_1e_1^*+3e_1^*\tilde{e}_1^2\eta\theta-3\tilde{e}_1^2-4\tilde{e}_1^3\eta\theta}{\left(1+\eta e_1^*\theta\right)} \end{vmatrix}}{\begin{vmatrix} \frac{\tilde{e}_1\eta\theta\left(1+\eta\tilde{e}_1\theta\right)^2-\left[\left(1+\eta\theta\tilde{e}_1\right)\left(1+e_1^*\eta\theta\right)+\eta\theta e_1^*\left(1+2e_1^*\eta\theta\right)\right]\left(1+\eta e_1^*\theta\right)}{\left(1+\eta e_1^*\theta\right)^2} & -\frac{\eta\theta\left\{e_1^*e_1^*\eta\theta+\tilde{e}_1\tilde{e}_1\eta\theta+2\left(e_1^*+\tilde{e}_1\right)\left(1+\eta\tilde{e}_1\theta\right)\right\}+\left(1+2\eta\tilde{e}_1\theta\right)}{\left(1+\eta e_1^*\theta\right)} \\ -\left(e_1^*\right)^2+\tilde{e}_1^2\frac{\left(1+\tilde{e}_1\eta\theta\right)^2}{\left(1+\eta e_1^*\theta\right)^2} & \frac{2\tilde{e}_1\left(e_1^*-\tilde{e}_1\right)\left(1+2\tilde{e}_1\right)}{\left(1+\eta e_1^*\theta\right)} \end{vmatrix}}$$

There, the numerator equals

$$-\frac{\eta\theta\left\{e_1^*e_1^*\eta\theta+\eta\tilde{e}_1\tilde{e}_1\theta+2\left(e_1^*+\tilde{e}_1\right)\left(1+\eta\tilde{e}_1\theta\right)\right\}+\left(1+2\eta\tilde{e}_1\theta\right)}{\left(1+\eta e_1^*\theta\right)}<0,$$

and the denominator

$$\frac{\tilde{e}_1\eta\theta\left(1+\eta\tilde{e}_1\theta\right)^2-\left(\eta\theta\tilde{e}_1+1\right)\left(1+e_1^*\eta\theta\right)^2-\eta\theta e_1^*\left(1+2e_1^*\eta\theta\right)\left(1+\eta e_1^*\theta\right)}{\left(1+\eta e_1^*\theta\right)^2}\left[2\left(e_1^*-\tilde{e}_1\right)\tilde{e}_1\frac{1+2\tilde{e}_1\eta\theta}{\left(1+\eta e_1^*\theta\right)}\right]$$
$$+\left(-\left(e_1^*\right)^2+\tilde{e}_1^2\frac{\left(1+\tilde{e}_1\eta\theta\right)^2}{\left(1+\eta e_1^*\theta\right)^2}\right)\frac{\eta\theta\left\{e_1^*e_1^*\eta\theta+\tilde{e}_1\tilde{e}_1\eta\theta+2\left(e_1^*+\tilde{e}_1\right)\left(1+\eta\tilde{e}_1\theta\right)\right\}+\left(1+2\eta\tilde{e}_1\theta\right)}{\left(1+\eta e_1^*\theta\right)},$$

which is negative because of $e_1^*>\tilde{e}_1$. Therefore,

$$\frac{de_1^*}{d\left(\delta\pi_2\right)}>0.$$

If $\delta\pi_1=0$, $b_1=0$, and π_1 is maximized by setting $w_1=w_2$, implying $e_1^*=e_2^*$. Therefore, $e_1^*>e_2^*$ given $\delta\pi_1>0$.

Moreover,

$$\frac{d\tilde{e}_1}{d(\delta\pi_2)} = \frac{\begin{vmatrix} \frac{\tilde{e}_1\eta\theta(1+\eta\tilde{e}_1\theta)^2-\left[(1+\eta\theta\tilde{e}_1)\left(1+e_1^*\eta\theta\right)+\eta\theta e_1^*\left(1+2e_1^*\eta\theta\right)\right]\left(1+\eta e_1^*\theta\right)}{\left(1+\eta e_1^*\theta\right)^2} & 0 \\ -(e_1^*)^2+\tilde{e}_1^2\frac{(1+\tilde{e}_1\eta\theta)^2}{\left(1+\eta e_1^*\theta\right)^2} & -1 \end{vmatrix}}{\begin{vmatrix} \frac{\tilde{e}_1\eta\theta(1+\eta\tilde{e}_1\theta)^2-\left[(1+\eta\theta\tilde{e}_1)\left(1+e_1^*\eta\theta\right)+\eta\theta e_1^*\left(1+2e_1^*\eta\theta\right)\right]\left(1+\eta e_1^*\theta\right)}{\left(1+\eta e_1^*\theta\right)^2} & -\frac{\eta\theta\left\{e_1^*e_1^*\eta\theta+\tilde{e}_1\tilde{e}_1\eta\theta+2\left(e_1^*+\tilde{e}_1\right)(1+\eta\tilde{e}_1\theta)\right\}+(1+2\eta\tilde{e}_1\theta)}{\left(1+\eta e_1^*\theta\right)} \\ -(e_1^*)^2+\tilde{e}_1^2\frac{(1+\tilde{e}_1\eta\theta)^2}{\left(1+\eta e_1^*\theta\right)^2} & \frac{2\tilde{e}_1\left(e_1^*-\tilde{e}_1\right)(1+2\tilde{e}_1)}{\left(1+\eta e_1^*\theta\right)} \end{vmatrix}}$$

This is negative, since the denominator is negative and the numerator, which equals

$$\frac{\left[(1+\eta\theta\tilde{e}_1)(1+e_1^*\eta\theta)+\eta\theta e_1^*(1+2e_1^*\eta\theta)\right](1+\eta e_1^*\theta)-\tilde{e}_1\eta\theta(1+\eta\tilde{e}_1\theta)^2}{\left(1+\eta e_1^*\theta\right)^2},$$

is positive.

Therefore,

$$\frac{dw_1}{d(\delta\pi_2)} < 0.$$

If $\delta\pi_1 = 0$, $b_1 = 0$, and π_1 is maximized by setting $w_1 = w_2$. Therefore, $w_1 < w_2$ given $\delta\pi_1 > 0$.

■

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