The More the Merrier? On the Optimality of Market Size Restrictions

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Abstract

This paper provides a novel rationale for the regulation of market size when heterogeneous firms compete. A regulator seeks to maximize total welfare by choosing the number of firms allowed to enter the market, e.g. by issuing a certain number of licenses. Opening up the market for more firms has a two-fold effect: it increases competition and thus welfare, but at the same time, it also attracts more cost-intensive firms, driving down average production efficiency. The regulator hence faces a trade-off between raising beneficial competition and detrimental costs. If goods are sufficiently substitutable, the latter effect can outweigh the former. It is then optimal to restrict the market size, rationalizing a limit to competition. This result holds even in the absence of entry costs, search costs or increasing returns to scale, which previous literature required.

Keywords: Regulation, Imperfect Competition, Oligopolies
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"You will agree, I'm sure, that there's nothing more destructive than a monopoly."

"Yes," said Taggart, "on the one hand. On the other, there's the blight of unbridled competition."

"That's true. That's very true. The proper course is always, in my opinion, in the middle. So it is, I think, the duty of society to snip the extremes, now isn't it?"

"Yes," said Taggart, "it is."

Ayn Rand, *Atlas Shrugged*

1 Introduction

Why do we observe markets where regulation restricts the number of firms allowed to compete, effectively creating oligopolies of a certain – at times seemingly arbitrary – size? After all, economic intuition suggests that in a market of profit-seeking firms, welfare is maximized by ensuring the highest possible degree of competition instead of limiting market entry. Even beyond the realm of economic theory, "competition is, in our system, a political and social desideratum" (McNulty, 1968, p. 639).

In some markets existing in practice, however, competition is limited in that only a number of firms are allowed to enter. In 2016, the Greek government cut the number of TV licenses granted to privately owned broadcasters from eight to four – only to change it again to five licenses two years later.[1] Similarly, the German football league (Bundesliga) raised the amount of licenses issued for live pay-TV broadcasts of its matches in 2016, claiming an improvement not only for fans and viewers, but also for the league's finances.[2] In New York City, the number of taxis in operation is regulated by the Taxi and Limousine Commission, which has barely varied the amount of "medallions" since the 1930s despite the city's rapid growth in population and size – while the socially optimal number of such medallions is estimated to be about 55% higher than its current value.[3] Further examples include the number of licenses given out to telecommunication companies in spectrum auctions or to betting agencies in sport wagering.

[1] See Iosifidis and Papathanassopoulos (2019) for a timeline.

[2] Heller and Sudaric (2019) describe the legal and procedural background. For the Bundesliga's statement, see `http://www.bundesliga.com/de/bundesliga/news/dfl-stellt-eckpunkte-der-ausschreibung-der-audiovisuellen-medienrechte-fuer-deutschland-ab-2017-18-vor-agmd29-2.jsp` (last retrieved August 17th, 2019).

[3] The historical development of the number of taxis granted to operate in NYC is outlined by Lagos (2003), while today's optimal number is computed by Fréchette et al. (2019).

Regulators are interested in both consumer surplus and industry profits in most markets, since the latter can (partly) be extracted using e.g. license auctions.[4] Theory suggests that, unless specific market assumptions are made (which we discuss in the literature review), utmost competition is optimal: welfare is maximized by having as many firms as possible compete with one another. While there may be practical reasons to restrict the market size in specific settings (technical constraints, health concerns, etc.), from an economic perspective "the limit appears to be rather arbitrary" (Borenstein, 1988, p. 357).

This paper explains restrictions to competition from an economic perspective and in a standard market absent specific assumptions required by previous work on entry regulation. We consider a two-stage model. First, a welfare-maximizing regulator specifies a market size, thus allowing a subset of finitely many interested firms to operate in a market. She does so e.g. by issuing licenses. Second, those firms to which access is granted compete à la Cournot. Crucially, firms differ in their marginal costs, with the costs unknown to the regulator. We show that raising market size by issuing more licenses has a two-fold effect: an increase in competition and a decrease in average production efficiency. We disentangle the two and study their welfare effects.

Intuitively, opening up the market gives rise to countervailing forces. On the one hand, it fosters competition. On the other hand, a greater market size also attracts less efficient firms: since firms with lower marginal costs generate higher profits, they enter the market first (as they place higher bids when licenses are auctioned off), while entrants arriving later produce at higher costs. To single out each effect, consider this market opening in two steps. First, add a new firm with production costs equal to the market average. This heats up competition while leaving production efficiency unchanged. As a result, prices are driven down, which is detrimental to firms but beneficial for consumers. Next, raise the entrant's cost to the value expected by the regulator. Surprisingly, this cost increase does not necessarily harm firms as a whole: it is possible that an increase in average marginal costs, keeping the total number of firms fixed, raises total firm profits. The more efficient firms can exploit the inefficiencies of their high-cost rivals, taking over their market share. The additional firm profits overcompensate the loss for consumers caused by the increase in production costs. When more firms are admitted to a market, we therefore observe a trade-off due to the two – potentially countervailing – effects: fostering competition always increases welfare, while the simultaneous decrease in production efficiency has an ambiguous impact on welfare. The regulator has to consider

[4]Kasberger (2018) describes the maximization of firm profits plus consumer surplus as the objective of "social efficiency" (p. 2).

both the competition and the cost effect in her choice of the market size.

Identifying the welfare maximizing market size, we first consider the benchmark scenario of homogeneous firms. Here, the standard intuition applies as utmost competition is optimal. The regulator does not impose any restrictions and allows all firms to enter. But with firms being heterogeneous, the regulator may face the aforementioned trade-off between competition and production costs, depending on market characteristics. It can now become optimal in terms of welfare to enforce an oligopoly, granting market access only to a limited number of firms. In this case, competition is desirable only to some extent such that the usual notion of "the more the merrier" does not apply. After establishing this result analytically, the paper then employs numerical methods to study how the optimal regulatory policy varies with the market specifications.

The paper proceeds as follows: the existing literature is reviewed in Section 2, while Section 3 presents the formal model. In Section 4, we first analyze the market equilibrium given a fixed market size. Subsequently, we solve the regulator's problem of choosing the optimal market size to maximize expected total welfare. Section 5 shows that the results are robust to changes to the model: we analyze both Cournot and Bertrand competition, i.e. firms setting either prices or quantities; heterogeneity is studied both in marginal costs and in quality levels; a regulator interested in auction revenue is considered. All proofs are in the Appendix. Accompanying Mathematica code is available upon request.

2 Related Literature

This paper builds on three strands of literature. First, it uses the framework of oligopolistic competition in a differentiated market. Second, it adds to the work on regulation and in particular on welfare-optimizing restrictions of competition. This line of research considers a certain market structure and asks what the optimal market size is. A third line takes the market size as given and varies the structure, showing that a decrease in production efficiency can lead to a surprising increase in welfare. We elaborate on each strand in detail.

In the analysis of the present's paper competition stage, we extend the model of a differentiated duopoly to the case of an arbitrary number of competing firms. The duopoly set-up is introduced in the seminal paper of Singh and Vives (1984). They study the competition of two firms who differ in their marginal costs and in their goods' value to consumers. In addition, there are substitutability effects between the goods. This differentiated market has been extended to the case of oligopolies of arbitrary size. Authors differ, however, in their modeling of firm heterogeneity: either via production costs or via quality levels. The first path is taken by Ledvina and Sircar (2011, 2012). Their model also is the closest to ours. However, they focus on the case of unregulated entry and analyze which firms *want* to be active. This paper, on the other hand, considers the question which firms *should* be active from a welfare perspective: if entry is attractive for all firms, how many of them should a regulator allow to operate in order to maximize total surplus? We therefore endogenize market size, which the authors take as given. The second path captures the heterogeneity of firms via quality rather than via production costs. Häckner (2000) presents such a model, assuming production costs of zero for all firms. He focuses on a comparison of equilibria under Cournot and Bertrand competition. We adopt this framework in an extension. In particular, we show that our results qualitatively carry over to this form of heterogeneity. Hsu and Wang (2005) also use this model and add a welfare analysis. Again, however, they consider a fixed market size, while we are interested in the regulator's optimal choice of this variable.

A vast literature has emerged on the optimal regulation of market sizes. In particular, this literature endogenizes the number of firms competing with one another and seeks to provide a rationale for limiting competition. A classic argument goes back to Schumpeter (1942), who argues that increasing returns to scale can outweigh the downsides of a monopoly. He thus requires decreasing marginal costs to justify a lack of competition. Stiglitz (1981) and Spence (1984) study a dynamic model where firms' choices of R&D lead to inefficiently low welfare levels: incumbents deter potential entrants by investing excessively in R&D in order to keep their productions costs low. While the authors do not explicitly study optimal regulatory policies, a limit to the market size would remedy the detrimental impact on welfare stemming from the firms' incentive to create entry barriers via wasteful investment levels. In a different approach, von Weizsäcker (1980) as well as Suzumura and Kiyono (1987) consider a static model with production levels being the firms' strategic choices. Their works, however, require the existence of fixed costs to explain an inefficiently high number of market entrants. Yet another path is taken by Stiglitz (1987): he considers search costs incurred by consumers who are looking for the preferred offer. Having too many firms operating in a market can cause inefficiently high search costs for only minor utility gains. In addition, there are works analyzing specific markets and the optimal regulation of their size, e.g. Kawakami (2017) who considers the case of securities trading. All of the literature in this strand therefore hinges on specific assumptions in order to rationalize a regulation of the market size. In particular, their analyzes require either fixed costs, search costs, decreasing marginal costs or R&D investments. None of them carry over to the case of constant marginal costs, which we consider. We thus add to this literature by providing a rationale for regulation in a more standard situation.

Finally, we turn to the third strand, analyzing how welfare is affected by production efficiency. This line of research considers a fixed market size and varies the production costs of some of the competing firms. Salop and Scheffman (1983) show that a firm may prefer to raise its competitor's costs – e.g. by inducing suppliers to boycott this rival – over predatory pricing. This competitor, however, obviously suffers, raising the question of the net effect of such behavior. Kimmel (1992) therefore studies the effect on total firm profits, arguing that the net effect may in fact be positive: if a small firm becomes less efficient, the benefit to its competitors from taking over the market share may outweigh the loss for the small firm. Yet, he does not consider consumers, whose utility decreases if any firm's production costs rise. The net effect thus remains unclear. Lahiri and Ono (1988) [henceforth LO] add consumers to the welfare analysis. They show that "helping" a small firm, e.g. by boosting its efficiency, may in fact lower total welfare. Our model differs mainly in three aspects. First, we allow for a differentiated market, while goods are necessarily homogeneous in the case of LO. We can therefore, second, explicitly study how market characteristics (degree of substitutability, quality of goods, size of the firm pool) affect the necessity for a regulator to intervene. Third, in the setup of LO, the regulator is perfectly informed about the firms' costs. As Baron and Myerson (1982) argue, "[t]his assumption is unlikely to be met in reality, since the firm would be expected to have better information about costs than would the regulator" (p. 911). We therefore model asymmetric information between firms and regulator. In particular, we assume firms' costs are random variables whose realization is unknown to the regulator. A more detailed analysis of the relevance of information in oligopolistic competition is conducted by Eliaz and Forges (2015). They consider the strategic interaction between a regulator and competing firms in an environment of asymmetric information. In their paper, however, it is the regulator who has an informational advantage and seeks to maximize welfare by choosing an optimal information structure for the firms. In our model, the asymmetry is to the regulator's disadvantage and she needs to maximize welfare by specifying the market size instead. All papers in this strand take the market size as given, while we endogenize it.

3 Model

We study a two-stage game. In the first stage, a welfare-maximizing regulator specifies the market size. She does so by choosing the number of firms allowed to enter, e.g. via issuing licenses. In the second stage, those firms that are granted access compete in a

Cournot style. Firms are heterogeneous in their marginal costs, where we refer to a firm with lower costs as "more efficient" throughout this paper.

Regulator: The regulator faces a pool of $m \in \mathbb{N}$ firms that seek to enter a market. She chooses the market size $n \leq m$, specifying the number of firms allowed to enter. Her goal is to maximize total welfare, given by consumer surplus and firm profits. The regulator is unaware of each firm's marginal costs. She knows only the cost distribution and the fact that given a market size n, the n firms with the lowest costs are going to enter the market.

We discuss this form of market regulation in more detail in Section 4.3. It is shown that the regulator could benefit from more sophisticated mechanisms allocating permits to enter the market. We will argue that entry of the most efficient firms – rather than selecting specific firms – is a political desideratum rather than an economic one in many regulatory settings. We construct a mechanism and equilibrium strategies implementing this goal, ensuring that more efficient firms indeed enter the market first.

In addition, we follow the empirical observation that the regulator *can* grant or deny market access to firms, but that she *cannot* prevent the firms' strategic profit-maximizing behavior once in the market. The socially optimal solution would require a regulation of production levels and prices to their first-best levels, which in practice is not available to the regulator in competitive markets. Our results extend to the case where the regulator's objective is not to maximize total welfare but rather the sum of consumer surplus and auction revenue generated from selling licenses for market entry, as shown in Section 5.3.

Firms: There are $m \in \mathbb{N}$ firms seeking to participate in the market. Each firm $i \in \{1, \ldots, m\}$ produces quantity q_i of a good at marginal cost c_i. Firms sell their goods at price p_i given by the consumers' inverse demand. A firm's profit function is given by

$$\pi_i(p_i, q_i) = q_i(p_i - c_i). \tag{1}$$

Firms are heterogeneous in marginal costs c_i. Their costs (or "types") are i.i.d. draws from $U[0,1]$.[5] Denote a cost profile by $\boldsymbol{c} = (c_1, \ldots, c_m) \in [0,1]^m$. Firms know each others' types, while the regulator does not. This information structure is motivated by producers having more detailed insights into the technical requirements of the specific

[5]The results presented in this paper qualitatively carry over to other cost distributions such as a binary one. However, even for these relatively "simple" distributions, closed form solutions do not exist in general. We therefore focus on the uniform case as opposed to more complex ones in order to identify more easily the different effects that come into play from the regulator's perspective.

market and the cost structure of their rivals than a government does. It allows us to focus on the regulator's optimal policy facing both imperfect competition and imperfect information without the additional complexity of firm's strategic choices made under uncertainty.[6] We later identify conditions such that indeed all firms find it profitable to engage in competition.

After the regulator has specified a market size $n \leq m$ in the first stage, the n firms with the lowest marginal costs enter, e.g. by having obtained a license. In the second stage, they engage in competition à la Cournot and choose their production levels $\boldsymbol{q} \equiv (q_1, \ldots, q_n)$.[7]

Consumers: There is a unit mass of consumers (or a single representative consumer, equivalently) with a standard utility function: utility is quadratic in the goods $\boldsymbol{q}$, while it is linear and separable in the numeraire good. With prices denoted by $\boldsymbol{p} \equiv (p_1, \ldots, p_n)$, the consumers' optimization problem is

$$\max_{\boldsymbol{q} \in \mathbb{R}^n_+} U(\boldsymbol{q}) - \sum_{i=1}^{n} p_i q_i, \tag{2}$$

yielding the inverse demand function $\boldsymbol{p}(\boldsymbol{q})$. We extend the standard model of a differentiated duopoly à la Singh and Vives (1984) to an oligopolistic setting, writing

$$U(\boldsymbol{q}) = \alpha \sum_{i=1}^{n} q_i - \frac{1}{2}\left(\sum_{i=1}^{n} q_i^2 + \gamma \sum_{i=1}^{n} \sum_{j \neq i} q_i q_j \right). \tag{3}$$

Here, γ specifies the degree of product differentiation: for $\gamma = 0$, goods are independent, while for $\gamma = 1$ they are perfect substitutes. We allow for arbitrary $\gamma \in [0, 1]$. The factor α can be interpreted as a common quality of all goods, which is sufficiently large such that market entry is profitable for all firms (see Lemma 2 below).[8]

[6] For a motivation for this information structure commonly used in the regulation of competitive markets, see e.g. by Vickers (1995). For an analysis of optimal information provision to competing firms, see Eliaz and Forges (2015).

[7] The results presented qualitatively carry over to a Bertrand market. This robustness is studied in Section 5.1. For a comparison of firm entry under Cournot and Bertrand competition see Cellini et al. (2004).

[8] Alternatively, firms could be heterogeneous in quality levels $(\alpha_1, \ldots, \alpha_m)$ rather than in costs. Again, the results presented in this paper a robust to such change of the model. See Section 5.2 for an analysis.

4 Analysis

To identify the optimal market size, we use backward induction. We first solve the consumers' optimization problem to obtain the inverse demand function $\boldsymbol{p}(\boldsymbol{q})$ given a profile of quantities $\boldsymbol{q}$ posted by the firms. Second, firms anticipate consumer demand and choose their production levels q_i as a function of the cost profile $\boldsymbol{c}$. This gives rise to equilibrium quantities and prices in a market of n firms. Third, we determine the regulator's choice of the market size n to maximize total welfare, where her choice determines the cost distribution of those firms active in the market.

The analysis is structured as follows: Sections 4.1 and 4.2 study the competitive market stage where firms and consumers interact. All results presented here are independent of the cost profile and hence do not hinge on the assumption of only the most efficient firms being active in the market. In Section 4.3, we turn to the mechanism governing market entry. We introduce and motivate an auction ensuring the entry of the n most efficient firms whilst also discussing superior mechanisms. Given this form of entry, Section 4.4 assesses the optimal market size and presents the main result, while Section 4.5 considers comparative statics.

4.1 Equilibrium Prices and Quantities

Consumers face the optimization problem given by eq. (2). Observing quantities $\boldsymbol{q}$ produced by the firms, consumers' inverse demand is given by

$$p_i = \alpha - q_i - \gamma \sum_{j \neq i} q_j. \tag{4}$$

Each firm i sets its production level q_i to maximize profit $\pi_i = q_i(p_i - c_i)$, given inverse demand and a cost profile $\boldsymbol{c}$. Taking into account its rivals' production levels q_j $(j \neq i)$, firm i thus chooses

$$q_i = \frac{1}{2}(\alpha - c_i - \gamma \sum_{j \neq i} q_j). \tag{5}$$

Summing over all firms and rearranging yields a firm's chosen quantity as a function of the cost profile. We define $\lambda = \{(2-\gamma)[2+\gamma(n-1)]\}^{-1} > 0$ and obtain:

$$q_i = \lambda \Big[\alpha(2-\gamma) - [2+\gamma(n-2)]c_i + \gamma \sum_{j \neq i} c_j\Big], \tag{6}$$

resulting in prices

$$p_i = \lambda\Big[\alpha(2-\gamma) + [2+\gamma(n-2)-\gamma^2(n-1)]c_i + \gamma\sum_{j\neq i} c_j\Big]. \tag{7}$$

Regarding the comparative statics of these equilibrium strategies,[9] consider an increase in firm i's marginal costs c_i: the firm reacts by producing a lower quantity, while its rivals increase production to take over the market share. This increase is more aggressive if goods are more substitutable as consumers are more easily persuaded to switch producers ($^{\mathrm{d}^2 q_j}/_{\mathrm{d}\gamma\,\mathrm{d}c_i} > 0$ for $j \neq i$).

Prices paid by consumers rise for *all* goods following a rise in firm i's marginal costs: with c_i increasing, the supply of good q_i is lowered. At this lower level of consumption, consumers' marginal benefit is greater for all goods, see eq. (8) below. Thus, also the firm whose costs have increased will enjoy a higher price for its goods. The willingness of consumers to pay higher prices to this specific firm, however, decreases in the substitutability of goods: consumers start purchasing some of the alternative goods instead ($^{\mathrm{d}^2 p_j}/_{\mathrm{d}\gamma\,\mathrm{d}c_i}$ is negative for $j = i$ and positive for $j \neq i$). As goods become less differentiated (more substitutable), the price increase accepted by consumers diminishes for the firm whose production costs have grown and becomes larger for the competitors' goods.

The following lemma summarizes these results and mirrors standard observations from Cournot competition in a differentiated market. It focuses on the comparative statics of equilibrium prices and quantities, while effects on profits, expenditure and welfare are studied in subsequent sections.

Lemma 1. *When the marginal costs of firm* i *<u>in</u>crease, ...*

a) this firm's production level <u>de</u>creases,

b) its competitors' production levels <u>in</u>crease,

c) all prices <u>in</u>crease.

The market equilibrium given by eqs. (6) and (7) resembles the results of Ledvina and Sircar (2012). We extend their analysis by also identifying conditions ensuring that production is profitable for every firm. Since we are interested in the regulator's choice of optimal market size, we want to focus on the case where each firm finds market entrance

[9] Omitting solutions where all production levels and prices are zero, this equilibrium is unique, which follows from applying the Poincaré-Hopf index theorem, see e.g. Vives (1999).

attractive. That is, we want to ensure that each firm *wants* to enter the market but *should* not necessarily do so from a welfare perspective. To this end we need to require a sufficiently large quality α of all goods. Intuitively, consider the marginal benefit to a consumer purchasing some (not necessarily optimal) quantity q_i,

$$\frac{\partial U(\boldsymbol{q})}{\partial q_i} = \alpha - q_i - \gamma \sum_{j \neq i} q_j. \tag{8}$$

For α large enough and low consumption levels, this marginal benefit outweighs the marginal costs p_i from additional consumption (note that $\frac{\partial p_i}{\partial \alpha} < 1$). If, on the other hand, the quality α is too low, the detrimental effects dominate. These effects stem from, first, diminishing marginal utility and, second, substitutability of goods. The equilibrium quantity given by eq. (6) would then prescribe negative consumption levels. The following lemma derives a lower bound on the quality ensuring positive supply levels and prices.

Lemma 2. *A sufficient condition for prices and quantities given by eqs.* (6) *and* (7) *to be weakly positive is* $\alpha \geq m$*, i.e.*

$$\alpha \geq m \Rightarrow p_i, q_i \geq 0 \quad \forall i, n, m, \boldsymbol{c}, \gamma \text{ s.t. } 1 \leq i \leq n \leq m, (\boldsymbol{c}, \gamma) \in [0,1]^{m+1}. \tag{9}$$

Throughout this paper, we assume that $\alpha \geq m$ is satisfied. If $\alpha < m$, the least efficient firms would instead choose a production level of zero rather than following eq. (6). Market entry would thus be unattractive for them in the first place. As seen from the equilibrium quantities, we have $q_i \geq q_j \Leftrightarrow c_i \leq c_j$. That is, only those firms with costs below a threshold $\bar{c}$ would choose their production according to eq. (6), where n is replaced by the number of firms in this set, i.e. $|\{i | c_i \leq \bar{c}\}|$. In this case, opening up the market and allowing more firms to enter does not have any effect: the cost distribution of firms with strictly positive production levels is truncated at $\bar{c}$ and all newly arriving firms affect neither prices nor quantities of the incumbents. The number of active firms, however, depends on the specific cost realization and is not known ex ante.

Note that $\alpha \geq m$ ensures strictly positive production levels for all firms apart from the polar case of $n = m = \alpha$, $\gamma = c_i = 1$ and $c_j = 0 \, \forall j \neq i$, where we obtain $q_i = 0$. However, all firms (including firm i) still produce according to eq. (6), and i can thus still be considered "active". The assumption $\alpha \geq m$ hence is sufficient to prevent discontinuities in production decisions.

Finally, it is readily verified from eq. (6) that for any market size n and any cost profile $\boldsymbol{c}$, the entry of an additional firm causes all incumbents to lower their production levels irrespective of the new rival's costs, while the total production level increases:

$$q_i(c_1, \ldots, c_n, c_{n+1}) \le q_i(c_1, \ldots, c_n) \ \forall i, n, \mathbf{c},$$
$$\sum_{i=1}^{n+1} q_i(c_1, \ldots, c_n, c_{n+1}) \ge \sum_{i=1}^{n} q_i(c_1, \ldots, c_n) \ \forall n, \mathbf{c}. \tag{10}$$

Having computed the equilibrium behavior of firms and consumers, we next turn to an analysis of their respective welfare levels.

4.2 Consumer Surplus, Firm Profits and Total Welfare

In the previous section we derived equilibrium prices and quantities as well as their changes following an increase in a firm's marginal costs. We now analyze how these effects carry over to the welfare of consumers, firms, and both sides taken together. That is, we still consider a fixed market size n with a known cost profile $\boldsymbol{c}$ and will only later turn to the regulator's optimal choice of n.

Given a profile of production levels $\boldsymbol{q}$, prices $\boldsymbol{p}$ and marginal costs $\boldsymbol{c}$, we can determine consumer surplus $\mathcal{W}_{\mathrm{C}}(n) = U(\boldsymbol{q}) - \sum_{i=1}^{n} p_i q_i$ as well as firm profits $\mathcal{W}_{\mathrm{F}}(n) = \sum_{i=1}^{n} \pi_i$. Together, they constitute total welfare,

$$\mathcal{W}_{\mathrm{tot}}(n) = \mathcal{W}_{\mathrm{C}}(n) + \mathcal{W}_{\mathrm{F}}(n) = [U(\boldsymbol{q}) - \sum_{i=1}^{n} p_i q_i] + \sum_{i=1}^{n} \pi_i. \tag{11}$$

To single out the effects caused by a change in marginal costs, we separate terms linear in the costs (c_i), squared (c_i^2) as well as mixed ($c_i c_j$) terms. This way, welfare expressions take the following form:

$$\mathcal{W}_{\mathrm{tot}}(n) = \eta_1 + \eta_2 \sum_{i=1}^{n} c_i + \eta_3 \sum_{i=1}^{n} c_i^2 + \eta_4 \sum_{i=1}^{n} \sum_{j \neq i} c_i c_j, \tag{12}$$

with the same form (but different coefficients η) if we only consider consumer surplus or firm profits. Each coefficient η is a function of quality α, product substitutability γ and market size n.[10] Due to the complexity of these expressions, we defer their explicit form to Appendix A, see eq. (22). Analyzing the effect of a change in marginal costs on welfare, however, is insightful for the regulator's decision studied below. The first set of observations comes at no surprise:

[10] The total number of firms, m, only enters through the regulator's *expectations* over marginal costs. Here, we consider a known cost profile $\boldsymbol{c}$. Therefore, eq. (12) is independent of m.

Lemma 3. *When the marginal costs of some firm* i *increase, …*

a) *this firm's profit decreases,*

b) *its rivals' profits increase,*

c) *consumer expenditure on this firm's good decreases,*

d) *consumer surplus decreases.*

For an intuition, recall the effect on equilibrium prices and quantities caused by an increase in marginal costs, as stated in Lemma 1: when firm i's costs increase, it responds by lowering its own output, while its rivals increase their quantities. At the same time, all prices rise. Thus, there are two countervailing effects for firm i: a detrimental cost increase and a beneficial price increase. As Lemma 3 shows, the former effect outweighs the latter (that is, we have $\frac{\mathrm{d}p_i}{\mathrm{d}c_i} < \frac{\mathrm{d}c_i}{\mathrm{d}c_i}=1$), so that the increase in costs drives down i's profits.

Next, consider the effect on consumer behavior. As the production level of good q_i is lowered, consumers' marginal benefit increases, see eq. (8). Hence, they are willing to pay an increased price. Regarding consumers' total spending on firm i's good, there thus also are two countervailing effects at work: a decrease in the quantity produced and an increase in the price paid. This resembles the impact of distortionary taxation under imperfect competition, where an exogenous change causes prices to rise and quantities to shrink.[11] Again, Lemma 3 shows that the former effect dominates and consumer spending on firm i's good, $q_i p_i$, decreases. Given this decrease in their expenditure, consumers substitute by purchasing more of the other goods. Since the prices of these other goods have also increased subsequent to the change in i's marginal costs, consumer surplus is lowered.

We now turn to the rivals of firm i. They benefit from the cost increase in a two-fold way. First, there is the consumers' reallocation of money just mentioned, enabling them to spend a larger fraction of their budget on all goods other than q_i. Second, quantities of these firms also increase: as firm i decreases production given its increased marginal costs, the rivals take over some of the market share. Both of these effects increase profits of the competitors whose cost have not changed. Furthermore, consumer expenditure on the goods of the competitors trivially increases, as both quantities and prices have grown.

As a final remark, note again the (standard) finding that firms with lower marginal costs have higher profits when participating in the market. This rationalizes our assump-

[11]See e.g. Anderson et al. (2001) for an analysis.

tion that more efficient firms enter the market first, e.g. because they accept higher prices for licenses permitting such entry, or because they place higher bids in an auction setting.

The effect on total welfare: An increase in a firm's marginal costs drives down consumer surplus but raises profits for all competitors of this firm, as just shown. This poses the question in which direction total welfare and total firm profits can change. In particular, is it possible that total welfare *increases* due to a firm's marginal costs having grown, ceteris paribus? (Note that if this is the case, total firm profits also necessarily increase.) In standard Cournot settings with homogeneous firms, total welfare unambiguously decreases in marginal production costs.[12] With heterogeneous costs, however, we find:

Proposition 1. *When a firm's marginal costs increase, total welfare may increase.*

We separate the effects on total welfare caused by a change in firm i's marginal costs, where we know the sign of each effect from Lemma 3. First, consumers suffer from an increase in marginal costs and their surplus decreases. Second, firm i's profit is driven down, and third, the profits of i's competitors rise. More formally, we have

$$\frac{\mathrm{d}\mathcal{W}_{\mathrm{tot}}}{\mathrm{d}c_i} = \underbrace{\frac{\mathrm{d}\mathcal{W}_{\mathrm{C}}}{\mathrm{d}c_i}}_{\substack{\leq 0 \\ \text{consumer effect}}} + \underbrace{\frac{\mathrm{d}\pi_i}{\mathrm{d}c_i}}_{\substack{\leq 0 \\ \text{firm effect}}} + \underbrace{\sum_{j\neq i} \frac{\mathrm{d}\pi_j}{\mathrm{d}c_i}}_{\substack{\geq 0 \\ \text{competitor effect}}} \gtreqless 0. \tag{13}$$

When does the positive competitor effect outweigh the negative consumer and firm effects? As shown in the proof of Proposition 1, this is the case if, first, firm i is relatively inefficient even before the increase in its costs (c_i large), and hence it has a small market share. Second, (most of) its competitors need to be relatively efficient, i.e. their costs c_j should be low. To shed light on these requirements, we look at higher order derivatives of total welfare. In particular, we analyze how the magnitude of each effect varies with the costs of firm i and its competitors.

We start with the competitor effect. Recall from eq. (6) that firm j has larger production levels if its own costs are low and those of its rival i are high. This production level q_j precisely drives the magnitude of the competitor effect: note we can write $\pi_j = q_j(p_j - c_j) = [q_j(\boldsymbol{c})]^2$ for firm profits and the competitor effect size is hence given by $2q_j \frac{\mathrm{d}q_j}{\mathrm{d}c_i}$. This term is large for firm j being efficient and i being inefficient, as such a cost profile drives up q_j according to eq. (6) and the derivative $\frac{\mathrm{d}q_j}{\mathrm{d}c_i}$ is independent of the

[12] This is shown e.g. by Corchón (2008) as well as by Lemma 5 below in the present paper.

costs. The positive impact on total welfare hence is amplified if the two requirements introduced above are met.

Next, consider the firm effect, analogously given by $2q_i \frac{\mathrm{d}q_i}{\mathrm{d}c_i}$. The effect size is small (in absolute terms) if firm i produces sparsely. This, again, is the case if i itself is inefficient while its rivals are efficient. If so, we therefore have a small negative effect on firm i's profits and a large positive effect on the competitors' profits. The detrimental impact of i's loss on total firm profits and on total welfare will be outweighed by the positive competitor effect. Before discussing the consumer effect, we capture this finding that the net effect on firm profits may in fact be positive, which directly follows from combining Proposition 1 and Lemma 3 d):

Corollary 1. *When a firm's marginal costs increase, total firm profits may increase.*

Finally, we turn to the consumer effect. From eq. (12) we know that it is given by $\frac{\mathrm{d}\mathcal{W}_\mathrm{C}}{\mathrm{d}c_i} = \eta_2^\mathrm{C} + 2\eta_3^\mathrm{C} c_i + 2\eta_4^\mathrm{C} \sum_{j \neq i} c_j$. As shown in the proof, it holds that $\eta_3^\mathrm{C} > 0$: although consumers suffer from the cost increase of firm i, they do so to a lesser extent if firm i's costs are high. Intuitively, given a high value of c_i, consumers purchase only little of q_i anyway. Now, if the costs c_i grow, the price p_i does so, too. But since consumption levels q_i are low, this affects consumers' net utility less than if i was more efficient and hence consumption levels were higher. Just as for the firm effect, the detrimental consequences of an increase in some firm's costs are diminished if this firm already is relatively inefficient in the first place.

To conclude, Proposition 1 shows that there are specific cost profiles where an *in*crease in a firm's marginal costs can cause a somewhat surprising *in*crease in total welfare. More specifically, this is the case if the firm whose costs increase is relatively small and inefficient, while the opposite is true for its rivals.[13]

4.3 Market Entry

So far, we have studied the competitive market stage where firms and consumers interact. We now turn to the regulator's task of governing market entry. While this section motivates her choice to admit only the most efficient firms and proposes an auction implementing this outcome, while the subsequent Section 4.4 analyzes the optimal number of efficient firms that should be allowed to compete *given* this specific form of regulation.

[13]This resembles findings presented by Lahiri and Ono (1988) for the effect on total welfare and by Kimmel (1992) for firm profits only.

Optimal regulatory mechanisms: Note first that regulation ensuring competition of only the most efficient firms is not necessarily optimal ex post. Denote a profile of the firms' realized costs by its order statistics $(c_{(1)}, \ldots, c_{(m)})$, where $c_{(i)} \leq c_{(i+1)}$ for all i. Having the $n \leq m$ most efficient firms enter the market will induce the profile $(c_{(1)}, \ldots, c_{(n)})$ at the competition stage. But from Proposition 1 we have learned that total welfare can increase if a small firm becomes less efficient. Hence, keeping the cost profile and the market size fixed, inducing firms $(c_{(1)}, \ldots, c_{(n-1)}, c_{(m)})$ to compete instead may be socially preferable.

The optimal composition of firms at the competition stage therefore depends on the realized cost profile. Any regulatory mechanism striving to maximize total welfare should thus seek to elicit this profile and then select individual firms to enter the market. Recall that each firm knows the marginal costs of all of its (potential) competitors. Since a firm's profit – and hence its valuation of entering the market – depends on the whole cost profile, the regulator faces a problem of interdependent valuations and symmetric information among the bidders. In the spirit of Crémer and McLean (1988), this information can easily and costlessly be extracted using a "shoot the liar mechanism", where each firm is asked for a report on the whole cost profile and deviations from the consensus are penalized. Having obtained the cost profile, the regulator then hand-picks the firms she finds preferable in her objective to maximize total welfare.

Practical considerations: In the light of this observation, why do we nevertheless focus on an entry of the firms with lowest production costs? The focus is motivated by a perception governing many procurement processes and license auctions in practice: when striving for efficient outcomes, "[e]fficiency [is] understood as putting the licenses into the hands of the bidders with the best business plans" (Binmore and Klemperer, 2002, p. C79). Rather than maximizing total welfare ex post, the auction merely "ensures that the object is allocated to that bidder that values it most" (van Damme, 2002, p. 7). Now recall that in the present model, a firm's profit decreases in its marginal costs irrespective of the rivals active in the market (Lemma 3). Hence, more efficient firms have a higher valuation of market entry independently of the firms they are going to compete with. An auction that causes the firms with lowest costs to enter in equilibrium therefore satisfies the requirement of granting access only to those with highest valuations and meets a political rather than a purely economic objective.[14] This requirement forms the

[14]An additional economic rationale for procurement to the most efficient firms is given by the incentives for R&D investments thus created: regulators want to induce firm innovations due to spillover effects and hence "reward" more efficient firms for their investments by choosing allocations accordingly. Che et al. (2016) analyze the provision of innovation incentives in public procurement.

constraint subject to which total welfare will be maximized in the next section: *given* that only the most efficient firms enter, how large should the market be?

Before turning to this question, we introduce an auction that does indeed implement the desired order of entry. For a fixed market size n, we can describe the process of allocating licenses by a multiunit auction in which n identical items are sold to m bidders with a cap of one item per bidder. The following lemma states that if licenses are auctioned off sequentially, with all bids being revealed after each round[15] and each round selling a single license to enter the market at a second price auction, then there is an equilibrium where indeed the n most efficient firms enter the market. In addition, firms bid their valuation truthfully on the equilibrium path.

Lemma 4. *Consider* n *sequential second-price single-unit open-bid auctions. There is an equilibrium in bidding strategies where each of the* n *most efficient firms wins one of the units for any cost profile.*

The equilibrium hinges on grim trigger strategies preventing more efficient firms from postponing their bids in hope of acquiring a license in later rounds at a cheaper price. More specifically, a selection of the remaining firms then starts bidding the monopoly profit, thus punishing the deviant, while also ensuring that overbidding is unattractive for the less efficient firms. For details, see the proof.

Note three remarks on the auction and the equilibrium discussed above. First, the allocation of licenses does not change if we allow for an aftermarket where firms can sell their licenses to those rivals who failed to place winning bids. Only less efficient firms have not yet obtained a license, but their valuation of holding one is lower than for each of the firms in possession of a license as long as the profile of competitors is unchanged. Bilateral exchange between firms therefore does not lead to a reallocation, although a centralized clearing house with side transfers could. Second, the equilibrium outcome of the n most efficient firms acquiring one item each is not unique. Consider the case of $m = 3$, $n = 1$ and $c_1 < c_2 < c_3$, implying $\pi_1 > \pi_2 > \pi_3$. The bidding profile (b_1, b_2, b_3) with $b_2 > \pi_1 > \pi_2 > \pi_3 > b_3 > b_1$ is an equilibrium where firm 2 instead of the more efficient firm 1 wins the only item. Third, an equilibrium of only the most efficient firms entering can also be implemented using a uniform-price auction selling all licenses at the same market-clearing price. This does, however, generate a much lower auction revenue.

To conclude, the focus on firms entering in the order of their efficiency as mandated in practice puts an additional constraint on the regulator's optimization problem. Rather than first eliciting the firms' privately held information and maximizing total welfare ex

[15] As shown in the proof, it suffices to reveal the winning bid and the winning bidder's identity.

post, she only regulates the number of firms allowed to enter and maximizes welfare ex ante, considering the expected costs of those firms who will compete.

4.4 Regulating the Market Size

We are now able to identify the optimal market size from the perspective of the welfare-maximizing regulator. Given a size n, she knows that the n most efficient firms will enter into competition – but she has to form expectations about the costs of the firms seeking to compete. Formally, her optimization problem is to find

$$n^* = \underset{n \in \{1,\dots,m\}}{\arg\max} \; \mathrm{E}_{\boldsymbol{c}}[\mathcal{W}_{\text{tot}}(n)]. \tag{14}$$

This section now presents the paper's key results. We first consider a benchmark scenario of homogeneous firms, all having the same marginal costs. These costs may or may not be known by the regulator. In this setting, the standard intuition of more competition being optimal does apply: welfare is always maximized by having all firms compete, as shown in Lemma 5. This result changes, however, if firms are heterogeneous: it can now be optimal to restrict market size, thus limiting the number of firms allowed to compete. An increase in competition, caused by opening up the market, no longer necessarily implies an increase in total welfare, as Proposition 2 shows. This result therefore justifies the restrictions to market sizes observed in practice.

We motivated the regulator's interest in *total* welfare by her possibility to (partly) extract firm profits via the sale of permits to participate in the market, e.g. by auctioning off licenses. Her objective hence is to maximize the sum of consumer surplus and firm profits. If she was only interested in the former, full competition would always be optimal and there would be no restriction at the optimum.

4.4.1 Homogeneous Firms

As a benchmark, we consider the pool of m firms to be homogeneous. That is, they all share the same marginal costs $c_i = c \in [0, 1]$. Their goods may still be differentiated, i.e. we still allow for any degree of product differentiation $\gamma \in [0, 1]$ as well as any quality level α satisfying the condition stated in eq. (9). The results presented here hold both if the regulator observes c and if she is unaware of the costs realized, knowing only that they are uniformly distributed.

As the following lemma states, an increase in competition always (weakly) increases total welfare if firms are homogeneous. That is, the regulator always finds it optimal to grant market access to all firms. There is no rationale to limit the market size.

Lemma 5. *If firms are homogeneous, it is always welfare-optimal to allow all firms to enter the market:*

$$\text{If } c_i = c \in [0,1]\ \forall i : \underset{n\in\{1,\dots,m\}}{\arg\max}\ \mathrm{E}_c[\mathcal{W}_{\text{tot}}(n)] = m \quad \forall \alpha, \gamma, m \text{ s.t. } 0 \le \gamma \le 1 \le m \le \alpha.$$

This result confirms the standard intuition that in a market where firms cannot be forced to set their prices or production levels in a welfare-maximizing way, it is optimal to have as many of them as possible compete with one another. Allowing more firms to enter the market increases competition and has a known two-fold effect. First, it reduces each firm's profits by lowering both production levels and prices. This is seen from recalling eqs. (6) and (7), where we can write $q_i = q$, $p_i = p\ \forall i$ if firms are homogeneous:

$$\frac{\mathrm{d}q}{\mathrm{d}n} = \frac{\mathrm{d}p}{\mathrm{d}n} = -\lambda^2\gamma(\alpha - c) \le 0. \tag{15}$$

Second, opening up the market increases consumer surplus: there is a larger variety of goods available and prices are lowered. In a setting of homogeneous firms, the latter effect outweighs the former: the benefit to consumers from having more competition overcompensates the loss to each firm. In consequence, total welfare is maximized by allowing all firms to enter.

As a final remark, note the result above is independent of the distribution of c. Unrestricted market access is optimal for any homogeneous cost realization $c \in [0,1]$ and even if the regulator knows the realized costs ex ante.

4.4.2 Heterogeneous Firms

Having considered a setting with symmetric firms, we now turn to a more realistic market with heterogeneous firms whose production costs may differ. We start with a simple numerical example. Let there be two potential entrants with marginal costs $c_1 = 0$, $c_2 = 1$. Their goods are perfectly differentiated and consumer utility is given by $U(q_1, q_2) = 4(q_1 + q_2) - \frac{1}{2}(q_1^2 + q_2^2)$. We compare total welfare in a monopoly of the more efficient firm 1 with an duopoly where both firms compete. In the monopoly, we find $q_{\text{mon}} = p_{\text{mon}} = 2$, resulting in a total welfare of $W_{\text{mon}}^{\text{tot}} = \pi_{\text{mon}} + W_{\text{mon}}^{\text{CS}} = 4 + 2 = 6$. In contrast, the duopoly setting yields $q_1 = p_1 = p_2 = \frac{5}{3}$, $q_2 = \frac{2}{3}$ and $W_{\text{duo}}^{\text{tot}} = \pi_1 + \pi_2 + W_{\text{duo}}^{\text{CS}} = \frac{25}{9} + \frac{4}{9} + \frac{49}{18} = \frac{107}{18} < 6$. Opening up the market to allow for more competition hence slightly lowers welfare.

We extend this example to the general model introduced above to identify market characteristics making restrictions to competition optimal. That is, we allow for an arbitrary degree of product differentiation γ, quality level α and firm pool size m. In addition, we consider asymmetric information on the firms' marginal costs, where the regulator only knows the cost distribution. Each firm i has a cost type $c_i \sim U[0,1]$ i.i.d. Recall from eq. (12) the form of the expression for total welfare: it contains terms which are linear in the costs (c_i), squared (c_i^2) as well as mixed ($c_i c_j$). We compute the expected value of these expressions, given a firm pool size m (exogenous) and a market size n (chosen by the regulator), in Appendix B.1.

As the following proposition shows, optimal market regulation can change dramatically if firms are heterogeneous. For homogeneous firms (see Section 4.4.1) we saw it is always preferable to have as many firms as possible compete with one another. This no longer holds true if production costs differ. Instead, total welfare may now be maximized by restricting market access, creating a setting which is neither monopolistic nor fully competitive. This provides a rationale for the seemingly arbitrary restriction of market access sometimes observed in practice.

Proposition 2. *If firms are heterogeneous, it can be welfare-optimal to limit competition:*

$$\exists \alpha, \gamma, m \text{ with } 0 \leq \gamma \leq 1 < m \leq \alpha \quad \text{s.t. } 1 < \underset{n \in \{1,\dots,m\}}{\arg\max}\ \mathrm{E}_{\boldsymbol{c}}[\mathcal{W}_{\text{tot}}(n)] < m.$$

When the regulator opens up the market by allowing more firms to enter, there are two forces affecting total welfare: a competition effect and a cost effect. The competition effect resemble the observation made in the setting of homogeneous firms. As the number of firms competing increases, prices and production levels are driven down. This harms each firm's profits, but raises consumer surplus. In addition, there now is an increase in variety: consumers benefit from a larger number of different goods, each yielding quality α, but they also face more substitution, lowering their utility. The net impact of this competition effect, however, is positive, as seen from Lemma 5: in a market with homogeneous firms, a larger number of entrants unambiguously improves total welfare.

Under firm heterogeneity, we now observe a second effect caused by a change to the market size: the cost effect. Since firms enter the market in the order of their efficiency, a newly arriving firm is known to be less efficient than those already in the market. This obviously changes the number of firms, captured by the competition effect. But it also changes the cost distribution, as it increases expected average production costs or, equivalently, decreases expected average efficiency. Intuitively, we can single out the latter effect by first adding a firm whose marginal costs are equal to the current market average (changing only the market size and thus creating a competition effect) and then decreasing the entrant's costs to the expected value given by the $(n+1)$-th order statistic.

In this second step, we effectively reduce the efficiency of a single firm and can hence recall Proposition 1, stating that such a reduction may in fact raise welfare. That is, the impact of the cost effect is ambiguous. It is hence not straightforward to see the net effect on total welfare if the market size is increased: the competition effect drives up total welfare, the cost effect may point in either direction.

As Proposition 2 shows, the cost effect caused by opening up the market can in fact be so detrimental to welfare that it outweighs the positive competition effect. Conversely, it can be beneficial to limit the market size: at the optimal market size n^*, the two opposing effects balance each other and welfare is maximized. As opposed to a setting of homogeneous firms, the net effect of an increase in competition no longer is necessarily positive. Instead, restrictions to market access are rationalized if firms are heterogeneous.

Total production level: The two opposing forces of competition and cost effect can also be observed when looking at the expected total quantity of goods produced in equilibrium. Again, opening up the market and allowing more firms to enter has two countervailing consequences. From a firm's individual quantity choice q_i in eq. (6) we obtain the total production level,

$$Q \equiv \sum_{i=1}^{n} q_i = \frac{n\alpha - \sum_{i=1}^{n} c_i}{2 + \gamma(n-1)}. \tag{16}$$

First consider the competition effect. We increase the number of firms while keeping costs constant, i.e. setting marginal costs $c_i = c$ for all firms. This yields

$$\frac{\mathrm{d}Q|_{c_i=c}}{\mathrm{d}n} = \frac{(2-\gamma)(\alpha - c)}{[2+\gamma(n-1)]^2} > 0, \tag{17}$$

driving up the production level. At the same time, a newly arriving firm lowers expected average efficiency. To single out this cost effect, let the number of firms n be fixed and increase average costs $\frac{1}{n}\sum_{i=1}^{n} c_i$ in eq. (16). This lowers the total level of goods produced and hence works against the competition effect. As the following lemma shows, the net effect is positive nevertheless:

Lemma 6. *If firms are heterogeneous, opening up the market always increases the expected level of total production.*

In a market with homogeneous firms, this result comes at no surprise: as more firms enter, the total production level moves from an inefficiently low level towards the efficient competitive level. Under heterogeneity, however, the net effect is not as obvious, as the increase in (expected) average costs drives down the overall quantity of goods. In the case of total welfare, this cost effect may outweigh the competition effect, as we saw in Proposition 2. Concerning production levels, however, the competition effect always dominates. In short, more firms always cause more production, but they may cause less welfare.

The optimal market size: So far, we have focused on the qualitative finding that some restriction to the market size may be optimal. On the quantitative side, one may be interested in, first, the actual size optimally chosen by the regulator and, second, the requirements that need to be met for such restriction to be optimal. The answers to both questions will depend on the market characteristics – that is, on the exogenous parameters: quality level α, degree of product differentiation γ, and size of the firm pool m.

However, an explicit solution to the regulator's optimization problem given by $\arg\max_{n\in\{1,\dots,m\}} \mathrm{E}_{\boldsymbol{c}}[\mathcal{W}_{\text{tot}}(n)]$ does in general not exist. This is because it is has the implicit form of a quintic polynomial equation to which there is no general algebraic solution in radicals.[16] Given a specific set of market characteristics, a solution can nevertheless be derived numerically. In the next section we therefore use simulations to analyze the way in which the market characteristics affect the regulator's optimal policy.

4.5 The Effect of Market Characteristics

Interaction of consumers and firms is determined by the characteristics of the market we study. In particular, the specific strategies chosen and outcomes implemented depend on the exogenous parameters observed by both sides. Market interaction therefore is driven by quality level α, degree of product differentiation γ, and size m of the firm pool. In this section, we analyze how each of these values affects the regulator's policy, i.e. their effect on the optimal market size n^*. We do so using computational methods, in particular numerical simulations. Observations stemming from such simulations will be referred to as *Results*, as opposed to the lemmata and propositions above, which were analytically proved.

In the following, we plot the optimal market size n^* as a function of the market characteristics α, γ, m. In each of the Figures 1 to 3, the solid bold line indicates where $n^* = m$ is just optimal. The line thus separates two parameter ranges: to one side of

[16] The impossibility to derive an explicit solution follows from the Abel–Ruffini Theorem, see e.g. Rosen (1995). Even for the simplifying assumption that $\alpha = m, \gamma = 1$, the first order condition cannot be solved analytically, while for $\alpha = m, \gamma = 0$, restriction is not optimal.

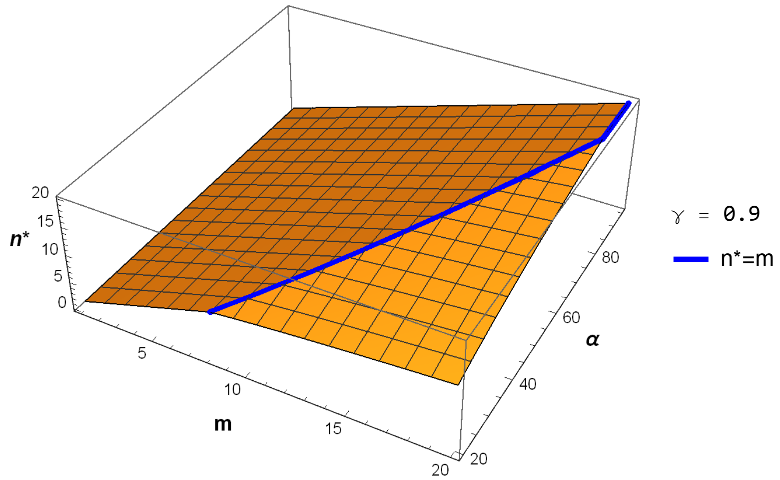

Figure 1: opt. market size n^* as a function of pool size m and quality level α

the line, the regulator optimally allows all firms to enter. This area is depicted by the diagonal plane in Figures 1 and 3 and by the horizontal plane in Figure 2. The regulator would even prefer a larger market size ($n > m$) but is restricted to the number of firms m available in the pool. To the other side, we observe the effect this paper rationalizes: the regulator finds it optimal to restrict access by setting some interior value $n^* < m$.

4.5.1 Quality Level α

The quality level α describes how much utility consumers derive from the consumption of some good *before* considering decreasing marginal benefits or substitution effects. If this quality is large, consumers greatly benefit from large consumption levels even in the presence of these detrimental effects. Formally, the value of α enters total welfare via the term αQ in eq. (11). Quality thus has a larger (positive) impact on welfare if production levels are high.

As more firms enter the market, the expected level of total production Q increases, as we know from Lemma 6. Such an increase in production level affects total welfare both positively and negatively: positively via the quality α as just mentioned and negatively, on the other hand, via decreasing marginal benefits to consumers as well as via substitution effects. In addition, total firm profits may decrease. For large quality levels α, the positive effect receives greater weight. The market can thus be opened further than if α was small. Only if the number of firms in the market becomes too large, the negative effects dominate and a restriction is optimal. We therefore expect the optimal market size n^* to increase with α: more quality puts more emphasis on large consumption levels

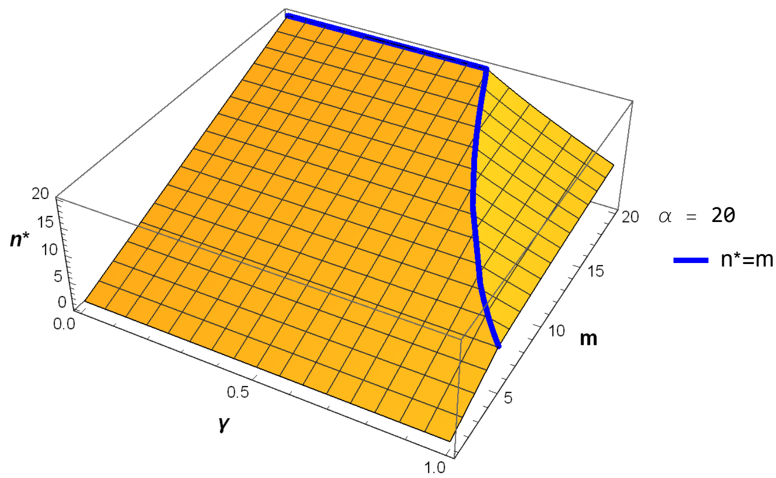

Figure 2: opt. market size n^* as a function of pool size m and product differentiation γ

and hence mandates a large market, despite the cost effects.

This expectation is confirmed by Figure 1: take a fixed degree of product differentiation γ. For small sizes m of the firm pool size, it is optimal to have all firms enter the market, irrespective of the quality level α. This is seen in the diagonal plane left of the solid line. Here, we have $n^* = m$ and ideally, the regulator would allow even more firms compete but she cannot do so as only m are available (we study the role of m in more detail in Section 4.5.3). At some point, however, increasing the market size – and thus total production – brings increasingly negative effects, such that $n^* < m$ becomes optimal. This tipping point is indicated by the solid line, to the right of which a restriction of the market size is optimal. Now we increase the quality level α: this shifts the tipping point – beyond which the negative effects dominate – further up. As the welfare gain from the level of production Q increases with α, the regulator wants more firms to enter. We therefore find:

Result 1. *Restricting the market size is optimal if the quality of goods is sufficiently low.*

4.5.2 Degree of Product Differentiation γ

We now turn to the question how the degree of product differentiation γ affects the optimal market size. Recall that goods are rather independent if γ is close to 0, and rather substitutable if γ is close to 1. For large values of γ, consumers' utility derived from the goods is greatly diminished by the substitutability effects. Formally, γ enters consumer surplus negatively via the term $\gamma \sum_{i=1}^{n} \sum_{j \neq i} q_i q_j$ in eq. (11), and it affects total welfare in the same way.

With a growing number of firms, this detrimental impact on welfare increases. There is a larger level of total production, and a larger number of different goods between which substitution effects arise. That is, we expect the negative consequences of substitutability to be more severe for larger values of n. At the same time, recall the beneficial competition effect stemming from an increase in market size. The larger γ is, the more will substitution outweigh competition, i.e. the sooner we will observe a welfare loss when allowing more firms to enter.

Figure 2 illustrates this finding. Again, the diagonal plane indicates where the regulator fully opens the market and would prefer to have even more firms in competition. At some point, however, we reach the tipping point, indicated by the solid line, beyond which more competition lowers welfare. This point is reached earlier for larger values of γ: substitution effects dominate and only a fraction of those firms in the pool is allowed to enter the market. If goods are too differentiated or the pool size is too small, the regulator will not want to limit competition. We summarize this finding:

Result 2. *Restricting the market size is optimal if the goods are sufficiently substitutable.*

So far, we have varied either the quality level α or the degree of product differentiation γ while keeping the other parameter fixed. Now, we also consider their interplay. Recall the threshold at which $n^* = m$ was *just* optimal, i.e. where the regulator grants market access to all firms in the pool, yet she would not want to admit more if the pool was larger. We have referred to this as the “tipping point” beyond which restrictions to the market size became optimal.

In Figure 3, we now fix some pool size m. If the goods are sufficiently differentiated (γ small) and have sufficiently high quality (α large), we expect no restriction to competition. This is observed in the left, flat part of the graph, where access is granted to all firms. As differentiation or quality decrease, the benefits of more competition shrink. By varying either of the two parameters, we reach the point where restriction starts becoming optimal. Figure 3 reveals that the less differentiated the goods are (i.e. the larger γ), the lower is the quality threshold *below* which the regulator limits market access. Conversely, the higher the quality of the goods (i.e. the larger α), the more substitutability is required for such a limiting policy to be optimal.

4.5.3 Pool Size m

Finally, we analyze the third market parameter and its effect on regulation: the number m of firms seeking to enter. It captures the number of independent draws from the uniform distribution, forming a profile of costs $\boldsymbol{c}$. This profile characterizes the pool of m

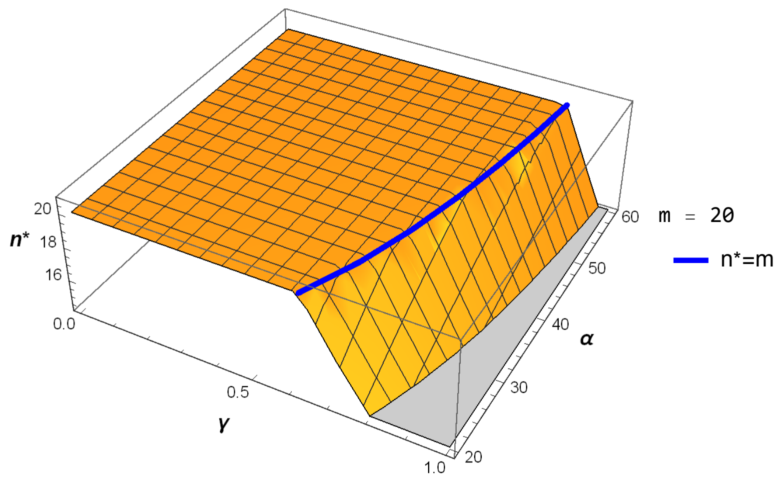

Figure 3: opt. market size n^* as a function of quality level α and product diff. γ

firms hoping to compete. When we compare different values of m, we therefore compare different sample sizes and ask how the regulator's policy changes if this size increases.

A larger value of this pool size has two effects. First, it simply allows the regulator to open up the market further, as she is constrained by her choice to set $n \leq m$. Second, however, for a fixed choice of n, the pool size also determines the cost distribution of those firms entering the market. For any market size, the n most efficient out of the m available firms enter. Since the firms' costs are uniformly distributed on the unit interval, the i-th most efficient firm has expected costs of $\frac{i}{m+1}$ (see Appendix B.1 for details). As the pool size increases by a single firm, this expectation changes to $\frac{i}{m+2}$. Similarly, the expected efficiency of all other firms grows.

When the regulator opens up the market by increasing n, we identified two countervailing effects: a gain in welfare due to increased competition, and a loss of welfare due to increased costs. The latter effect now is diminished if the pool size has increased, as each firm has become more efficient. The competition effect dominates for larger values of n and we expect the regulator to allow more firms to enter. Only as n gets very large, the detrimental cost effect becomes more relevant.

Figures 1 and 2 show this interplay. For low values of m, the regulator wants all firms to engage in competition. Increasing m hence increases the optimal market size n^* by the same amount, as observed in the diagonal plane in both plots. At some point, the detrimental cost effect mentioned above dominates, however, and the regulator starts restricting access. But even then, a further increase in m has a positive effect on n^*: instead of a flat plane beyond the "tipping point" indicated by the solid line, we still observe a growth of n^*, even though it remains below m. This growth stems from the fact that as the pool size increases to $m+1$, the expected efficiency of the m original firms rises – both of those that were operating in the market and of those still seeking entry. The cost effect driving down welfare is reduced and the regulator allows some additional firms to enter.

The above analysis explains why the optimal market size increases in the pool size. First and foremost, however, we restate the main observation: only if sufficiently many firms are available, the regulator finds it optimal to deny access to some of them. For a small pool size, the competition effect still dominates the cost effect and she prefers to have all firms compete.

Result 3. *Restricting the market size is optimal if there are many firms seeking to compete.*

This observation can equivalently be explained in terms of statistical sampling: the average costs of the n most efficient firms decrease in the sample size m. Only if this size is sufficiently large does the regulator prefer to have a subset of the sample compete rather than all firms. The advantage of a cost-efficient yet oligopolistic market then outweighs a more competitive alternative with higher average costs.

As a final remark, recall the results from Sections 4.5.1 and 4.5.2. We just stated that the pool size m needs to reach some threshold beyond which the regulator limits market access. Combining this finding with the previous sections, we can determine the effect of the other market parameters on this threshold of m. First, it increases with the quality level α, see Figure 1: if the quality of goods is large, consumers derive large utility values from high production levels. Unrestricted access becomes more beneficial and is only dominated if the pool size increases even further. Second, the threshold decreases in the degree of product differentiation γ, as seen in Figure 2. If goods are rather substitutable (large γ), opening up the market hampers welfare early on. Even for small pool sizes, the regulator therefore only allows a fraction of firms to enter.

This section has shed some light on the market characteristics and their effect on the regulator's optimal policy. Given the lack of an explicit solution for the optimal market size, we have resorted to numerical analyzes. These allowed us to see when restriction becomes optimal from a total welfare perspective, given the parameter ranges for quality levels, product differentiation and pool size.

5 Robustness & Extensions

The model introduced in Section 3 is a standard extension of the differentiated duopoly setting. So far, we have been focusing on a *Cournot* market with firms which are heterogeneous in their *marginal costs*. In such a market it can be optimal for a welfare-maximizing regulator to restrict competition by limiting the market size, as Proposition 2 states.

In this section, we show that the result is robust to changes in the setup. First (Section 5.1), we study the move to a Bertrand market, where firms compete in prices rather than in quantities. Second (Section 5.2), we adapt the model such that firms' heterogeneity stems from the quality α_i (which we so fare assumed to be the same value α for all firms) of their goods rather than from their marginal costs c_i. Finally (Section 5.3) we change the regulator's objective function. Up until now, we have assumed she maximizes total welfare, given by the sum of consumer surplus and firm profits. She might, however, be more interested in the auction revenue generated from selling the licenses rather than the actual industry performance. We therefore extend our findings to the case of a regulator maximizing consumer surplus plus auction revenue.

5.1 Bertrand Competition

In a Cournot market studied above, firms were setting their quantities q_i strategically, with consumers responding via an inverse demand function. If we move to a Bertrand market, firms instead choose their prices, expecting consumers' consumption levels. Again, we can derive equilibrium prices $\boldsymbol{p}$ and quantities $\boldsymbol{q}$ as a function of the cost profile $\boldsymbol{c}$: consumers face the optimization problem given by eq. (2), which firms anticipate. Each firm therefore chooses its profit-maximizing price equal to $\arg\max_{p_i} q_i(p_i - c_i)$. We defer the derivation of the equilibrium to the Appendix given the complexity of the expressions, see eqs. (29) and (30).

Just as in the case of Cournot competition, we want to focus on the case where all firms *want* to participate in the market but *should* not necessarily do so from a total welfare perspective. We therefore need to identify conditions such that the equilibrium yields positive prices, demand and profits for all firms, irrespective of the particular cost profile. Under Bertrand competition with perfect substitutability of goods ($\gamma = 1$), however, we have the standard "winner takes it all" outcome, where the most efficient firm can always undercut the zero-profit price of its competitors. We hence focus on product differentiation levels $\gamma \in [0, 1)$. In addition, to ensure positivity and uniqueness of the equilibrium, we again need to assume a lower bound on the joint quality level α, analogously to eq. (9).

Lemma 7. *If firms compete in a Bertrand market and the quality α of their goods is sufficiently high, there exists an equilibrium in which each firm finds it profitable to enter the market.*

More formally speaking, Lemma 7 states that for any pool size $m \in \mathbb{N}_+$ and any degree of product differentiation $\gamma \in [0, 1)$, there exists a lower bound $\underline{\alpha}(m, \gamma)$ on the quality such that, if $\alpha > \underline{\alpha}$, there is an equilibrium with $(\boldsymbol{q}, \boldsymbol{p}) \in \mathbb{R}_+^{2n}$ and $\pi_i > 0\, \forall i$ for every market size $n \in \{1, \ldots, m\}$ and every cost profile $\boldsymbol{c} \in [0, 1]^m$. As opposed to the simple threshold we derived in the case of Cournot competition, where Lemma 2 required $\alpha \geq m$, the lower bound is more complex in the case of Bertrand. In particular, we have $\lim_{\gamma \to 1} \underline{\alpha} = \infty$. As shown in the proof of Lemma 7, positivity of the equilibrium in a Bertrand setting neither implies nor requires positivity in a Cournot setting. Omitting solutions where firms set excessively large prices ($p_i \to \infty$) and consumers respond with zero demand, the equilibrium we derive is unique, which follows follows from the Poincaré-Hopf index theorem just as for the Cournot case.

In analogy to the Cournot setting, we want to ascertain that more efficient firms enter the market first. That is, firms with lower marginal costs are supposed to generate higher profits. The regulator, unaware of the firms' respective costs, can then expect those with lower cost realizations to place higher bids in a license auction, allowing them to be among the restricted set of market participants. We show at the end of the proof of Lemma 7 that indeed each firm's profit decreases in its own marginal costs.

Having ensured that all firms find market participation profitable, and that they enter in the order of their efficiency, we now turn to the regulator's problem. She specifies the market size, seeking to maximize expected total welfare. In particular, we ask whether she may again find it optimal to limit market access by setting a market size n smaller than the firm pool size m. The answer is positive:

Proposition 3. *If firms compete à la Bertrand, it can be welfare-optimal to limit competition:*

$$\exists \alpha, \gamma, m \text{ with } 0 \leq \gamma < 1 < m,\ \alpha \geq \underline{\alpha} \text{ s.t. } 1 < \operatorname*{arg\,max}_{n \in \{1,\ldots,m\}} \mathrm{E}_{\boldsymbol{c}}[\mathcal{W}_{\text{tot}}^{\text{B}}(n)] < m.$$

The qualitative finding of firm heterogeneity rationalizing restrictions to market size therefore extends to a Bertrand setting. While the computations are more involved given the more complex market equilibrium $(\boldsymbol{q}, \boldsymbol{p})$, the intuition is the same as in the Cournot case, see the discussion in Section 4.4.2.

5.2 Quality Heterogeneity

So far, we have studied firms which are heterogeneous in marginal costs c_i but share the quality α which their goods have for consumers. Instead, firms could be heterogeneous in quality, which we analyze in this section. Let each firm's quality level α_i be separable into a common "base" quality α and a private margin $x_i \sim U[0,1]$, such that $\alpha_i = \alpha - x_i$. We denote a quality profile by $\boldsymbol{x} = (x_1, \ldots, x_m)$. With such individual values for quality, consumer utility now is given by

$$U(\boldsymbol{q}) = \sum_{i=1}^{n} \alpha_i q_i - \frac{1}{2}\Big(\sum_{i=1}^{n} q_i^2 + \gamma \sum_{i=1}^{n} \sum_{j \neq i} q_i q_j\Big). \tag{18}$$

For simplicity, we assume all firms' marginal cost to be zero, i.e. $c_i = 0\, \forall i$. Häckner (2000) derives firm prices and consumer demand under Cournot competition:

$$p_i = q_i = \frac{[\gamma(n-2)+2]\alpha_i - \gamma \sum_{j \neq i} \alpha_j}{(2-\gamma)[\gamma(n-1)+2]} \tag{19}$$

Again, we want to ascertain that all firms find market entrance profitable. We can ensure positivity of all prices and quantities by requiring a sufficiently high base quality α:

Lemma 8. *A sufficient condition for prices and quantities given by eq.* (19) *to be positive is $\alpha \geq m$, i.e.*

$$\alpha \geq m \Rightarrow p_i, q_i \geq 0 \quad \forall i, n, m, \gamma \text{ s.t. } 1 \leq i \leq n \leq m, (\boldsymbol{x}, \gamma) \in [0,1]^{m+1}. \tag{20}$$

With zero marginal costs, firm profits are given by $\sum_{i=1}^{n} \pi_i = \sum_{i=1}^{n} q_i p_i$. Total welfare $\mathcal{W}_{\text{tot}}^{\alpha_i}$ with heterogeneous quality levels α_i thus simply equals $U(\boldsymbol{q})$. The following proposition states that there again are markets where the regulator finds it optimal to restrict entry in order to maximize welfare.

Proposition 4. *If firms are heterogeneous in quality, it can be welfare-optimal to limit competition:*

$$\exists \alpha, \gamma, m \text{ with } 0 \leq \gamma \leq 1 < m \leq \alpha \text{ s.t. } 1 < \underset{n \in \{1,\ldots,m\}}{\arg\max}\, \mathrm{E}_{\boldsymbol{x}}[\mathcal{W}_{\text{tot}}^{\alpha_i}(n)] < m.$$

5.3 A Regulator Interested in Auction Revenue

Up until now, the regulator's objective was to maximize total welfare given by consumer surplus plus firm profits. Suppose now the regulator is not intrinsically interested in industry performance but rather in the extraction of firm profits by auctioning off licenses for market entry. Does she still find it optimal to restrict the market size?

To answer this question, we recall Lemma 4, which stated that in n sequential second-price single-unit open-bid auctions, there is an equilibrium where firms bid their valuations – i.e. their profits from entering the market – truthfully. Auction revenue in the first round thus is given by the profits of the second most efficient firm, in the second round by the third most efficient firm, and so forth to the n-th round, where the revenue is equal to the profit the $(n+1)$-th most efficient firm would generate if it entered the market instead of the n-th most efficient one. Using the bidding strategies and notation introduced in the proof of Lemma 4, we can compute the total auction revenue R and compare it to total firm profits:

$$\begin{aligned} R(n) &= \sum_{i=2}^{n} \pi_{(i)}(c_{(1)}, \ldots, c_{(n)}) + \pi_{(n+1)}(c_{(1)}, \ldots, c_{(n-1)}, c_{(n+1)}) \\ &= \mathcal{W}_{\mathrm{F}} - \pi_{(1)}(c_{(1)}, \ldots, c_{(n)}) + \pi_{(n+1)}(c_{(1)}, \ldots, c_{(n-1)}, c_{(n+1)}). \end{aligned} \tag{21}$$

The regulator now chooses the market size n^* maximizing the expected sum of consumer surplus and auction revenue, $\mathrm{E}_{\boldsymbol{c}}[\mathcal{W}_{\mathrm{C}}(n) + R(n)]$. The following proposition considers the above auction, the equilibrium bidding strategies and the regulatory objective function to conclude that it can again be optimal to restrict the market size:

Proposition 5. *If the regulator maximizes expected consumer surplus plus auction revenue, it can be welfare-optimal to limit competition:*

$$\exists \alpha, \gamma, m \text{ with } 0 \leq \gamma < 1 < m,\ \alpha \geq \underline{\alpha} \text{ s.t. } 1 < \underset{n \in \{1,\ldots,m\}}{\arg\max}\ \mathrm{E}_{\boldsymbol{c}}[\mathcal{W}_{\mathrm{C}}(n) + R(n)] < m.$$

6 Conclusion

This paper presents a novel explanation for restrictions to the market size. We show that a regulator seeking to maximize total welfare may find it optimal to grant market access only to a limited number of firms, e.g. by issuing a certain number of licenses. Considering a two-stage game, the regulator faces a finite number of firms seeking to enter a market. Firms differ in their marginal costs, with the regulator knowing only the distribution. In the first stage, the regulator specifies a market size, allowing the most efficient firms to enter. In the second stage, these firms engage in Cournot competition.

If firms are homogeneous, the standard notion of more competition being better does apply. That is, the regulator finds it welfare-optimal to allow unrestricted market entrance. But if firms are heterogeneous, this paper rationalizes regulation. In particular, expected welfare can be maximized by choosing a certain oligopolistic market size.

We identified a two-fold effect driving this result. When the market is opened up and more firms are admitted, there is a change to, first, the degree of competition and, second, the distribution of production costs in the market. The competition effect harms firms and benefits consumers, with a positive net effect on total welfare. The cost effect is ambiguous: new entrants are less efficient and hence drive up average production costs, but firm profits and total welfare may nevertheless rise – despite a decrease in consumer surplus. This stems from the observation that if a small firm becomes less efficient, its more efficient rivals may take over some of its market share. The loss to this small firm is overcompensated by the additional profit of its competitors, causing overall firm profits and total welfare to increase.

When the regulator opens up the market, she thus faces a trade-off between beneficial competition and a potential decrease in production efficiency. The latter effect may outweigh the former, such that she optimally limits the number of firms allowed to operate. This result does not hinge on the existence of entry costs, search costs or decreasing returns to scale, which previous literature required.

The two-fold effect and the optimality of regulation are robust to changes to the model. We looked at both Cournot and Bertrand competition, at firms being heterogeneous either in their production costs or in the quality of their goods, and at a regulator interested in the revenue generated from auctioning off licenses for market entry. The paper thus rationalizes the regulatory practice observed in many real-world markets, where competition is limited by restricted market access.

Appendix A Proofs

Proof of Lemma 1. The lemma directly follows from differentiating the equilibrium price and quantity values with respect to marginal costs. Let $i, j \in \{1, \ldots, n\}$. From eq. (7), we obtain

$$\frac{\mathrm{d}q_i}{\mathrm{d}c_j} = \begin{cases} -\lambda[2 + \gamma(n-2)] & < 0 \text{ if } i = j \\ \lambda\gamma & \geq 0 \text{ if } i \neq j \end{cases}$$

proving part a) and b). Differentiating eq. (7) yields

$$\frac{\mathrm{d}p_i}{\mathrm{d}c_j} = \begin{cases} \lambda[2 + \gamma(n-2) - \gamma^2(n-1)] & > 0 \text{ if } i = j \\ \lambda\gamma & \geq 0 \text{ if } i \neq j \end{cases}$$

and thus proves part c). □

Proof of Lemma 2. Since $p_i \geq q_i$, it suffices to show positivity of q_i. The denominator of eq. (6) is strictly positive while the numerator is decreasing in c_k for $k = i$ and increasing for $k \neq i$. Hence, consider the cost profile $c_i = 1, c_j = 0$. The numerator becomes $\alpha(2-\gamma) - [2 + \gamma(n-2)]$, which decreases in γ for $n \geq 2$ (for $n = 2$, q_i is trivially positive). Hence, let $\gamma = 1$. We have $q_i \geq \{(2-\gamma)[2 + \gamma(n-1)]\}^{-1}(\alpha - n)$. Since $n \leq m$, weak positivity is ensured if $\alpha \geq m$. Note that positivity is strict unless $\alpha = m = n, \gamma = 1, c_i = 1, c_j = 0$. □

Proof of eq. (12). We use the equilibrium prices and quantities $(\boldsymbol{q}, \boldsymbol{p})$ derived in Section 4.1 to compute the total surplus given a profile of costs $\boldsymbol{c}$, where we continue to write $\lambda = \{(2-\gamma)[2 + \gamma(n-1)]\}^{-1}$. Here and in subsequent proofs we make use of several auxiliary computations derived for arbitrary equilibria $(\boldsymbol{q}, \boldsymbol{p})$ in Appendix B.2.

$$\begin{aligned} \mathcal{W}_{\text{tot}} &= \alpha \sum_{i=1}^{n} q_i - \frac{1}{2} \sum_{i=1}^{n} q_i^2 - \frac{\gamma}{2} \sum_{i=1}^{n} \sum_{j \neq i} q_i q_j - \sum_{i=1}^{n} q_i c_i \\ &= \frac{\lambda^2}{2} \Big(\quad \alpha^2 (2-\gamma)^2 [\gamma(n-1) + 3] n \\ &\qquad - 2\alpha(2-\gamma)^2 [\gamma(n-1) + 3] \sum_{i=1}^{n} c_i \\ &\qquad - [\gamma^3 (n-2)(n-1) - \gamma^2 \{n(3n-13) + 13\} - 12\gamma(n-2) - 12] \sum_{i=1}^{n} c_i^2 \\ &\qquad - \gamma[\gamma\{\gamma + (3-\gamma)n - 6\} + 8] \sum_{i=1}^{n} \sum_{j \neq i} c_i c_j \Big). \end{aligned} \tag{22}$$

This yields the four factors η_1, η_2, η_3 and η_4 of eq. (12). Note that $\eta_1, \eta_3 > 0$, while $\eta_2, \eta_4 \leq 0$. For expressions of consumer surplus $\mathcal{W}_\text{C}$ and firm profits $\mathcal{W}_\text{F}$, we use the same form and obtain the respective coefficients. Consumer surplus is given by

$$\mathcal{W}_\text{C} = U(\boldsymbol{q}) - \sum_{i=1}^{n} p_i q_i = \eta_1^\text{C} + \eta_2^\text{C} \sum_{i=1}^{n} c_i + \eta_3^\text{C} \sum_{i=1}^{n} c_i^2 + \eta_4^\text{C} \sum_{i=1}^{n} \sum_{j \neq i} c_i c_j, \tag{23}$$

with

$$\begin{aligned}
\eta_1^\text{C} &= \tfrac{\lambda^2}{2} \alpha^2 (2-\gamma)^2 [\gamma(n-1)+1] n && > 0 \\
\eta_2^\text{C} &= -\lambda^2 \alpha (2-\gamma)^2 [\gamma(n-1)+1] && < 0 \\
\eta_3^\text{C} &= \tfrac{\lambda^2}{2} \{\gamma[-\gamma^2(n-2)(n-1) + \gamma(n-7)n + 7\gamma + 4n - 8] + 4\} && > 0 \\
\eta_4^\text{C} &= \tfrac{\lambda^2}{2} \gamma^2 [\gamma(n-1) - n + 2] && \gtreqless 0.
\end{aligned}$$

For firm profits, we find:

$$\mathcal{W}_\text{F} = \sum_{i=1}^{n} \pi_i = \sum_{i=1}^{n} (p_i - c_i) q_i = \eta_1^\text{F} + \eta_2^\text{F} \sum_{i=1}^{n} c_i + \eta_3^\text{F} \sum_{i=1}^{n} c_i^2 + \eta_4^\text{F} \sum_{i=1}^{n} \sum_{j \neq i} c_i c_j, \tag{24}$$

with

$$\begin{aligned}
\eta_1^\text{F} &= \lambda^2 \alpha^2 (2-\gamma)^2 n && > 0 \\
\eta_2^\text{F} &= -2\lambda^2 \alpha (2-\gamma)^2 && < 0 \\
\eta_3^\text{F} &= \lambda^2 [\gamma\{3\gamma + n[\gamma(n-3)+4] - 8\} + 4] && > 0 \\
\eta_4^\text{F} &= -\lambda^2 \gamma [\gamma(n-2) + 4] && \leq 0.
\end{aligned}$$

□

Proof of Lemma 3. To compute the profits of firm i given a profile of costs $\boldsymbol{c}$, we again consider the equilibrium quantities and prices $(\boldsymbol{q}, \boldsymbol{p})$ derived Section 4.1.

$$\begin{aligned}
\pi_i &= q_i (p_i - c_i) \\
&= q_i \lambda \{\alpha(2-\gamma) + [2 + \gamma(n-2) - \gamma^2(n-1) - \lambda^{-1}] c_i + \gamma \sum_{j \neq i} c_j\} \\
&= q_i^2.
\end{aligned}$$

Differentiation with respect to marginal costs yields

$$\frac{\mathrm{d}\pi_i}{\mathrm{d}c_j} = 2 q_i \frac{\mathrm{d}q_i}{\mathrm{d}c_j}.$$

Recall from Lemma 2 that for $\alpha \geq m$ we have $q_i \geq 0\,\forall i$. Invoke Lemma 1 for the sign of $\frac{\mathrm{d}q_i}{\mathrm{d}c_j}$. Combining, we obtain

$$\frac{\mathrm{d}\pi_i}{\mathrm{d}c_i} \leq 0, \quad \frac{\mathrm{d}\pi_i}{\mathrm{d}c_j} \geq 0 \text{ if } i \neq j,$$

which proves parts a) and b) of the lemma, respectively.

For part c), consider

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}c_i} q_i p_i = {} & q_i \frac{\mathrm{d}p_i}{\mathrm{d}c_i} + p_i \frac{\mathrm{d}q_i}{\mathrm{d}c_i} \\
= {} & \lambda^2 \{\alpha(2-\gamma) - [2+\gamma(n-2)]c_i + \gamma \sum_{j\neq i} c_j\}[2+\gamma(n-2) - \gamma^2(n-1)] \\
& - \lambda^2 \{\alpha(2-\gamma) + [2+\gamma(n-2) - \gamma^2(n-1)]c_i + \gamma \sum_{j\neq i} c_j\}[2+\gamma(n-2)] \\
= {} & -\lambda^2 \{\alpha(2-\gamma)\gamma^2(n-1) + 2[2+\gamma(n-2)][2+\gamma(n-2) - \gamma^2(n-1)]c_i \\
& + \gamma^3(n-1)\sum_{j\neq i} c_j\} \\
\leq {} & 0
\end{aligned}$$

Finally, for part d), we need to show that the derivative of consumer surplus with respect to any firm's marginal costs is negative. From eq. (23), we obtain

$$\frac{\mathrm{d}\mathcal{W}_{\mathrm{C}}}{\mathrm{d}c_i} = \eta_2^{\mathrm{C}} + 2\eta_3^{\mathrm{C}} c_i + 2\eta_4^{\mathrm{C}} \sum_{j\neq i} c_j.$$

Recall that $\eta_2^{\mathrm{C}} < 0$ and $\eta_3^{\mathrm{C}} > 0$, while the sign of η_4^{C} is ambiguous. Since the derivative is linear in c_j, we consider the case of both $c_j = 0$ and $c_j = 1$. For $\eta_4^{\mathrm{C}} > 0$, we have

$$\begin{aligned}
\frac{\mathrm{d}\mathcal{W}_{\mathrm{C}}}{\mathrm{d}c_i} \leq {} & \eta_2^{\mathrm{C}} + 2\eta_3^{\mathrm{C}} + 2\eta_4^{\mathrm{C}}(n-1) \\
= {} & -\lambda^2(\alpha-1)(2-\gamma)^2[1+\gamma(n-1)] \leq 0.
\end{aligned}$$

If, on the other hand, $\eta_4^{\mathrm{C}} \leq 0$, it holds true that

$$\begin{aligned}
\frac{\mathrm{d}\mathcal{W}_{\mathrm{C}}}{\mathrm{d}c_i} & \leq \eta_2^{\mathrm{C}} + 2\eta_3^{\mathrm{C}} \\
& = \lambda^2\{4[1+\gamma(n-2)] + \gamma^2[7 - 2\gamma + n\{3\gamma - 7 + n(1-\gamma)\}] - \alpha(2-\gamma)^2[1+\gamma(n-1)]\} \\
& \leq -\lambda^2(n-1)\{4 + \gamma[\{4 - \gamma(5-2\gamma)\}n + \gamma(7-2\gamma) - 8]\} \\
& \leq 0,
\end{aligned}$$

where we have used that $\alpha \geq n$ at the second inequality. This proves that consumer surplus $\mathcal{W}_{\text{C}}$ is weakly decreasing in every firm's marginal costs. □

Proof of Proposition 1. From eq. (22) for total welfare $\mathcal{W}_{\text{tot}}$ we obtain

$$\frac{\mathrm{d}\mathcal{W}_{\text{tot}}}{\mathrm{d}c_i} = \eta_2 + 2\eta_3 c_i + 2\eta_4 \sum_{j \neq i} c_j,$$

where $\eta_2, \eta_4 \leq 0$, $\eta_3 > 0$. Hence, we consider the cost profile $c_i = 1$, $c_j = 0$ $\forall j \neq i$ and assume $\gamma = 1$, yielding

$$\left.\frac{\mathrm{d}\mathcal{W}_{\text{tot}}}{\mathrm{d}c_i}\right|_{\substack{\gamma = c_i = 1, \\ c_j = 0}} = 2n^2 + n(2 - \alpha) - 2\alpha - 1.$$

Since we need $\alpha \geq m$ by Lemma 2 and $n \leq m$ by assumption, we consider $n = \alpha$. Here, the derivative takes the form $n^2 - 1$, which is strictly positive unless there is a monopoly. We thus have identified market situations where, given the model at hand, an increase in some firm's marginal costs can cause total welfare to rise. □

Proof of Lemma 4. We introduce the following notation. We refer to the firms by the order statistics of their marginal costs $c_{(1)} \leq \ldots \leq c_{(m)}$, such that the index (i) signifies the i-th most efficient firm. Denote by $[\boldsymbol{c}]^n$ the set of subsets with n elements, i.e. the possible combinations of firms which can be active in a market of size n. $\pi_{(i)}(\boldsymbol{c}_n)$ is the profit of firm i in a market where the firms $\boldsymbol{c}_n \in [\boldsymbol{c}]^n$ compete. Note that this profit is equal to zero if $c_{(i)} \notin \boldsymbol{c}_n$ (i.e. if the firm is not active in the market) and that it is strictly positive otherwise by Lemma 2 apart from the polar case discussed at the end of Section 4.1, when it is zero. Recall that this polar case as well as a cost realization with $c_{(i)} = c_{(j)}$ for $i \neq j$ have zero probability mass due to m being finite and can be covered using a tie breaking rule. Finally, for each of the $k \in \{1, \ldots, n\}$ sequential auctions, denote by $w(k) \in \{1, \ldots, m\}$ the firm winning this auction, identified by the firm's order statistic (i). Since bids are public and a second-price auction is played each round, a bidding strategy for round k can be contingent on $w(k')$ for all $k' < k$.

We now construct a profile of bidding strategies as follows: in equilibrium, the k-th auction will be won by the k-th most efficient firm, i.e. $w(k) = k$ for all $k \in \{1, \ldots, n\}$, such that the n most efficient firms will indeed each hold one item ("license") each. In each round k, we therefore check whether for all past auctions $k' < k$ the k'-th round was won by the k'-th most efficient firm, i.e. whether $w(k') = k'$ for all $k' < k$.

On-path strategies: Let all firms $i \leq n$ bid their on-equilibrium market valuation given by $\pi_{(i)}(c_{(1)}, \ldots, c_{(n)})$, while the remaining firms $i > n$ bid the off-equilibrium valuation of them entering the market instead of firm (n), i.e. instead of the least efficient active firm: $\pi_{(i)}(c_{(1)}, \ldots, c_{(n-1)}, c_{(i)})$. Finally, since there is a cap of one item per bidder, firms having won an item (i.e. in round k all firms i s.t. $\exists k' < k$ with $w(k') = i$) in a previous round bid zero in the future.

Recall that a firm's profit decreases in its marginal costs and hence for all $\boldsymbol{c}_n$ we have $\pi_{(i)}(\boldsymbol{c}_n) \geq \pi_{(j)}(\boldsymbol{c}_n)$ if and only if $i \leq j$. For the same reason it holds that for all $\mathbf{c}$ and $i > n$: $\pi_{(n)}(c_{(1)}, \ldots, c_{(n)}) \geq \pi_{(i)}(c_{(1)}, \ldots, c_{(n-1)}, c_{(i)})$. Note that in the latter statement we compare two different profiles of active firms while in the first the firms are the same. Combining these inequalities we conclude that, ceteris paribus, a more efficient firm always has a higher valuation and places a higher bid if it follows the strategies above. Using these strategies, the k-th auction will be won by the k-th most efficient firm and the desired outcome is implemented.

Off-path strategies: We now need to ensure that more efficient firms have no incentive to initially lower their bids in order to wait for rounds when only very inefficient rivals are still bidding, providing them with much lower second-highest bids of their rivals. The most efficient firm, for example, could obtain the n-th license at a price much lower than the first license without running the risk of not obtaining a license at all. To avoid this, we now define bids for the case where in some past round k' a different firm than the k'-th most efficient has placed the highest bid, i.e. if in round k $\exists k' < k$ s.t. $w(k') \neq k'$. Using the notation introduced above, $\pi_{(1)}(c_{(1)})$ denotes the monopoly profit of the most efficient firm. Given a cost profile $\boldsymbol{c}$, this value is an upper limit for the possible profits across firms i and market market sizes n.[17] Now for every round after some $k' \neq w(k')$, all firms with $i < n$ who do not yet hold a license bid $\pi_{(1)}(c_{(1)})$, while all firms with $i > n$ bid zero. Firm n bids the monopoly profit if the deviant is among the more efficient firms, i.e. if $w(k') < n$, while it bids zero otherwise, i.e. if a firm that was not supposed to enter the market in equilibrium has done so.

This grim trigger strategy deters deviations in two ways. First, it prevents more efficient firms ($i < n$) from postponing their bids in order to win later, cheaper rounds. If they do not acquire their item in the round they are supposed to, they will only be able to do so at a price equal to the monopoly profit, making a later entry unattractive. Second, it also prevents overbidding of less efficient firms ($i > n$). Initially, none of these firms have an incentive to acquire a license: in any round $k \leq n$, a firm $i > n$ would be

[17]This follows from noting that for any market size n, any cost profile $\boldsymbol{c}_n$ and any firm i we have $\pi_{(i)}(\boldsymbol{c}_n) = [q_{(i)}(\boldsymbol{c}_n)]^2$ from eq. (6). Since the production level $q_{(i)}$ decreases if an additional firm is added irrespective of the entrant's costs, the profit of i also decreases.

paying a price $\pi_{(k)}(c_{(1)}, \ldots, c_{(n)})$, which is greater than its own profit *if* the profile of competitors remained unchanged. However, if *all* firms – instead of only the remaining $n-k$ most efficient ones – would then move to monopoly bids, the deviating firm i could hope that a tie breaking rule will cause less efficient firms ($j > n$) to enter, increasing its own profit beyond the price initially paid. The off-path strategies above ensure that if a less efficient firm enters the market it will nevertheless only face the $n-1$ most efficient rivals, making overbidding unattractive. □

Proof of Lemma 5. First consider the case where the costs are not known by the regulator. With $c_i = c \sim U[0,1]\ \forall i$, we have $\mathrm{E}[c_i] = \frac{1}{2}$, $\mathrm{E}[c_i^2] = E[c_i c_j] = \frac{1}{3}$. Plugging this into eq. (22) for total welfare yields

$$\begin{aligned}\mathrm{E}_c[\mathcal{W}_{\text{tot}}] =&\ \mathrm{E}_c[\eta_1 + \eta_2 \sum_{i=1}^{n} c_i + \eta_3 \sum_{i=1}^{n} c_i^2 + \eta_4 \sum_{i=1}^{n} \sum_{j \neq i} c_i c_j] \\ &= \eta_1 + \frac{1}{2} n\eta_2 + \frac{1}{3}[n\eta_3 + n(n-1)\eta_4] \\ &= \frac{n[1 + 3\alpha(\alpha-1)][3 + \gamma(n-1)]}{6[2+\gamma(n-1)]^2}.\end{aligned}$$

To analyze the effect of an increase in the market size n on expected welfare, we take the derivative:

$$\frac{\mathrm{d}}{\mathrm{d}n}\,\mathrm{E}_c[\mathcal{W}_{\text{tot}}] = \frac{[1+3\alpha(\alpha-1)]\{6+\gamma[n(1-\gamma)+\gamma-5]\}}{6[2+\gamma(n-1)]^2},$$

which is strictly positive $\forall \alpha, \gamma, n, m$ s.t. $0 \leq \gamma \leq 1 \leq n \leq m \leq \alpha$. Next, consider the case where the costs are $c_i = c\ \forall i$ with c being public. We have

$$\begin{aligned}\mathcal{W}_{\text{tot}} &= \eta_1 + n\eta_2 c + [n\eta_3 + n(n-1)\eta_4]c^2 \\ &= \frac{n(\alpha-c)^2[3+\gamma(n-1)]}{2[2+\gamma(n-1)]^2}, \\ \Rightarrow\ \frac{\mathrm{d}}{\mathrm{d}n}\mathcal{W}_{\text{tot}} &= \frac{(\alpha-c)^2\{6+\gamma[n(1-\gamma)+\gamma-5]\}}{2[2+\gamma(n-1)]^3},\end{aligned}$$

which again is weakly positive $\forall \alpha, \gamma, n, m, c$ s.t. $1 \leq n \leq m \leq \alpha$, $(c, \gamma) \in [0,1]^2$ and equal to zero only for the polar case of a monopoly with $\alpha = c = 1$ (recall that $n \leq \alpha$). □

Proof of Proposition 2. We combine eq. (22) and Appendix B.1 for the computation of total welfare with expectations over marginal costs:

$$\begin{aligned}
\mathrm{E}_{\boldsymbol{c}}[\mathcal{W}_{\text{tot}}] &= \eta_1 + \eta_2 \sum_{i=1}^{n} \mathrm{E}_{\boldsymbol{c}}[c_i] + \eta_3 \sum_{i=1}^{n} \mathrm{E}_{\boldsymbol{c}}[c_i^2] + \eta_4 \sum_{i=1}^{n} \sum_{j \neq i} \mathrm{E}_{\boldsymbol{c}}[c_i c_j] \\
&= \eta_1 + \eta_2 \frac{n(n+1)}{2(m+1)} + \eta_3 \frac{n(n+1)(n+2)}{3(m+1)(m+2)} + \eta_4 \frac{n(n-1)(n+1)(n+2)}{4(m+1)(m+2)} \\
&= \; n \cdot \{24(2-\gamma)^2(m+1)(m+2)[2+\gamma(n-1)]^2\}^{-1} \\
&\quad \cdot \Big[12\alpha(2-\gamma)^2(m+2)[3+\gamma(n-1)][\alpha(m+1)-(n+1)] \\
&\quad + 4(n+1)(n+2)\{12+\gamma[\gamma(13-2\gamma)+(3-\gamma)n[4+\gamma(n-3)]-24]\} \\
&\quad - 3\gamma(n-1)(n+1)(n+2)\{8+\gamma[\gamma+(3-\gamma)n-6]\} \Big], \qquad (25)
\end{aligned}$$

for arbitrary parameters satisfying $0 \leq \gamma \leq 1 \leq n \leq m \leq \alpha$. The numerator is a polynomial of fifth degree in n, the denominator one of second degree. We hence cannot find a general solution for the welfare-maximizing value of n by the Abel–Ruffini Theorem. To prove existence of interior solutions, consider the case of $\alpha = m = 10, \gamma = 1$. We obtain $\mathrm{E}_{\boldsymbol{c}}[\mathcal{W}_{\text{tot}}] = [n(n+2)(156\,965 + n\{(2n+7)n - 1\,430\})][3\,168(n+1)^2]^{-1}$, which can be maximized numerically.[18] There are five values for n^* satisfying $\frac{\mathrm{d}}{\mathrm{d}n} \mathrm{E}_{\boldsymbol{c}}[\mathcal{W}_{\text{tot}}]|_{n^*} = 0$. Ignoring the solutions $n_1^* < 0$, $n_2^* > m$ as well as complex values n_3^*, n_4^* s.t. $\mathrm{Im}(n_3^*)$, $\mathrm{Im}(n_4^*) \neq 0$, we obtain the interior optimum $n_5^* \approx 5.38$, noting that $1 < n_5^* < m$. Computing the second order derivative, it is readily verified that n_5^* is indeed a maximizer.

To verify that consumers benefit from an increase in competition, consider $\frac{\mathrm{d}}{\mathrm{d}n} \mathrm{E}_{\boldsymbol{c}}[\mathcal{W}_{\mathrm{C}}(n)]$ and note that it is increasing in both α and m. Hence, let $\alpha = m = n$ to see that the derivative is positive for all $0 \leq \gamma \leq 1 \leq n$. □

Proof of Lemma 6. The expected level of total production is given by

$$\mathrm{E}_{\boldsymbol{c}}[Q] = \frac{n\alpha - \sum_{i=1}^{n} \mathrm{E}[c_i]}{2 + \gamma(n-1)} \; . \qquad (26)$$

Using $\sum_{i=1}^{n} \mathrm{E}[c_i] = \frac{n(n+1)}{2(m+1)}$ from Appendix B.1, we obtain

$$\frac{\mathrm{d}\,\mathrm{E}_{\boldsymbol{c}}[Q]}{\mathrm{d}n} = \frac{\lambda^2(2-\gamma)^2}{2(m+1)} \{-\gamma n^2 - 2(2-\gamma)n + (2-\gamma)[2\alpha(m+1) - 1]\}. \qquad (27)$$

Since we have $n \leq m$ by assumption and $m \leq \alpha$ from Lemma 2, the positive term in

[18] The computations using MATHEMATICA are available upon request.

$\{\ldots\}$ dominates the negative ones and the whole derivative is positive. $\square$

Proof of Lemma 7. From the consumers' optimization problem, eq. (2), we derive their consumption levels. Differentiating and summing over all firms yields demand q_i given prices $\boldsymbol{p}$:

$$q_i = \xi_1 - \xi_2 p_i + \xi_3 \sum_{j \neq i} p_j, \tag{28}$$

with coefficients

$$\xi' = \{(1-\gamma)[1+\gamma(n-1)]\}^{-1}, \quad \xi_1 = \xi'\alpha(1-\gamma), \quad \xi_2 = \xi'[1+\gamma(n-2)], \quad \xi_3 = \xi'\gamma.$$

Firms anticipate this demand when setting their prices by maximizing $\pi_i = q_i(p_i - c_i)$. Considering eq. (28), their optimization problem is given by

$$\max_{p_i \in \mathbb{R}_+} (\xi_1 - \xi_2 p_i + \xi_3 \sum_{j \neq i} p_j)(p_i - c_i).$$

Summing the respective first order conditions over all firms and rearranging yields

$$\begin{aligned} p_i &= \frac{1}{(2\xi_2+\xi_3)[2\xi_2-(n-1)\xi_3]}\left[\xi_1(2\xi_2+\xi_3) + \xi_2[2\xi_2-(n-2)\xi_3]c_i + \xi_2\xi_3 \sum_{j \neq i} c_j\right] \\ &= \xi_4 + \xi_5 c_i + \xi_6 \sum_{j \neq i} c_j, \end{aligned} \tag{29}$$

with coefficients

$$\begin{aligned} &\xi'' = \{[2+\gamma(n-3)][2+\gamma(2n-3)]\}^{-1}, \quad &&\xi_4 = \xi''\alpha(1-\gamma)[2+\gamma(2n-3)], \\ &\xi_5 = \xi''[1+\gamma(n-2)][2+\gamma(n-2)], &&\xi_6 = \xi''\gamma[1+\gamma(n-2)]. \end{aligned}$$

Plugging eq. (29) into eq. (28), we obtain consumer demand given $\boldsymbol{c}$:

$$q_i = \xi_7 - \xi_8 c_i + \xi_9 \sum_{j \neq i} c_j, \tag{30}$$

with coefficients

$$\xi_7 = \xi'\xi_4[1+\gamma(n-2)], \quad \xi_8 = \xi''\xi_2\{2+\gamma(-6+5\gamma+n[3+\gamma(n-2)])\}, \quad \xi_9 = \xi'\xi_6[1+\gamma(n-2)].$$

We now turn to positivity of the equilibrium. Note that all coefficients $\xi_1, \ldots, \xi_9$ are weakly positive. Since $\xi_4 > 0$ (recall that $\gamma < 1$ in the Bertrand setting), prices are trivially strictly positive. To ensure positivity of demand, note that $q_i \geq \xi_7 - \xi_8 \,\forall \boldsymbol{c}$. Hence, a sufficient condition is $\frac{\xi_7}{\xi_8} > 1$, which is equivalent to

$$\alpha \geq \frac{2 + \gamma(-6 + 5\gamma + n[3 + \gamma(n-2)])}{(1-\gamma)[2 + \gamma(2n-3)]} \equiv \alpha^*(n, \gamma).$$

Note that $\frac{\partial \alpha^*}{\partial n} \geq 0 \,\forall \gamma \in [0, 1)$. Since the model requires $n \leq m$, it suffices to have $\alpha > \alpha^*(m, \gamma) \equiv \underline{\alpha}$. We hence conclude that

$$\begin{aligned} \forall m \in \mathbb{N}_+, \gamma \in [0,1) \; \exists \underline{\alpha} > 0 \text{ s.t.:} \quad & \alpha > \underline{\alpha} \Rightarrow p_i, q_i > 0 \\ & \forall i, n, \boldsymbol{c} \text{ s.t. } 1 \leq i \leq n \leq m, \boldsymbol{c} \in [0,1]^m, \\ \text{with} \quad & \underline{\alpha} = \frac{2 + \gamma(-6 + 5\gamma + m[3 + \gamma(m-2)])}{(1-\gamma)[2 + \gamma(2m-3)]}. \end{aligned} \tag{31}$$

We make several remarks. First, let $\alpha = \underline{\alpha}$. All firms still post strictly positive prices and have strictly positive demand, unless the polar case $c_i = 1, c_j = 0 \,\forall j \neq i$ and $n = m$ occurs. Here, the demand for firm i goes to zero. Second, note there is no finite lower bound on α ensuring positive quantities for *all* degrees of product differentiation, as opposed to the case of Cournot competition. This follows from $\frac{\partial \underline{\alpha}}{\partial \gamma} \geq 0$ and $\lim_{\gamma \to 1} \underline{\alpha} = \infty$. Third, sufficiency in a Bertrand setting neither implies nor requires sufficiency in a Cournot setting. That is, $\alpha \geq \underline{\alpha}$ does not necessarily imply $\alpha \geq m$ or vice versa. This is readily seen from $\underline{\alpha}|_{\gamma=0} = 1$ and $\lim_{\gamma \to 1} \underline{\alpha} = \infty \,\forall m \in \mathbb{N}_+$.

Next, we turn to firm profits. In particular, we want to ascertain that $\pi_i = q_i(p_i - c_i)$ is positive for every firm and every profile of costs. We have already identified a condition ensuring that $q_i > 0$. Hence, we only need to check whether $p_i \geq c_i$ holds for all $\boldsymbol{c} \in [0,1]^m$. Recalling the equilibrium price from eq. (29), this is equivalent to

$$\xi_4 + (\xi_5 - 1)c_i + \xi_6 \sum_{j \neq i} c_j \geq 0.$$

Noting that $\xi_5 - 1 \leq 0$ and $\xi_6 \geq 0$, this expression is bounded below by setting $c_i = 1$ and $c_j = 0 \,\forall j \neq i$. The requirement thus is satisfied if $\frac{\xi_4}{1-\xi_5} \geq 1$, which holds true if $\alpha \geq \underline{\alpha}$. To see that each firm's profit decreases in its marginal costs, consider

$$\begin{aligned} \frac{\mathrm{d}\pi_i}{\mathrm{d}c_i} &= \frac{\mathrm{d}q_i}{\mathrm{d}c_i}(p_i - c_i) + q_i(\frac{\mathrm{d}p_i}{\mathrm{d}c_i} - 1) \\ &= -\xi_8(p_i - c_i) + q_i(\xi_5 - 1) \leq 0, \end{aligned}$$

using $\xi_8 \geq 0$, $p_i - c_i \geq 0$, $q_i \geq 0$ and $\xi_5 - 1 \leq 0$ from above for $\alpha \geq \underline{\alpha}$. □

Proof of Proposition 3. The proof mirrors the Cournot case. Only differences are the market equilibrium $(\boldsymbol{q}, \boldsymbol{p})$, where prices and quantities are now given by eqs. (29) and (30), as well as the quality threshold $\underline{\alpha}$ now defined by eq. (31). We compute total welfare given a profile of costs $\boldsymbol{c}$:

$$\begin{aligned}
\mathcal{W}^{\mathrm{B}}_{\mathrm{tot}} = \tfrac{1}{2}\{&(1-\gamma)[2+\gamma(n-3)][1+\gamma(n-1)[2+\gamma(2n-3)]\}^{-2} \\
&\cdot \Big(n\alpha^2(1-\gamma)^2[3+\gamma(n-4)][1+\gamma(n-2)][1+\gamma(n-1)][2+\gamma(2n-3)]^2 \\
&- 2\alpha(1-\gamma)^2[3+\gamma(n-4)][1+\gamma(n-2)][1+\gamma(n-1)][2+\gamma(2n-3)]^2\sum_{i=1}^{n} c_i \\
&+ (1-\gamma)[1+\gamma(n-2)][1+\gamma(n-1)]\Big\{\gamma^4[n(n[n(3n-28)+94]-132)+66] \\
&\quad + \gamma^3(n-2)[85+n(18n-85)] + 3\gamma^2[55+n(13n-55)] + 36\gamma(n-2)+12\Big\}\sum_{i=1}^{n} c_i^2 \\
&- \gamma(1-\gamma)[1+\gamma(n-1)][1+\gamma(n-2)]^2\big\{3\gamma^2[5+n(n-5)] \\
&\quad + 11\gamma(n-2)+8\big\}\sum_{i=1}^{n}\sum_{j\neq i} c_i c_j\Big).
\end{aligned} \tag{32}$$

Applying the expected order statistics from Appendix B.1, we obtain

$$\begin{aligned}
\mathrm{E}_{\mathbf{c}}[\mathcal{W}^{\mathrm{B}}_{\mathrm{tot}}] = \{&24(1-\gamma)(m+1)(m+2)[2+\gamma(n-3)]^2[1+\gamma(n-1)][2+\gamma(2n-3)]^2\}^{-1} \\
&\cdot n[1+\gamma(n-2)]\Big[3n^6\gamma^4 + n^5\gamma^3(30-31\gamma) \\
&+ n^4\gamma^2(\gamma(64\gamma+48\alpha(\gamma-1)(m+2)-175)+99) \\
&- n^3\gamma\Big\{\gamma\big(\gamma(69-151\gamma)+48\alpha^2(\gamma-1)\gamma(m+1)(m+2) \\
&\quad + 48\alpha(\gamma-1)(6\gamma-5)(m+2)+192\big)-120\Big\} \\
&+ n^2\Big\{\gamma\big(\gamma((965-379\gamma)\gamma-723)+48\alpha^2(\gamma-1)\gamma(7\gamma-5)(m+1)(m+2) \\
&\quad + 12\alpha(\gamma-1)(\gamma(29\gamma-60)+28)(m+2)+96\big)+48\Big\} \\
&- 3n\Big\{(4\gamma-3)(\gamma(\gamma(7\gamma+20)-40)+16)+4\alpha^2(\gamma-1)\gamma(3\gamma-2)(19\gamma-14)(m+1)(m+2) \\
&\quad - 4\alpha(\gamma-1)(3\gamma-2)(\gamma(7\gamma+3)-6)(m+2)\Big\} \\
&+ 2(3\gamma-2)\Big\{\gamma(\gamma(58\gamma-129)+96)+6\alpha^2(\gamma-1)(3\gamma-2)(4\gamma-3)(m+1)(m+2) \\
&\quad - 6\alpha(\gamma-1)(3\gamma-2)(4\gamma-3)(m+2)-24\Big\}\Big].
\end{aligned} \tag{33}$$

Again, no explicit solution exists for general parameters $0 \le \gamma \le 1 \le n \le m \le \alpha$. To show the existence of an interior solution $1 < n^* < m$, we considers the case $\alpha = 90$, $\gamma = 0.9$ and $m = 20$. Computing the extreme values numerically and excluding all those which are either below 1, above m or whose imaginary part is nonzero, we obtain $n^* \approx 16.898$, with the second order derivative verifying that it is a maximum indeed. □

Proof of Lemma 8. We have $\gamma \in [0,1]$ and $m \geq n \geq 1$. For $n > 1$, the denominator of eq. (19) is concave in γ and positive for $\gamma \in \{0,1\}$, thus for all $\gamma \in [0,1]$. For $n = 1$, positivity is trivial.

The numerator is increasing in α_k for $k = i$ and decreasing for $k = j \neq i$. By assumption, we know $\alpha \geq m$ and $x_i \in [0,1]$. Hence

$$\begin{aligned}
[\gamma(n-2)+2]\alpha_i - \gamma\sum_{j\neq i}\alpha_j &= (2-\gamma)\alpha - [\gamma(n-2)+2]x_i + \gamma\sum_{j\neq i}x_j \\
&\geq (2-\gamma)\alpha - [\gamma(n-2)+2] \\
&\geq (2-\gamma)m - [\gamma(m-2)+2] \\
&\geq 0,
\end{aligned}$$

with equality only if $n = m = \alpha$. □

Proof of Proposition 4. Using $\alpha_i = \alpha - x_i$, we can rewrite the production levels q_i in the market equilibrium given by eq. (19). Recall the definition $\lambda = \{(2-\gamma)[2+\gamma(n-1)]\}^{-1}$ to obtain

$$q_i = \lambda\Big[\alpha(2-\gamma) - [2+\gamma(n-2)]x_i + \gamma\sum_{j\neq i}x_j\Big], \tag{34}$$

which mirrors the equilibrium output in the case of cost heterogeneity from eq. (6) with costs c_i replaced by marginal quality x_i. Total welfare in the case of quality heterogeneity is given by

$$\mathcal{W}_{\text{tot}}^{\alpha_i} = U(\boldsymbol{q}) = \alpha\sum_{i=1}^{n}q_i - \frac{1}{2}\sum_{i=1}^{n}q_i^2 - \frac{\gamma}{2}\sum_{i=1}^{n}\sum_{j\neq i}q_iq_j - \sum_{i=1}^{n}q_ix_i, \tag{35}$$

which in turn mirrors the expression for total welfare in the case of cost heterogeneity from eq. (22). But since x_i and c_i are both i.i.d. draws from $U[0,1]$, the distributions of their order statistics coincide. Hence, all results for the expected value $\mathrm{E}_{\boldsymbol{c}}[\mathcal{W}_{\text{tot}}]$ directly carry over to $\mathrm{E}_{\boldsymbol{x}}[\mathcal{W}_{\text{tot}}^{\alpha_i}]$ and we can apply Proposition 2. That is, from the regulator's perspective, expected total welfare is given by eq. (25) and the same interior maxima exist. □

Proof of Proposition 5. Recall that for all firms i and all cost profiles $\boldsymbol{c}$ it holds that $\pi_i(\boldsymbol{c}) = [q_i(\boldsymbol{c})]^2$. We can therefore rewrite the profit of the most efficient firm by separating the effect of c_1, $(c_2, \ldots, c_{n-1})$ and c_n:

$$\begin{aligned}
\pi_{(1)}(c_{(1)}, \ldots, c_{(n)}) = \lambda^2\{ \quad & \alpha^2(2-\gamma)^2 - 2\alpha(2-\gamma)[2+\gamma(n-2)]c_{(1)} + 2\alpha(2-\gamma)\gamma c_{(n)} \\
& + 2\alpha(2-\gamma)\gamma \sum_{i=2}^{n-1} c_{(i)} + [2+\gamma(n-2)]^2 c_{(1)}^2 + \gamma^2 c_{(n)}^2 + \gamma^2 \sum_{i=2}^{n-1} c_{(i)}^2 \\
& - 2[2+\gamma(n-2)]\gamma c_{(1)} c_{(n)} - 2[2+\gamma(n-2)]\gamma c_{(1)} \sum_{i=2}^{n-1} c_{(i)} \\
& + 2\gamma^2 c_{(n)} \sum_{i=2}^{n-1} c_{(i)} + \gamma^2 \sum_{i=2}^{n-1} \sum_{\substack{j=2 \\ j \neq i}}^{n-1} c_{(i)} c_{(j)} \} \qquad (36)
\end{aligned}$$

The bid of the $(n+1)$-th most efficient firm is the second highest in the n-th auction and hence is paid by the n-th most efficient firm for obtaining the last license. Firm $(n+1)$ bids the profit it *would* generate if it was to enter the market instead of firm (n), the other rivals' costs remaining the same. We directly obtain $\pi_{(n+1)}(c_{(1)}, \ldots, c_{(n-1)}, c_{(n+1)})$ from eq. (36) by replacing $c_{(1)}$ with $c_{(n+1)}$ and $c_{(n)}$ with $c_{(1)}$. This allows us to write the total auction revenue:

$$\begin{aligned}
R(n) &= \mathcal{W}_{\mathrm{F}} - [\pi_{(1)}(c_{(1)}, \ldots, c_{(n)}) - \pi_{(n+1)}(c_{(1)}, \ldots, c_{(n-1)}, c_{(n+1)})] \\
&= \mathcal{W}_{\mathrm{F}} - \lambda^2\{ \quad 2\alpha(2-\gamma)[2+\gamma(n-2)](c_{(n+1)} - c_{(1)}) + 2\alpha(2-\gamma)\gamma(c_{(n)} - c_{(1)}) \\
&\qquad - [2+\gamma(n-2)]^2(c_{(n+1)}^2 - c_{(1)}^2) + \gamma^2(c_{(n)}^2 - c_{(1)}^2) \\
&\qquad + 2[2+\gamma(n-2)]\gamma[(c_{(n+1)} - c_{(n)})c_{(1)} + (c_{(n+1)} - c_{(1)})\sum_{i=2}^{n-1} c_{(i)}] \\
&\qquad + 2\gamma^2(c_{(n)} - c_{(1)}) \sum_{i=2}^{n-1} c_{(i)} \} \qquad (37)
\end{aligned}$$

We combine eq. (23) for consumer surplus with eq. (37) for auction revenue and use the order statistics from Appendix B.1 to obtain:

$$
\begin{aligned}
&\mathrm{E}_{\boldsymbol{c}}[\mathcal{W}_{\mathrm{C}}(n) + R(n)] \\
&= \{24(2-\gamma)^2(m+1)(m+2)[2+\gamma(n-1)]^2\}^{-1} \\
&\cdot\Big[n^5\gamma^2(3-\gamma) + n^4\gamma^2(8+3\gamma) + n^3\gamma[144 - 12\alpha(2-\gamma)^2(m+2) - \gamma(83-11\gamma)] \\
&+ n^2\{240 + 12\alpha^2\gamma(2-\gamma)^2(m+1)(m+2) \\
&\qquad - 12\alpha(2-\gamma)(6+\gamma)(m+2) + \gamma[48 - \gamma(140+3\gamma)]\} \\
&+ 2n\{312 - 6\alpha(7-\gamma)(2-\gamma)^2(m+2) \\
&\qquad + 6\alpha^2(3-\gamma)(2-\gamma)^2(m+1)(m+2) - \gamma[528 - \gamma(214-5\gamma)]\} \\
&+ 48(2-\gamma)[\alpha\gamma(m+2) - 2 + \gamma]\Big],
\end{aligned} \tag{38}
$$

for arbitrary parameters satisfying $0 \leq \gamma \leq 1 \leq n \leq m \leq \alpha$. Just as in the proof of Proposition 2, we observe that the numerator is a polynomial of fifth degree in n, the denominator one of second degree. We hence cannot find a general solution for the welfare-maximizing value of n by the Abel–Ruffini Theorem. To prove existence of interior solutions, again consider the case of $\alpha = m = 10, \gamma = 1$. We obtain $\mathrm{E}_{\boldsymbol{c}}[\mathcal{W}_{\mathrm{C}}+R(n)] = [2n^5 + 11n^4 - 1\,368n^3 + 148\,465n^2 + 308\,146n + 5712][3\,168(n+1)^2]^{-1}$, which can be maximized numerically. There are five values for n^* satisfying $\frac{\mathrm{d}}{\mathrm{d}n}\mathrm{E}_{\boldsymbol{c}}[\mathcal{W}_{\mathrm{C}}(n) + R(n)]|_{n^*} = 0$. Ignoring the solutions $n_1^* < 0$, $n_2^* > m$ as well as complex values n_3^*, n_4^* s.t. $\mathrm{Im}(n_3^*)$, $\mathrm{Im}(n_4^*) \neq 0$, we obtain the interior optimum $n_5^* \approx 5.10$, noting that $1 < n_5^* < m$. Computing the second order derivative, it is readily verified that n_5^* is indeed a maximizer. □

Appendix B Auxiliary Computations

Appendix B.1 Order Statistics

To compare the welfare for different values of the market size n, the regulator has to form expectations over the marginal costs c_i of those firms that will be active in the market. In particular, we need to compute expectations of the terms appearing in eq. (12). Recalling Lemma 3, more efficient firms – i.e. those with lower marginal costs – can be expected to enter the market first as they generate higher profits.

Thus, from the regulator's perspective, we are interested in the expected costs of the n most efficient firms out of m, with firm costs $c_1, \ldots, c_m$ being an i.i.d. sample from $U[0,1]$. That is, we need to compute expectations for the first n order statistics.

Incorporating the cases of heterogeneity in both costs (c_i) and quality (α_i), we follow the notation of David and Nagaraja (2003) in that we write X for the random variable ("cost type") and denote by $X_{(i:m)}$ the i-th order statistic of a sample with size m. That is, the variables are ordered such that

$$X_{(1:m)} \leq \ldots \leq X_{(n:m)} \leq \ldots \leq X_{(m:m)}.$$

To simplify notation, we write $X_{(i)}$ whenever the sample size is m. Realizations are denoted by lowercase letters. The following paragraphs closely follow David and Nagaraja (2003), Chapter 2.

Distributions: Let $F(x)$ be the cumulative distribution function (cdf) of the *unordered* random variables X_k and $F_{(i)}(x)$ the cdf of the i-th order statistic $X_{(i)}$. Verbally, the latter denotes the probability that at least i of the m (unordered) X_k are less than or equal to x:

$$F_{(i)}(x) = \sum_{k=i}^{m} \binom{m}{k} [F(x)]^k [1 - F(x)]^{m-k}. \tag{39}$$

Differentiating and rearranging yields the probability density function (pdf):

$$\begin{aligned}
f_{(i)}(x) &= \frac{\mathrm{d}}{\mathrm{d}x} F_{(i)}(x) \\
&= f(x) \Big[\sum_{k=i}^{m} \frac{m!}{k!(m-k)!} k [F(x)]^{k-1} [1 - F(x)]^{m-k} \\
&\qquad - \sum_{k=i}^{m} \frac{m!}{k!(m-k)!} (m-k) [F(x)]^{k} [1 - F(x)]^{m-k-1} \Big] \\
&= f(x) \Big[\sum_{k=i-1}^{m-1} \frac{m!}{k!(m-k-1)!} [F(x)]^{k} [1 - F(x)]^{m-k-1} \\
&\qquad - \sum_{k=i}^{m-1} \frac{m!}{k!(m-k-1)!} [F(x)]^{k} [1 - F(x)]^{m-k-1} \Big] \\
&= m f(x) \binom{m-1}{i-1} [F(x)]^{i-1} [1 - F(x)]^{m-i}
\end{aligned} \tag{40}$$

For the third equality, we have shifted the index of the first sum while at the second sum we note that the last term (where $k = m$) equals zero.

Using the specific distribution $X \sim U[0, 1]$, the pdf of the i-th order statistic for

$x \in [0,1]$ is given by:

$$f_{(i)}(x) = \frac{m!}{(i-1)!(m-i)!} x^{i-1}(1-x)^{m-i}. \tag{41}$$

Since we will encounter "mixed terms" when computing expected types, we also need to consider the joint pdf of $X_{(i)}, X_{(j)}$. For $i \leq j$ and $x_i \leq x_j$, it holds that

$$f_{(i,j)}(x_i, x_j) = \frac{m!}{(i-1)!(j-i-1)!(m-j)!} x_i^{i-1}(x_j - x_i)^{j-i-1}(1-x_j)^{m-j}. \tag{42}$$

For a derivation, see e.g. David and Nagaraja (2003) eq. (2.2.2).

Expected values: To compute expectations of the order statistics $x_{(i)}$ as well as squared and mixed terms, we make use of the Beta function defined by

$$\begin{aligned} B(i,j) &= \int_0^1 x^{i-1}(1-x)^{j-1}\,\mathrm{d}x && \text{for general } i,j > 0 \\ &= \frac{(i-1)!(j-1)!}{(i+j-1)!} && \text{for } i,j \in \mathbb{N}_+. \end{aligned} \tag{43}$$

This yields the expected marginal cost of the i-th most cost efficient firm:

$$\begin{aligned} \mathrm{E}[x_{(i)}] &= \int_0^1 x f_{(i)}(x)\,\mathrm{d}x \\ &= \frac{1}{B(i, m-i+1)} \int_0^1 x^{(i+1)-1}(1-x)^{(m-i+1)-1}\,\mathrm{d}x \\ &= \frac{B(i+1, m-i+1)}{B(i, m-i+1)} \\ &= \frac{i}{m+1} \end{aligned}$$

The following equalities will prove useful. They are computed in a similar way.

$$\begin{aligned} \mathrm{E}[x_{(i)}^2] &= \frac{i(i+1)}{(m+1)(m+2)} \\ \mathrm{E}[x_{(i)}x_{(j)}] &= \frac{i(j+1)}{(m+1)(m+2)} \quad \text{for } i < j \end{aligned}$$

Summing over the first n out of m order statistics – that is, the n most efficient firms – allows us to compute the expectations of the terms appearing in eq. (12):

$$\sum_{k=1}^{n} \mathrm{E}[x_{(k)}] = \frac{n(n+1)}{2(m+1)}$$

$$\sum_{k=1}^{n} \mathrm{E}[x_{(k)}^2] = \frac{n(n+1)(n+2)}{3(m+1)(m+2)}$$

$$\sum_{k=1}^{n} \sum_{l \neq k} \mathrm{E}[x_{(k)} x_{(l)}] = \frac{n(n-1)(n+1)(n+2)}{4(m+1)(m+2)} = 2 \sum_{k=1}^{n} \sum_{l=1}^{k-1} \mathrm{E}[x_{(k)} x_{(l)}]$$

Appendix B.2 Summations

In the proofs of Appendix A, we computed welfare for the default scenario of Cournot competition with cost heterogeneity as well as for extensions with Bertrand competition, quality heterogeneity and auction revenue maximization. For these computations, general expressions for arbitrary equilibrium profiles $(\boldsymbol{q}, \boldsymbol{p})$ in quantities and prices were used, both in the analytical derivation and in the computational implementation. This section briefly presents summations frequently occurring in these derivations.

Since market equilibria differ across the scenarios, consider an arbitrary profile $(\boldsymbol{q}, \boldsymbol{p})$ with $q_i = \mu_1 + \mu_2 c_i + \mu_3 \sum_{j \neq i} c_j$ and $p_i = \nu_1 + \nu_2 c_i + \nu_3 \sum_{j \neq i} c_j$ for $i \in \{1, \ldots, n\}$. We find:

$$\sum_{i=1}^{n} q_i = n\mu_1 + [\mu_2 + (n-1)\mu_3] \sum_{i=1}^{n} c_i,$$

$$\sum_{i=1}^{n} c_i q_i = \mu_1 \sum_{i=1}^{n} c_i + \mu_2 \sum_{i=1}^{n} c_i^2 + \mu_3 \sum_{i=1}^{n} \sum_{j \neq i} c_i c_j,$$

$$\sum_{i=1}^{n} q_i^2 = n\mu_1^2 + [2\mu_1\mu_2 + 2(n-1)\mu_1\mu_3] \sum_{i=1}^{n} c_i + [\mu_2^2 + (n-1)\mu_3^2] \sum_{i=1}^{n} c_i^2 + [(n-2)\mu_3^2 + 2\mu_2\mu_3] \sum_{i=1}^{n} \sum_{j \neq i} c_i c_j,$$

$$\sum_{i=1}^{n} q_i p_i = n\mu_1\nu_1 + [\mu_1\nu_2 + (n-1)\mu_1\nu_3 + \mu_2\nu_1 + (n-1)\mu_3\nu_1] \sum_{i=1}^{n} c_i + [\mu_2\nu_2 + (n-1)\mu_3\nu_3] \sum_{i=1}^{n} c_i^2 + [\mu_2\nu_3 + \mu_3\nu_2 + (n-2)\mu_3\nu_3] \sum_{i=1}^{n} \sum_{j \neq i} c_i c_j.$$

$$\sum_{i=1}^{n}\sum_{j\neq i} q_i q_j = (n-1)n\mu_1^2 + 2(n-1)\mu_1[\mu_2 + \mu_3(n-1)]\sum_{i=1}^{n} c_i$$
$$+ (n-1)\mu_3[2\mu_2 + \mu_3(n-2)]\sum_{i=1}^{n} c_i^2$$
$$+ [\mu_2^2 + 2(n-2)\mu_2\mu_3 + \mu_3\{1 + n^2 + 2\mu_3 - n(2+\mu_3)\}]\sum_{i=1}^{n}\sum_{j\neq i} c_i c_j.$$

The summations above allow us to derive general expressions for total welfare and its components – consumer surplus and firm profits – given different equilibria. We make use of these equations in Appendix A.

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